Network models for ponding on sea ice

Michael Coughlan, Sam Howison, Ian Hewitt, Andrew Wells (University of Oxford)

Image courtesy of NASA

Motivation

- Sea ice surface melts in summer
- Ponds grow in hollows → interact → join → drain

- Process creates individual ponds AND connected systems

- Model as a network
  - Ponds as nodes
  - Channels/fluxes as edges
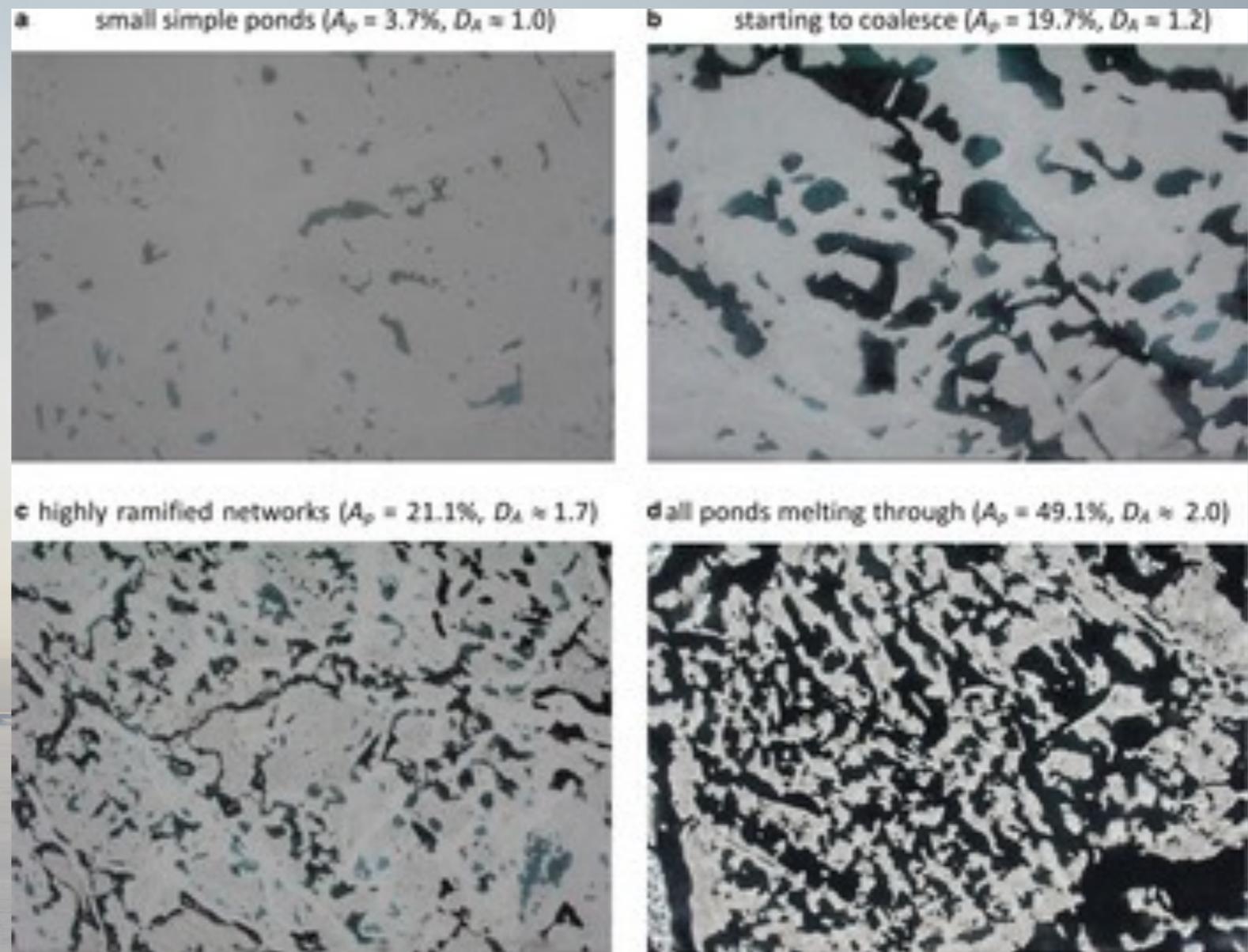- Network can model development of percolation process

Variety of models of melt pond area have been developed:

- Cellular automaton (e.g. Luthje et al 2006, Scott et al 2010, Skyllingstad et al 2015)
- Level sets of accumulated melt (Bowen et al 2016),
- Statistical physics models (Ma et al 2019)
- Climate model schemes (e.g. Flocco et al, 2010)

a small simple ponds (A_p = 3.7%, D_A = 1.0)   b starting to coalesce (A_p = 19.7%, D_A = 1.2)

c highly ramified networks (A_p = 21.1%, D_A ≈ 1.7)   d all ponds melting through (A_p = 49.1%, D_A ≈ 2.0)

Image from Huang 2016 doi:10.1017/aog.2016.30

- Ponds show clear perimeter-area scaling law
- Transition of scaling at Area~30 m^2
  - Ponds suddenly become densely interconnected → "Percolation transition"
- Suggested that ponds systems tend to this point
- Shows behaviour in `catchments'
- May lead to cheaper schemes to parameterise ponds in larger models

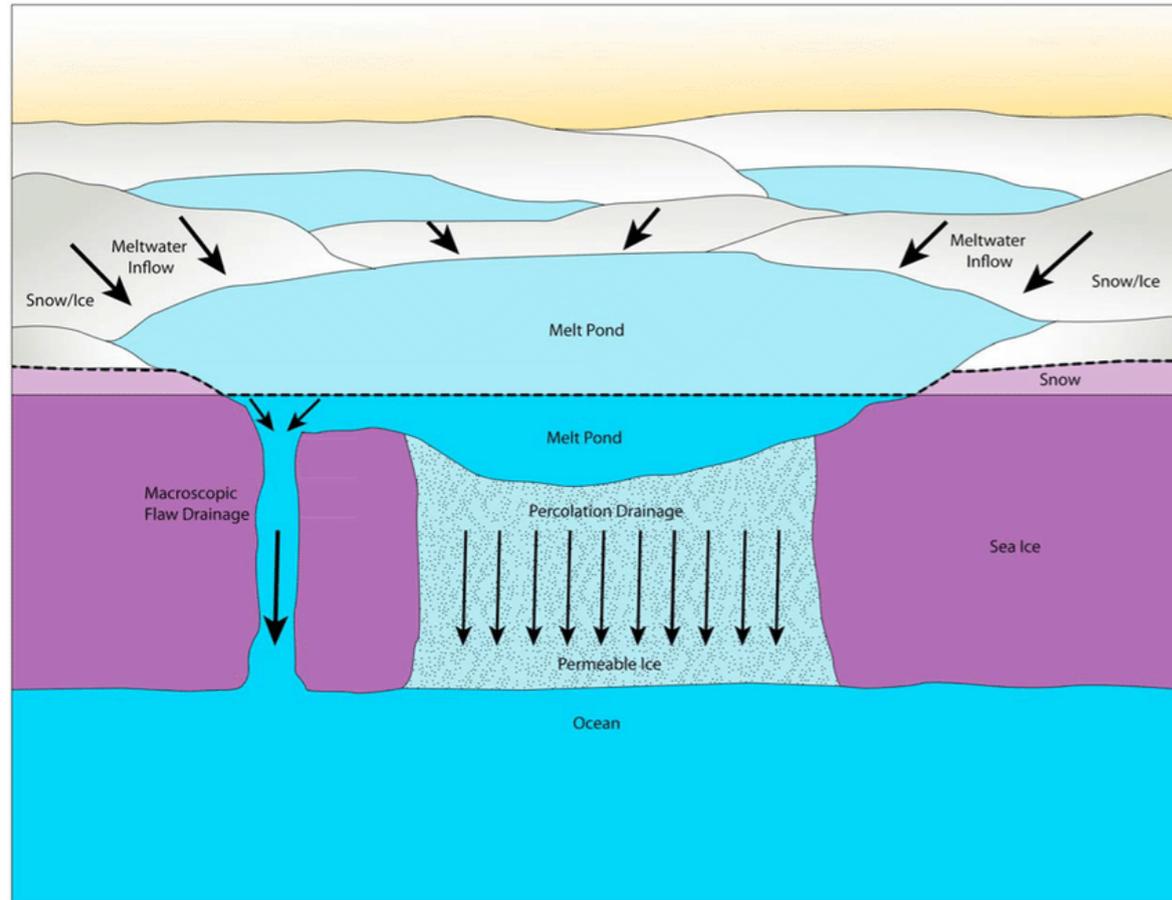# 1. Model a single pond growing in a catchment

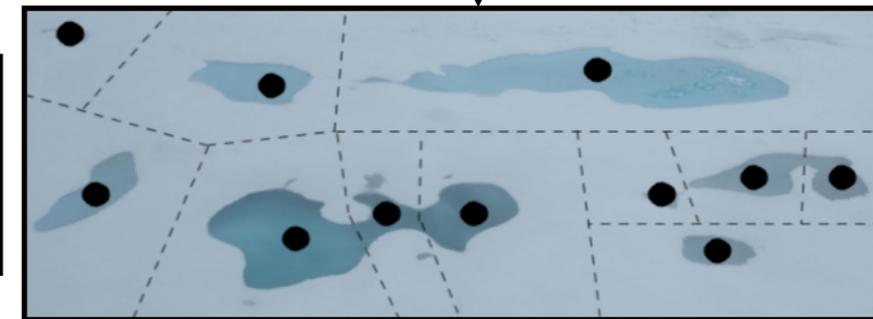Image C. Chris Polashenski doi:10.1029/2011JC007231

1. Incoming solar flux causes surface melt
2. Water formed collects at lowest point in catchment
3. Neighbouring catchments may eventually join
4. Possibility of drainage

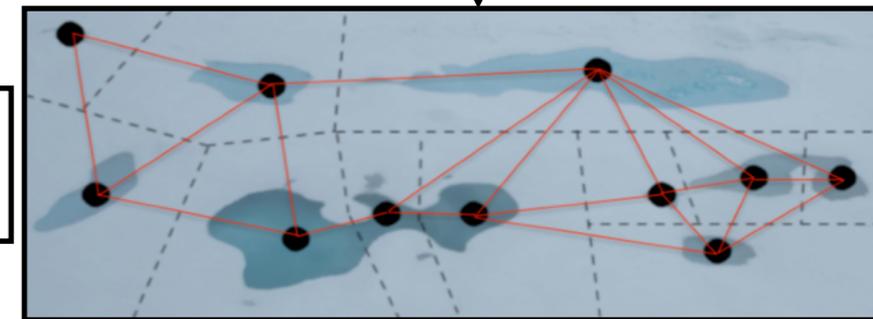# 2. Modelling collective behaviour as pond network

1. Create topography

2. Define catchment areas for water

3. Construct network of catchments
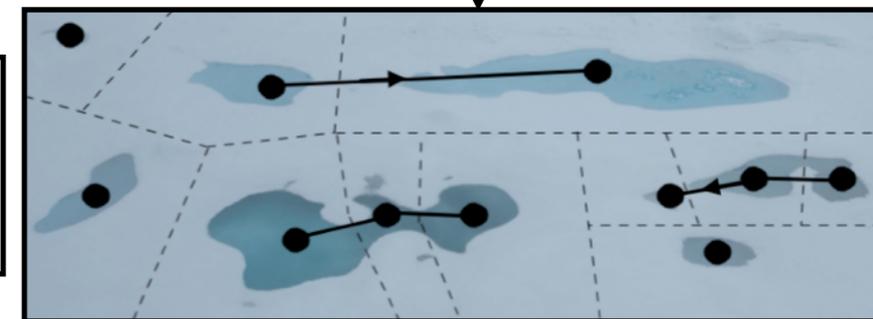

4. Evolve system using rules for node and edge behaviour

Image C. Peng Lu doi:10.5194/tc-12-1331-2018

The model - node behaviour

1. $\frac{\partial h}{\partial t}(\mathbf{x}, t) = \begin{cases} -v_b(t), & h(\mathbf{x}, t) \geq H(t) \\ -v_w(t), & h(\mathbf{x}, t) < H(t) \end{cases}$

Melt rate on bare ice

Ice surface height

Melt rate in pond

Water level

Ice surface height evolves according to melt rates, with enhanced melting in ponds

Volume of melt

2(a). $\dot{V}_m = \rho_{iw} \iint_{a_0} -\frac{\partial h(\mathbf{x}, t)}{\partial t} \, \mathrm{d}A$ Calculate total melt in catchment

Catchment area

Fluxes between pond i and j

3. $\dot{V}_{Ai} = \dot{V}_{mi} - \sum_j q_{i,j}$

Volume accumulated

Water accumulates with water level $H(t)$

2(b). $V_A(t) = \iint_a H(t) - h(\mathbf{x}, t) \, \mathrm{d}A$

Pond area

4. Node behaviour (water level) along with hypsometry

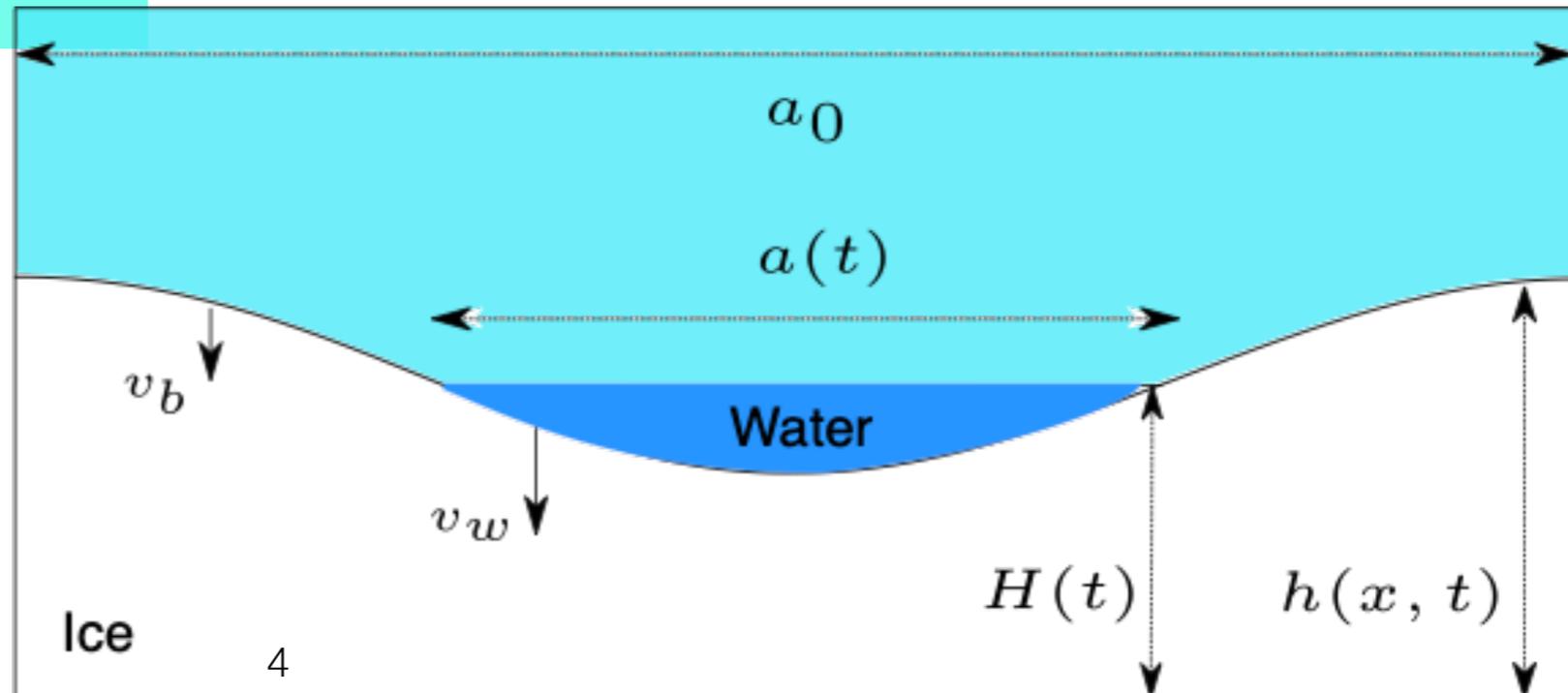$a_i \dot{H}_i = \rho_{iw} v_b a_0 - [v_w - \rho(v_w - v_b)]a_i - \sum_j q_{i,j}$

hypsometry
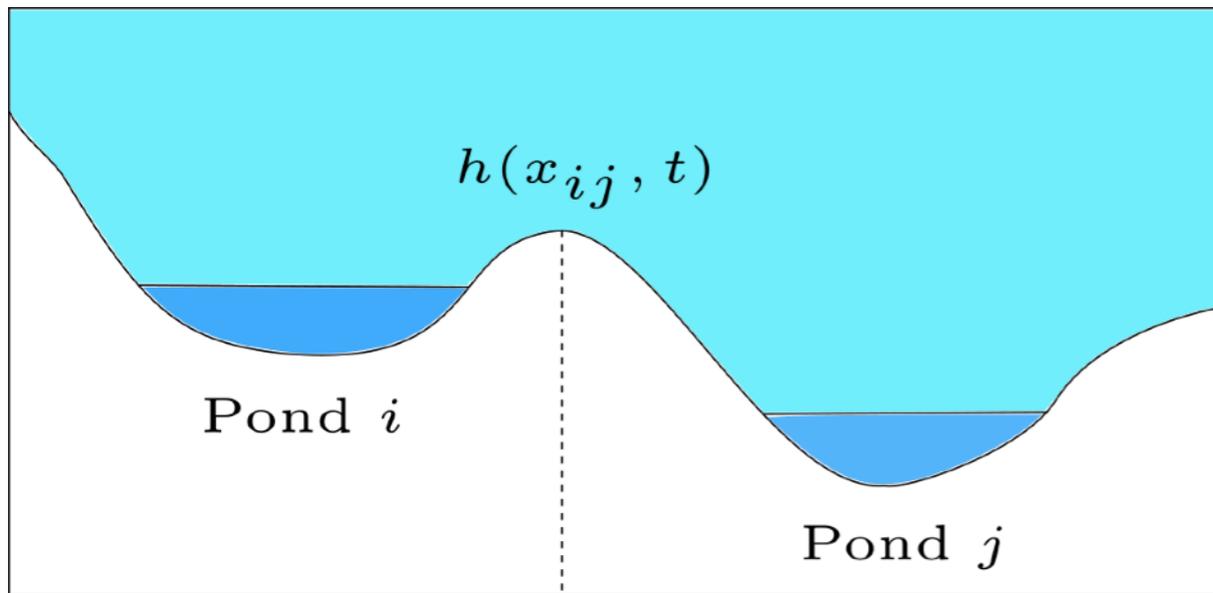
$a_i = f_i(H_i, t)$

Density parameter (taken to be 0.9)
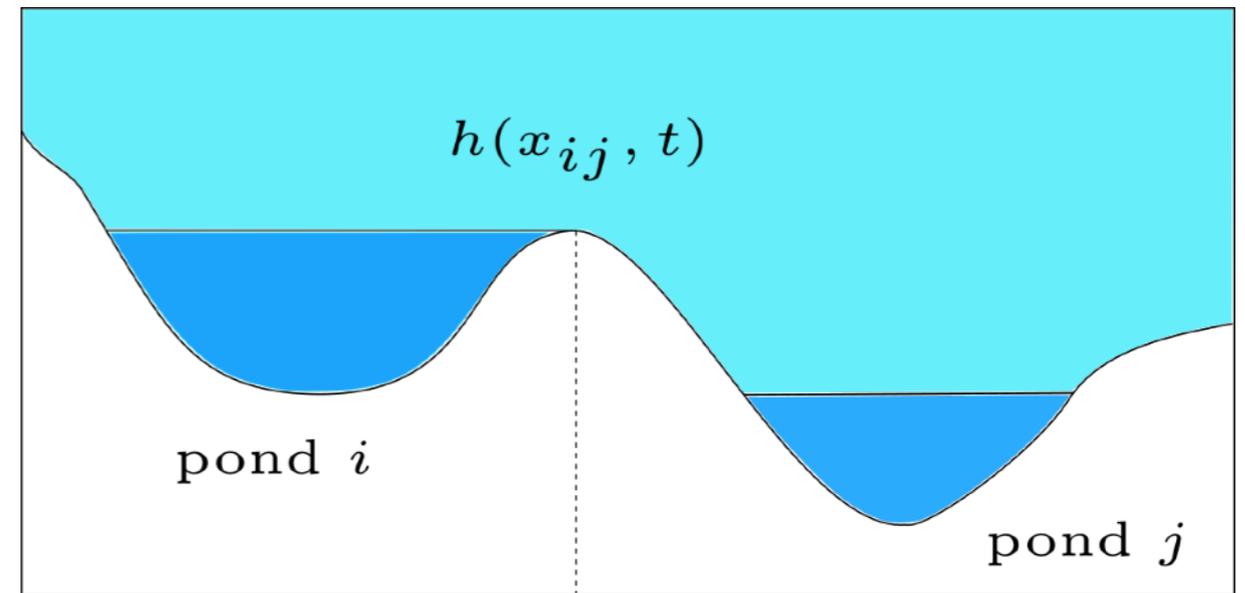
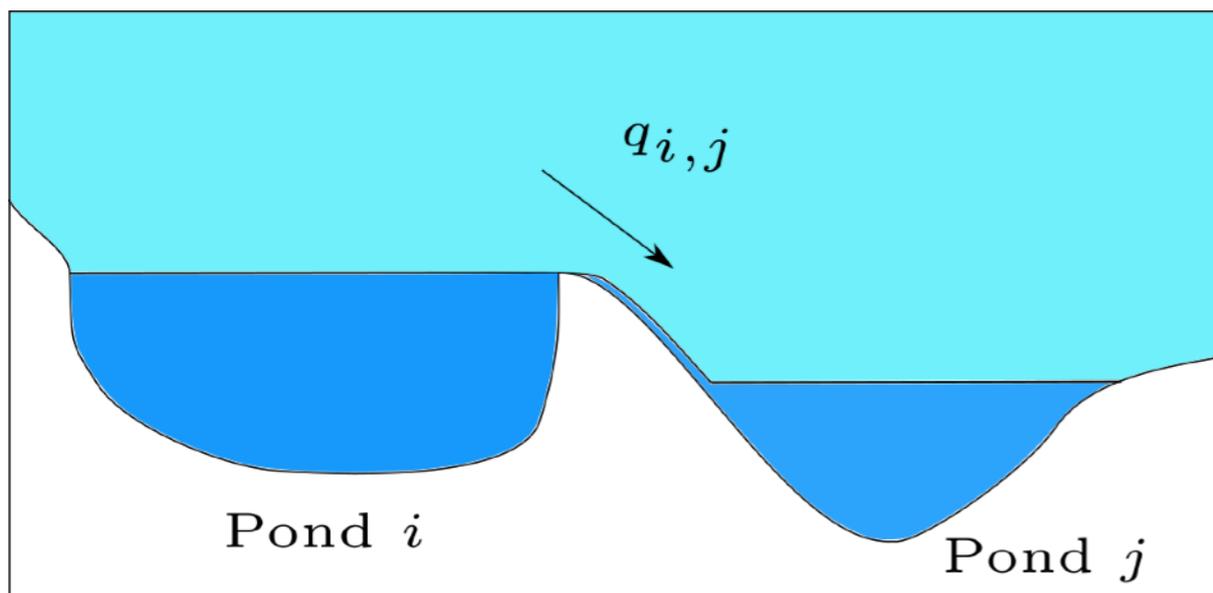$\rho_{iw} = \frac{\rho_{ice}}{\rho_{water}}$

Melt ratio

$v = \frac{v_w}{v_b}$
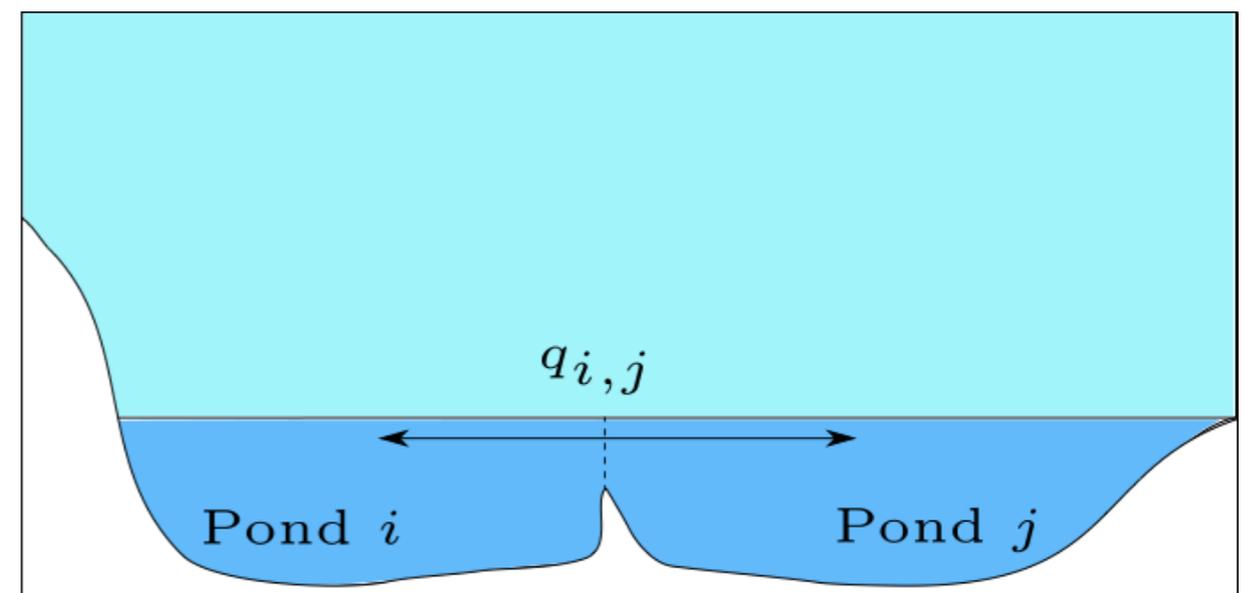

4

Edge behaviour - what does it look like?

1. Inactive

2. Overflow begins

3. Overflow

4. Joining

Modelling - edge behaviour

$$a_i \dot{H}_i = \rho_{iw} v_b a_0 - [v_w - \rho(v_w - v_b)]a_i - \sum_j q_{i,j}$$

Solve system of equations for fluxes q, subject to constraints across cases of edges

State	Condition	Constraint
1. No flux if both ponds below saddle	$H_i < h_{i,j}$ & $H_j < h_{i,j}$	$q_{i,j} = 0$
2. Overflow (left to right)	$H_i \geq h_{i,j}$ & $H_j < h_{i,j}$	$\dot{H}_i = -v_b$
3. Overflow (right to left)	$H_i < h_{i,j}$ & $H_j \geq h_{i,j}$	$\dot{H}_j = -v_b$
4. Join if both ponds higher than saddle	$H_i \geq h_{i,j}$ & $H_j \geq h_{i,j}$	$H_i = H_j$

Saddle height between ponds i and j

Fluxes

Connectivity matrix
weighted by area

$$\mathbf{Aq = b}$$

Forcing (melt)
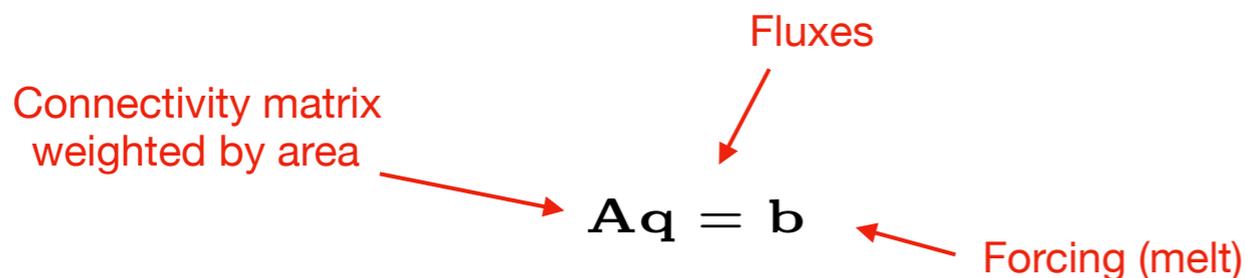
Use conditions and constraints to write connectivity
matrix and forcing vector, then solve for fluxes

Porous drainage and buoyancy

Catchment behaviour - same as before except every node has a possible drainage flux

$$\dot{H}_i = \frac{\rho v_b a_{0i} - \sum_j q_{ij} - q_{is}}{a_i} - [v_w - \rho_{iw}(v_w - v_b)]$$

Drainage flux

Pond fluxes

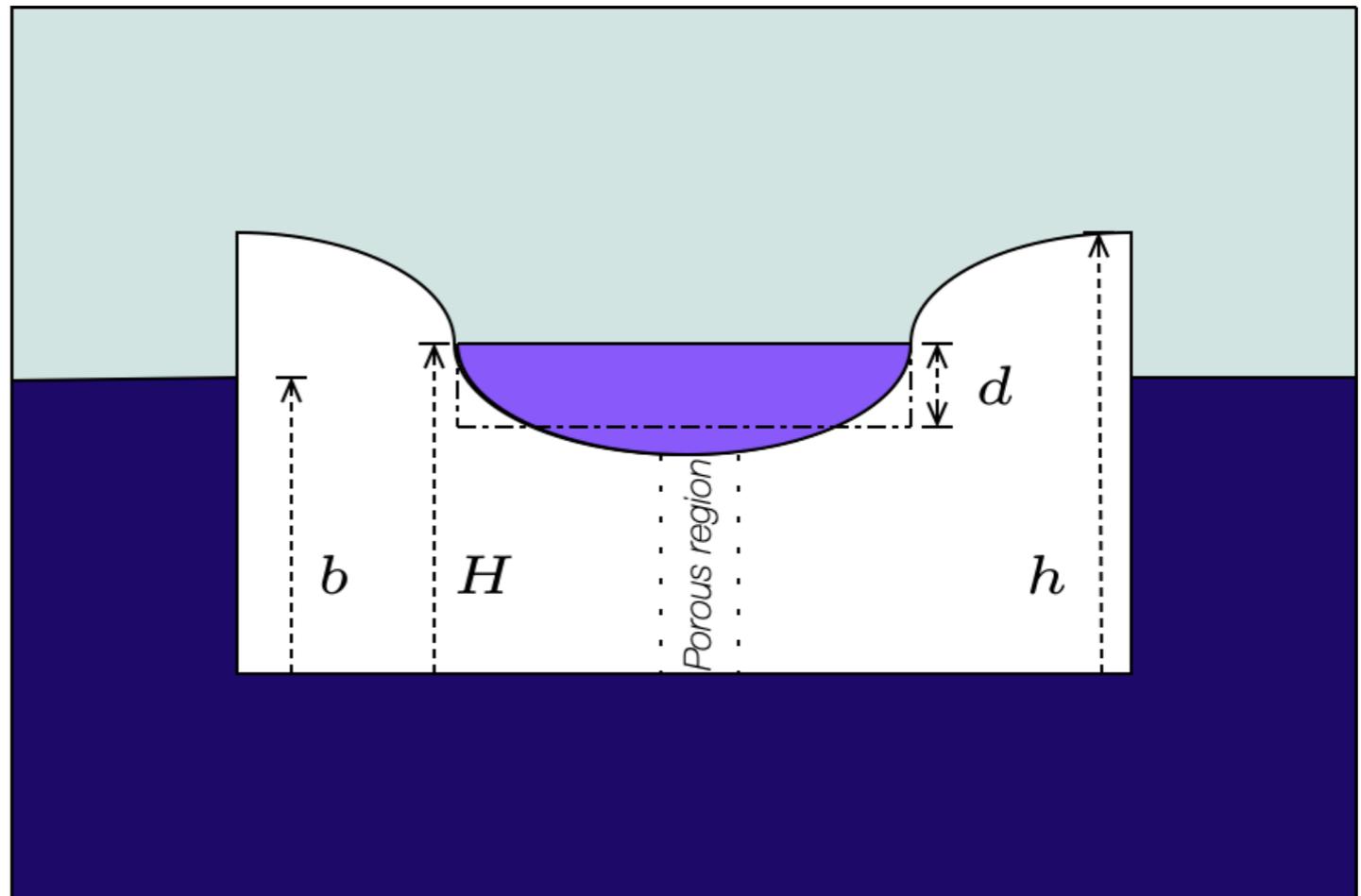## Add extra 'ocean' node

Thickness at which drainage begins

Prescribe Darcy flux for drainage

$$q_{is} = \begin{cases} 0, & h_{ci} > h_{crit} \\ K(H_i - \frac{\rho_s}{\rho_w}b), & h_{ci} \leq h_{crit}. \end{cases}$$
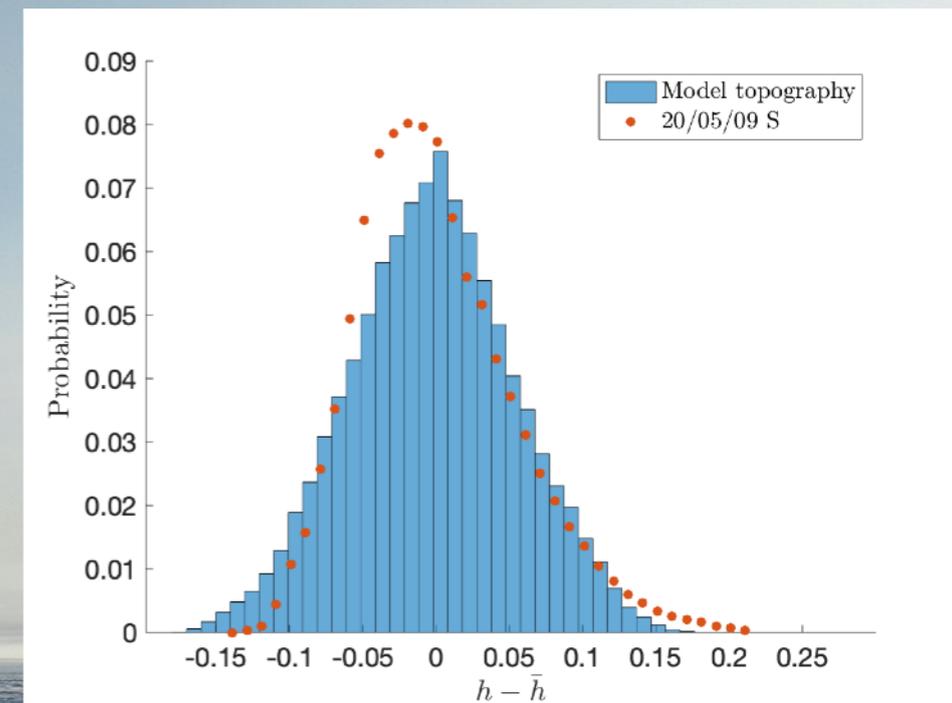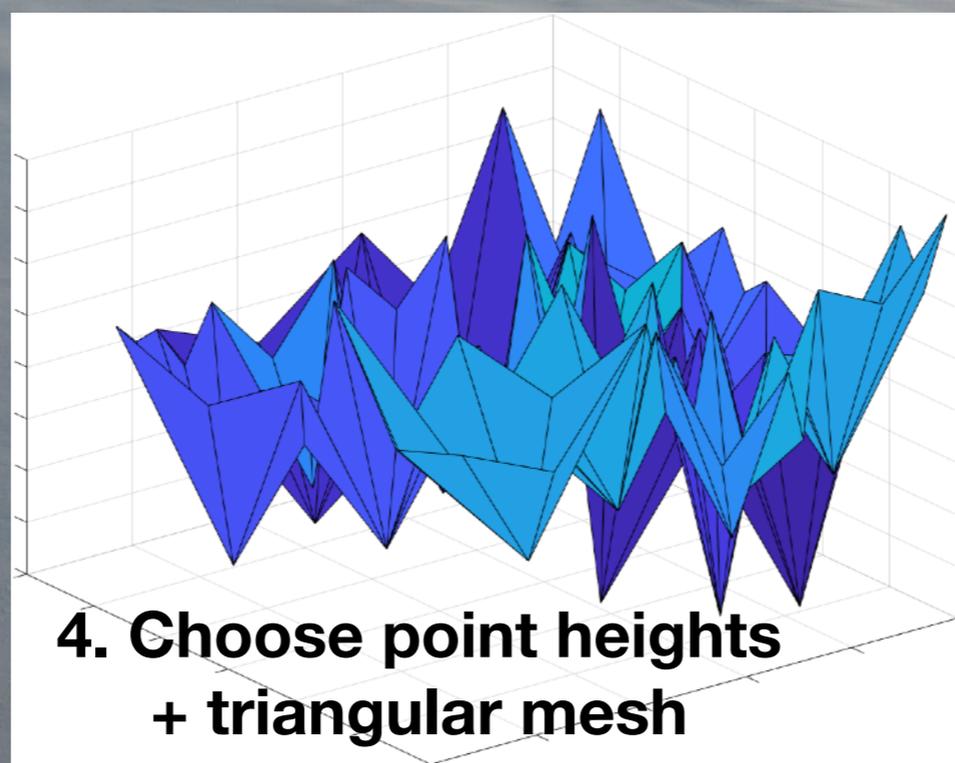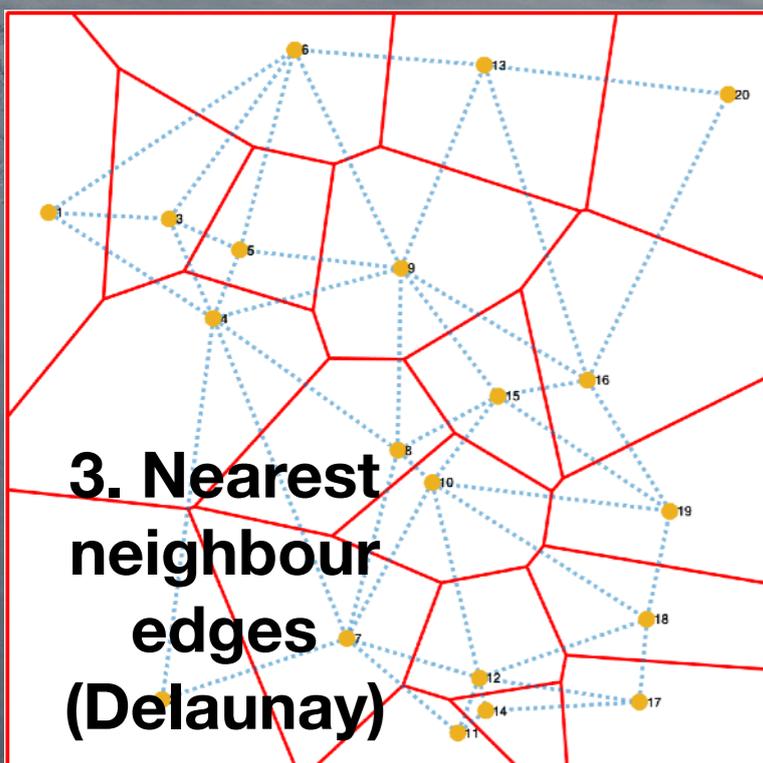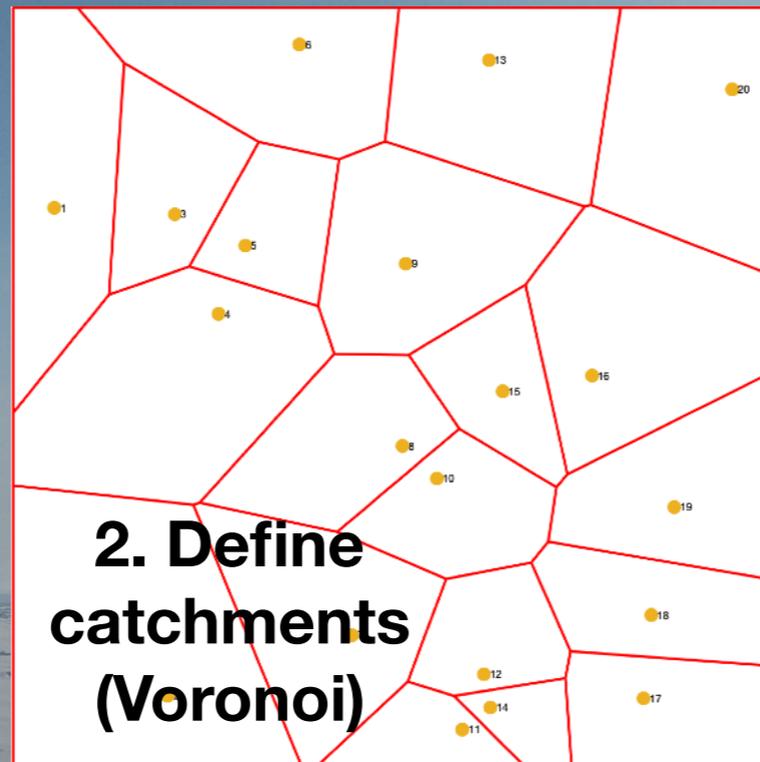
Drainage parameter

Lowest point on pond floor

Archimedes

$$\rho_s g a_0 b(t) = \rho_i g \iint h(\mathbf{x}, t)\mathrm{d}a + \rho_w g \iint [H(t) - h(\mathbf{x}, t)]\mathrm{d}a$$

Upthrust

Ice head

Water head

Draft

Draft equation

$$\dot{b} = -\frac{\sum_i q_{is}}{\rho_{ws} \sum a_0}$$

Building a floe in 2-D

1. Points (centres) at random

2. Define catchments (Voronoi)

Comparison with Polashenski et al 2012

3. Nearest neighbour edges (Delaunay)

4. Choose point heights + triangular mesh
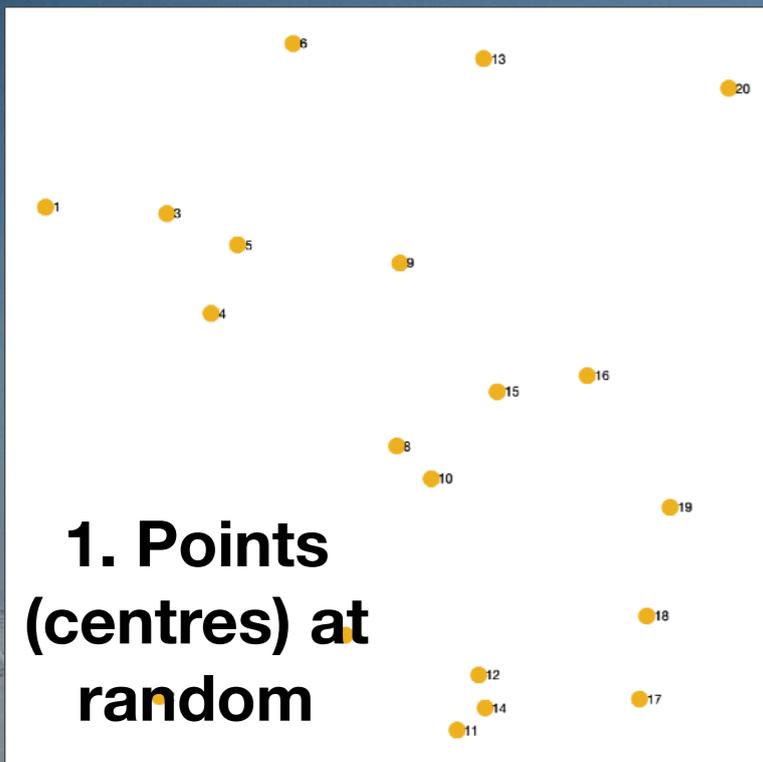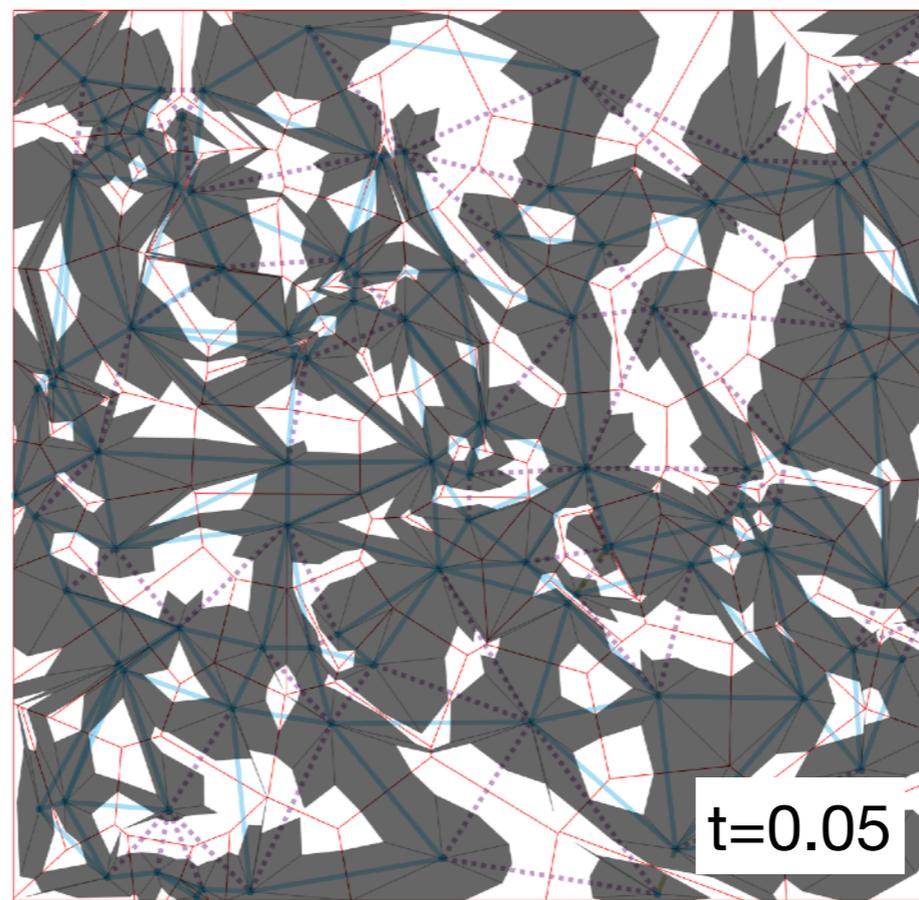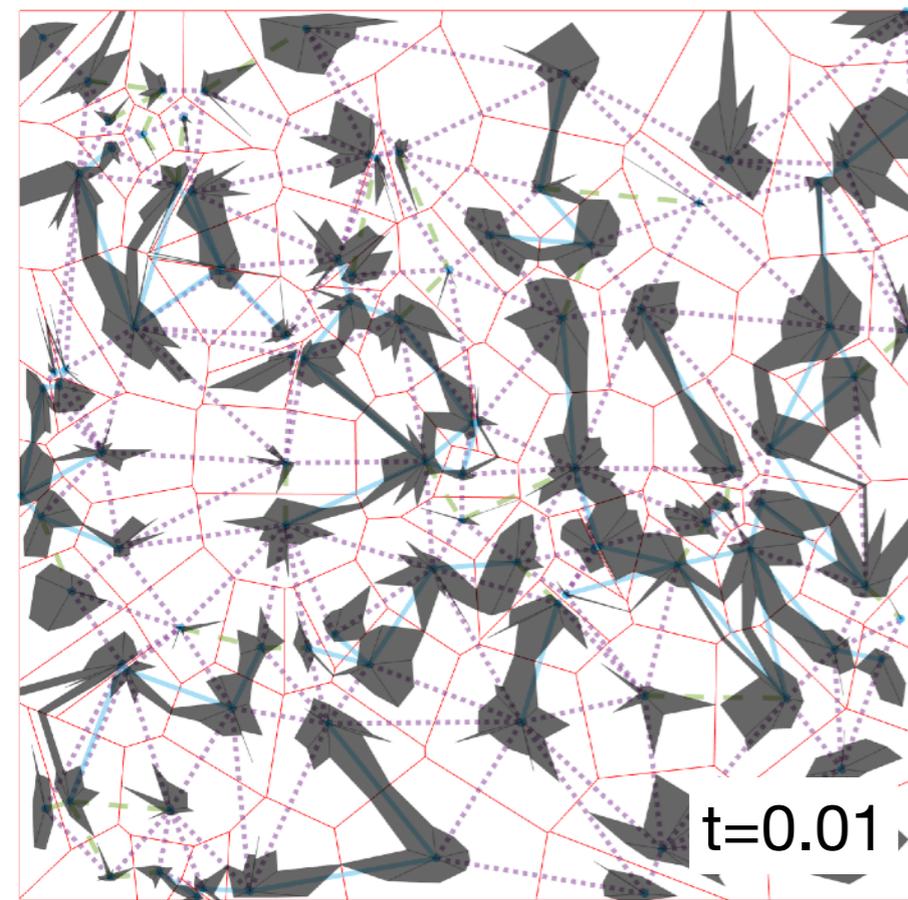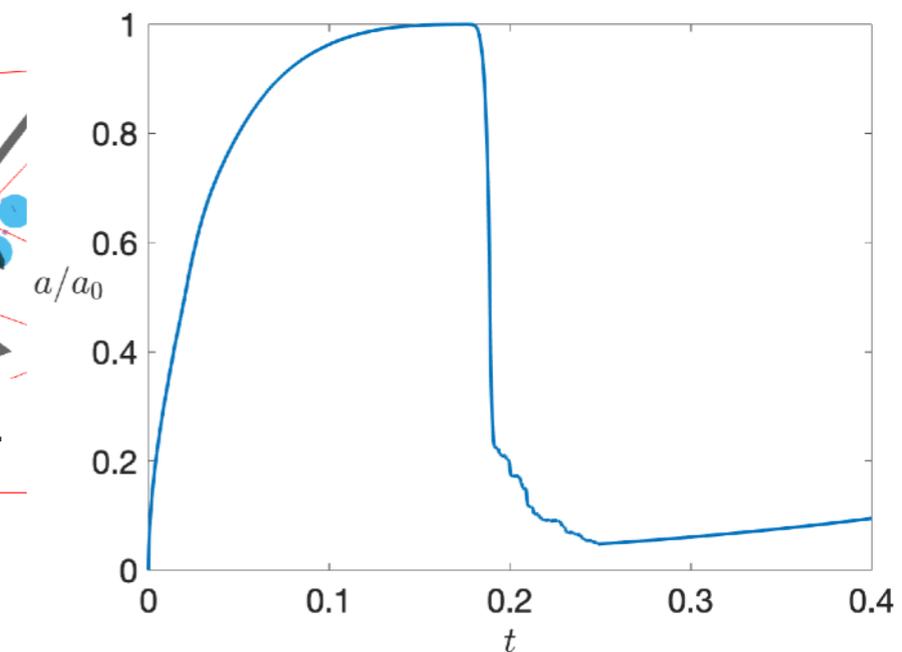
Topographic variables:
- Number of catchments
- How centres are chosen (random or ordered)
- How point heights are chosen (distribution)
- Amplitude of variation of heights

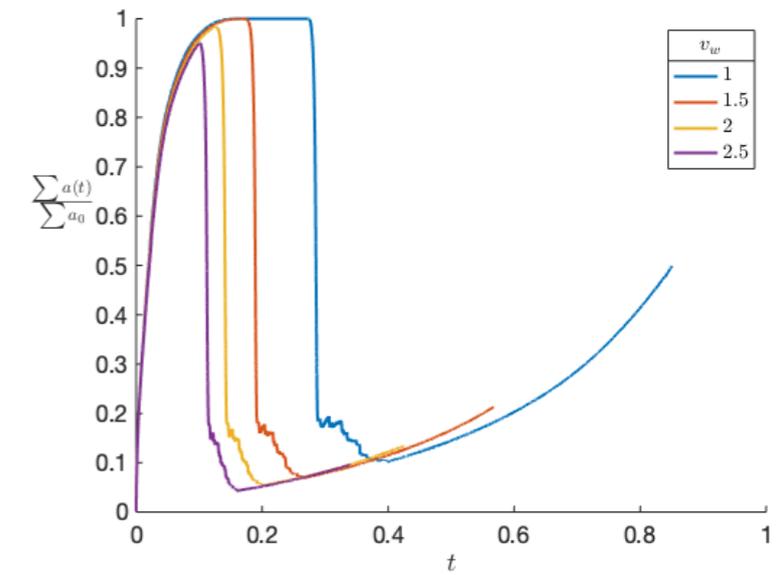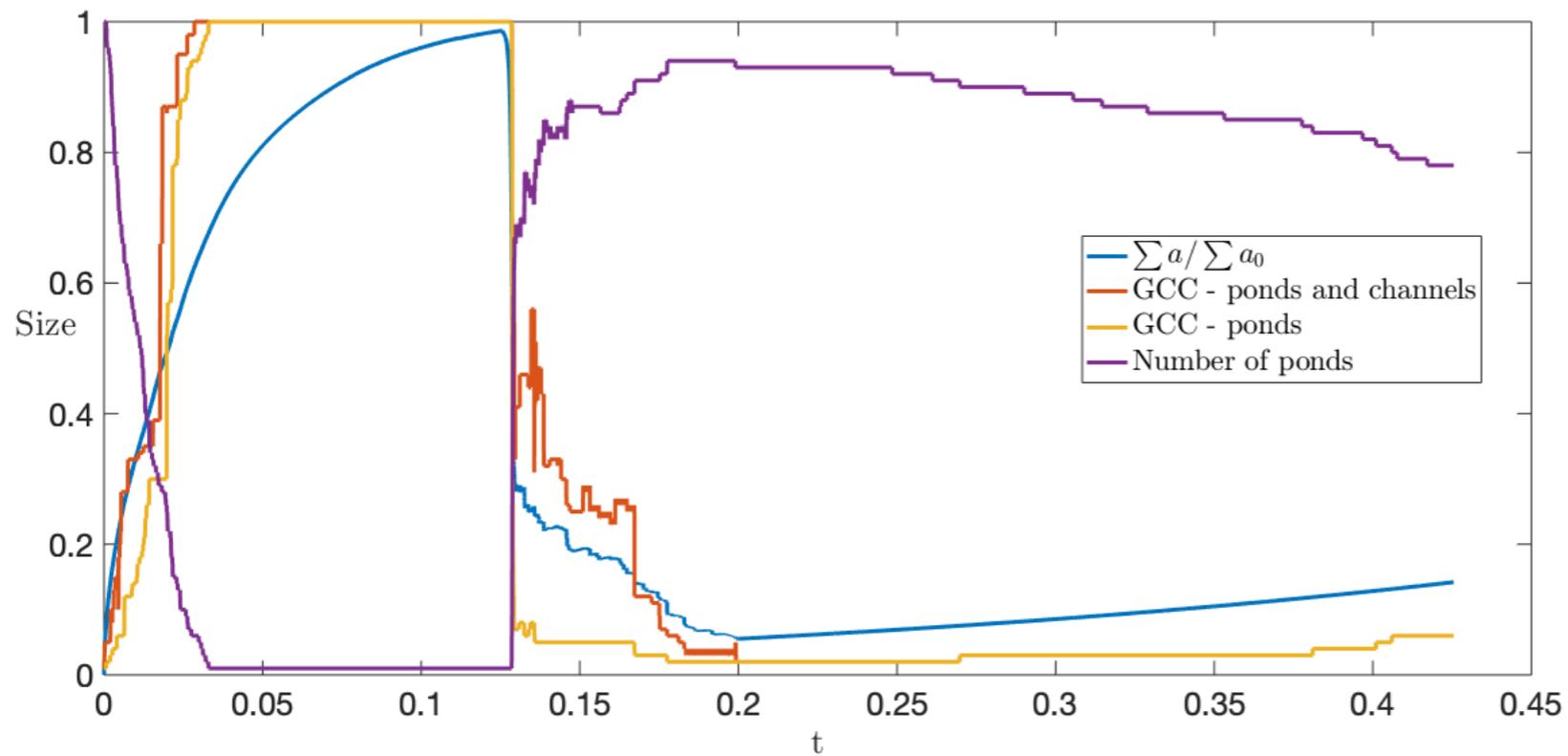# Simulations - overhead and network view
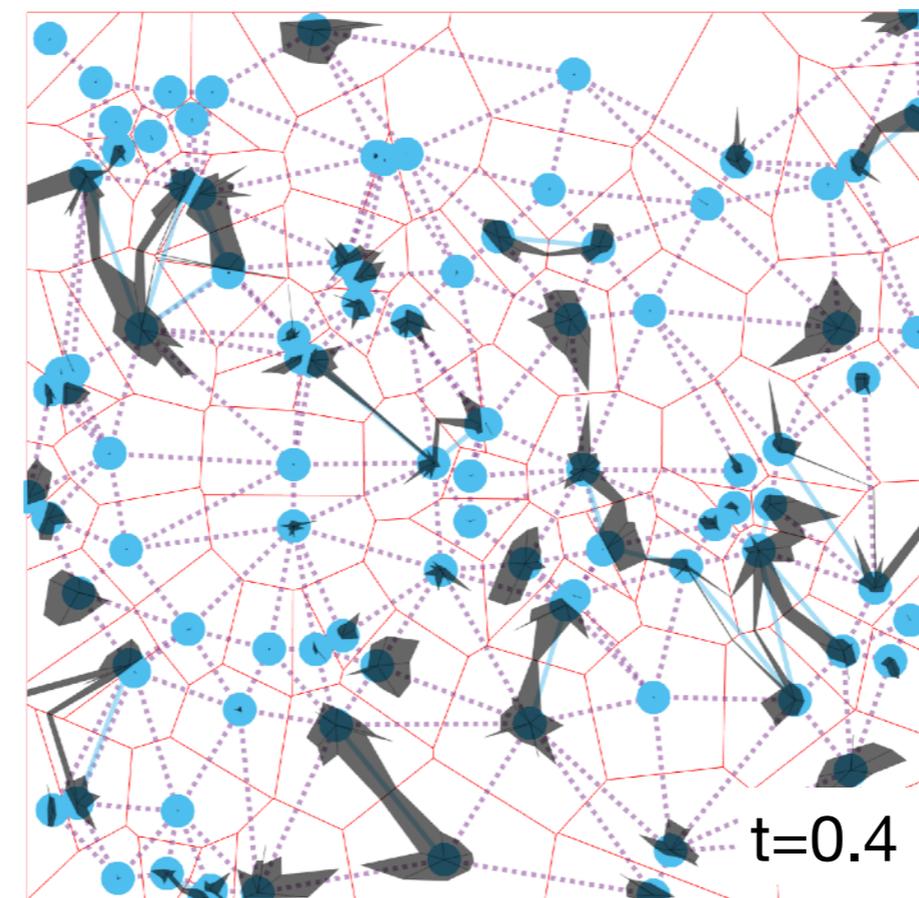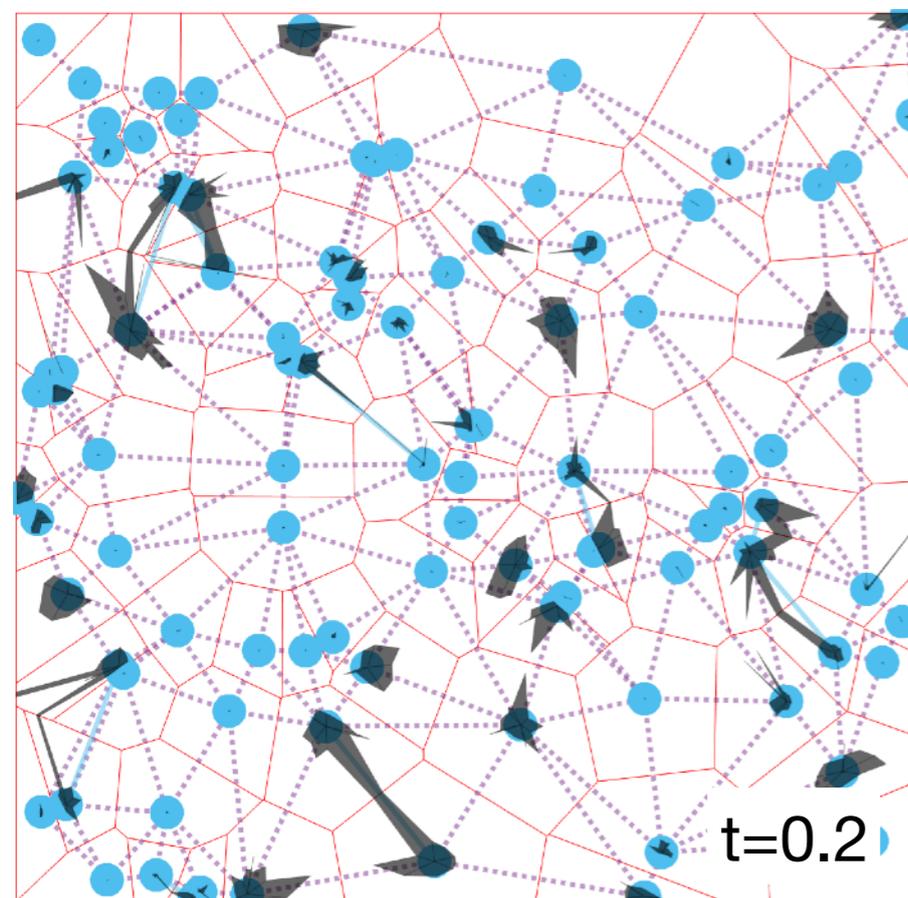
- Grey areas ponded
- Grey lines denote catchments
- Dotted lines: inactive edges
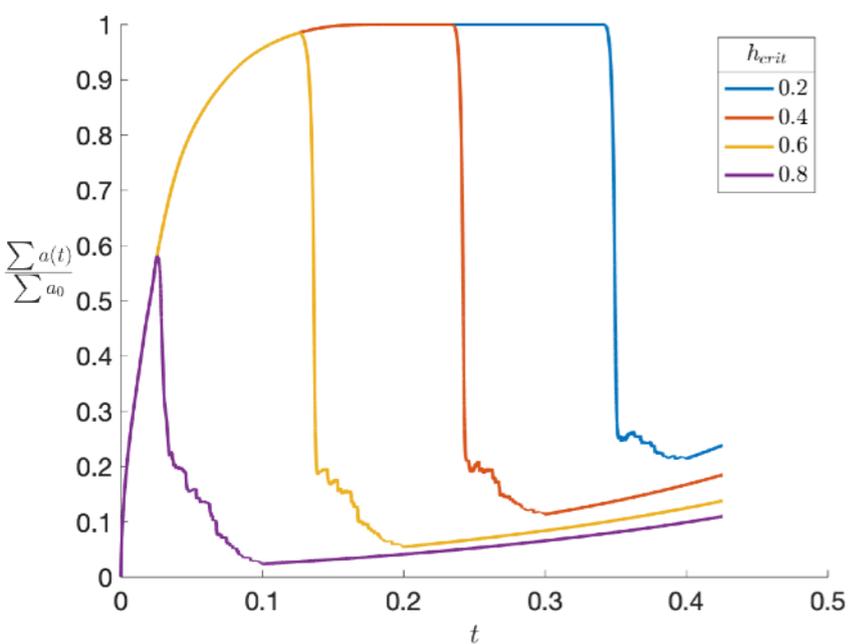- Dashed lines: overflows
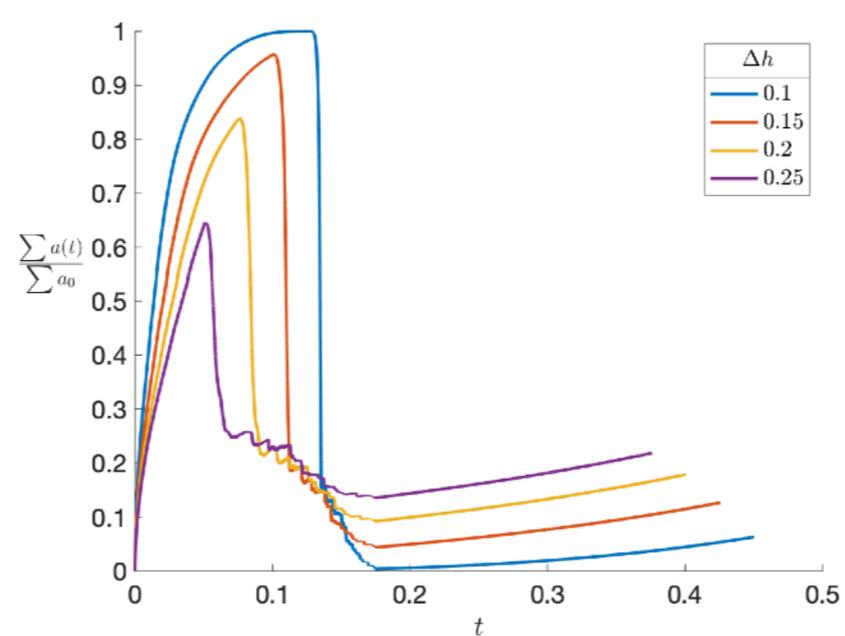- Solid blue: joins

Pond area fraction

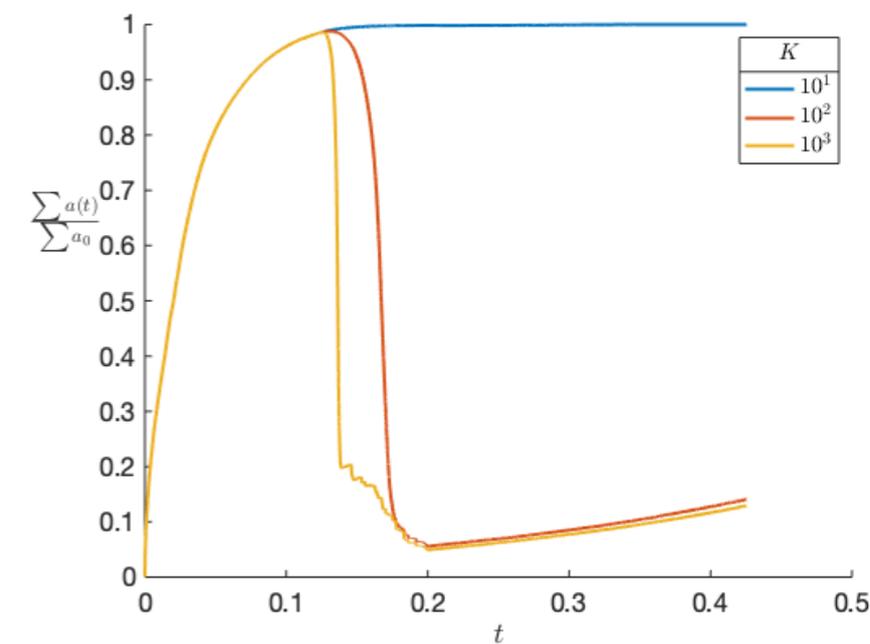# Porous drainage - dependence on parameters

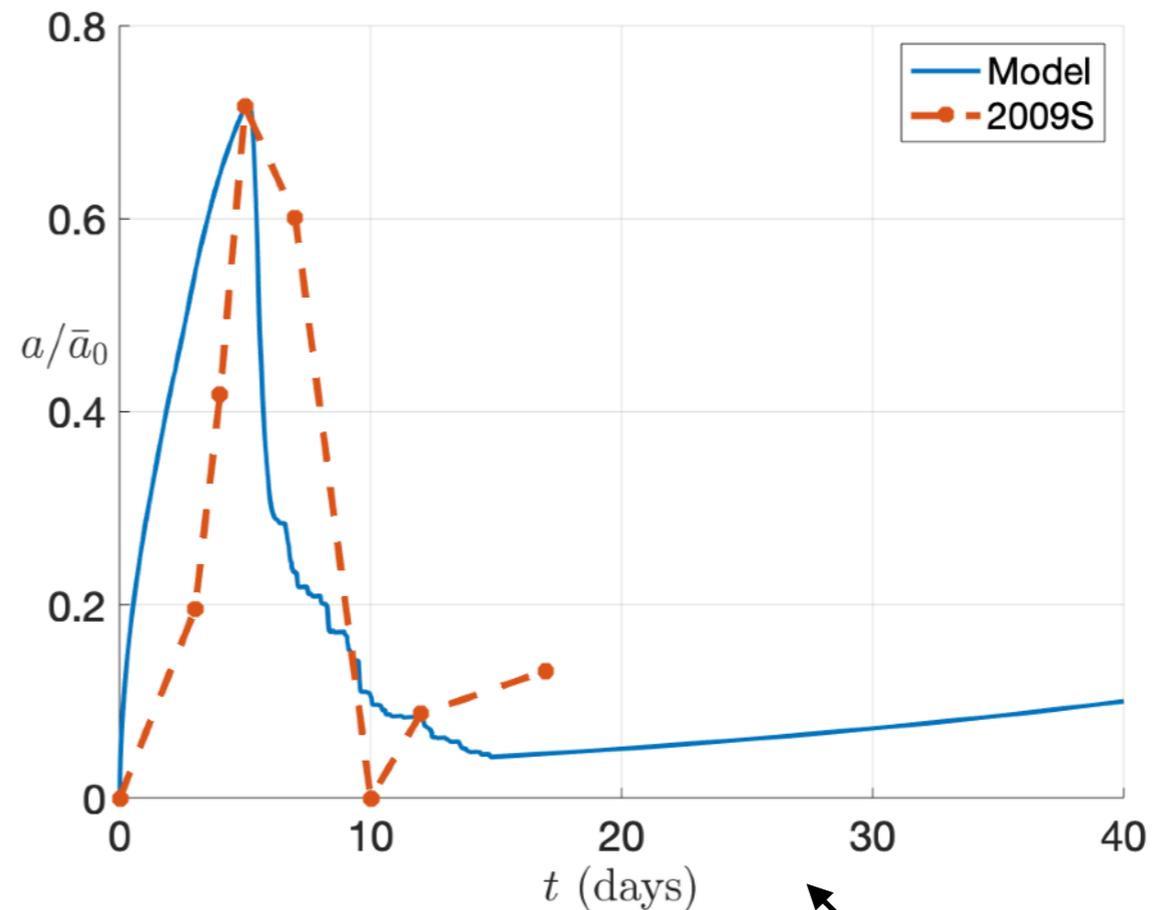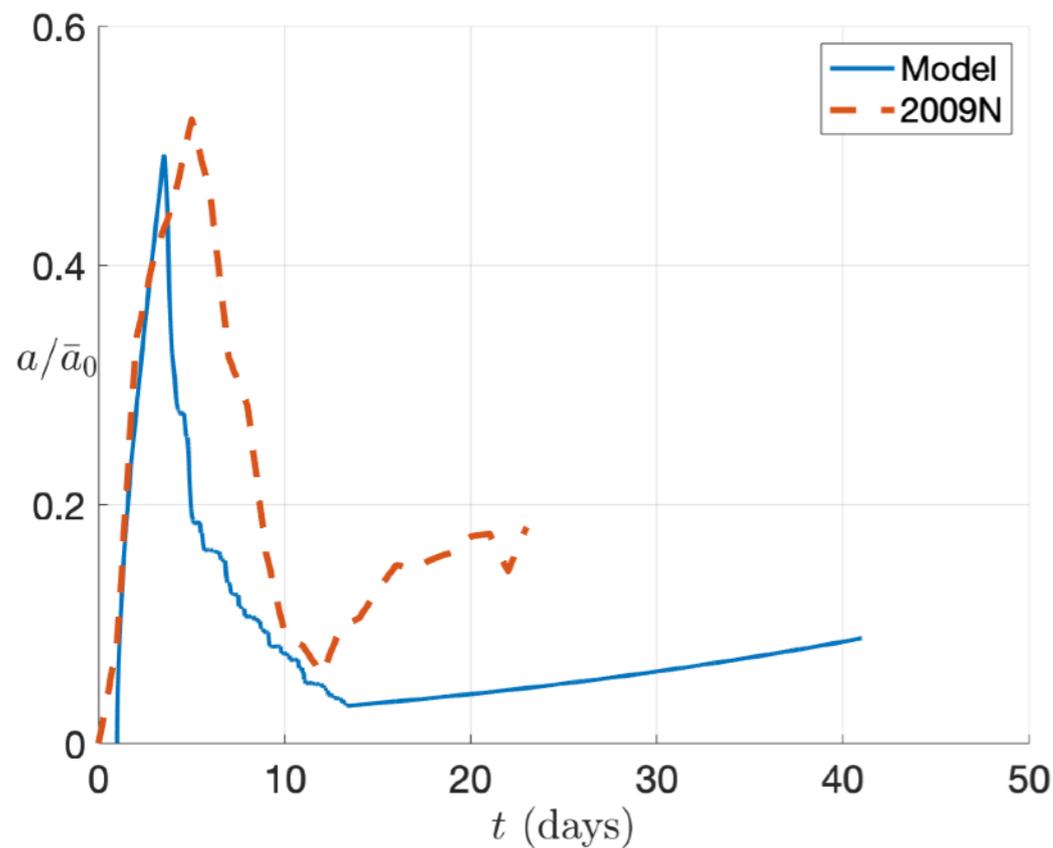
Melt ratio - small effect

Onset of drainage - big effect

Surface roughness - big effect

Permeability - changes path but not final state (except for impermeable ice)
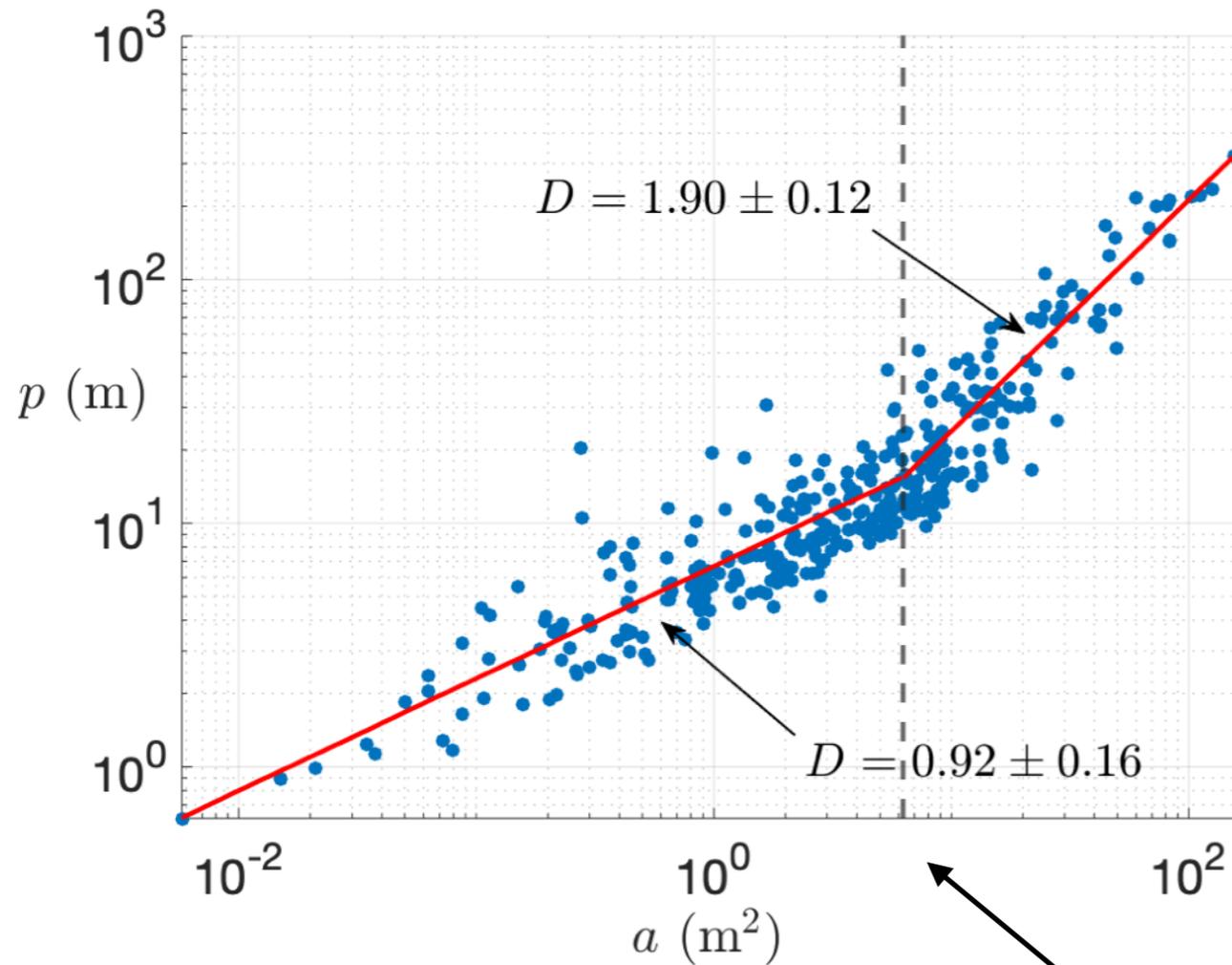
Comparison with data

North site

Days from onset of ponding

South site

- Comparison with data from Polashenski et al. 2012
- Scale lengths taken from data
- Threshold for drainage estimated ~ 0.7 floe thickness
- Model captures all important behaviour
- Underestimates pond fraction in mid-summer

Model results   vs.   Observations

$D = 1.90 \pm 0.12$

$D = 0.92 \pm 0.16$

p (m)

a (m^2)

D~2

General trend line D~1

FYI

Perimeter/(m)

Area/(m^2)

Changes in perimeter-area scaling

- Distinct change in perimeter-area relationship → ponds beginning to connect
- Change implies transition closely aligned with percolation transition of network

Change in scaling found at area where, on average, pond in catchment expected to join to a neighbour
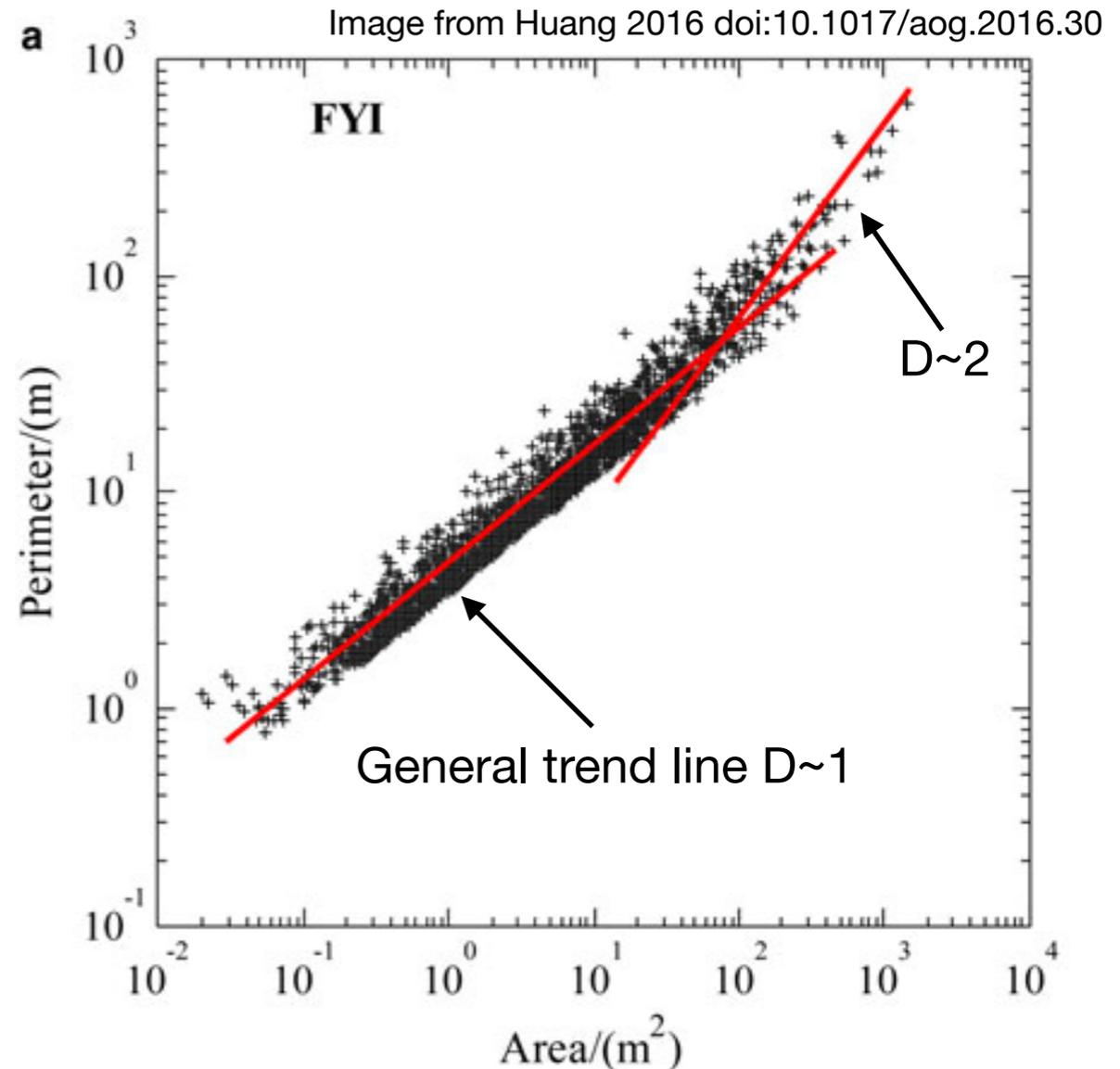
Perimeter-area scaling

$$p = ka^{D/2}$$

Conclusions

- Network model represents physics and geometry, from formation to break-up/refreezing

- Model recreates pond statistics qualitatively

- Once drainage begins, pond area is very sensitive to surface roughness and depth at which permeability becomes large