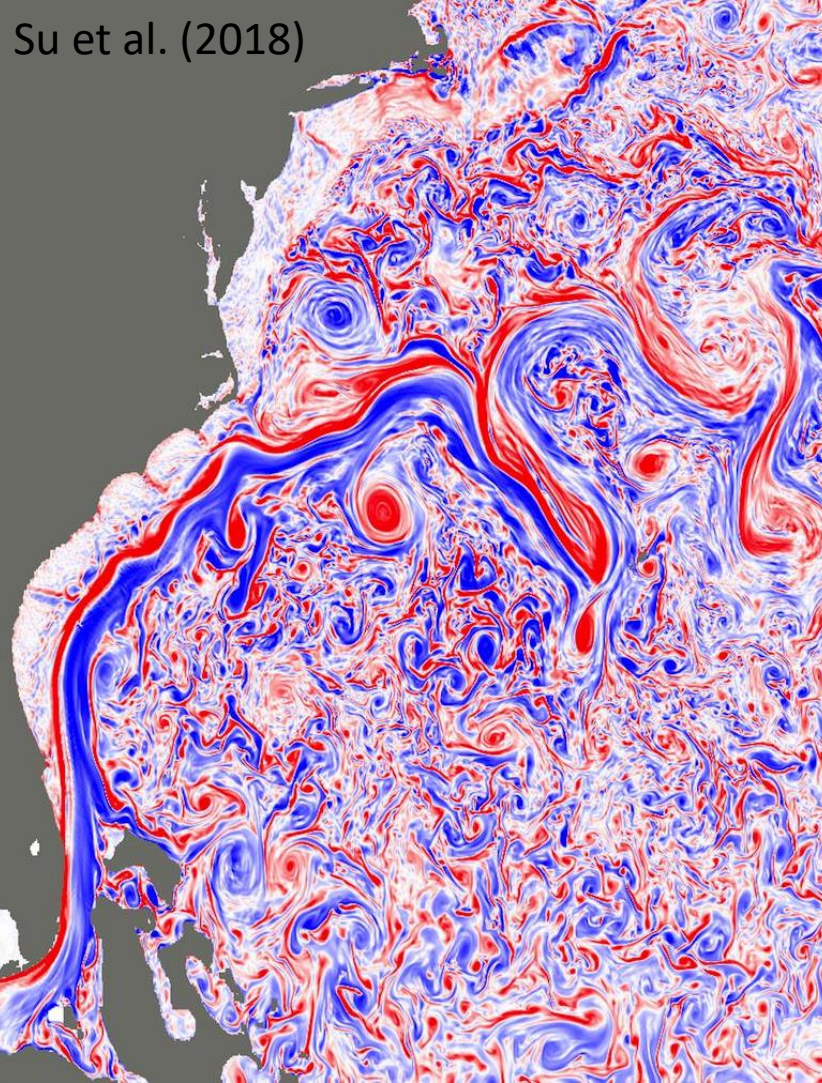




The barotropic vorticity budget of the Weddell Gyre

Andrew Styles

Department of Physics | University of Oxford



DOCTORAL TRAINING PARTNERSHIP
**ENVIRONMENTAL
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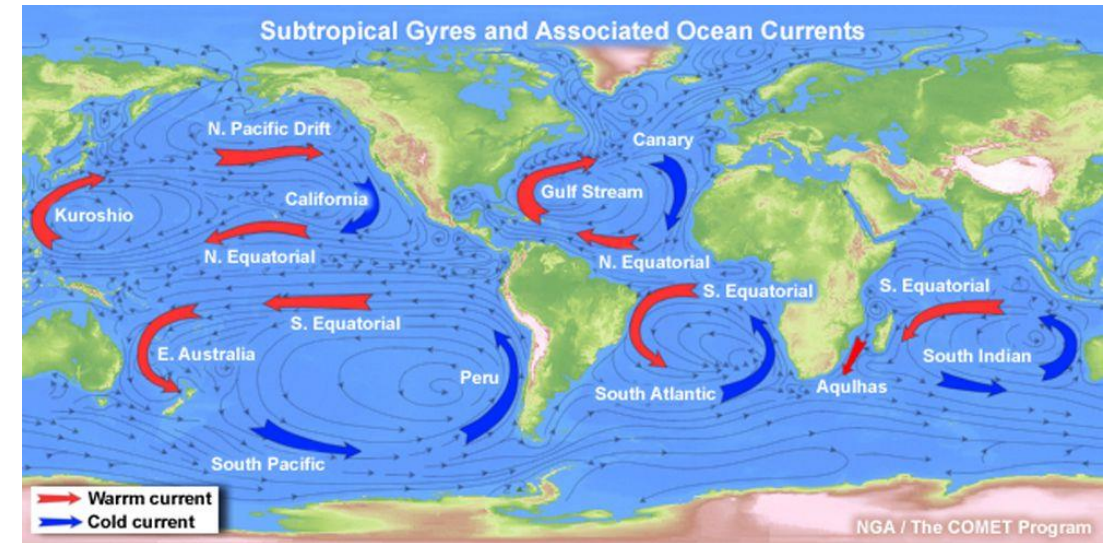


Gyres

Gyres are basin scale circulations in the world ocean

Features

- Significant meridional transport (~ 10 Sv)
- Closed by intense western boundary currents
- Often driven by a wind stress curl



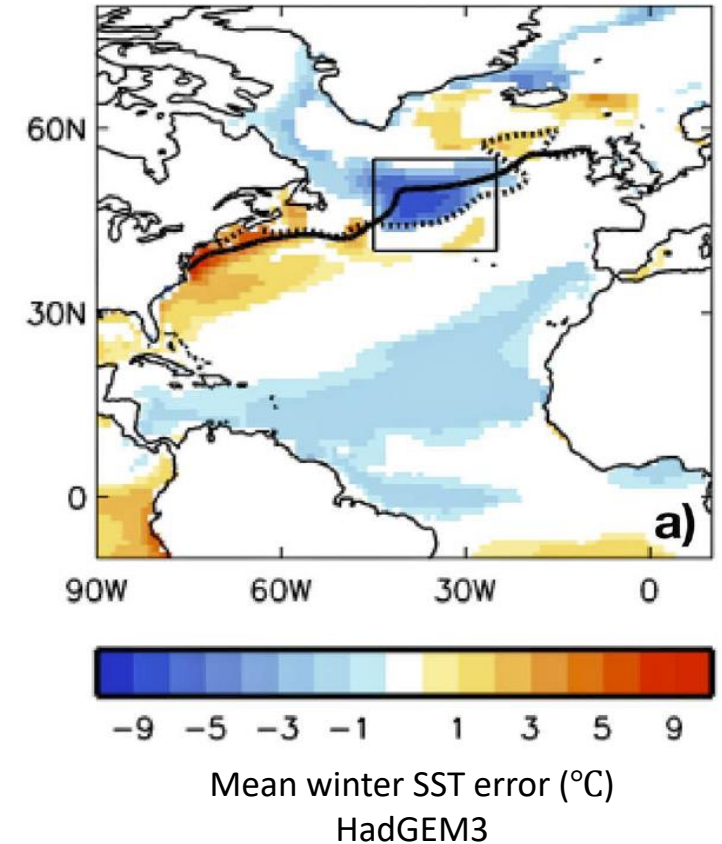
COMET - <http://meted.ucar.edu/>

Error in HadGEM3-GC3.1

In the North Atlantic (Subtropical)

- Systematic error off Western coast
- $\sim 7^{\circ}\text{C}$ error in 1 degree HADGEM3 model
- Error is reduced in higher resolution models
- Becomes highly sensitive to bathymetry - *'bathymetric steering'*

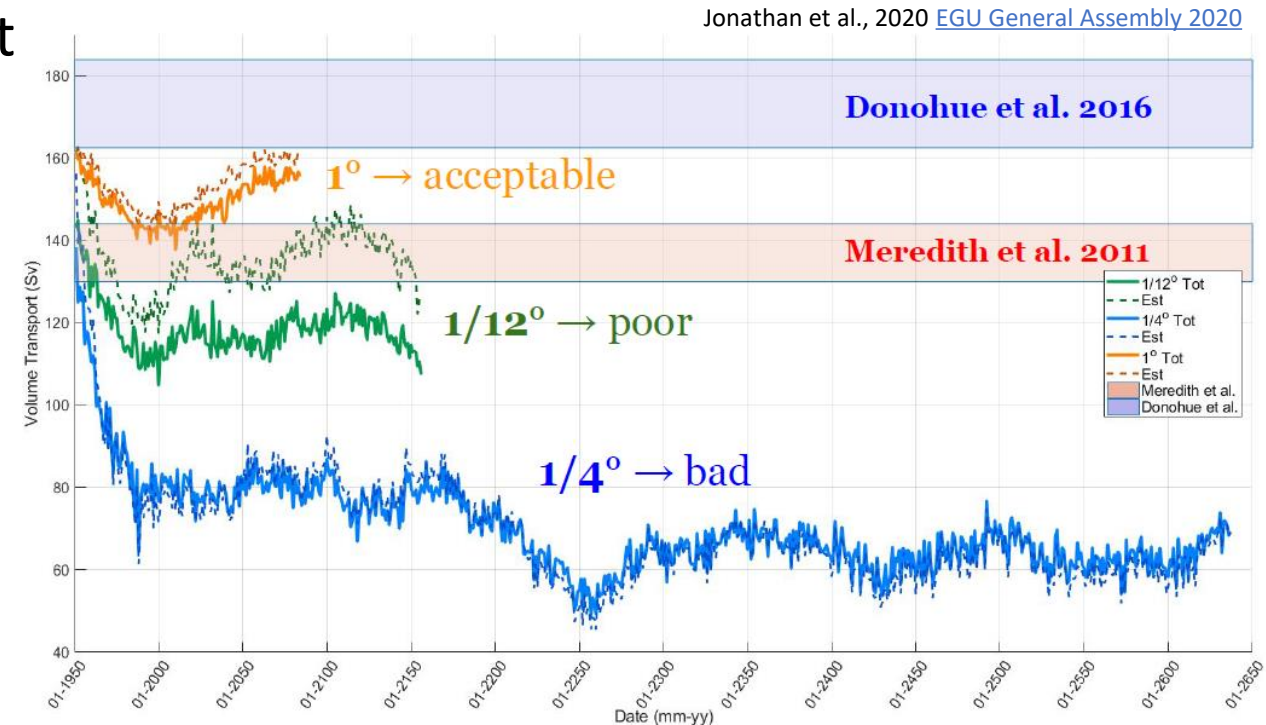
Scaife et al. 2011



Error in HadGEM3-GC3.1

In the Southern Ocean (Subpolar)

- Poor representation of ACC transport at higher resolutions
- ACC transport is weakest at $1/4^\circ$
- Attributed to **slumping of isopycnals** near southern boundary
- Consequently related to **strengthening of the Weddell Gyre (WG)**



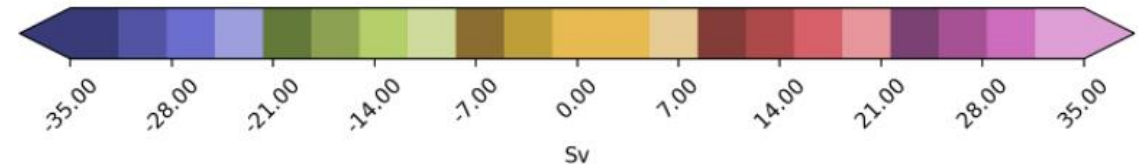
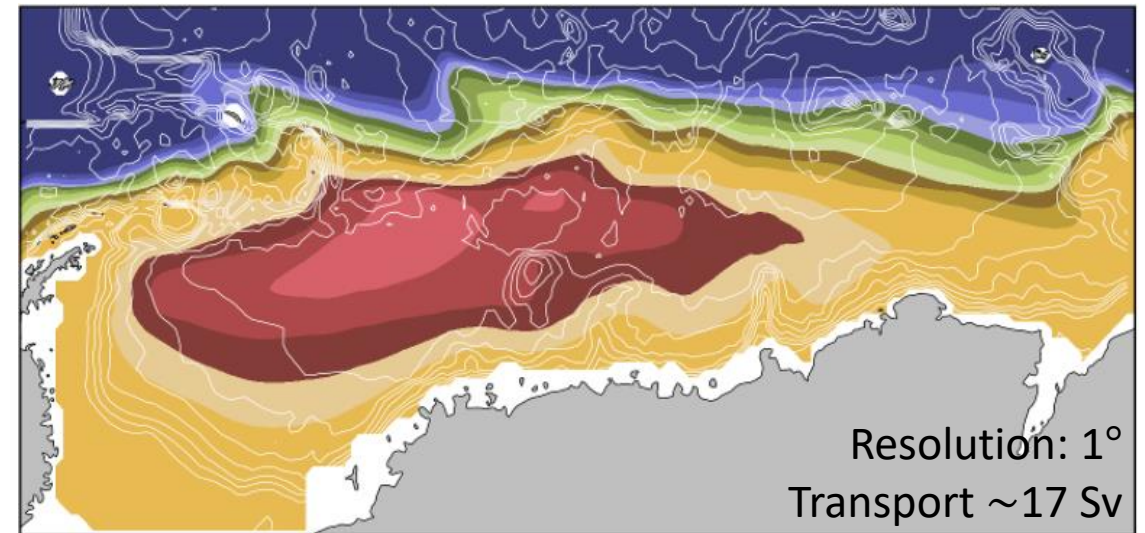
Evolution of ACC transport for R1, R4 and R12 HadGEM3-GC3.1 models

Error in HadGEM3-GC3.1

In the Southern Ocean (Subpolar)

- WG strengthening occurs in the **uncoupled** global model also.
- **WG transport increases** with resolution
- **WG extent increases** with resolution
- WG expands into **Drake passage** at $1/4^\circ$ and $1/12^\circ$ resolution

Figure from [D. Storkey](#)



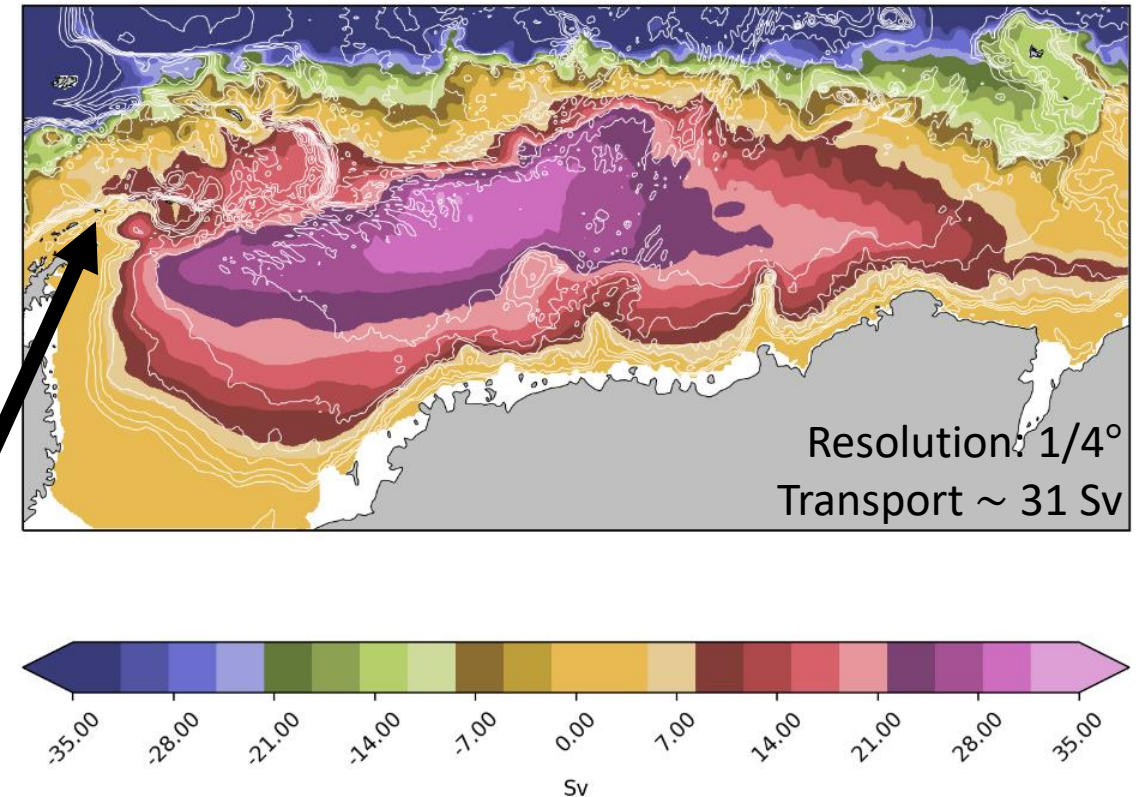
Stream function of Weddell Gyre in uncoupled model (10 yr average)
White contours show bathymetry (every 700 m)

Error in HadGEM3-GC3.1

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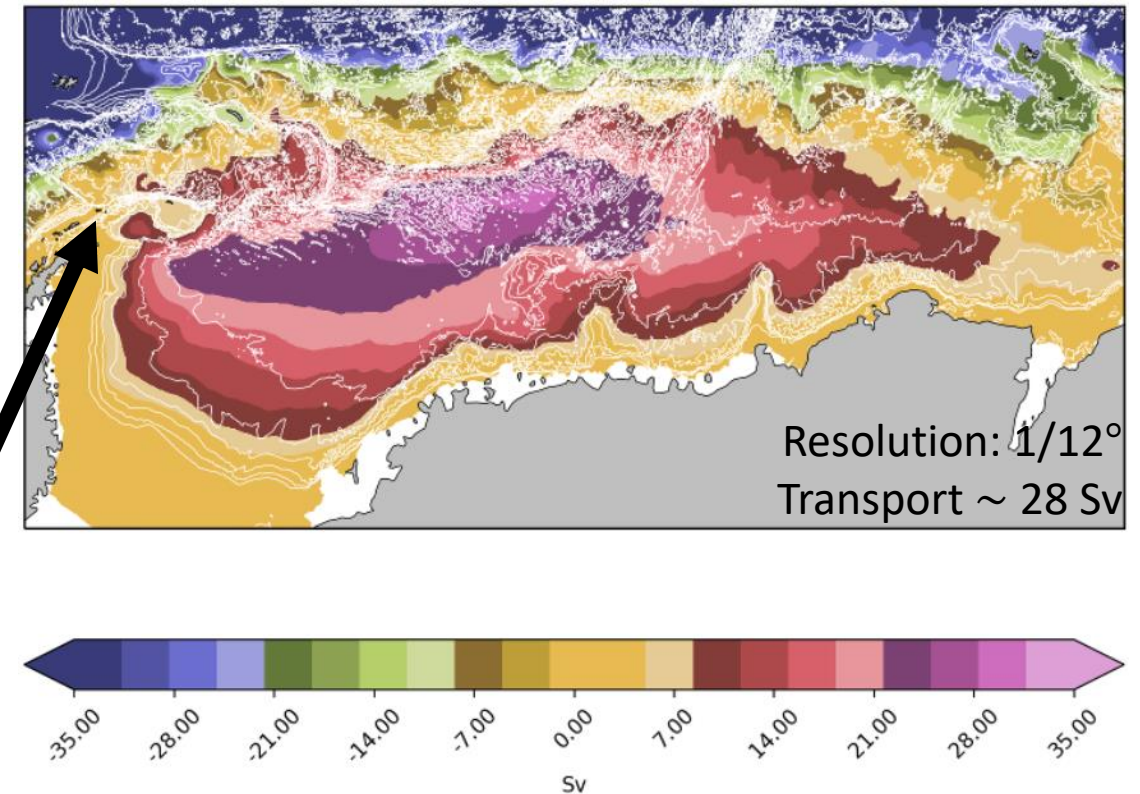
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Error in HadGEM3-GC3.1

In the Southern Ocean (Subpolar)

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Figure from [D. Storkey](#)



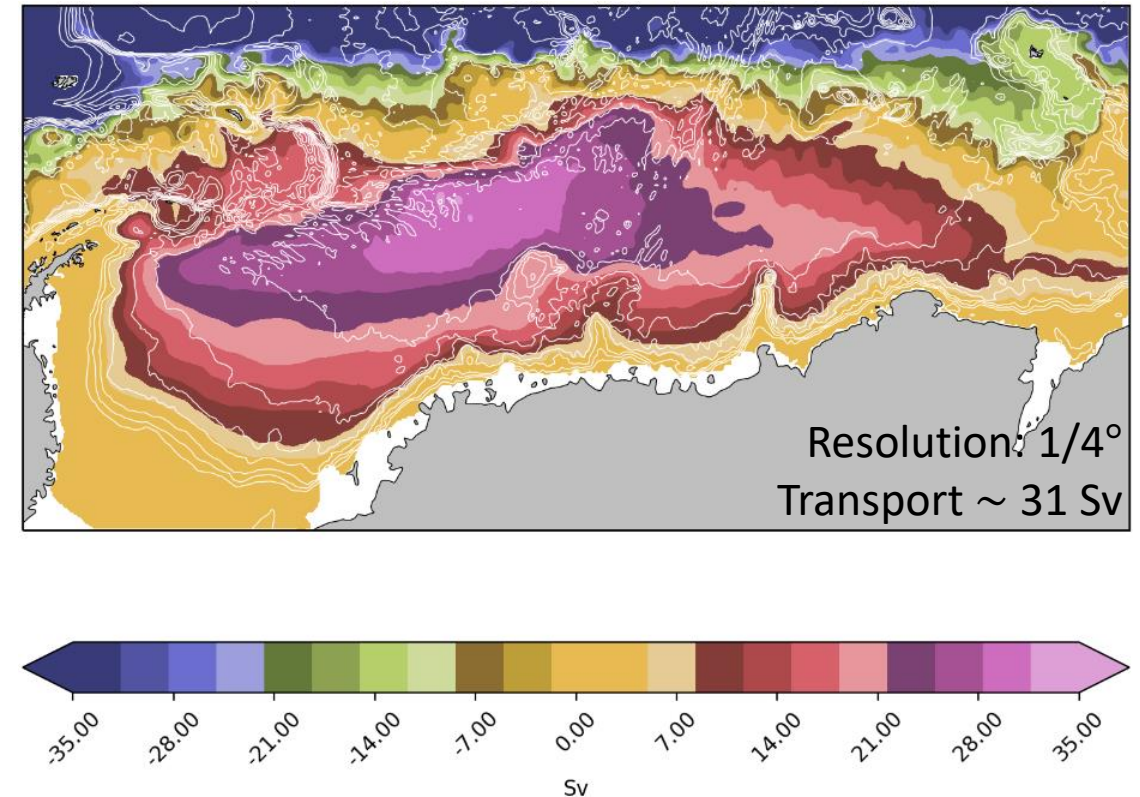
Stream function of Weddell Gyre in uncoupled model (10 yr average)
White contours show bathymetry (every 700 m)

Error in HadGEM3-GC3.1

Additional model behaviours

- Adding **partial slip** boundary condition helps to reduce WG transport
- Using **Gent-McWilliams** reduces the transport but reduces eddy energies
- Adjusting the scheme for the **Coriolis force** affects the WG transport

Figure from [D. Storkey](#)



Project Aims

Project aims

- Study the vorticity budget of the Weddell Gyre
 - Compare results with a simple box model (GYRE PISCES)
- 
- 
- Identify any increased forcing or reduced dissipation at higher resolutions

Main focus of this presentation

Vorticity and Gyres

Introducing the **barotropic vorticity** equation

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot (f\mathbf{U}) - \nabla_h \cdot (\zeta\mathbf{U}) + \nabla \times \left(\frac{\tau_s}{\rho H} \right) \cdot \hat{k} - r\zeta + \kappa \nabla^2 \zeta + \frac{1}{\rho_0} \nabla P \times \nabla H$$

Obtained by taking the curl of the **barotropic momentum** equation

$\mathbf{U}$ is the **depth-integrated** velocity

ζ is the **depth-integrated** relative vorticity

Vorticity and Gyres

Introducing the **barotropic vorticity** equation

$$\underbrace{\frac{\partial \zeta}{\partial t}}_{\text{Time evolution of relative vorticity}} = \underbrace{-\nabla_h \cdot (f\mathbf{U})}_{\text{Coriolis forcing } = \beta V \text{ for incompressible flow}} + \underbrace{-\nabla_h \cdot (\zeta\mathbf{U})}_{\text{Advection of relative vorticity}} + \underbrace{\nabla \times \left(\frac{\tau_s}{\rho H} \right) \cdot \hat{k}}_{\text{Wind stress curl}} - \underbrace{r\zeta}_{\text{Bottom drag (linear)}} + \underbrace{\kappa \nabla^2 \zeta}_{\text{Lateral diffusion}} + \underbrace{\frac{1}{\rho_0} \nabla P \times \nabla H}_{\text{Bottom pressure torque}}$$

TOT	PVO	ADV	WND	FRC	LDF	HPG
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Corresponding codes for model vorticity trends

Vorticity and Gyres

Different balances characterise different types of Gyres

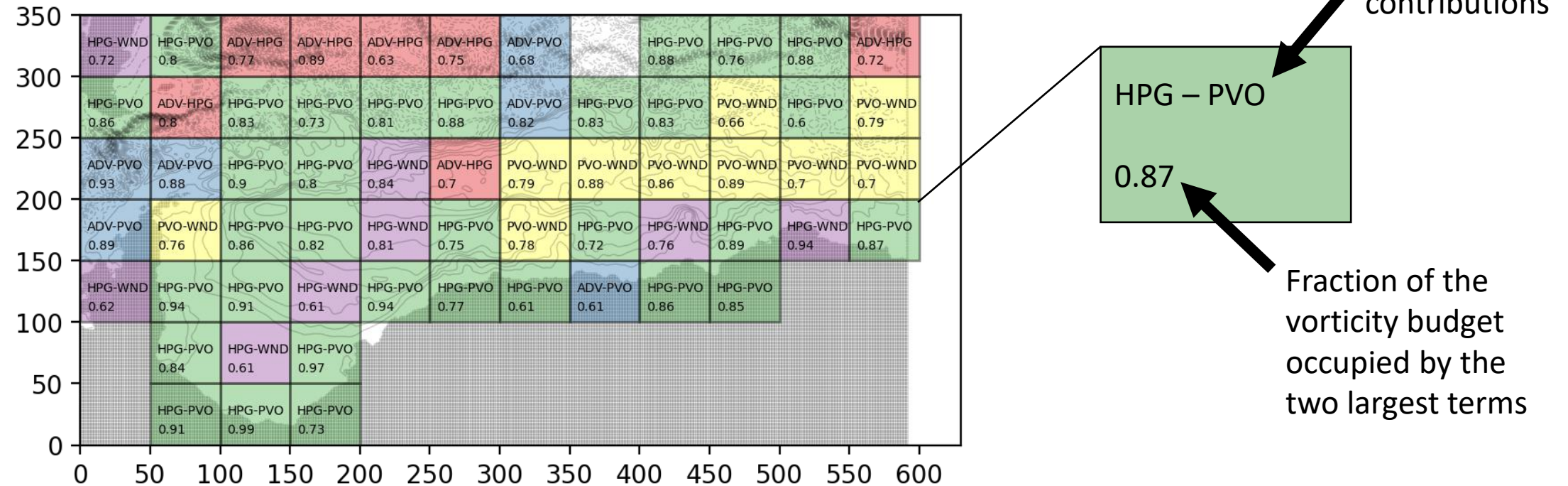
$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot (f\mathbf{U}) - \nabla_h \cdot (\zeta\mathbf{U}) + \nabla \times \left(\frac{\tau_s}{\rho H} \right) \cdot \hat{k} - r\zeta + \kappa \nabla^2 \zeta + \frac{1}{\rho_0} \nabla P \times \nabla H$$

For example a **steady, linear** flow in the absence of **bathymetry** and **diffusion**

This is the vorticity balance of the Stommel (1948) gyre.

Local balances

We can identify the leading order balances in local areas
(we will come back to this plot later)



Contour integrations

Local balances are useful but it can be difficult to determine how the gyre transport is affected.

Instead then, we choose to integrate over the area **enclosed by the streamlines**.

$$I(\psi) = \iint_{A_\psi} \underbrace{(\nabla \times \mathbf{M}) \cdot \hat{\mathbf{k}}}_{\text{Vorticity diagnostic}} dA = \oint_{\psi} \underbrace{\mathbf{M} \cdot d\mathbf{l}}_{\text{Work done per unit mass}}$$

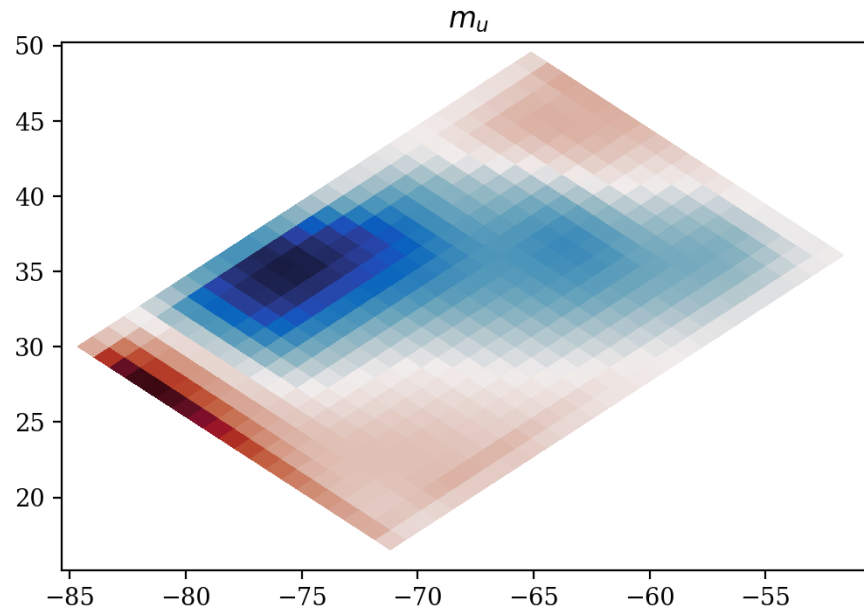
Vorticity diagnostic associated
with momentum diagnostic
(force per unit mass) $\mathbf{M}$

Work done per unit mass on a
fluid column circulating the
gyre once

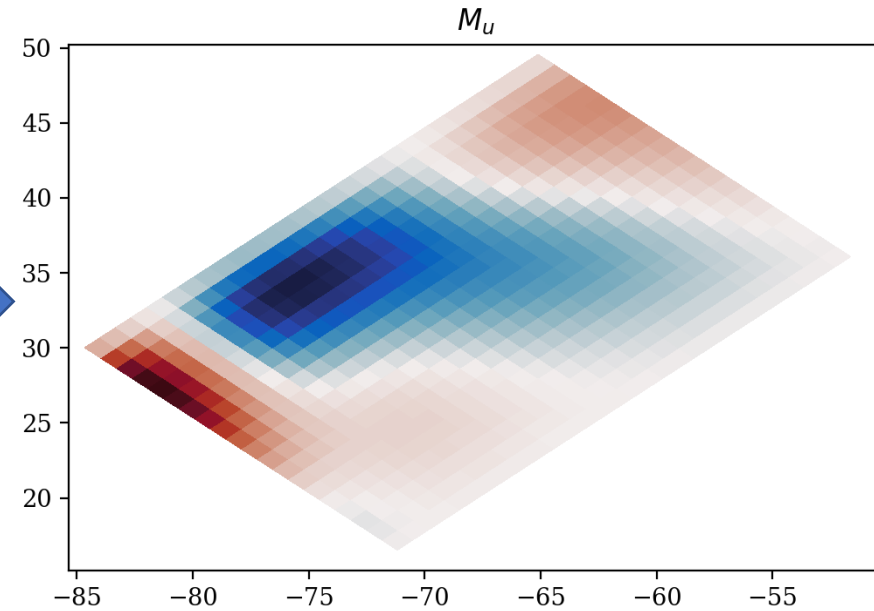
Contour integrations

Before we get to results, let's summarise the full procedure

1. Depth integrate momentum trend m



Depth
integrate

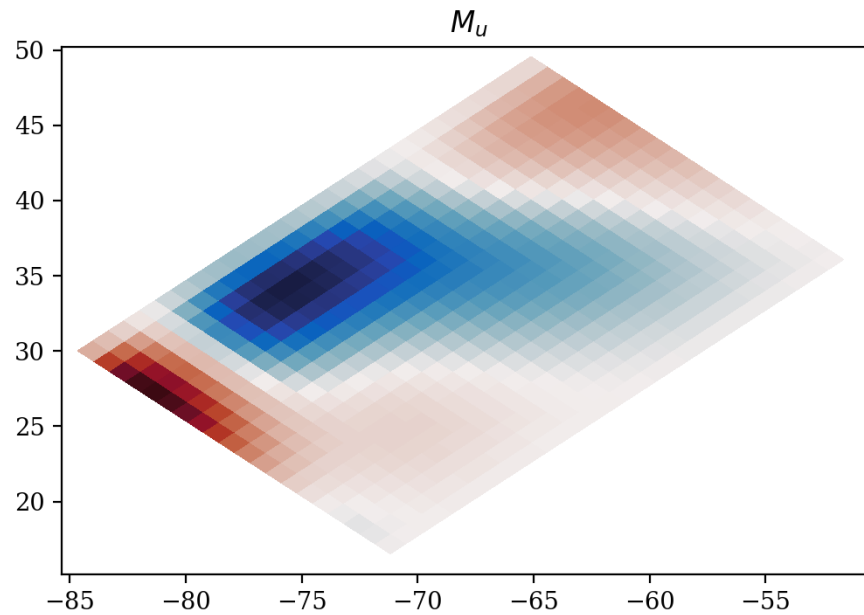


GYRE box model is used for illustrative purposes

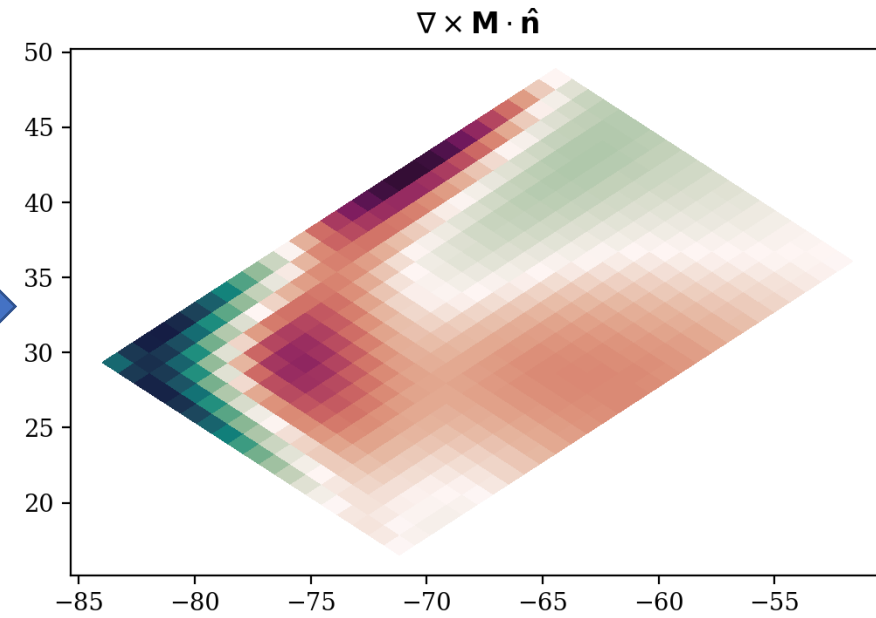
Contour integrations

Before we get to results, let's summarise the full procedure

2. Take the curl of $\mathbf{M}$ using M_u and M_v



Curl

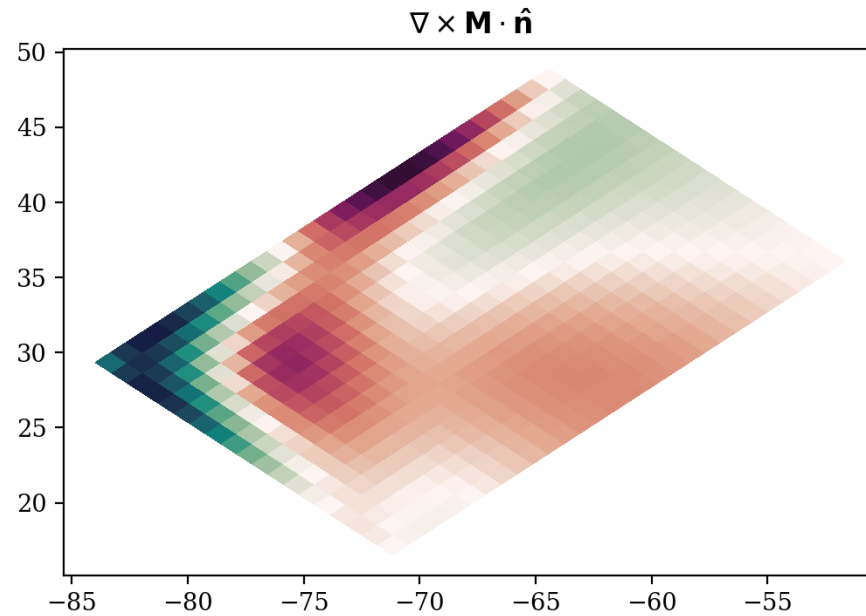


GYRE box model is used for illustrative purposes

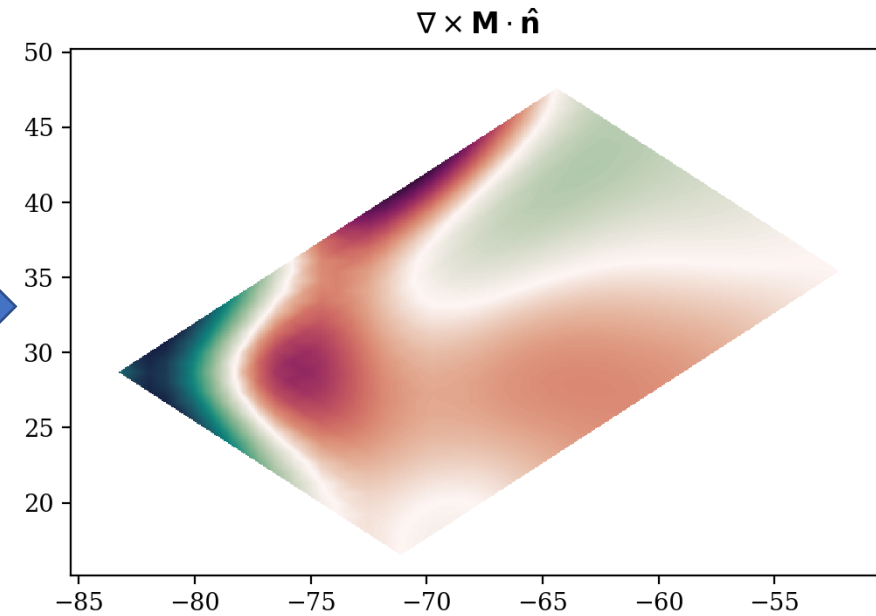
Contour integrations

Before we get to results, let's summarise the full procedure

3. Interpolate $\nabla \times \mathbf{M} \cdot \hat{\mathbf{k}}$ to a fine grid to minimise edge effects when integrating



Interpolate
to $1/12^\circ$

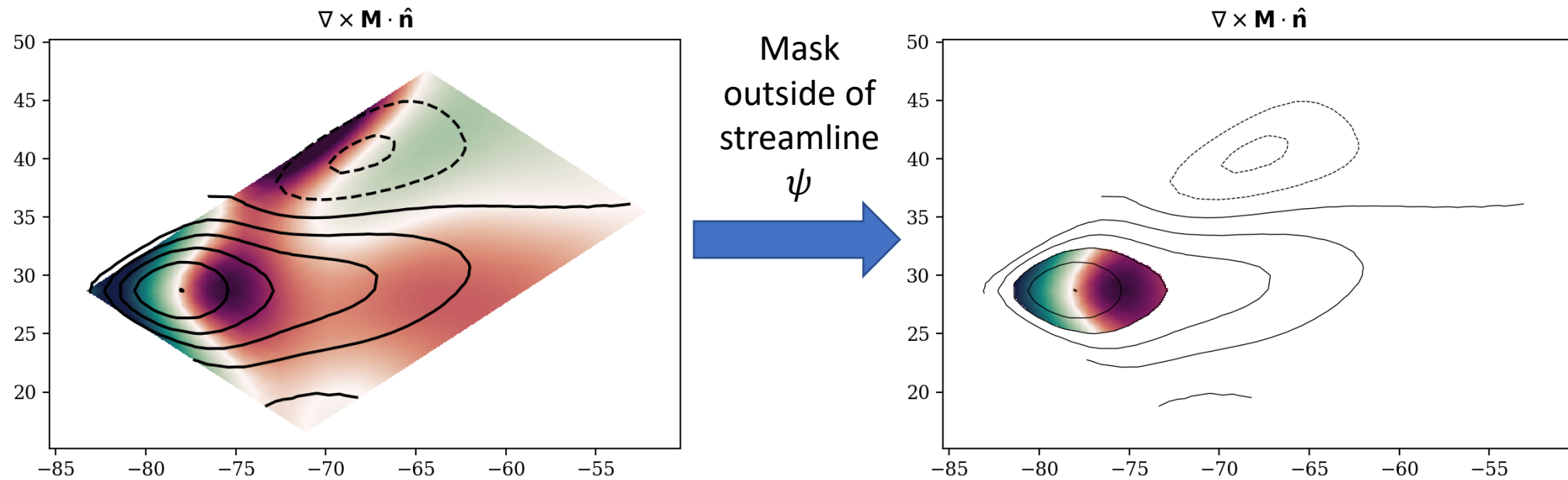


GYRE box model is used for illustrative purposes

Contour integrations

Before we get to results, let's summarise the full procedure

4. Mask values outside of a given streamline ψ (using Scikit-image)

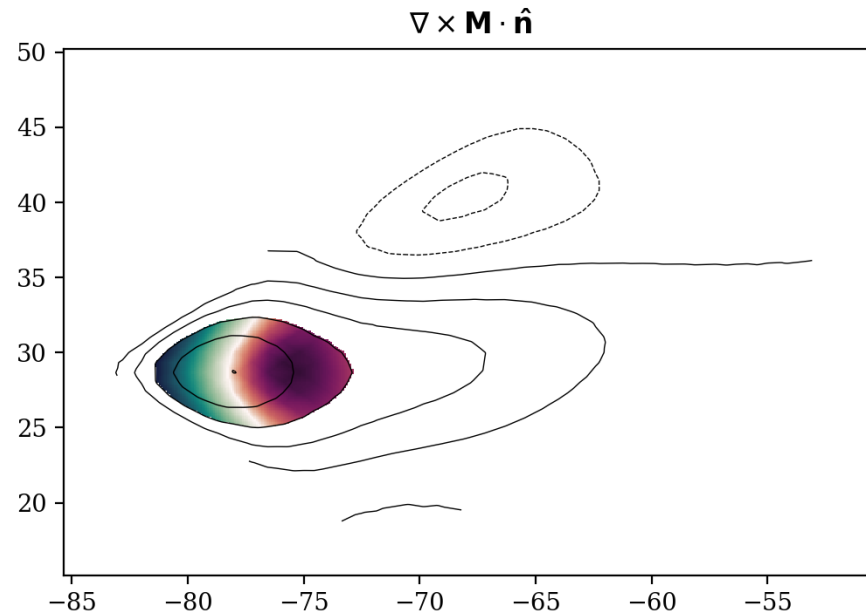


GYRE box model is used for illustrative purposes

Contour integrations

Before we get to results, let's summarise the full procedure

5. Integrate over enclosed area



Integrate



$$I(\psi)$$

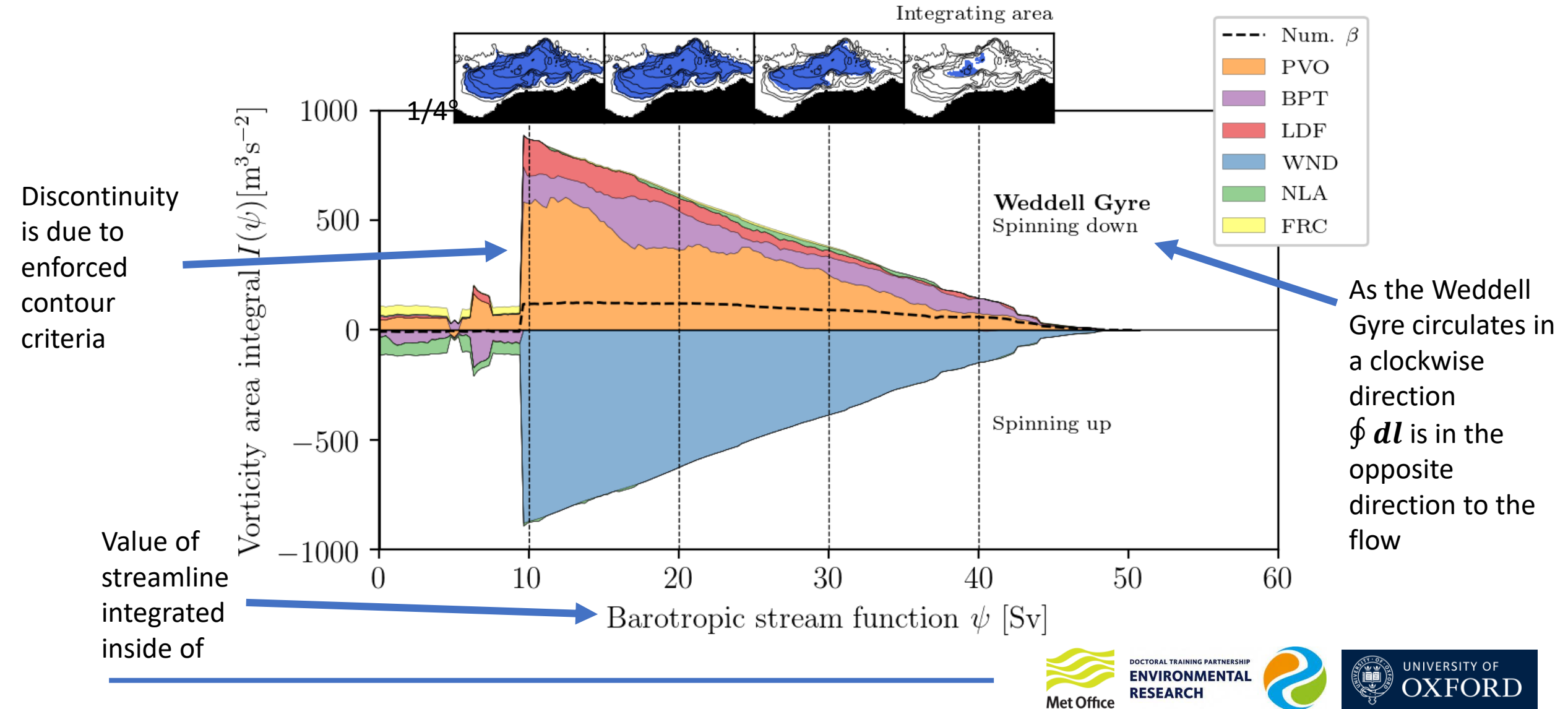
GYRE box model is used for illustrative purposes

Contour integrations

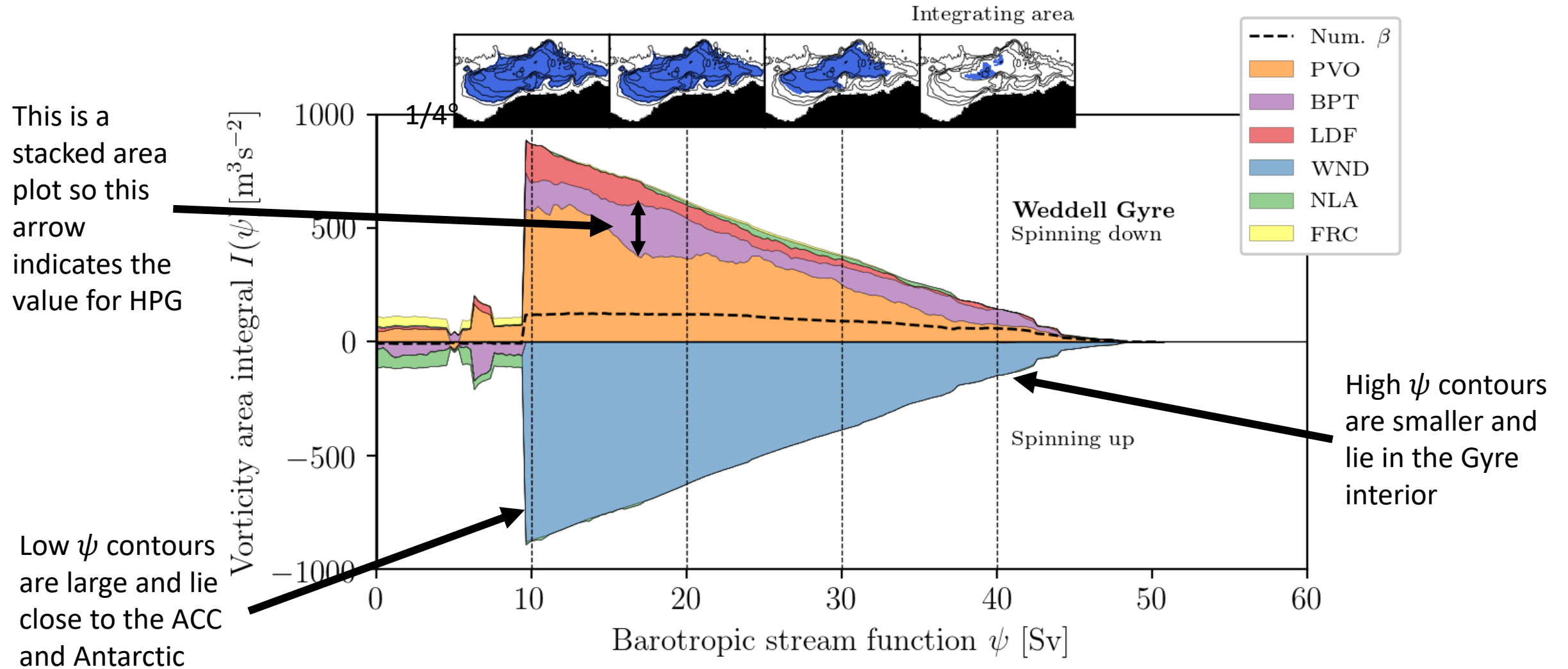
Before we get to results, let's summarise the full procedure

1. Depth integrate momentum trend $\mathbf{m}$
2. Take the curl of $\mathbf{M}$ using M_u and M_v
3. Interpolate $\nabla \times \mathbf{M} \cdot \hat{\mathbf{k}}$ to a fine grid to minimise edge effects when integrating
4. Mask values outside of a given streamline (using Scikit-image)
5. Integrate over enclosed area

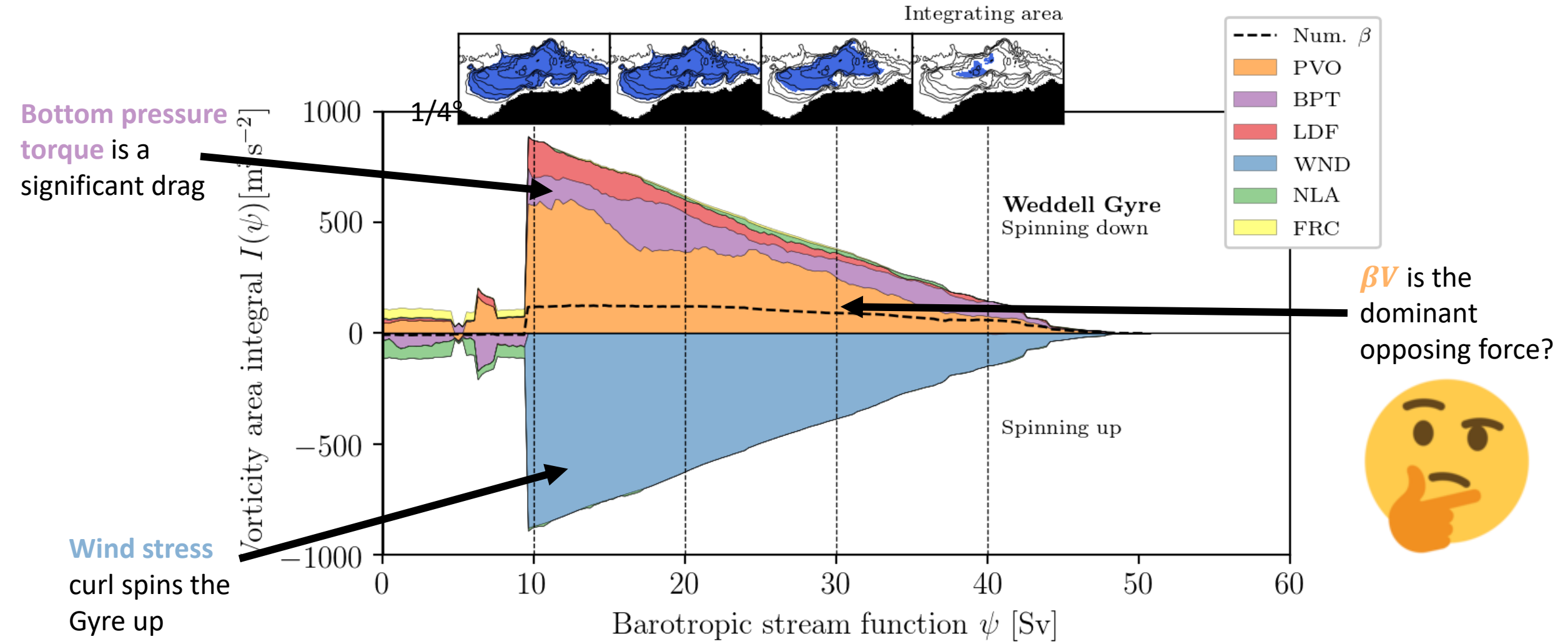
Weddell Gyre results



Weddell Gyre results



Weddell Gyre results



Why is PVO so big?

Analytically we would expect the Coriolis contribution to be **zero**

$$I_{PVO}(\psi) = \iint_{A_\psi} \nabla \times (-f \hat{\mathbf{k}} \times \mathbf{U}) \cdot \hat{\mathbf{k}} = - \iint_{A_\psi} \nabla_h \cdot (f \mathbf{U}) dA$$

Using the divergence theorem we would expect

$$I_{PVO}(\psi) = - \oint_{\psi} f \mathbf{U} \cdot \hat{\mathbf{n}} ds = 0$$

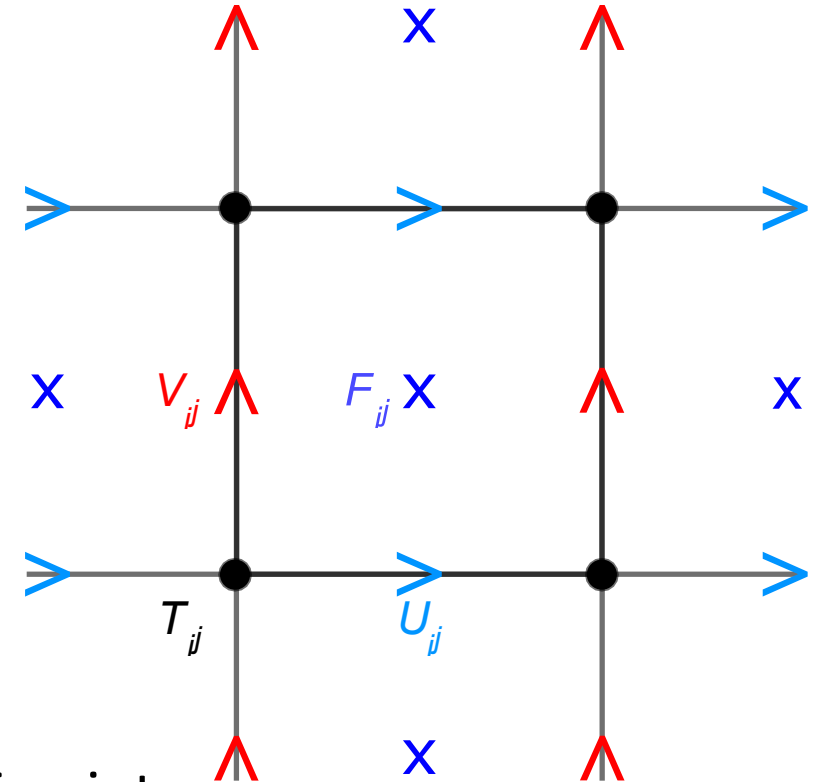
As the flow is parallel to the streamlines

Why is PVO so big?

On the **model (C) grid** would we expect the same?

Data points lie on a staggered grid

- Scalar values (e.g. SSH) on T points
- Velocities on U and V points
- Vorticities and stream functions (e.g. f and ψ) on F points



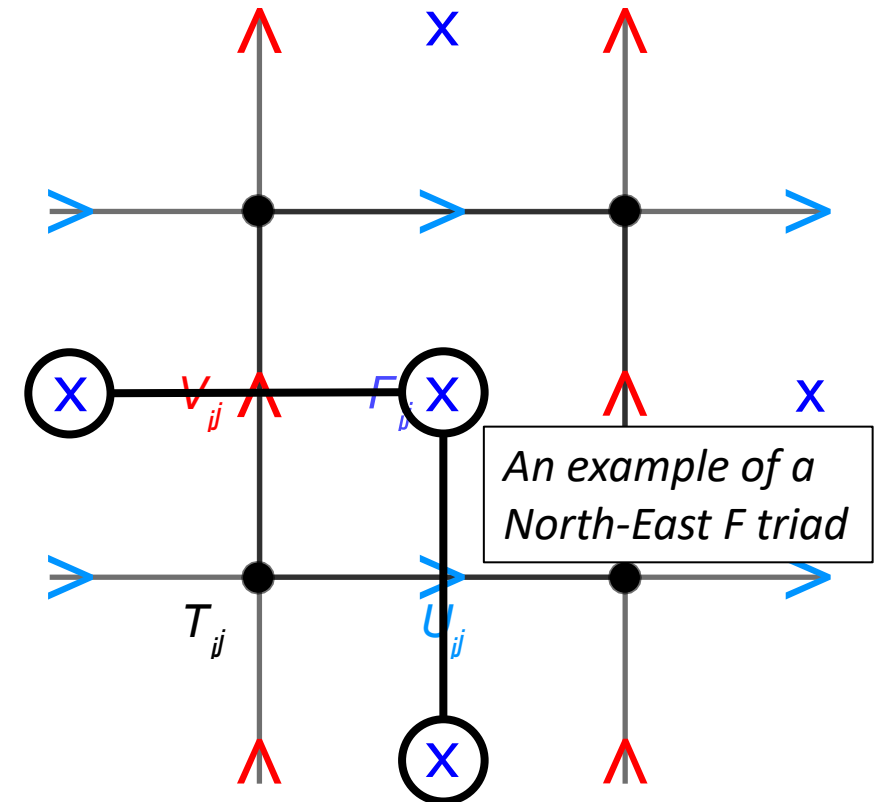
Why is PVO so big?

Calculating the Coriolis force is not trivial

The exact method depends on which **Coriolis scheme** is used.

In this case the **EEN** (energy and enstrophy conserving scheme) is used.

EEN relies on **thickness weighted triads** of F points.



Why is PVO so big?

The **curl** of the **depth-integrated** Coriolis force is not simple ...

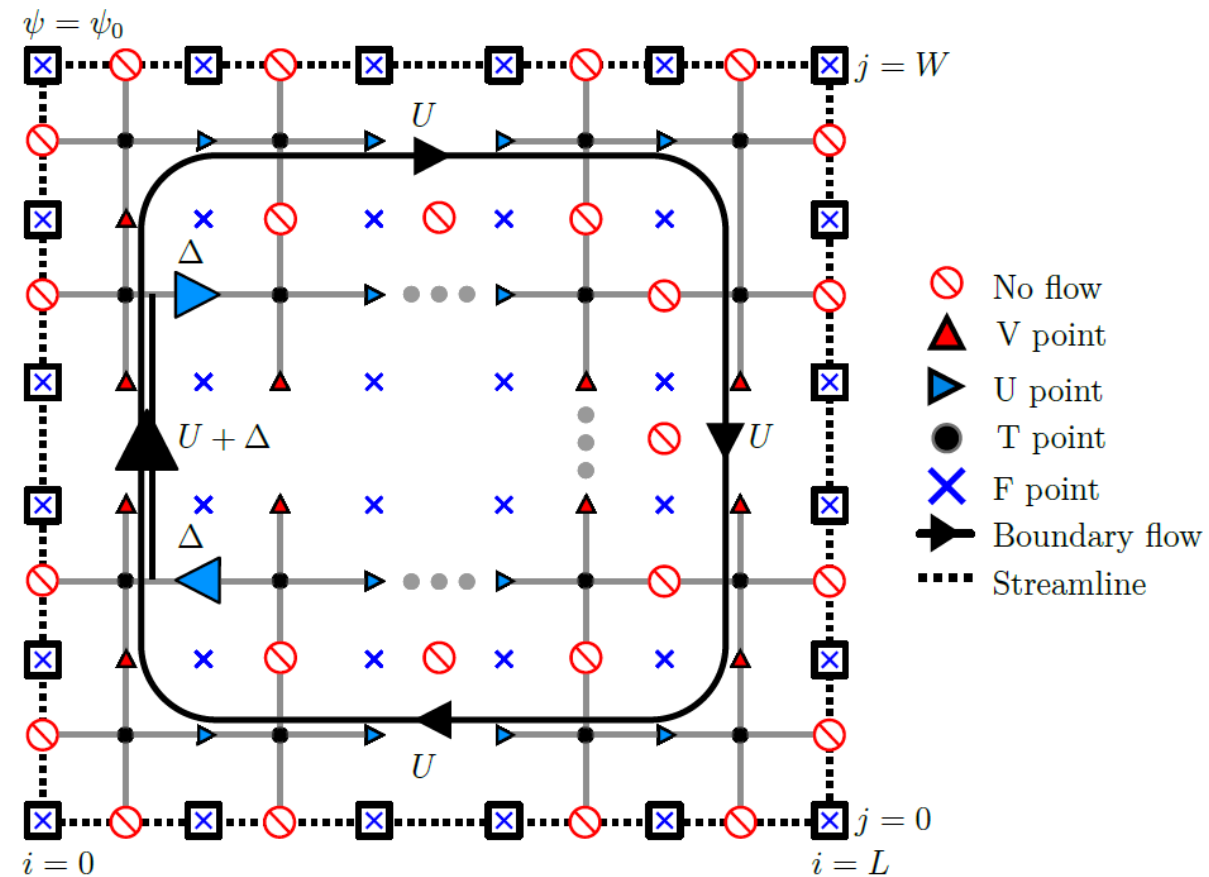
$$\begin{aligned}
 = & \frac{\sum_k}{12(e^{1f}e^{2f})} \left[-F_{i,j+1,k}^{NE} (ve^{3v}e^{1v})_{i,j+1} e_{i,j+1,k}^{3u} - F_{i+1,j+1,k}^{NW} (ve^{3v}e^{1v})_{i+1,j+1} e_{i,j+1,k}^{3u} + F_{i,j,k}^{SE} (ve^{3v}e^{1v})_{i,j-1} e_{i,j,k}^{3u} \right. \\
 & + F_{i+1,j,k}^{SW} (ve^{3v}e^{1v})_{i+1,j-1} e_{i,j,k}^{3u} - F_{i+1,j+1,k}^{SE} (ue^{3u}e^{2u})_{i+1,j+1,k} e_{i+1,j,k}^{3v} - F_{i+1,j,k}^{NE} (ue^{3u}e^{2u})_{i+1,j,k} e_{i+1,j,k}^{3v} \\
 & + F_{i,j+1,k}^{SW} (ue^{3u}e^{2u})_{i-1,j+1,k} e_{ijk}^{3v} + F_{ijk}^{NW} (ue^{3u}e^{2u})_{i-1,j,k} e_{ijk}^{3v} + (ve^{3v}e^{1v})_{i+1,j,k} (F_{i+1,j,k}^{NW} e_{ijk}^{3u} - F_{i+1,j+1,k}^{SW} e_{i,j+1,k}^{3u}) \\
 & + (ve^{3v}e^{1v})_{ijk} (F_{ijk}^{NE} e_{ijk}^{3u} - F_{i,j+1,k}^{SE} e_{i,j+1,k}^{3u}) + (ue^{3u}e^{2u})_{i,j+1,k} (F_{i,j+1,k}^{SE} e_{ijk}^{3v} - F_{i+1,j+1,k}^{SW} e_{i+1,j,k}^{3v}) \\
 & \left. + (ue^{3u}e^{2u})_{ijk} (F_{ijk}^{NE} e_{ijk}^{3v} - F_{i+1,j,k}^{NW} e_{i+1,j,k}^{3v}) \right]
 \end{aligned}$$

Look at two simple cases to interpret this

Simple case 1: Flat topography

With **flat topography** (no partial cells)

- Integrated over a “rectangular” stream function
- Velocity equals U everywhere on all sides except western boundary
- Simple intensification on western boundary, $U + \Delta$



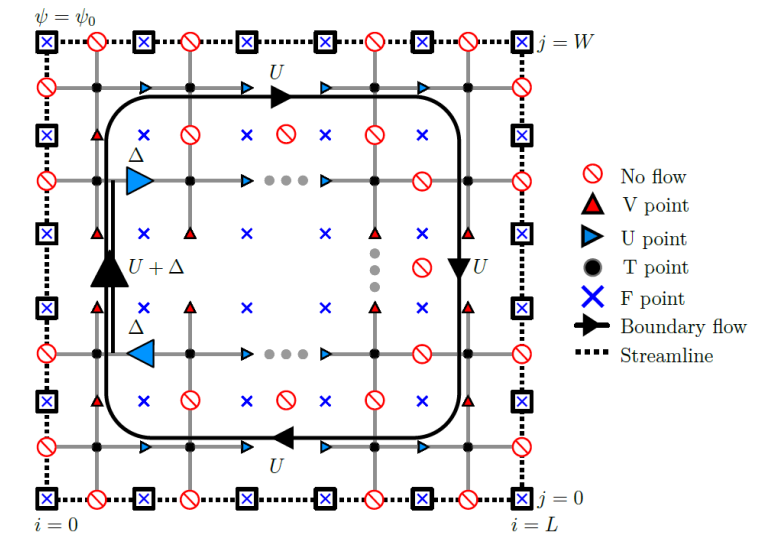
Simple case 1: Flat topography

In this case

$$I_{PVO} = \frac{1}{12} [(f_{i_0+1, j_0+M-1}^{NE} - f_{i_0+1, j_0+2}^{NE}) + (f_{i_0+1, j_0+M-1}^{SE} - f_{i_0+1, j_0+2}^{SE})] \Delta$$

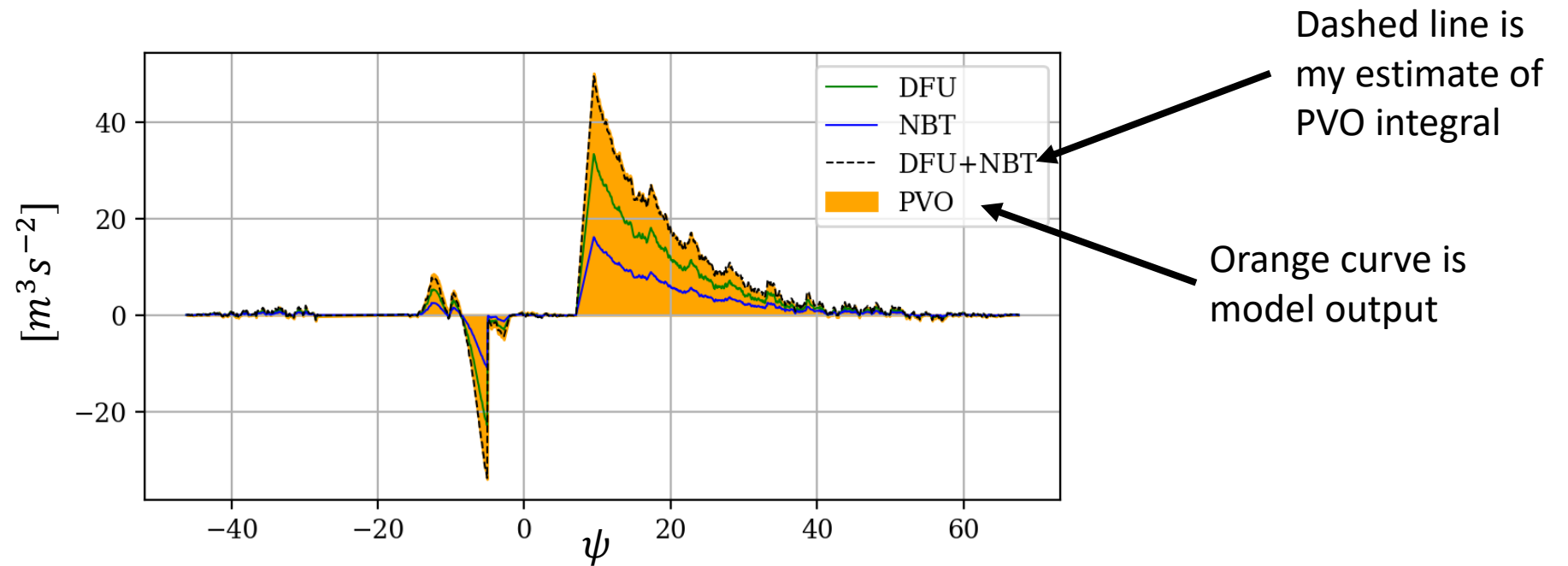
Additional β effect on intensified current

An example of a significant non-zero integral
($\sim 10 \text{ m}^3 \text{ s}^{-2}$)



Simple case 1: Flat topography

Verified in the GYRE box model

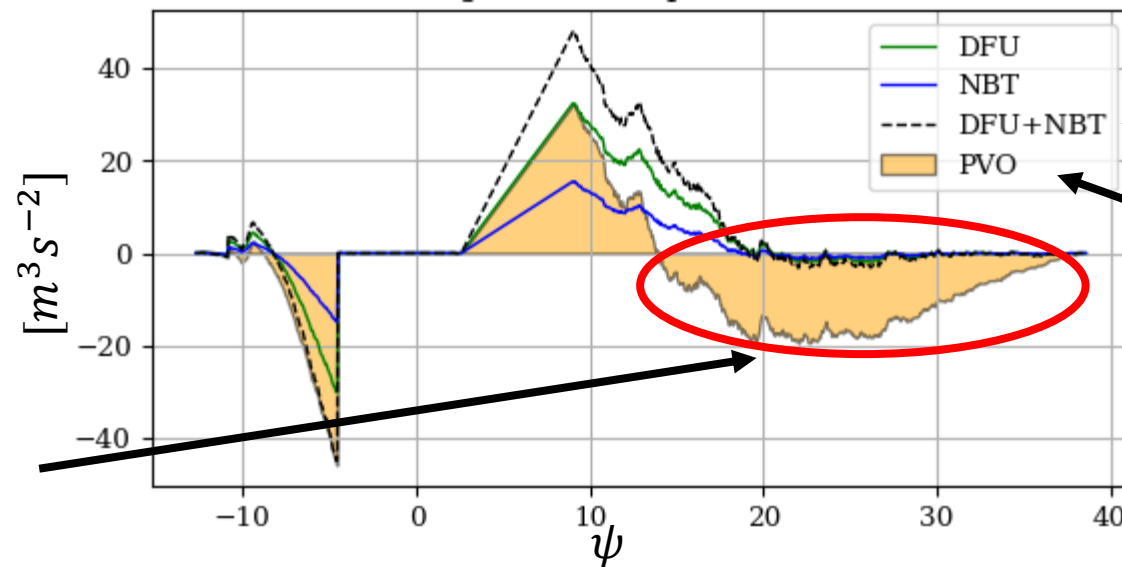


Decomposition of PVO contour integral in flat GYRE model. Derived form of PVO matches model output exactly

Simple case 1: Flat topography

This breaks down when topography is introduced

Estimate is missing
a feature for the
subtropical gyre



Dashed line is
my estimate of
PVO integral

Orange curve is
model output

Decomposition of PVO contour integral in sloped GYRE model. Derived form of PVO deviates severely

Simple case 2: F plane with topography

F plane ($\beta = 0$) with varying topography (partial cells)

Assuming a gentle slope ($\alpha \ll 1$)

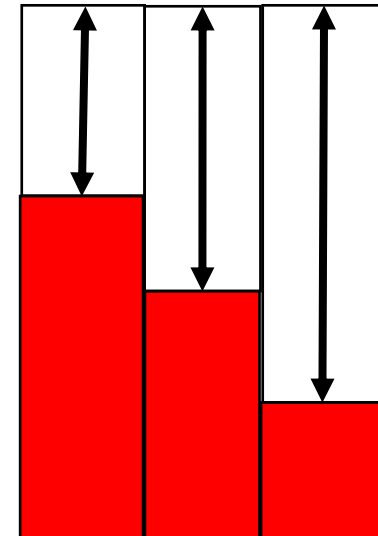
In this case the PVO vorticity trend is

$$= \frac{f_0 \alpha}{3} [(w_{i+1,j,0} + w_{i+1,j+1,0}) - (w_{i,j,0} + w_{i,j+1,0})] \sim 10^{-10} \text{ m s}^{-2}$$

A **numerical vortex stretching** on down-slope
gradients in w

Cell thicknesses at lowest level

$$e^3(1 - \alpha) \quad e^3 \quad e^3(1 + \alpha)$$



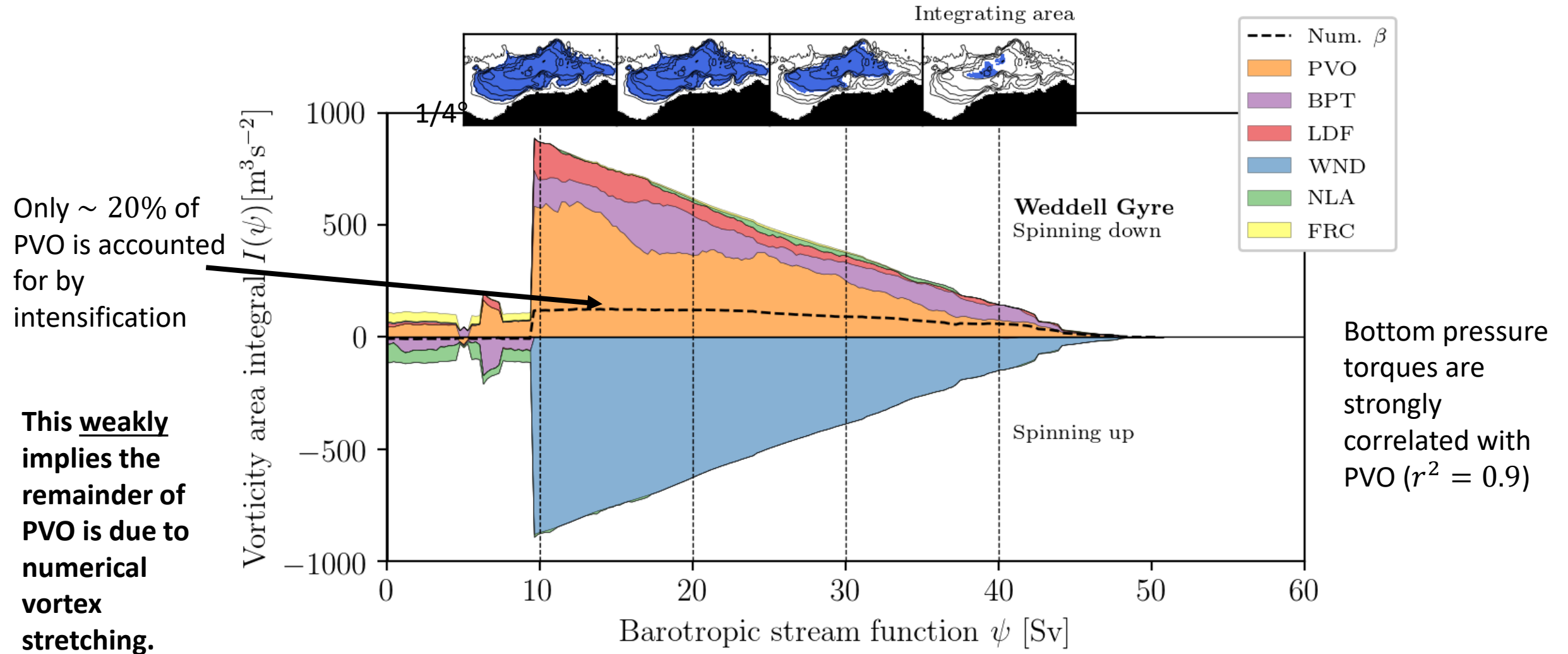
Why is PVO so big?

The PVO integral has two contributions:

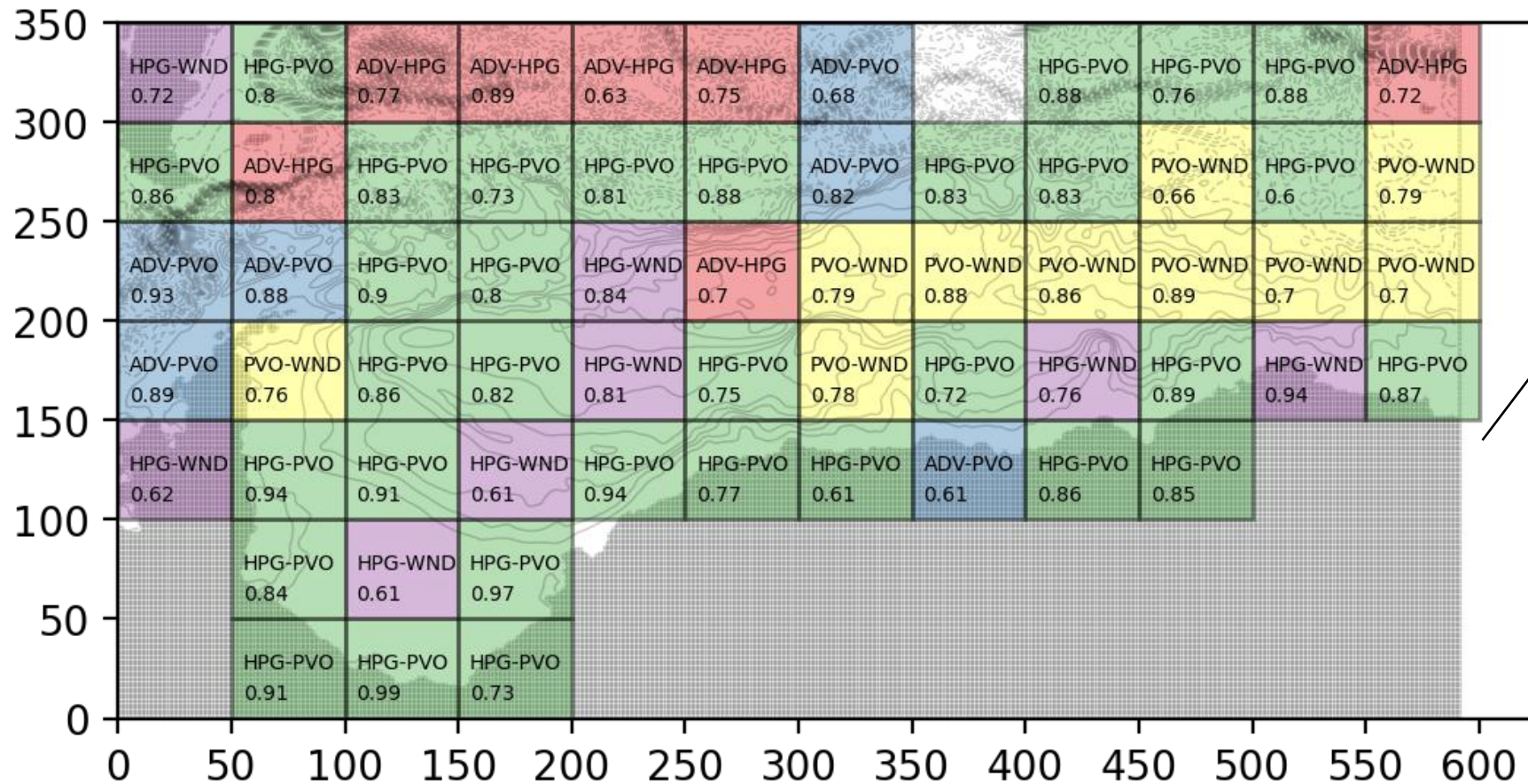
- Numerical beta effect on intensified currents
Can calculate explicitly
- Numerical vortex stretching of down-slope gradients in w
Currently unable to calculate accurately

Which of these dominates in the Weddell Gyre?

Num. Vortex stretching in the Weddell Sea



Num. Vortex stretching in the Weddell Sea



Two largest contributions

HPG – PVO

0.87

Fraction of the vorticity budget occupied by the two largest terms

Summary

The Weddell Gyre has a strong dependence on model resolution

Studying the barotropic vorticity balance at $1/4^\circ$ identifies **Wind stress curl** and **PVO** as the leading order balance.

The PVO contribution has at least two numerical contributions due to

- Intensification of the current

- Numerical vortex stretching

In the $1/4^\circ$ model it appears numerical vortex stretching dominates

Future work

- Analyse vorticity balances of Weddell Gyre at 1° and $1/12^\circ$
- Attempt to recreate the numerical vortex stretching contribution
- Look at similar data sets with roughened/smoothed bathymetry
- Determine if these numerical factors are “real” and affecting the circulation.

Extra Slides



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Weddell Results (R4)

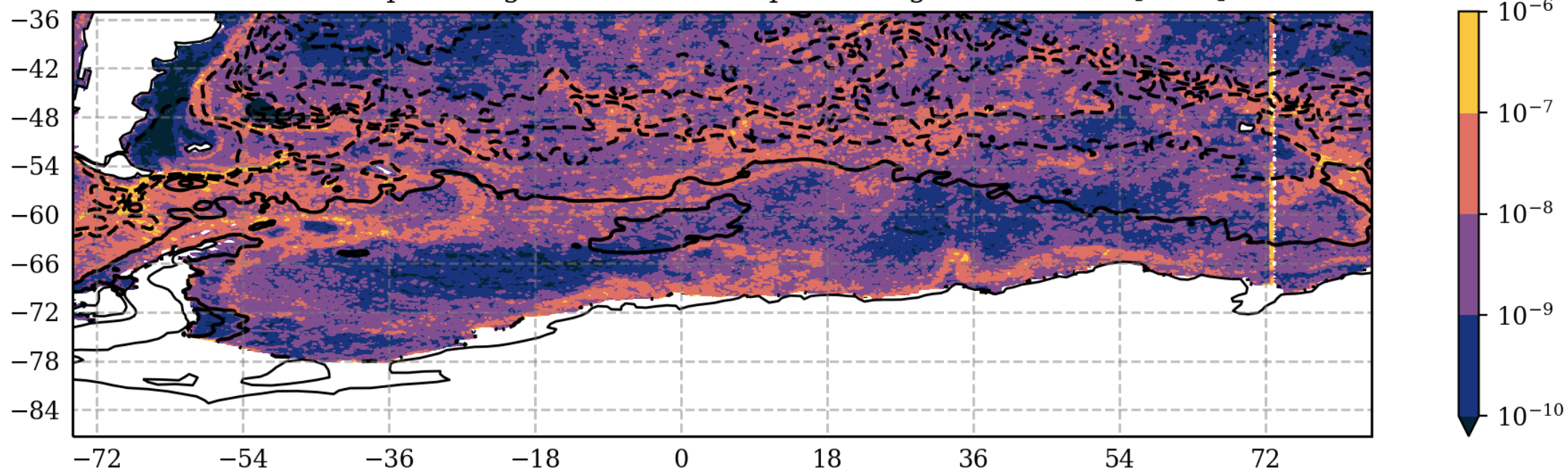


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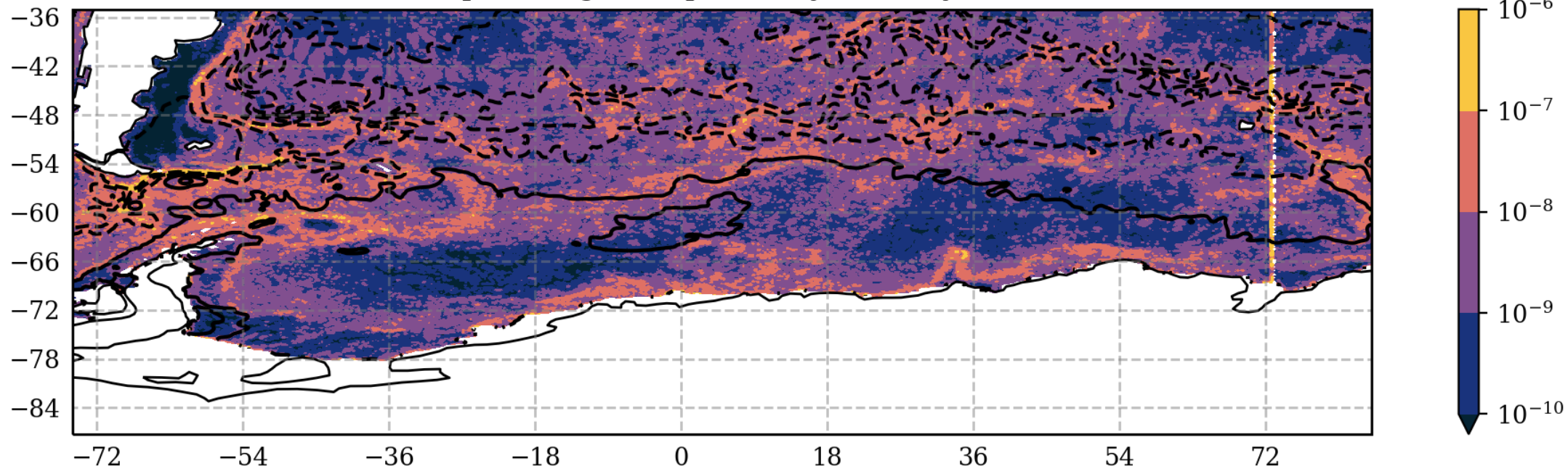
k-curl of depth-integrated horizontal pressure gradient trend [m/s²]

HPG



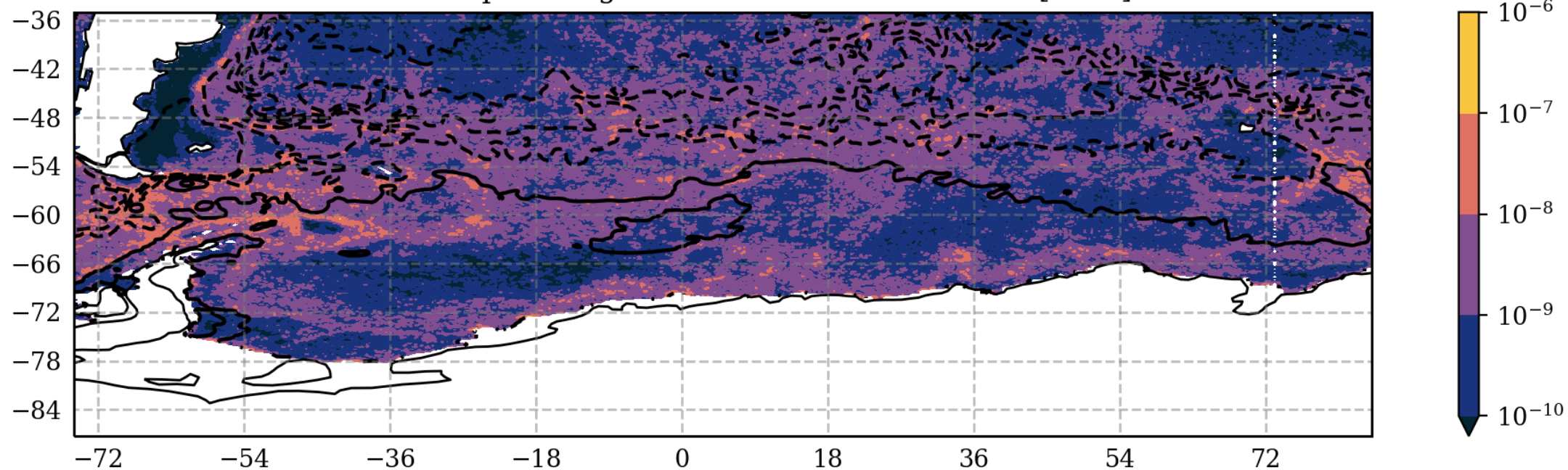
k-curl of depth-integrated planetary vorticity trend [m/s²]

PVO



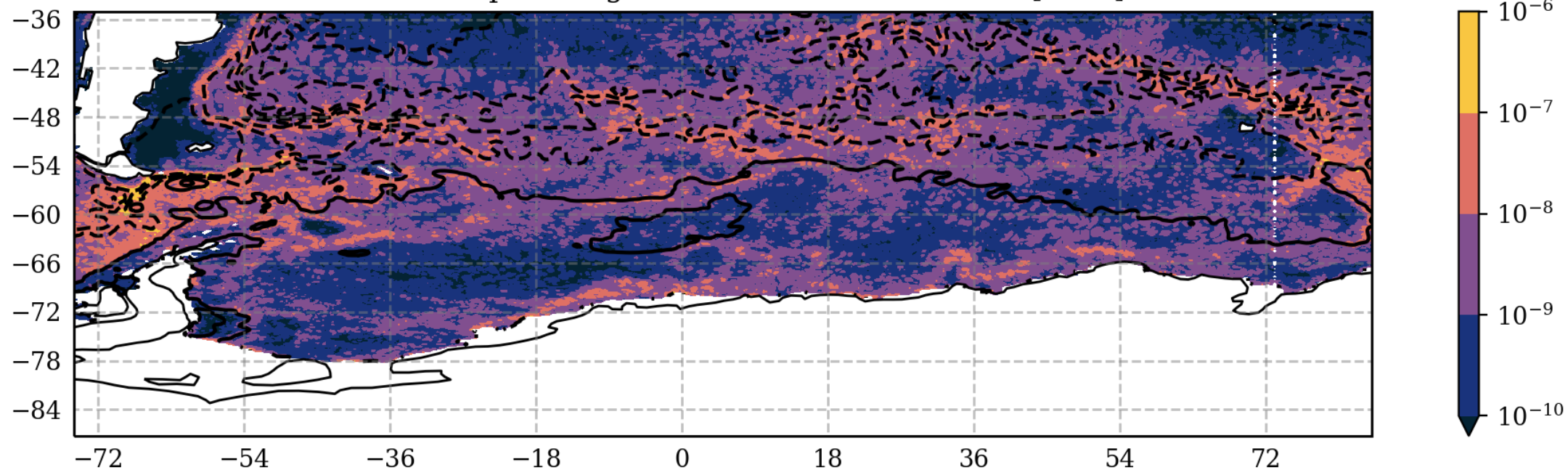
k-curl of depth-integrated lateral diffusion trend [m/s²]

LDF



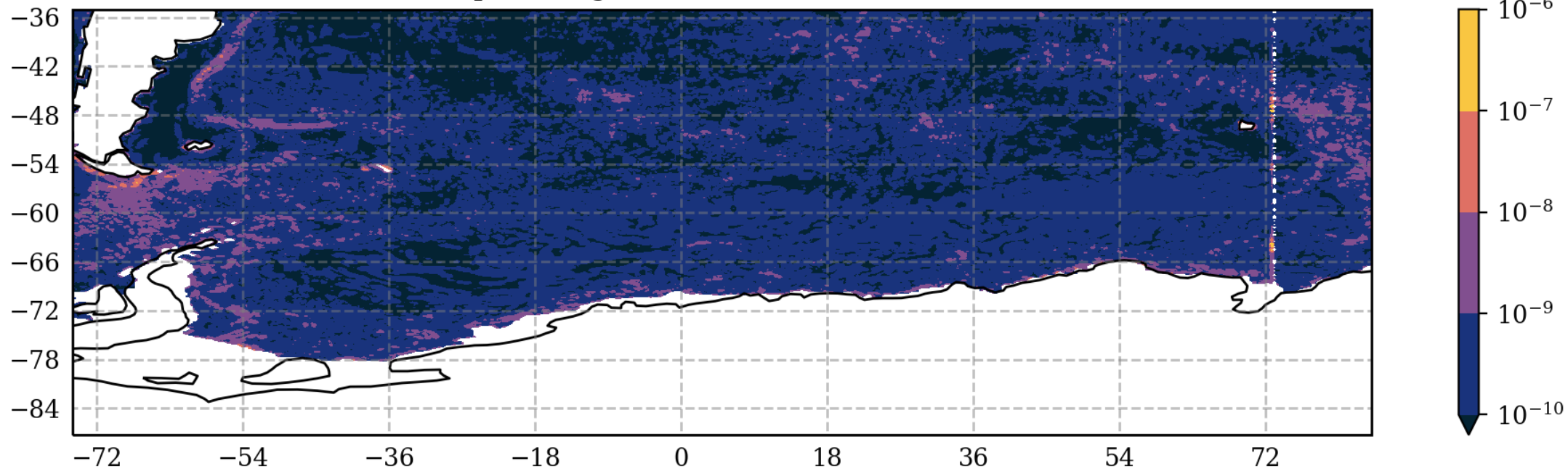
k-curl of depth-integrated total advection trend [m/s²]

ADV



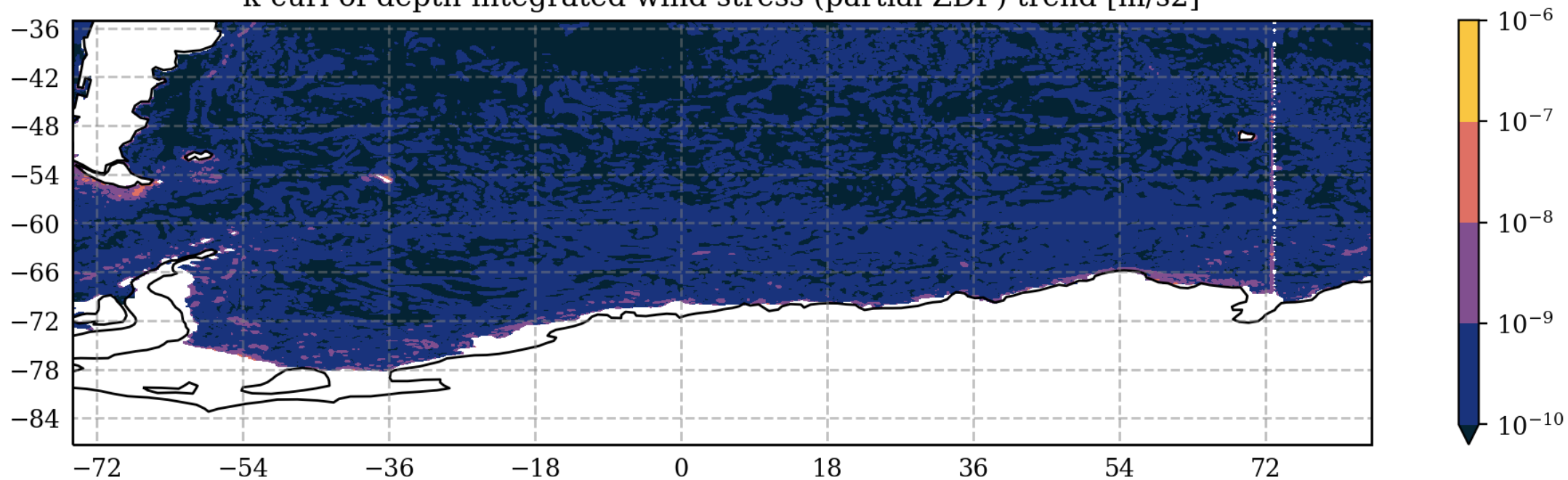
k-curl of depth-integrated vertical diffusion trend [m/s²]

ZDF



k-curl of depth-integrated wind stress (partial ZDF) trend [m/s²]

WND
(upper
100 m
of ZDF)



GYRE results



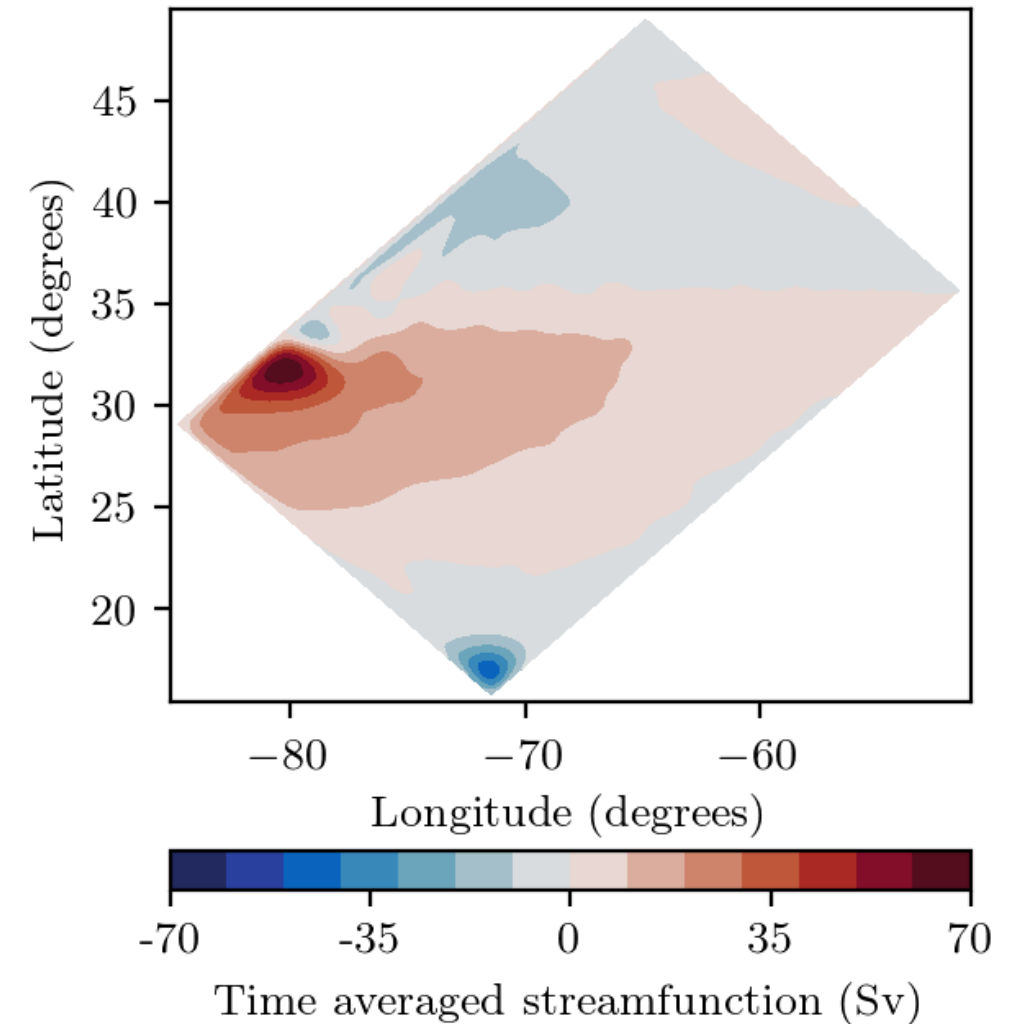
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Gyre results

Flat GYRE configuration R4

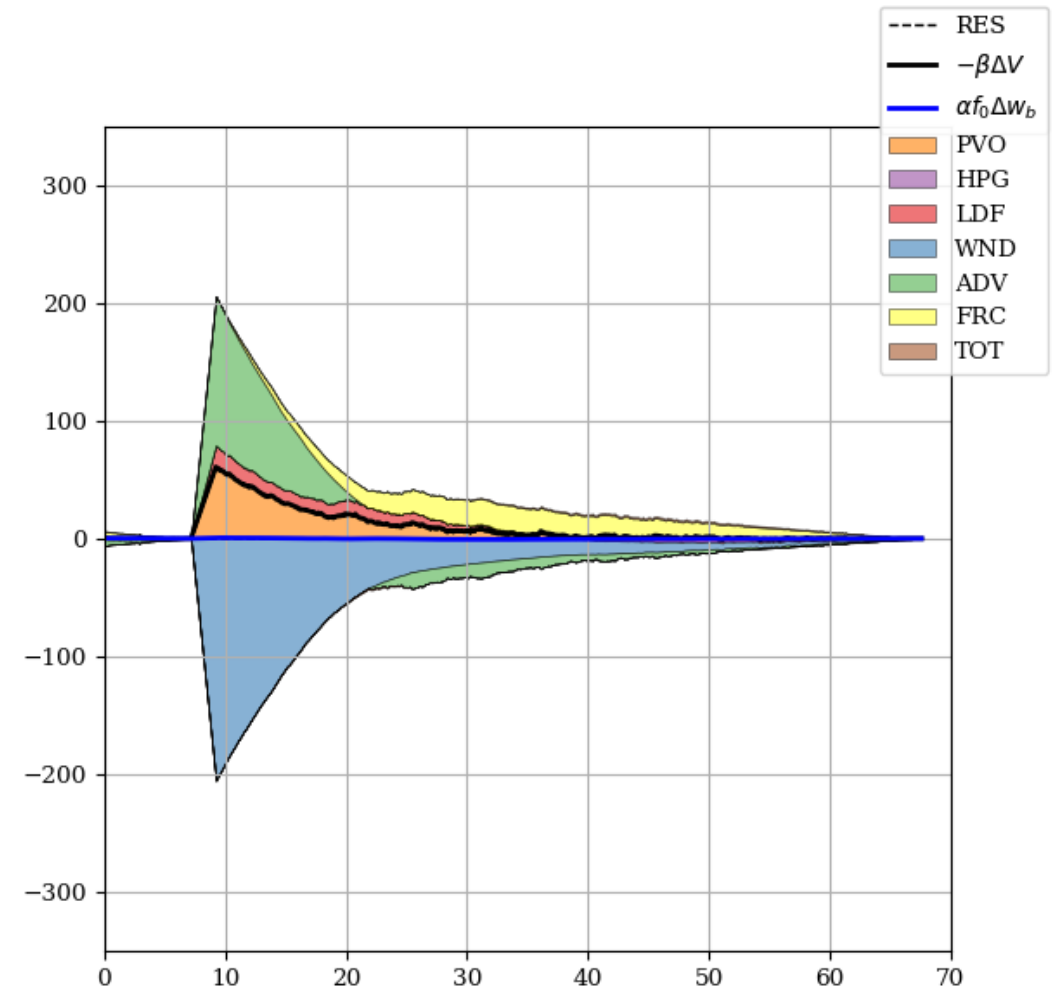
- Rectangular double gyre configuration
- Rotated grid
- ENE Coriolis scheme



Gyre results

Flat GYRE configuration R4

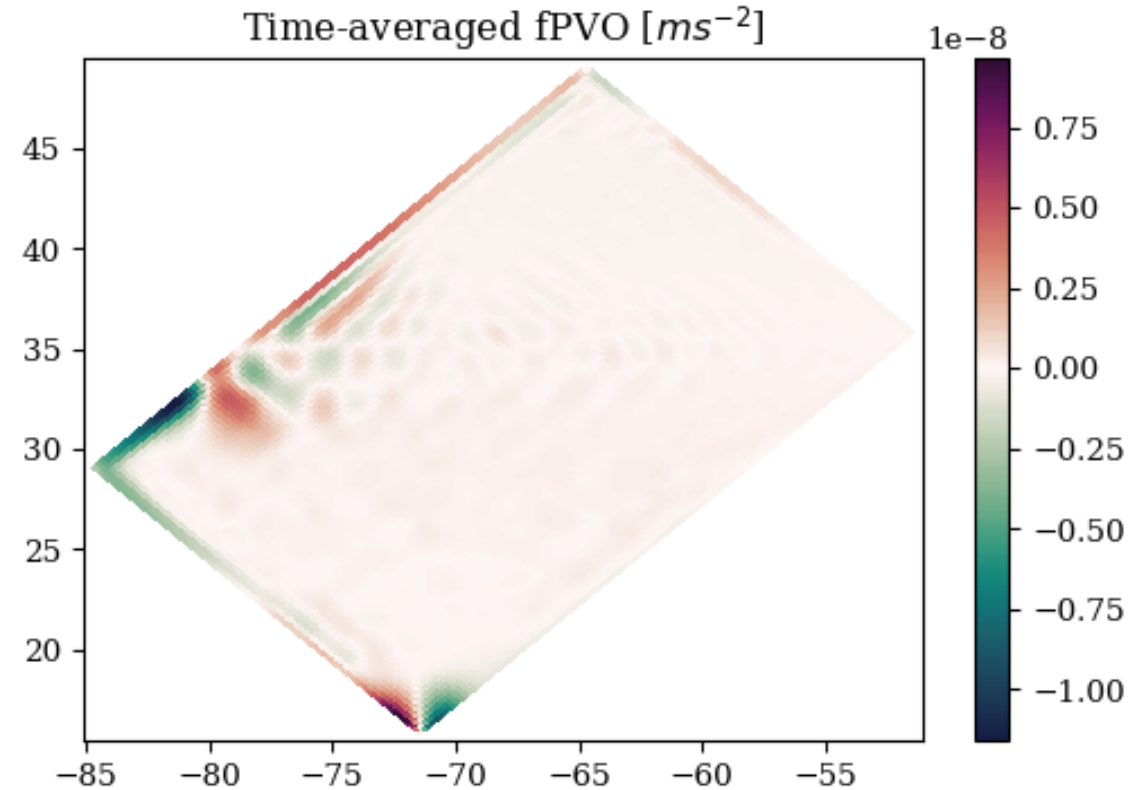
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Gyre results

Flat GYRE configuration R4

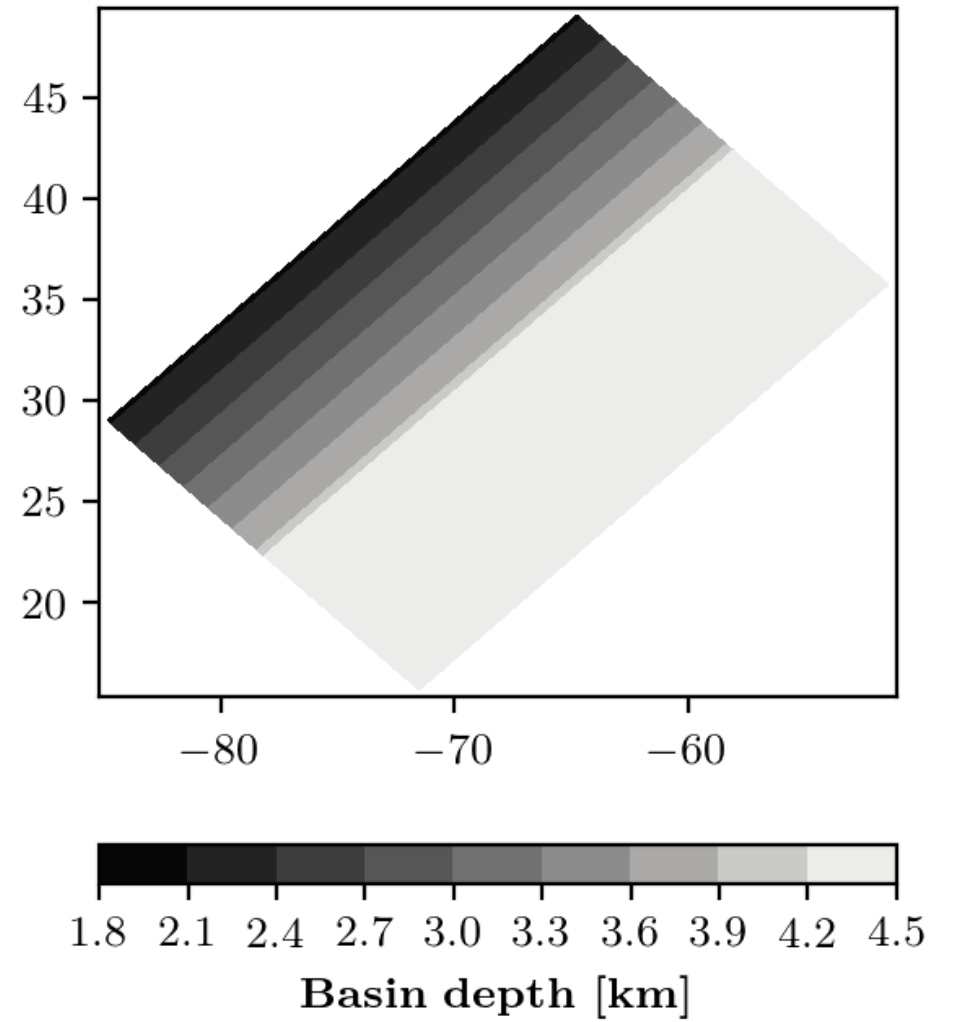
- Rectangular double gyre configuration
- Rotated grid
- ENE Coriolis scheme



Gyre results

Sloped GYRE configuration R4

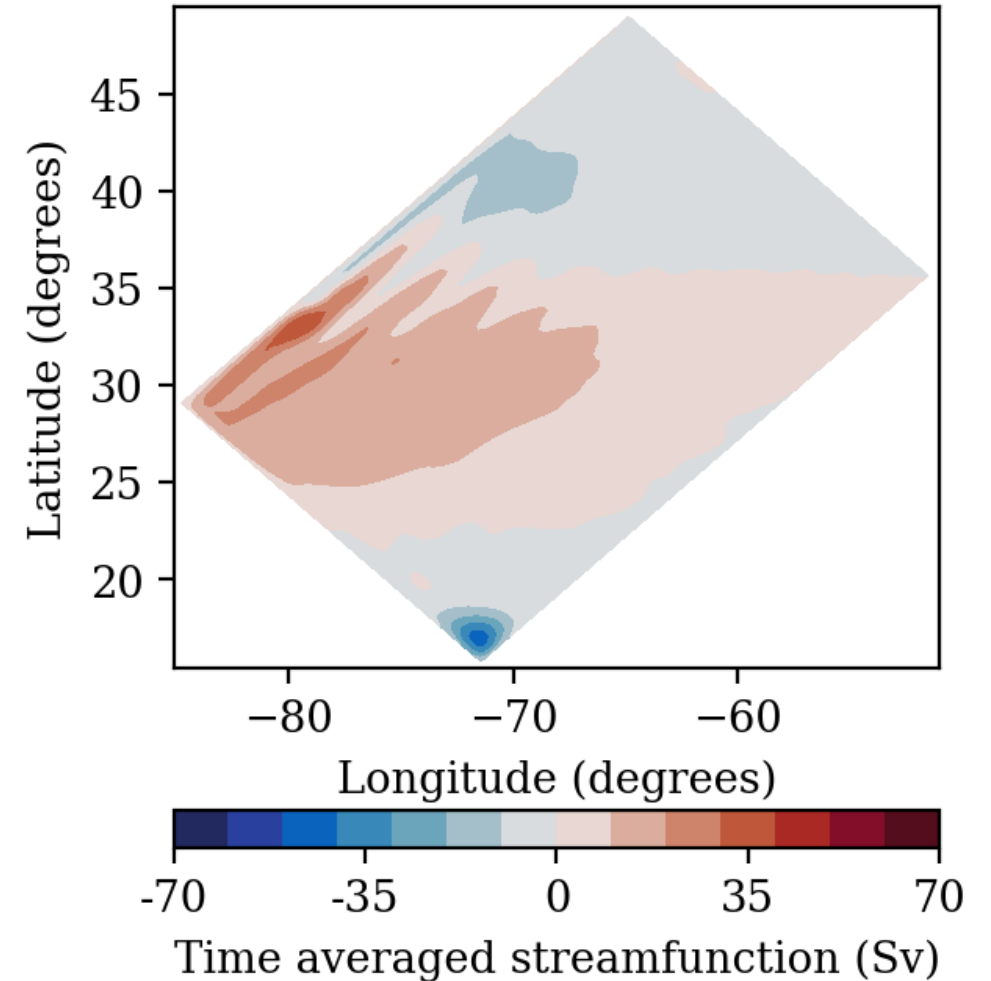
- Modified with simple slope (partial cells)
- Rectangular double gyre configuration
- Rotated grid
- ENE Coriolis scheme



Gyre results

Sloped GYRE configuration R4

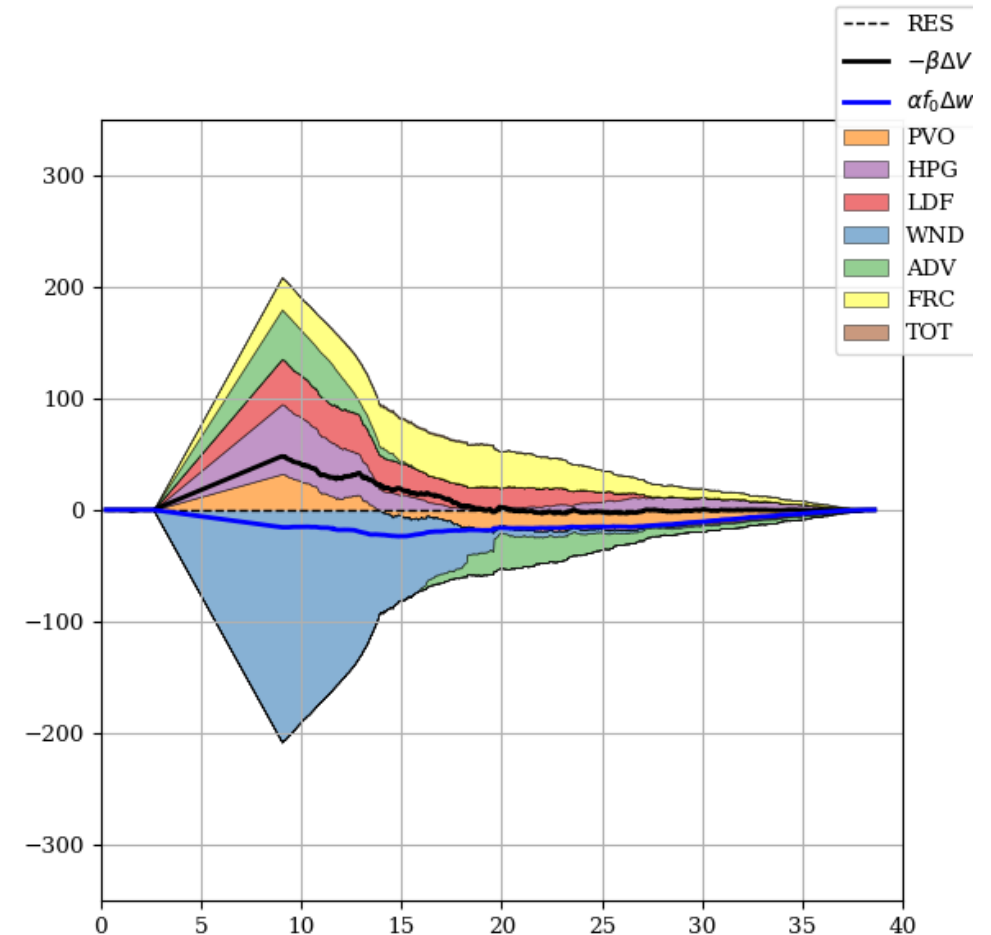
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Gyre results

Sloped GYRE configuration R4

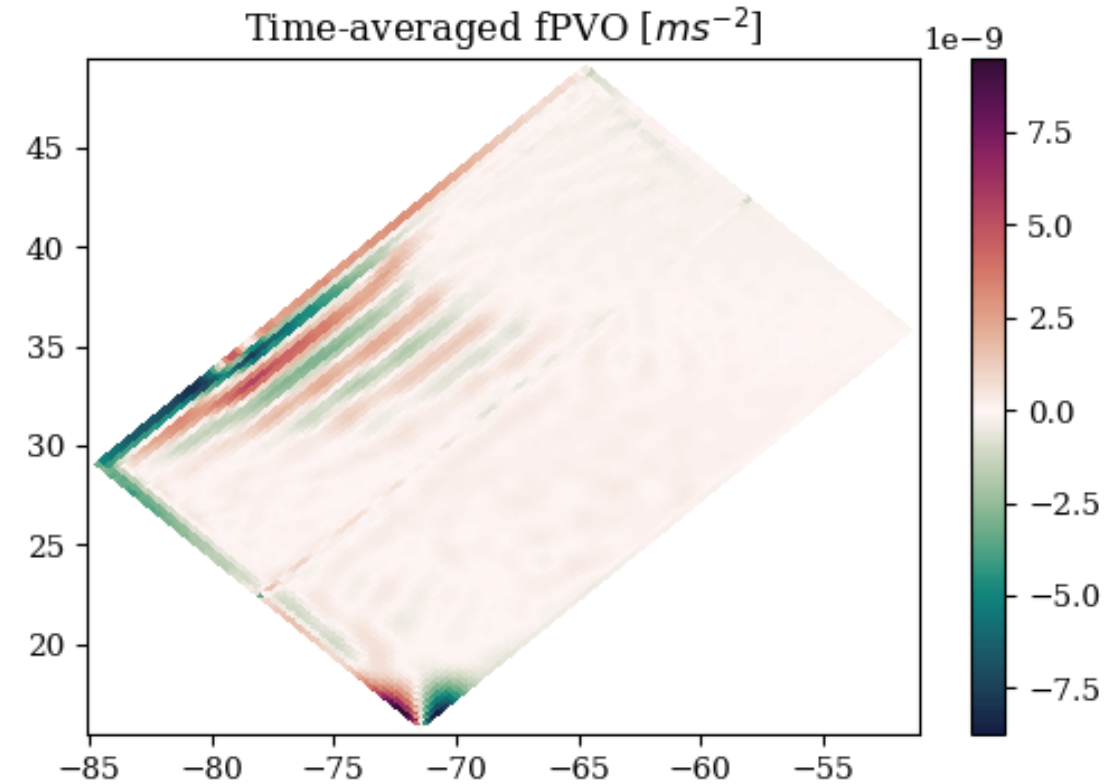
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Gyre results

Sloped GYRE configuration R4

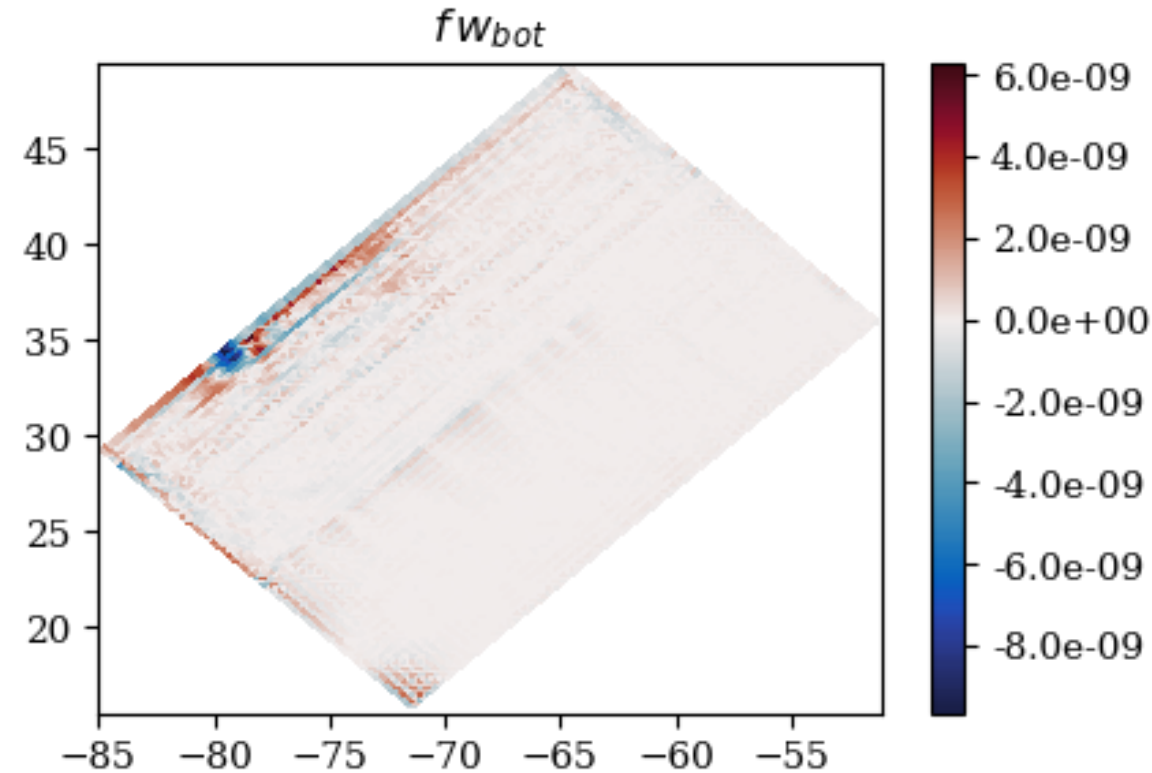
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Gyre results

Sloped GYRE configuration R4

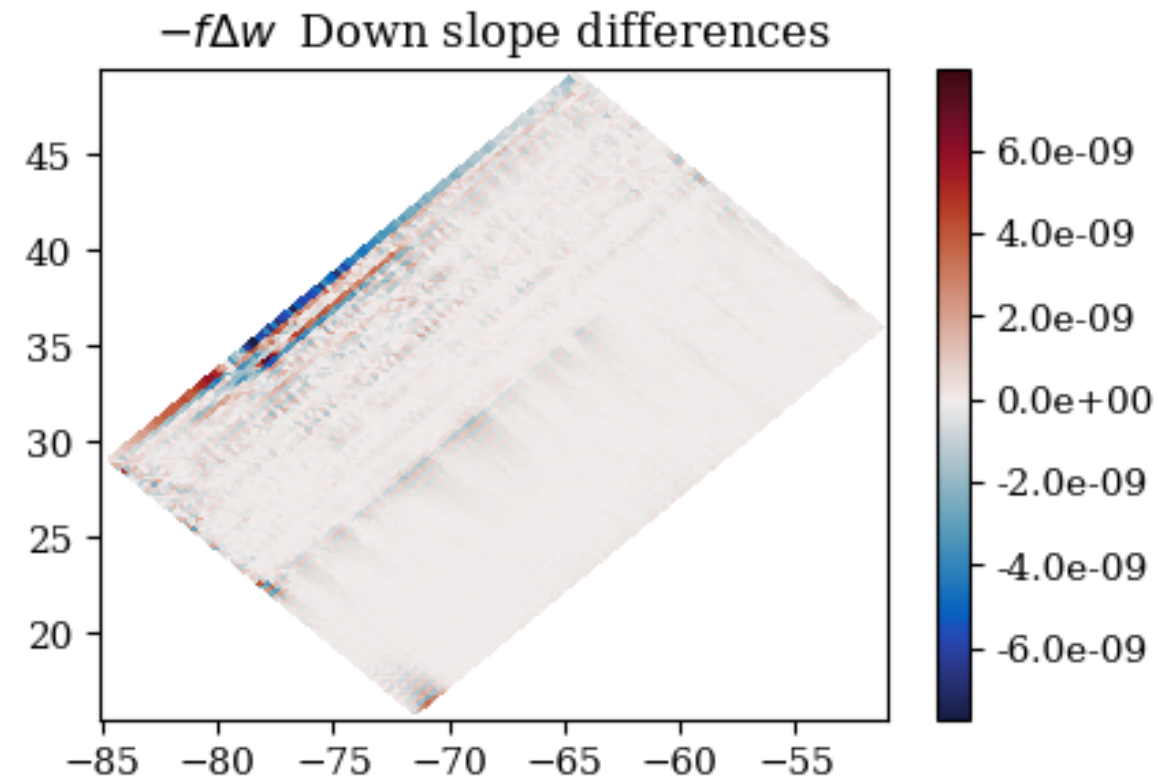
- Modified with simple slope (partial cells)
- Rectangular double gyre configuration
- Rotated grid
- ENE Coriolis scheme



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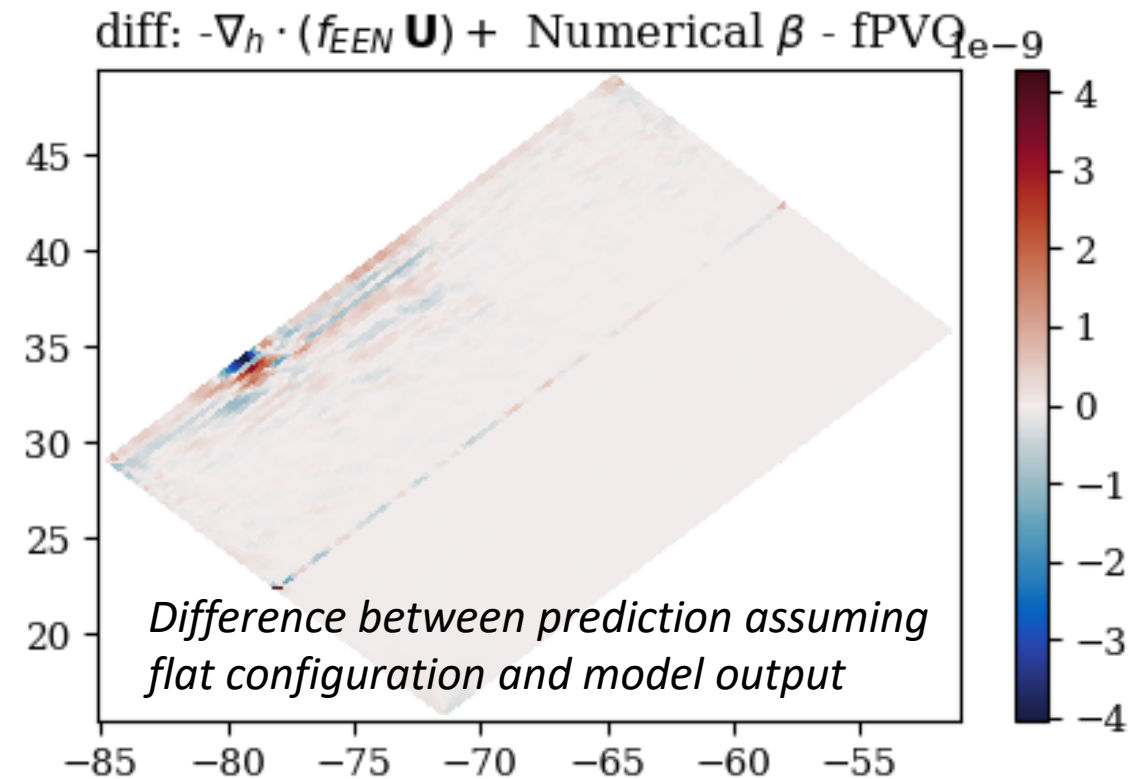
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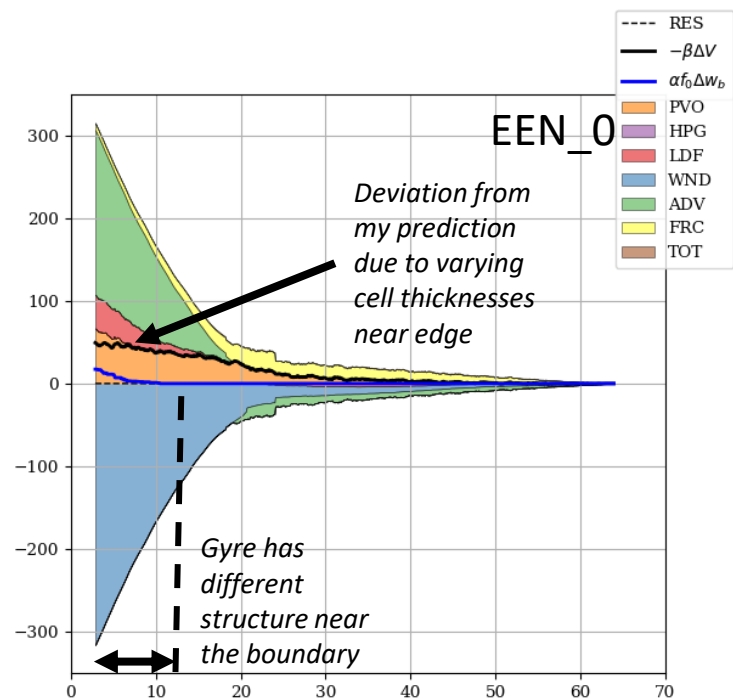


Comparing Coriolis Schemes



DOCTORAL TRAINING PARTNERSHIP
**ENVIRONMENTAL
RESEARCH**





Flat Gyre

Contour integrations under 4 different Coriolis schemes

EEN_0 – Energy and enstrophy conserving scheme

Used in the **Global** model

Calculates f cell thickness via $\sum(e3t * tmask)/4$

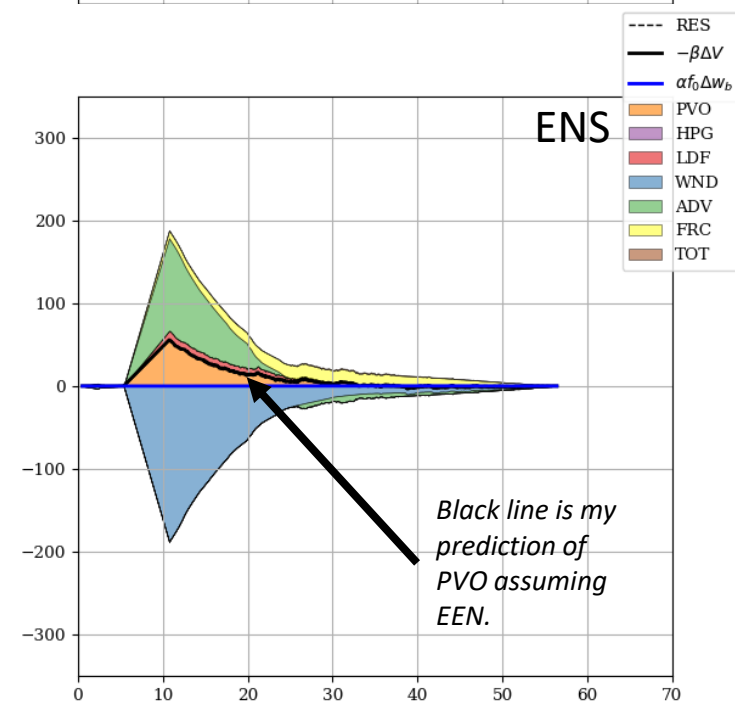
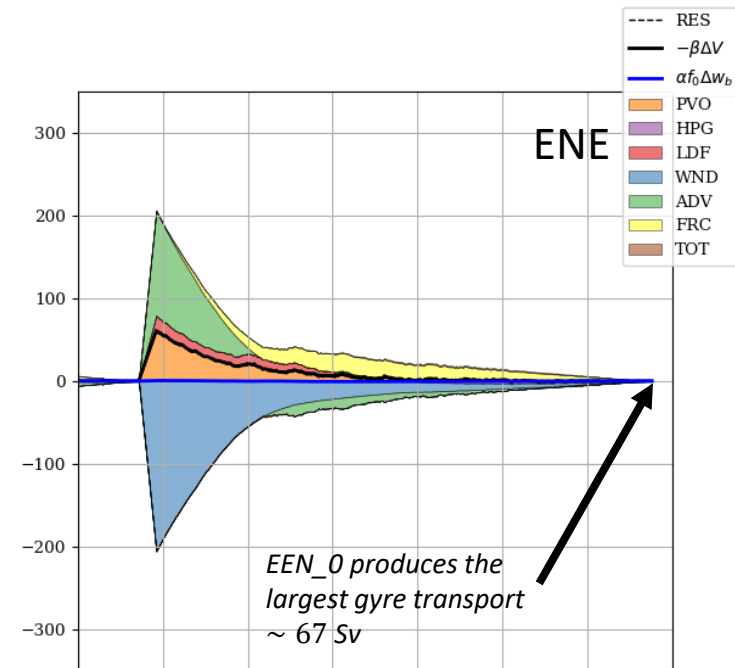
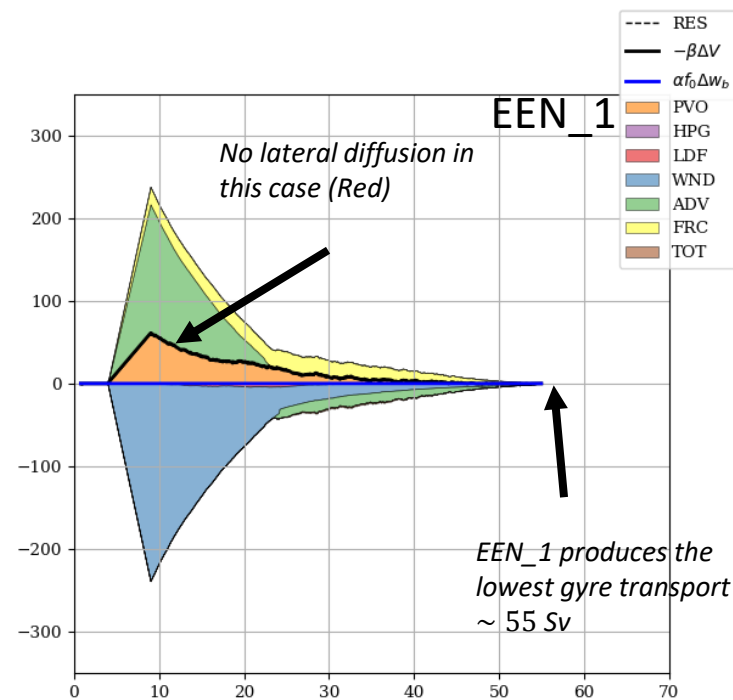
EEN_1 – Energy and enstrophy conserving scheme

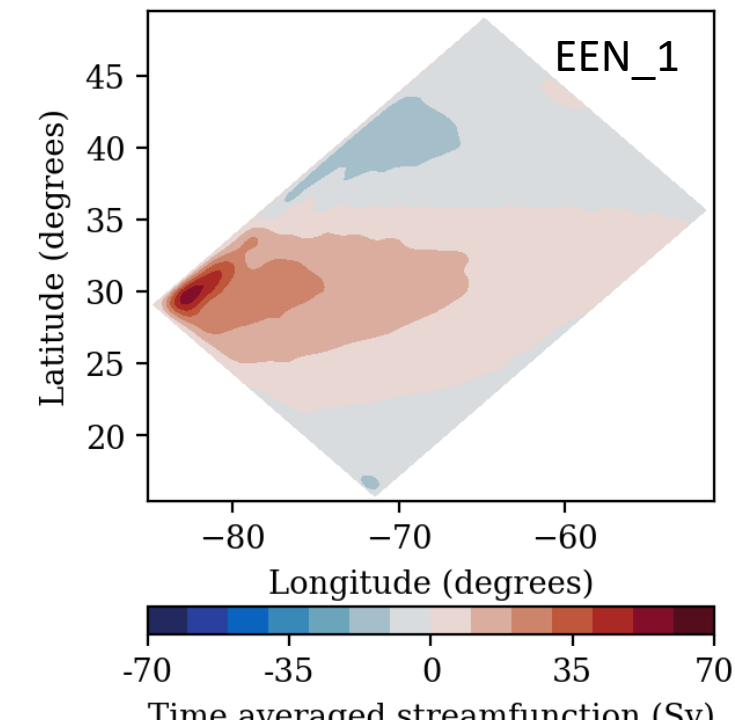
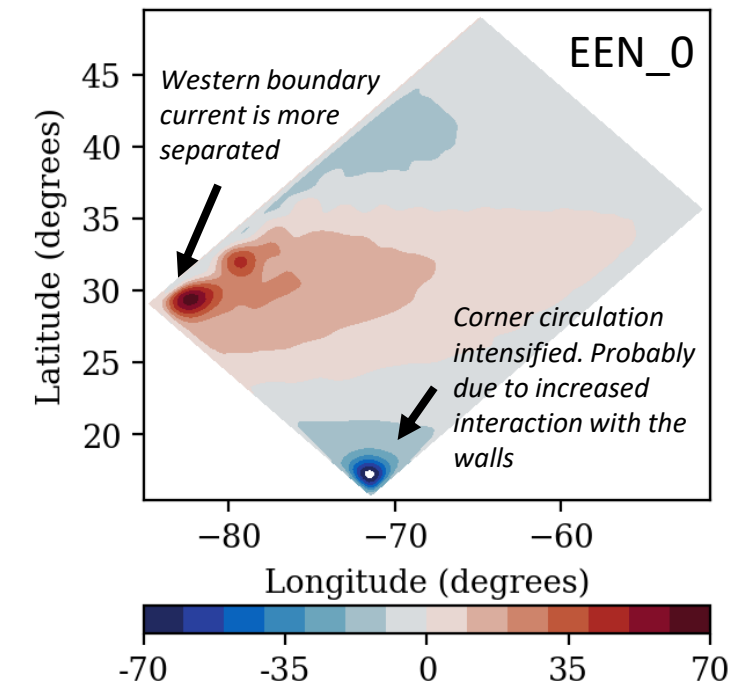
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Default for the **GYRE** model

ENS – Enstrophy conserving scheme





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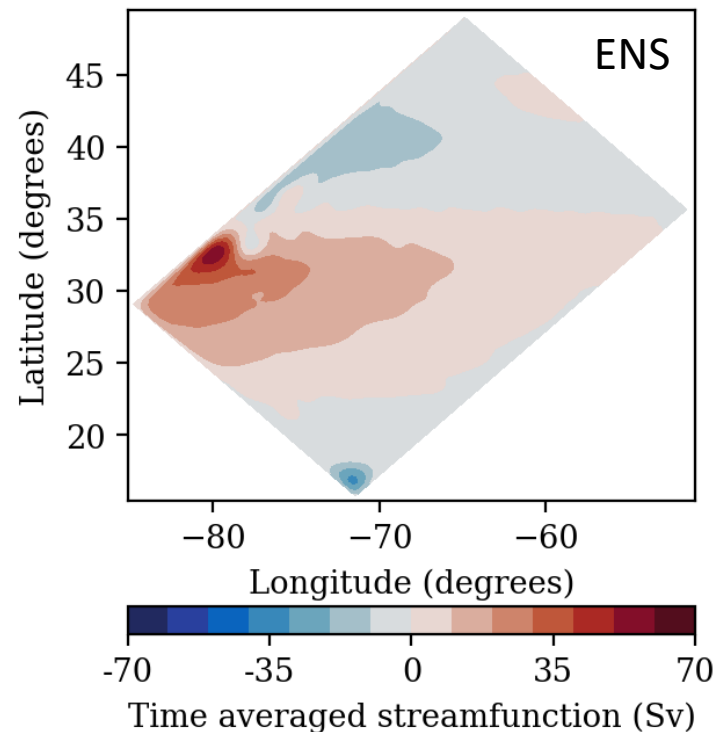
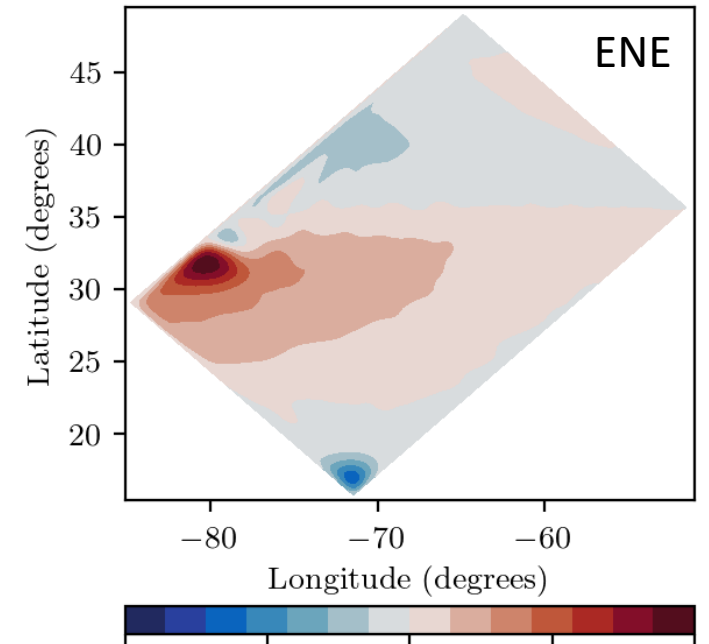
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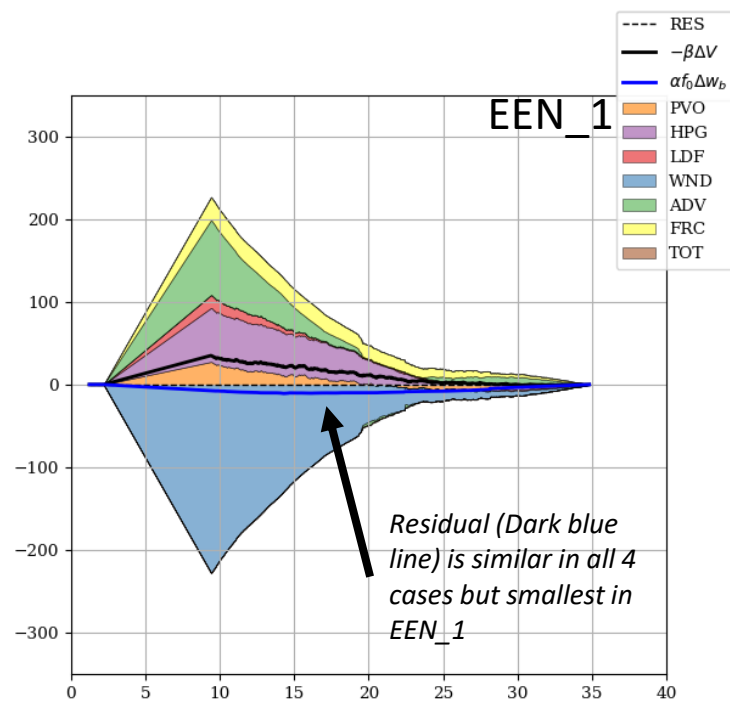
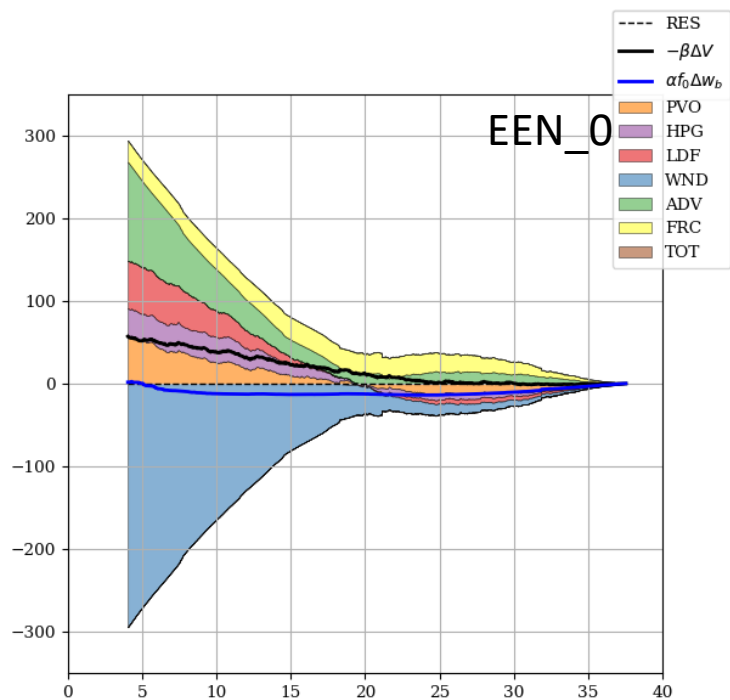
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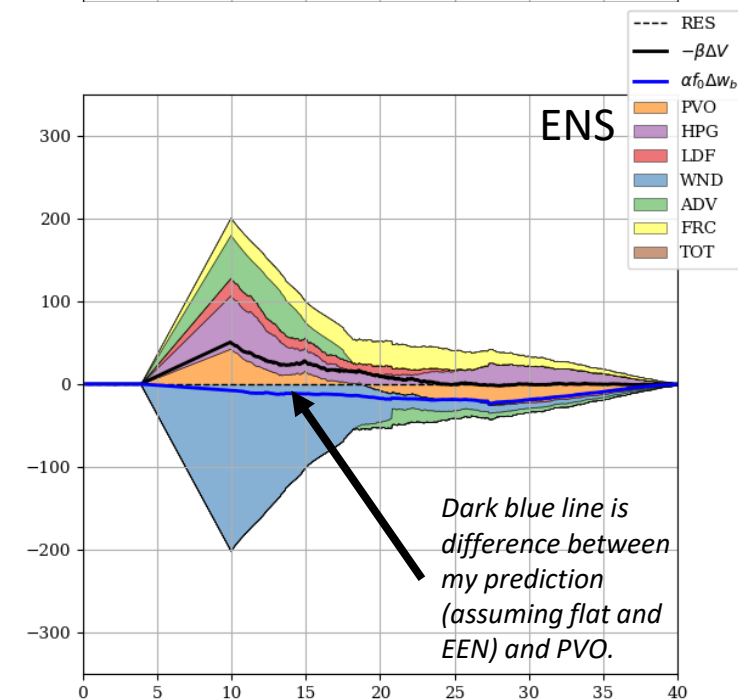
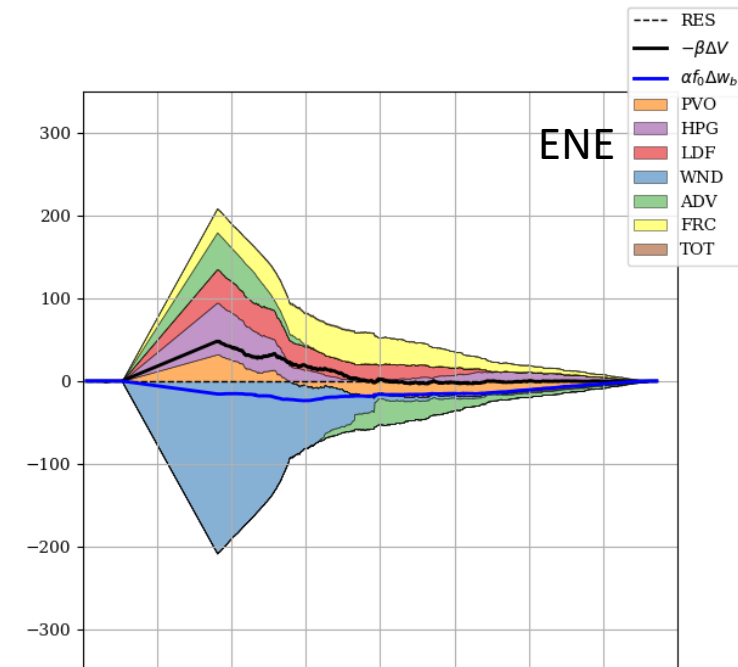
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