



THE UNIVERSITY
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Interrogating Tomographic Uncertainties for Subsurface Structural Information

Andrew Curtis, Xuebin Zhao and Xin Zhang

*“Interrogation Theory”, Arnold & Curtis 2018, Geophys. J. Int., **214***

Outline

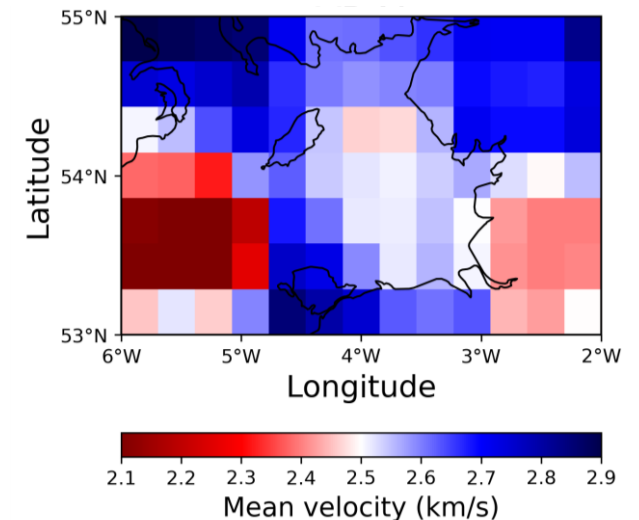
1. Interrogation Theory
2. Synthetic Test of Travel Time Tomography
3. Field Data Test of the East Irish Sea Basin
4. Conclusion

Interrogation Theory

Answer low dimensional questions by interrogating data

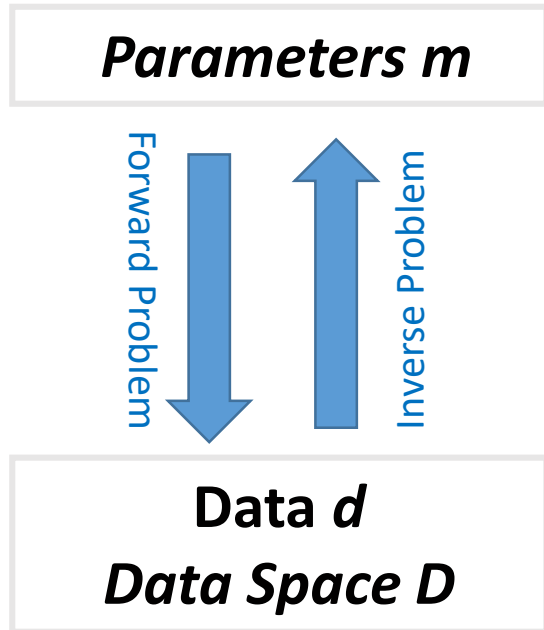
- What is the volume of a particular subsurface reservoir? $\rightarrow X \text{ km}^3$
- Is there a geothermal plume beneath this area? $\rightarrow Y = \text{Yes/No}$
- At what depth does a mantle plume originate? $\rightarrow Z \text{ km}$

Answers often lie within high dimensional models



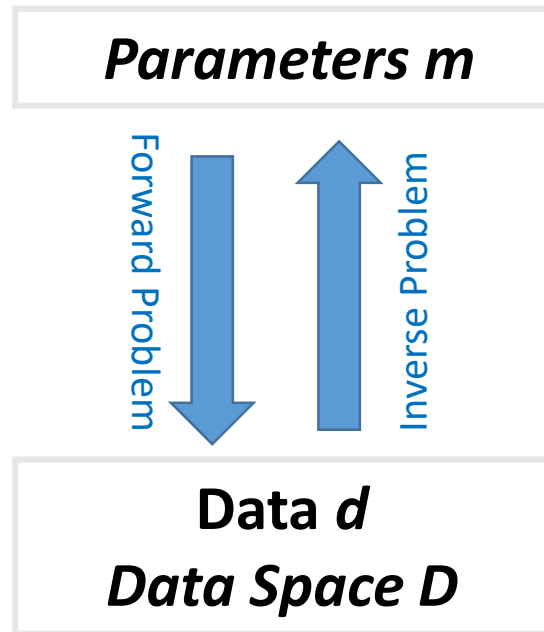
Inverse Theory and Interrogation Theory

Is there a geothermal plume beneath this area?

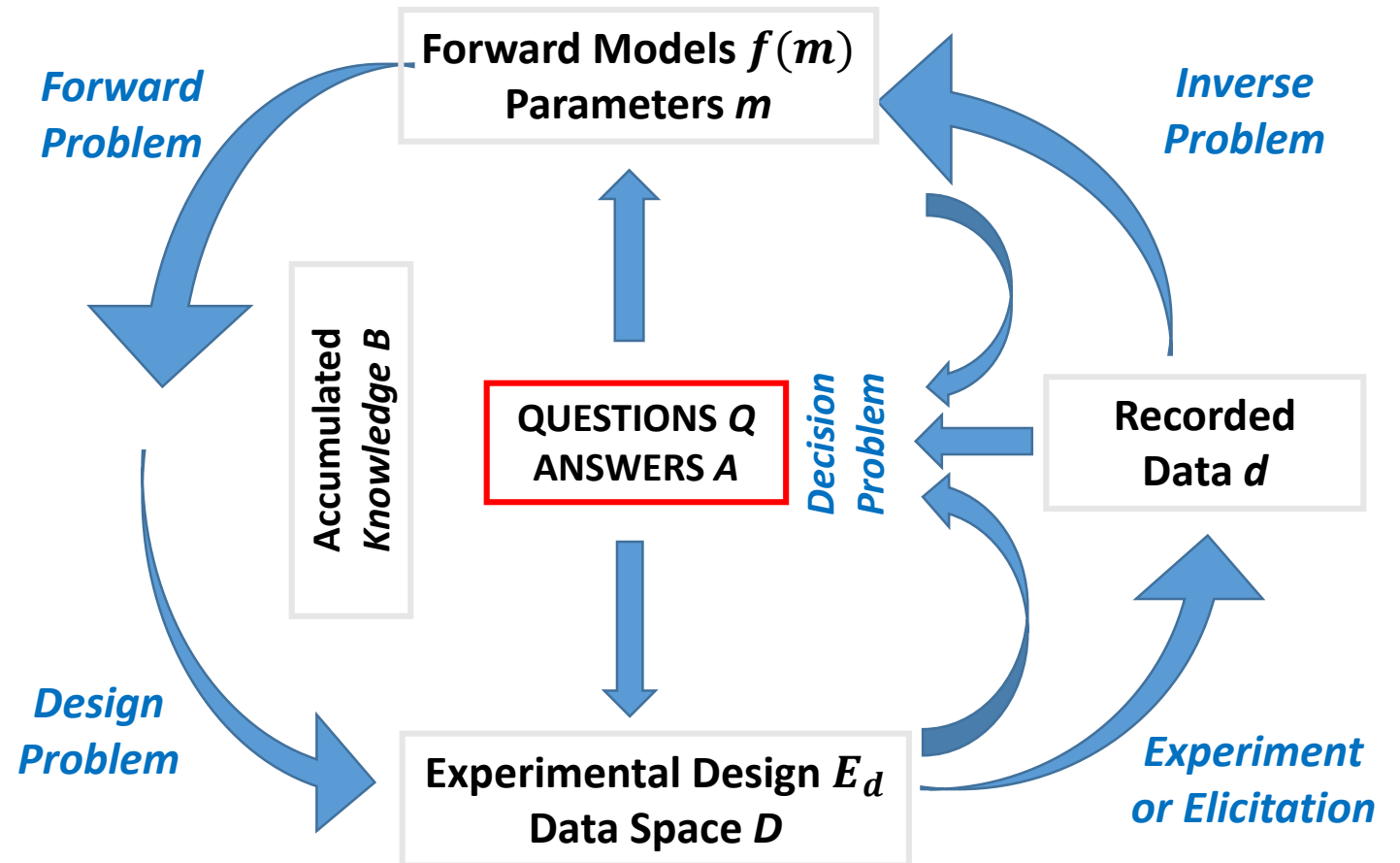


Inverse Theory

Inverse Theory and Interrogation Theory



Inverse Theory

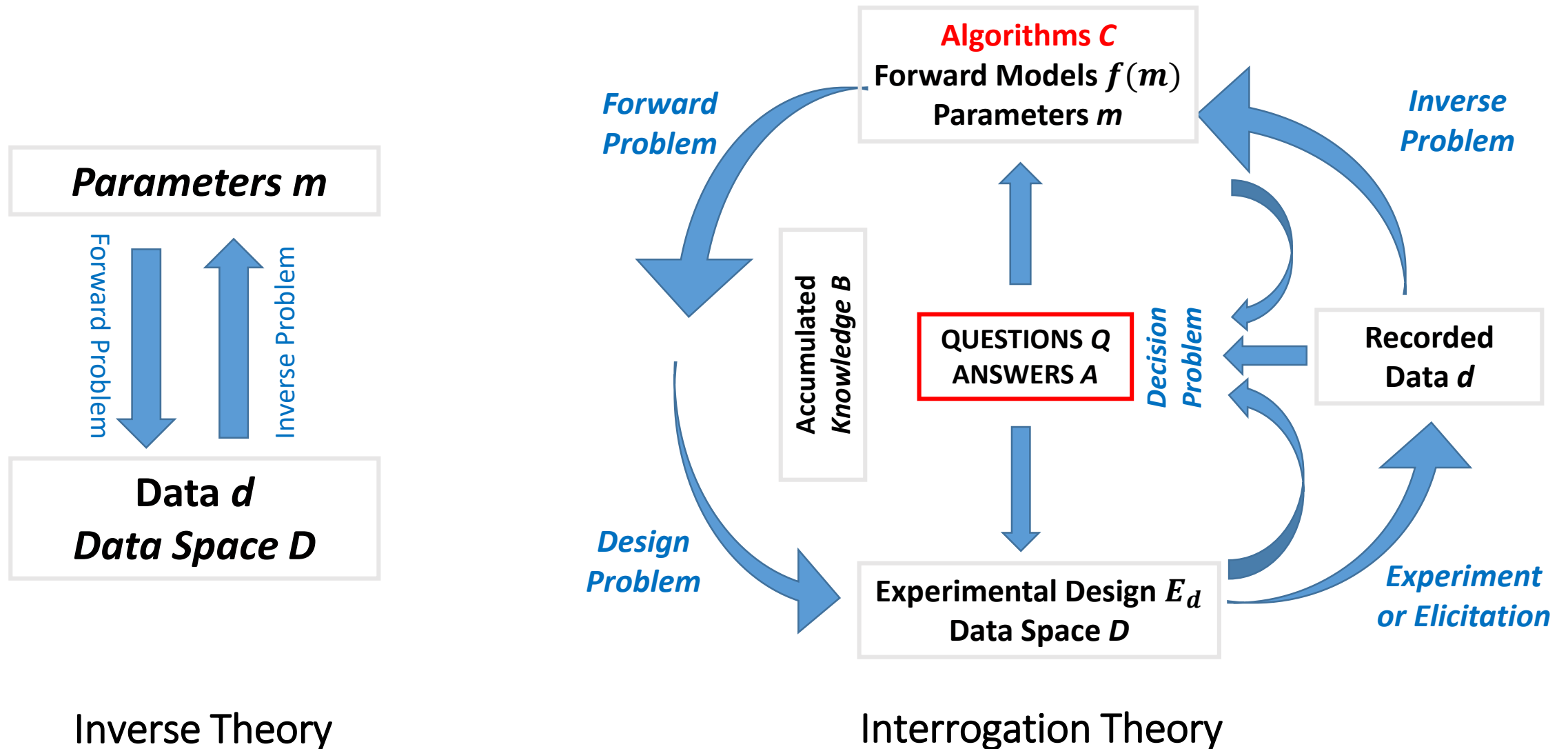


Interrogation Theory

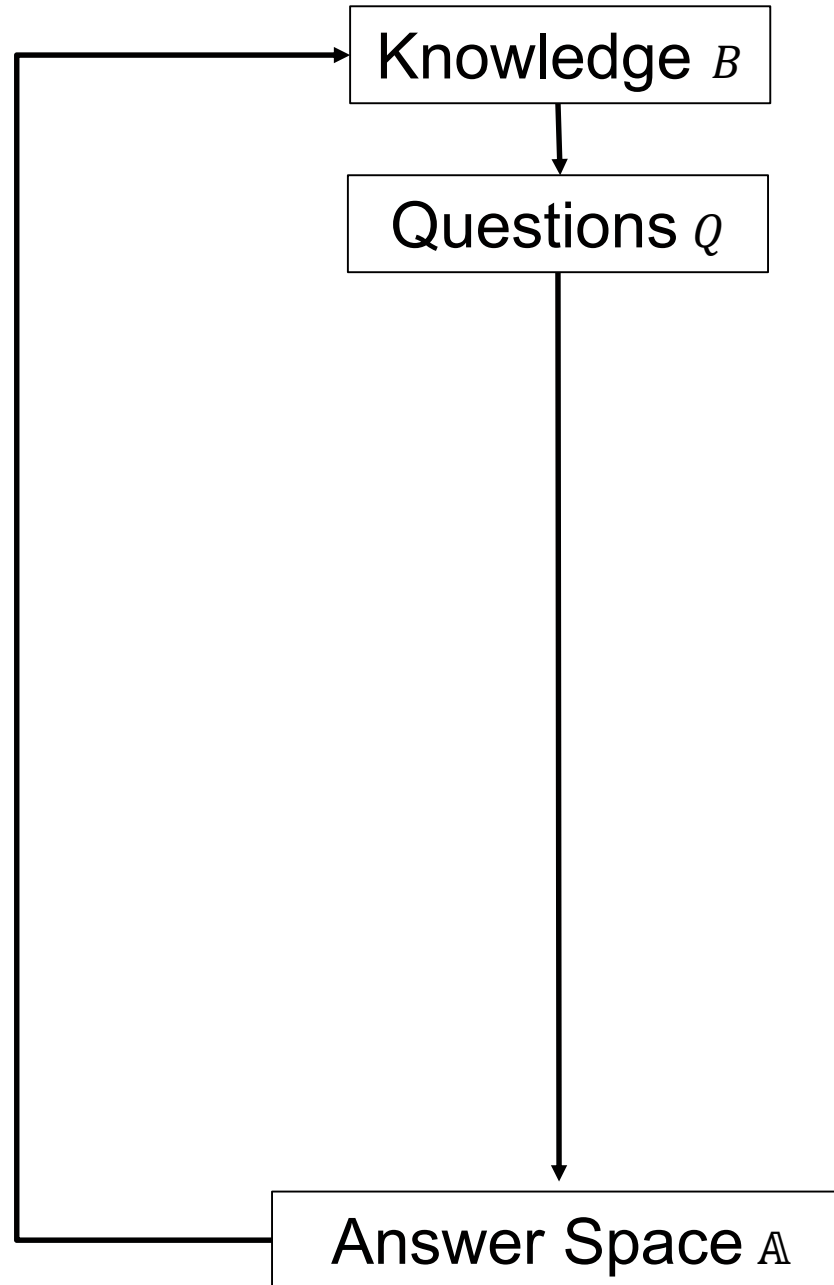
Arnold & Curtis (2018)

Geophys. J. Int.

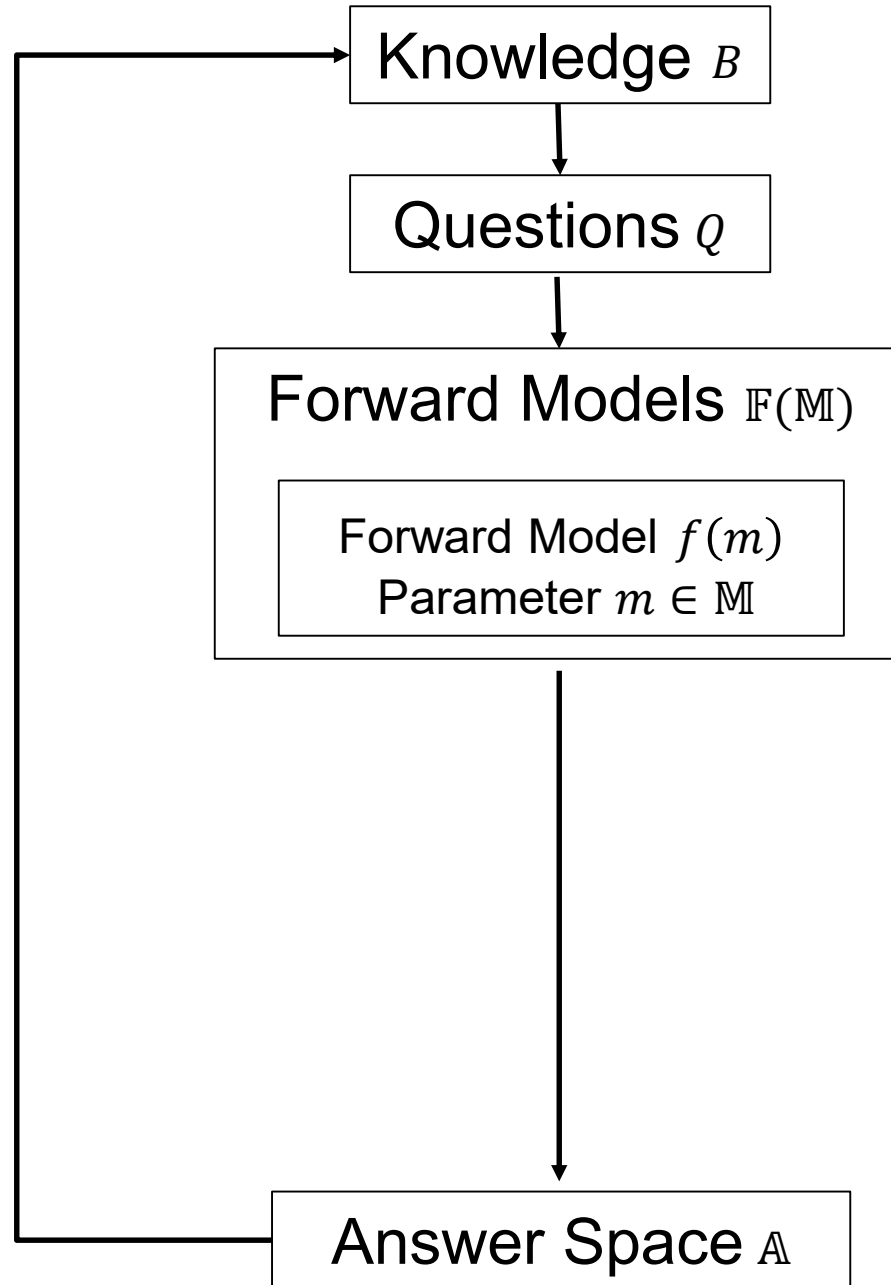
Inverse Theory and Interrogation Theory



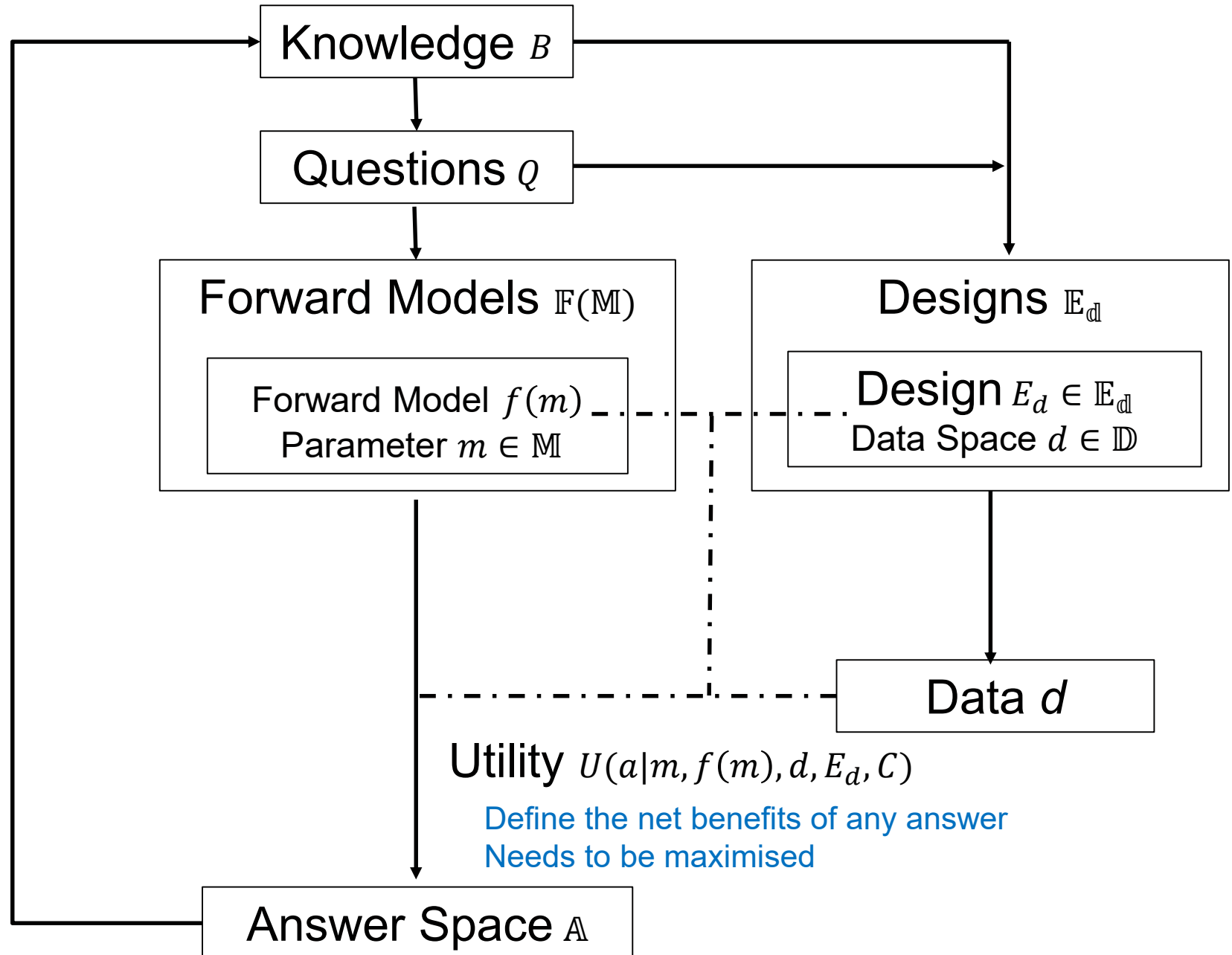
Interrogation Theory



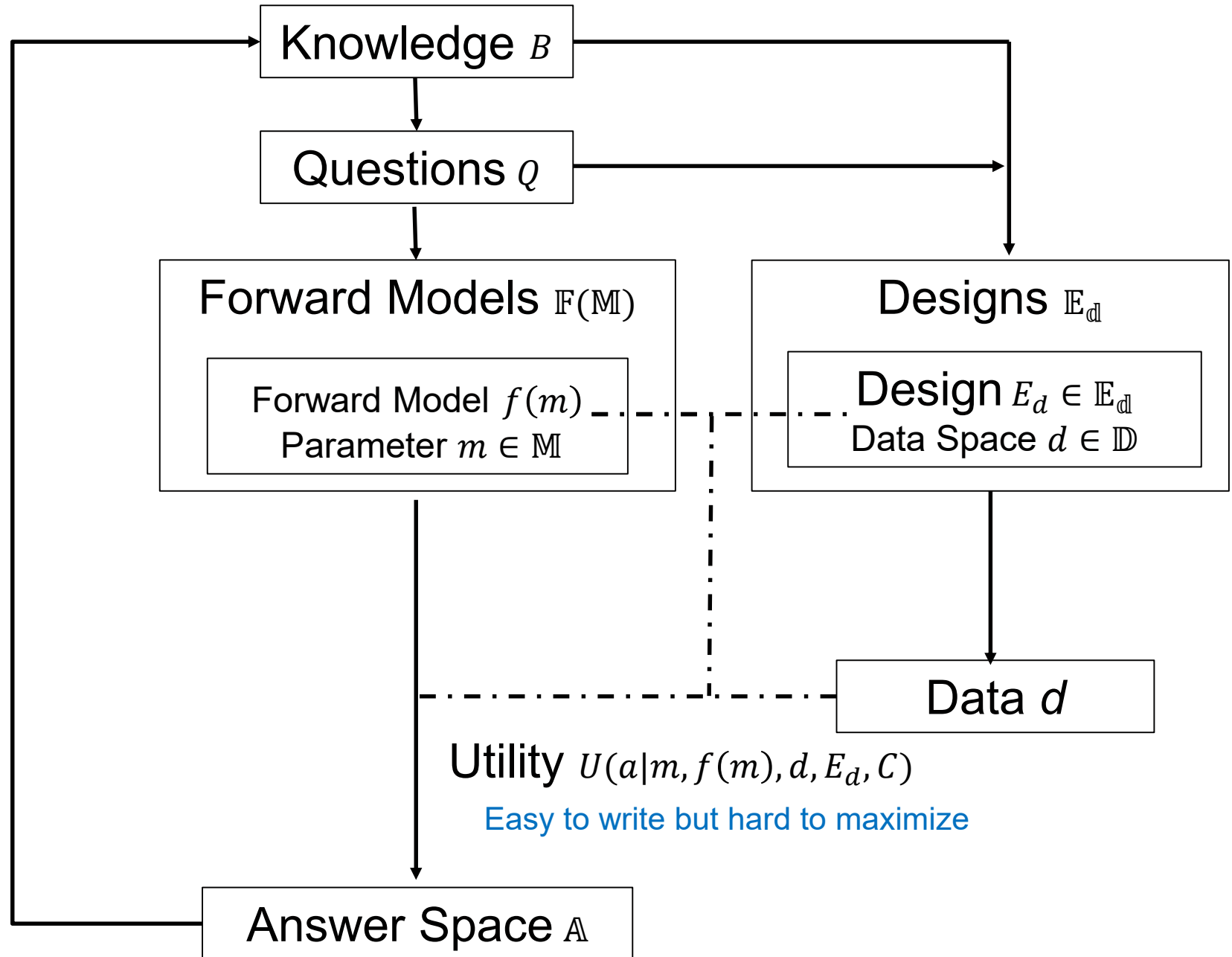
Interrogation Theory



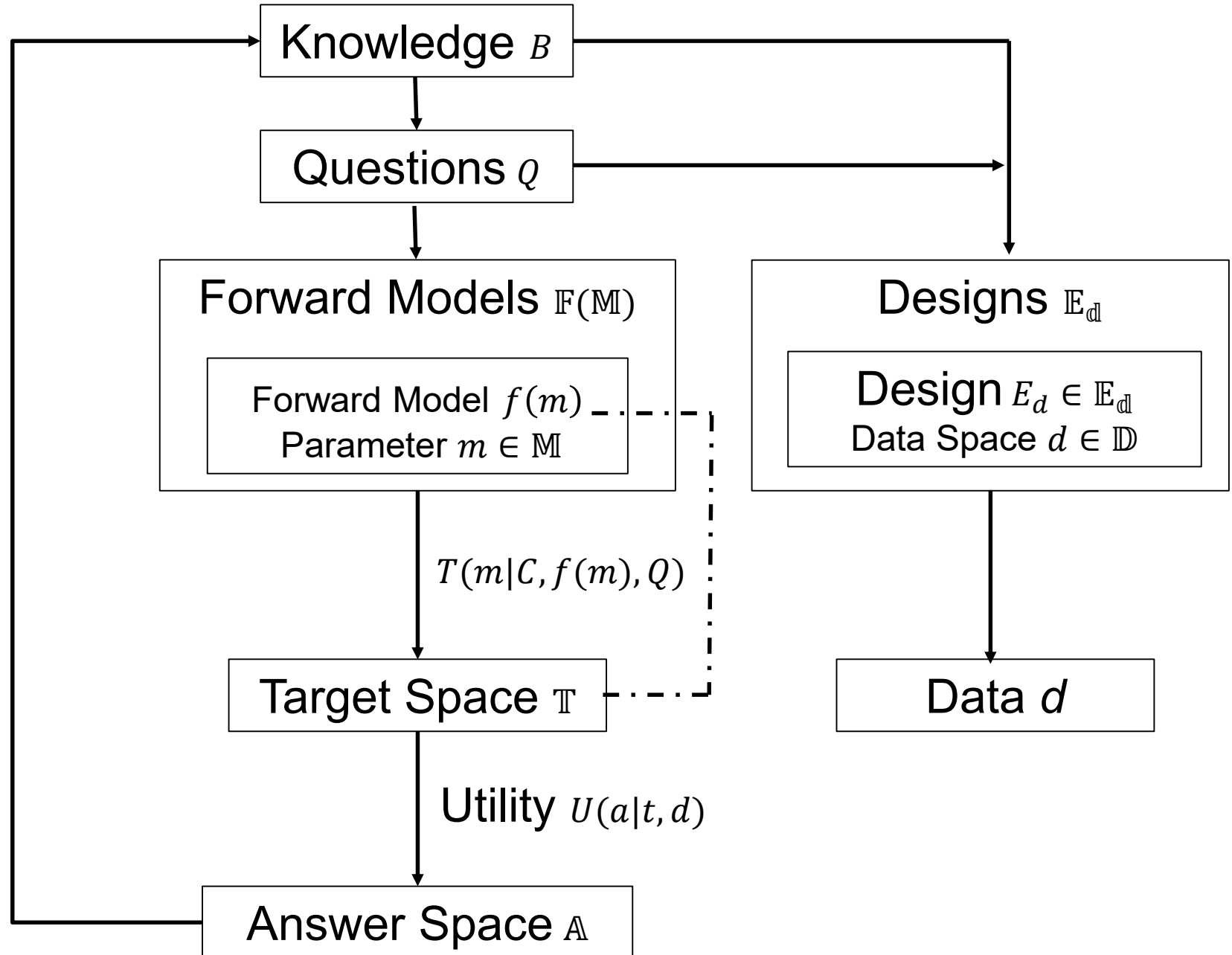
Interrogation Theory



Interrogation Theory



Interrogation Theory



Interrogation Theory

True answer

$$\text{Utility: } U(a|t, d) = -(a - \boxed{t})^2$$

→ Arnold & Curtis (GJI – 2018)

$$\text{Optimal Answer: } a^* = \mathbb{E}[T|E_d, d]$$

$$= \sum_{f(m)} \int_m T(m|f(m)) p(m, f(m)|E_d, d) dm$$

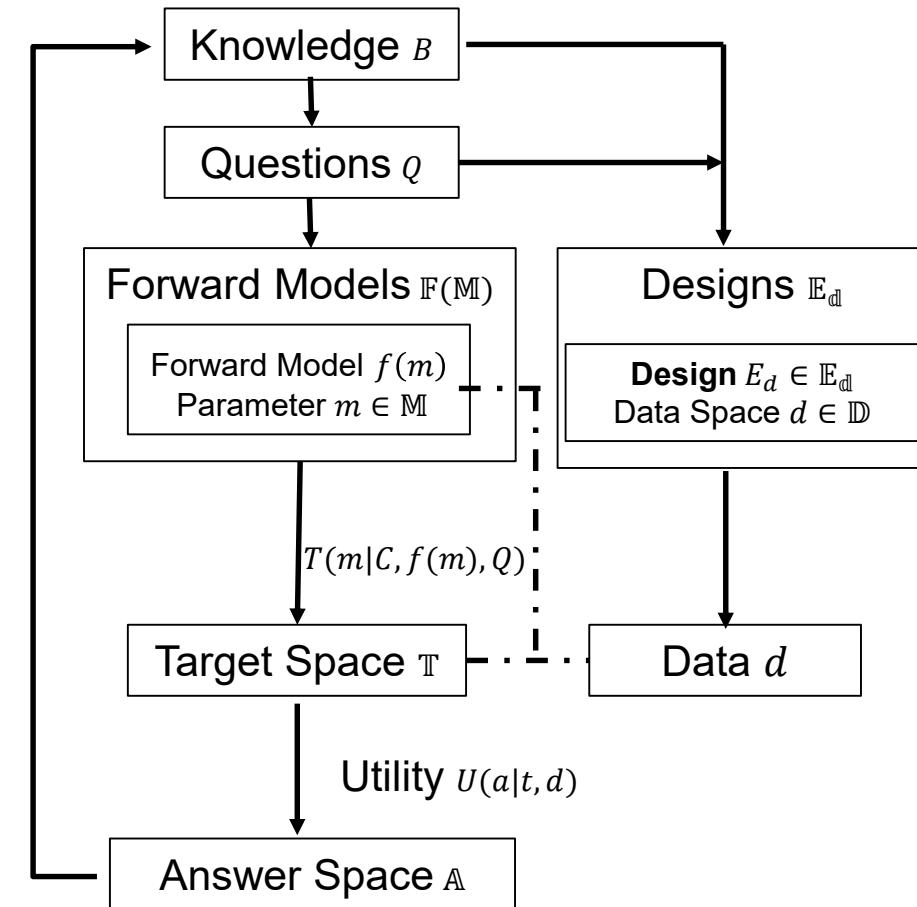


$$\text{Optimal Answer: } a^* = \mathbb{E}[T|E_d, d]$$

$$= \sum_{C, f(m)} \int_m T(m|C, f(m)) \textcolor{red}{p(m, C, f(m)|E_d, d)} dm$$

$$= \sum_{C, f(m)} \underline{\textcolor{red}{p(C|E_d, d)p(f(m)|E_d, d)}} \int_m T(m|C, f(m)) \underline{\textcolor{red}{p(m|C, f(m), E_d, d)}} dm$$

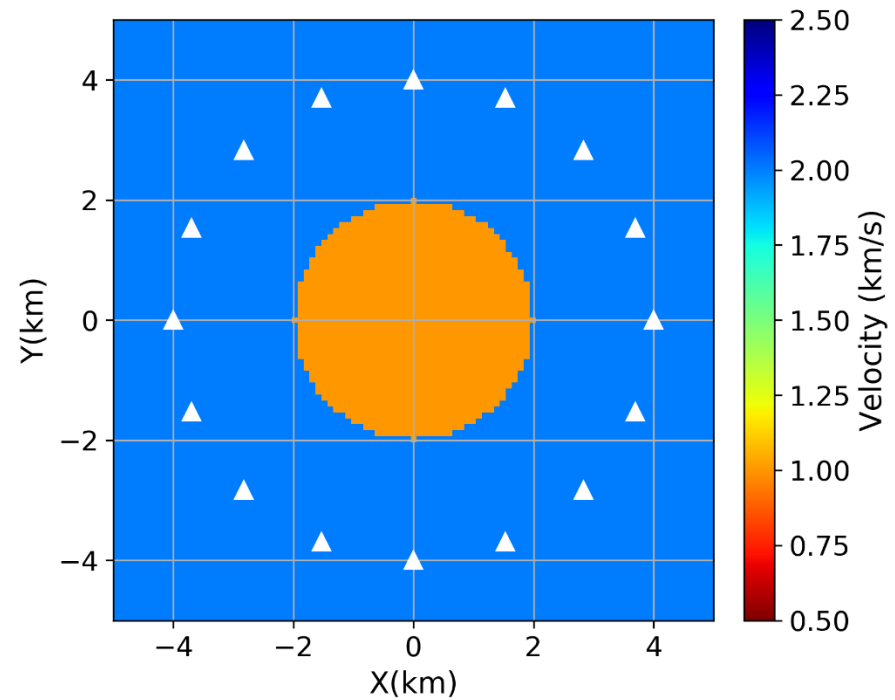
$p[C/f(m)|E_d, d]$: The probability that $C/f(m)$
would give **TRUE** posterior pdf



Posterior pdf

Synthetic Test of Travel Time Tomography

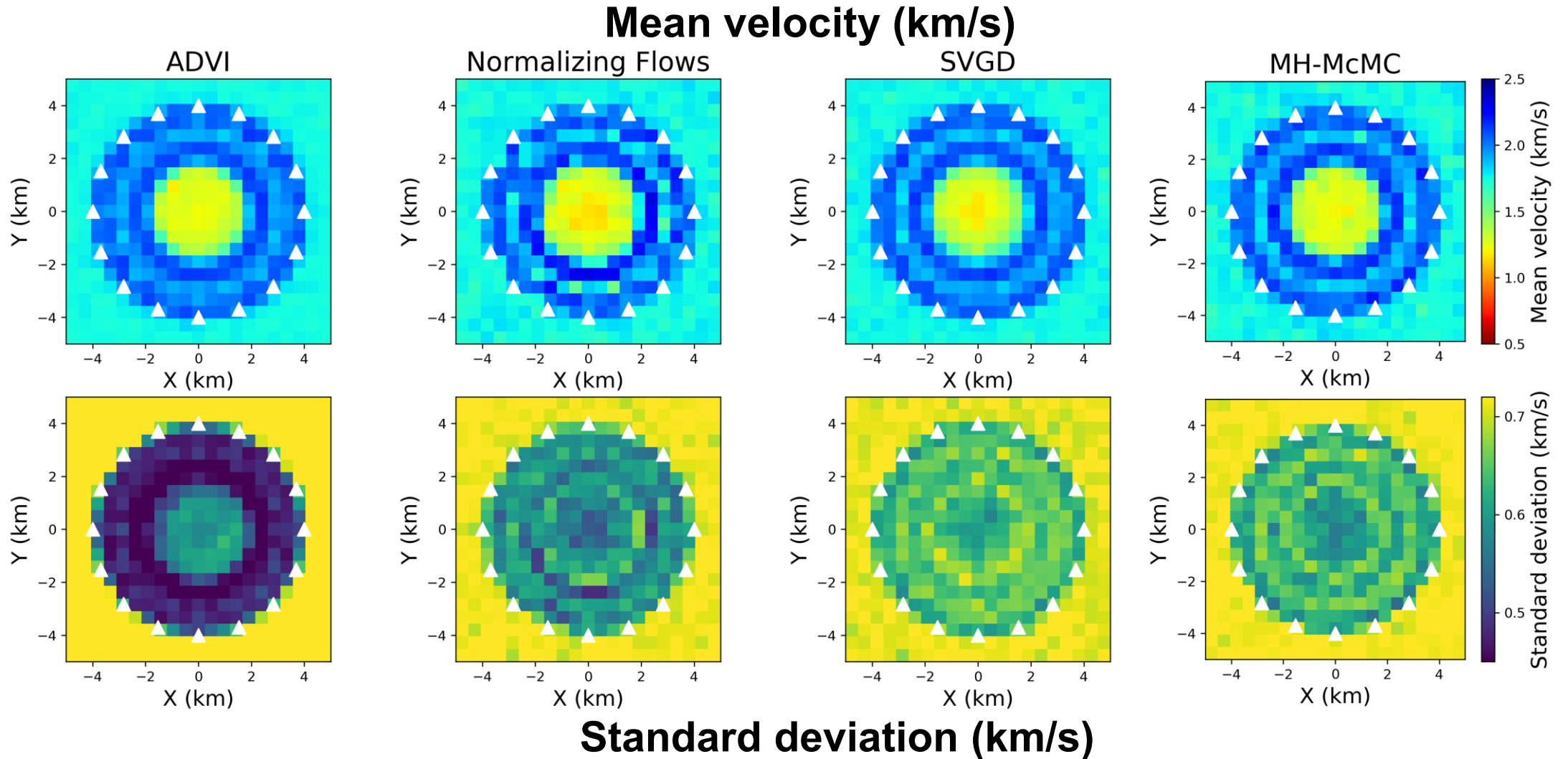
Travel Time Tomography



True Velocity Model

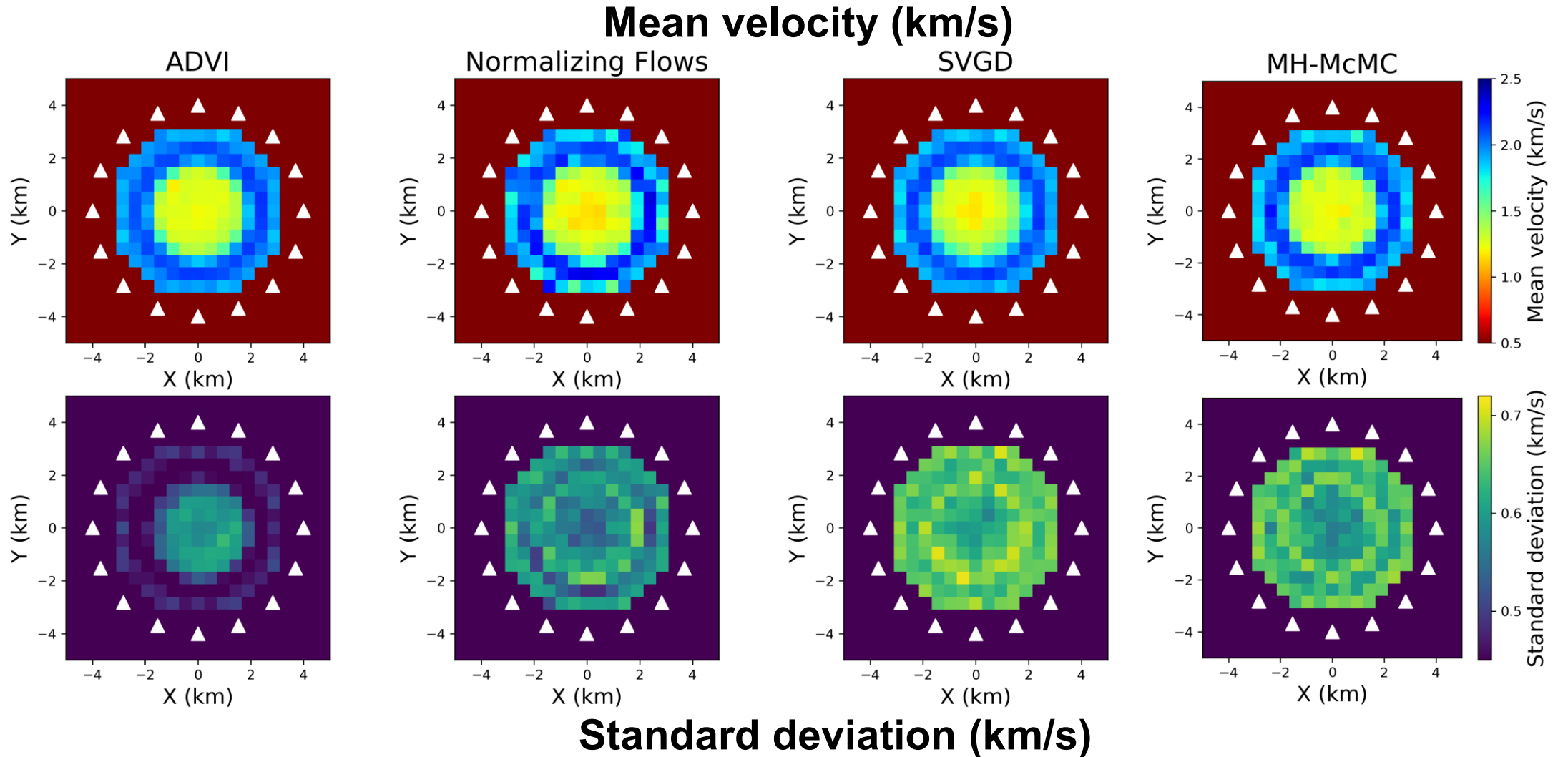
Application of Interrogation Theory

Question: what is the area of the low velocity anomaly?



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Application of Interrogation Theory

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Forward Model $f(m)$: fast marching method

Parametrization m : seismic velocity, uniform in each cell

Design E_d : Source-receiver station locations

Data d : Source-receiver travel-time data

Algorithms C and **prior** $p[C|E_d, d]$:

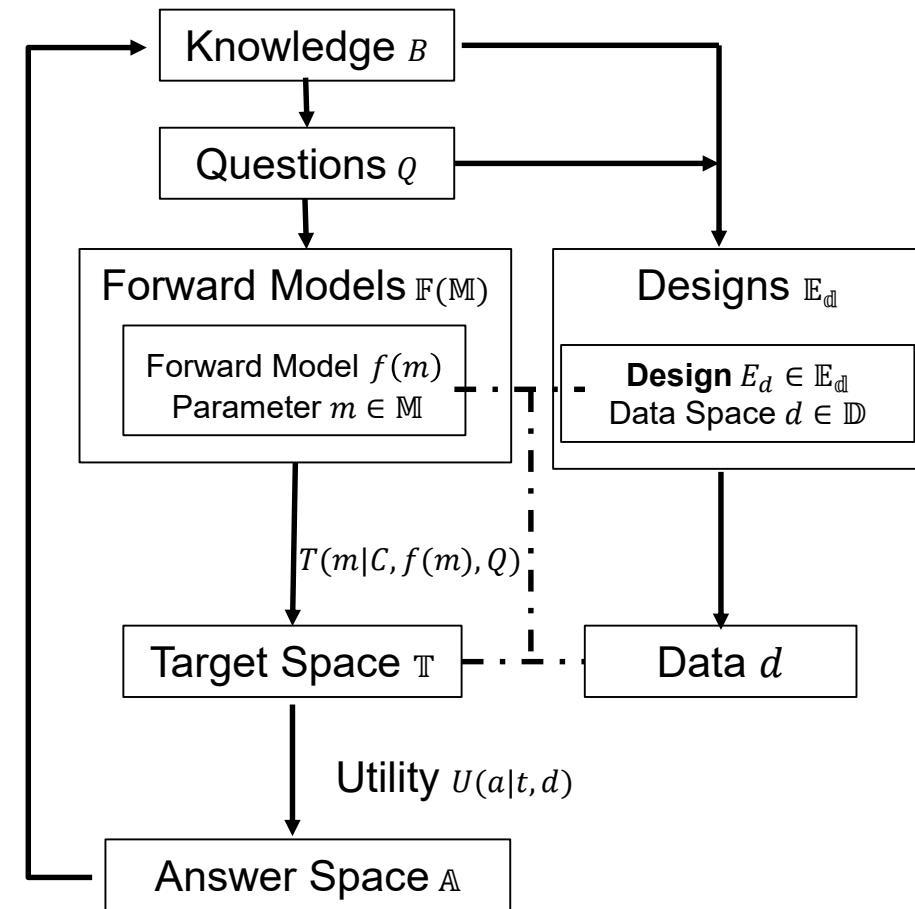
ADVI: 0.1, SVGD: 0.3, MH-McMC: 0.3 and Normalizing flows: 0.3

Target function $T(m)$: transforms m into area of low velocity

Utility function: $U(a|t) = -(a - t)^2$

squared error loss utility in Arnold & Curtis (2018)

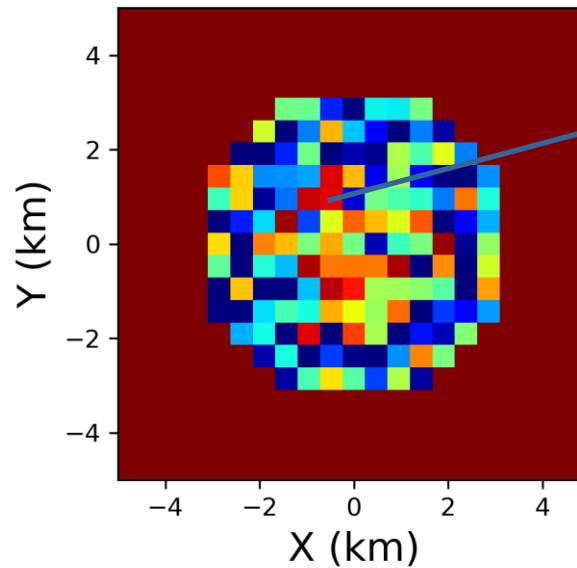
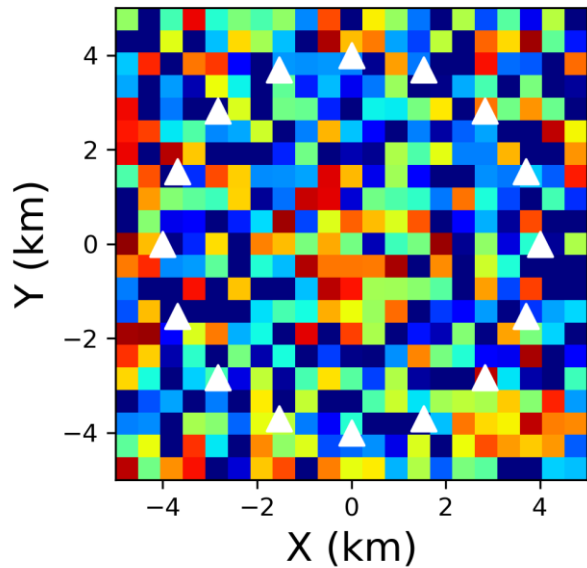
Optimal Answer: $a^* = \mathbb{E}[T|E_d, d]$



Application of Interrogation Theory

Question: what is the area of the low velocity anomaly?

Optimal Answer: $a^* = E[T|E_d, d]$ $\longrightarrow$ $T(m|f(m)) = \text{Area of low velocity body}$

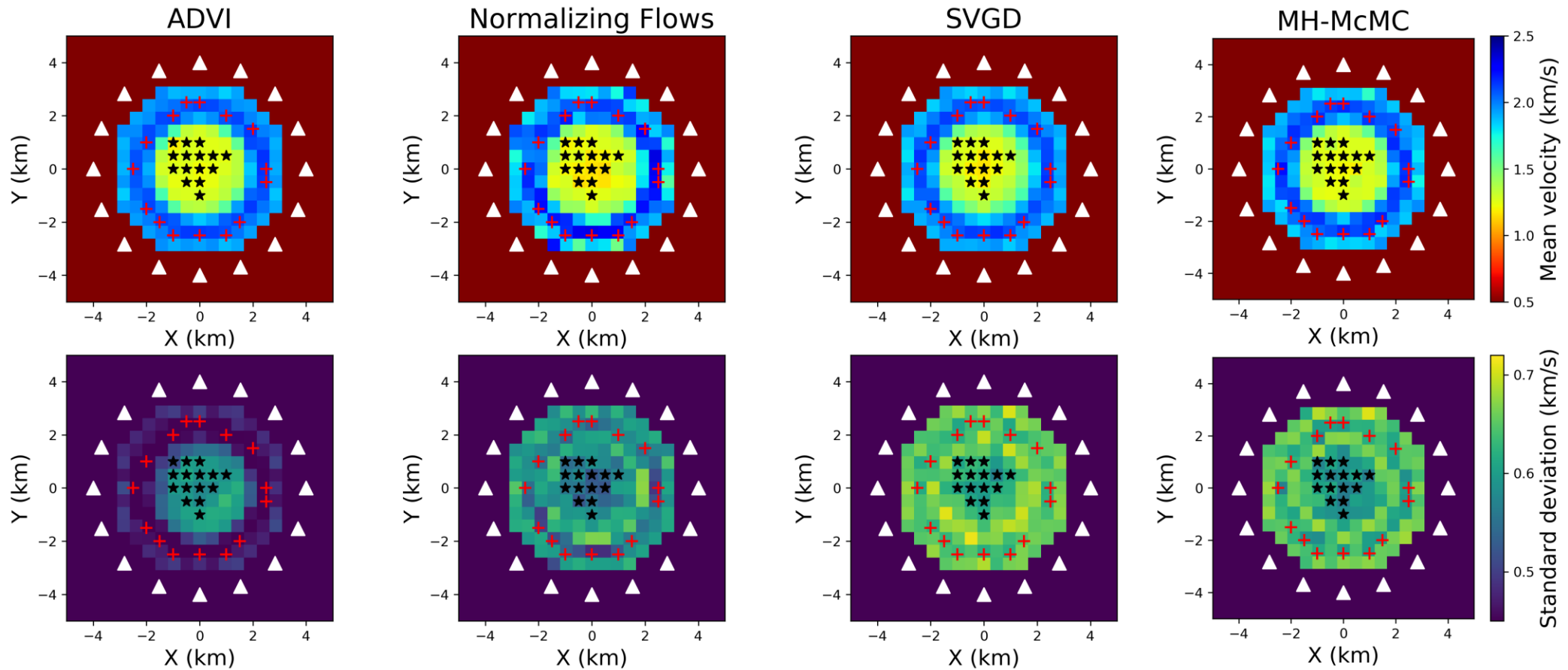


Sub-question: what is the best threshold to discriminate low from high velocities with minimal bias?

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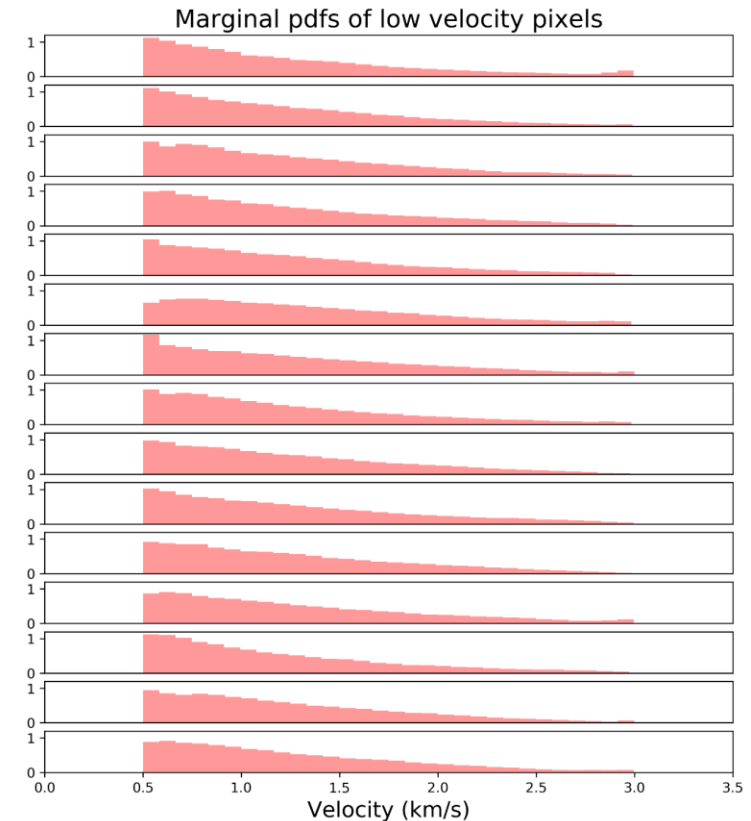
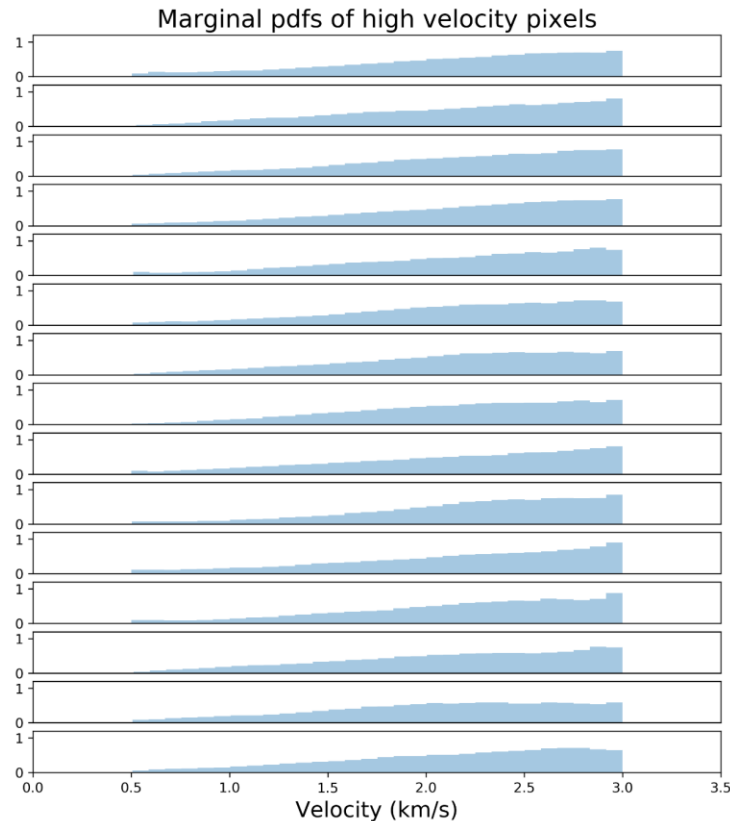
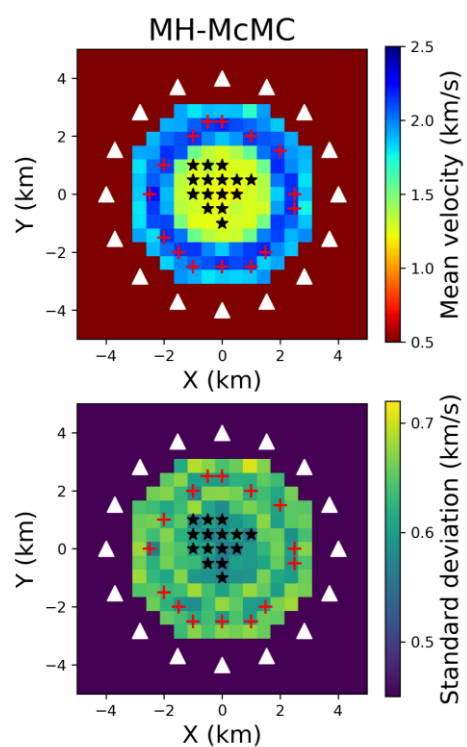
1. Find grid cells that are most likely to be low and high velocities



Application of Interrogation Theory

Sub-question: what is the best threshold to discriminate low from high velocities with minimal bias?

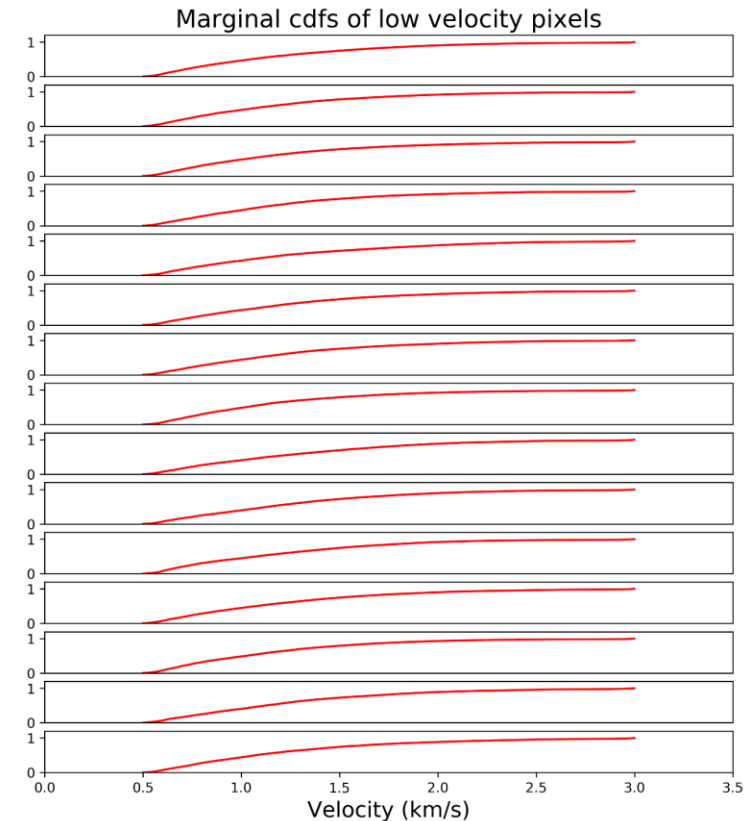
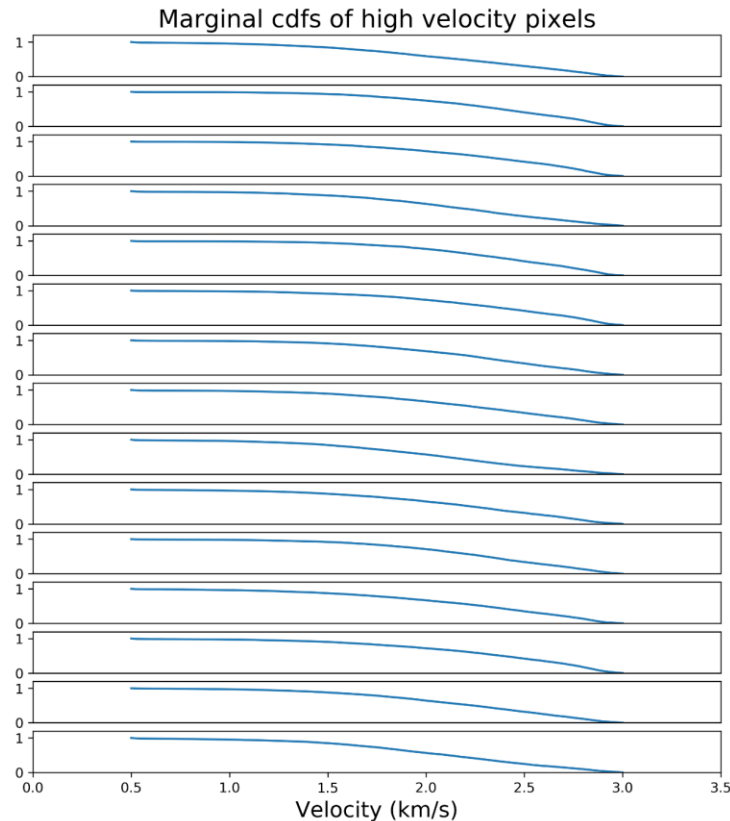
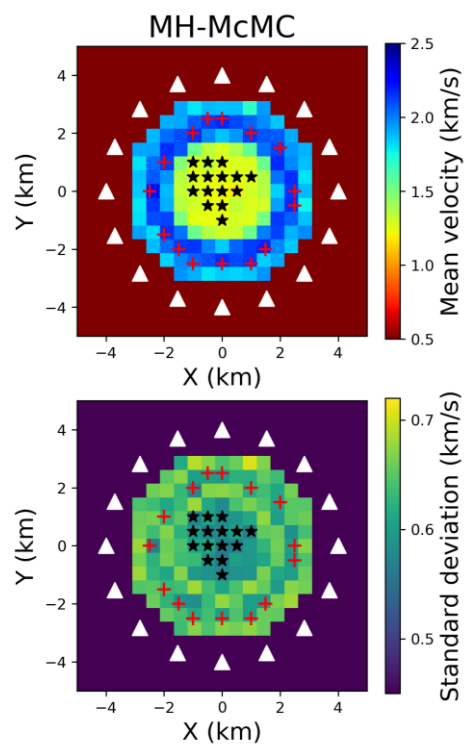
1. Find grid cells that are most likely to be low and high velocities
2. Calculate the cumulative density function (cdf) for low and high velocity cells



Application of Interrogation Theory

Sub-question: what is the best threshold to discriminate low from high velocities with minimal bias?

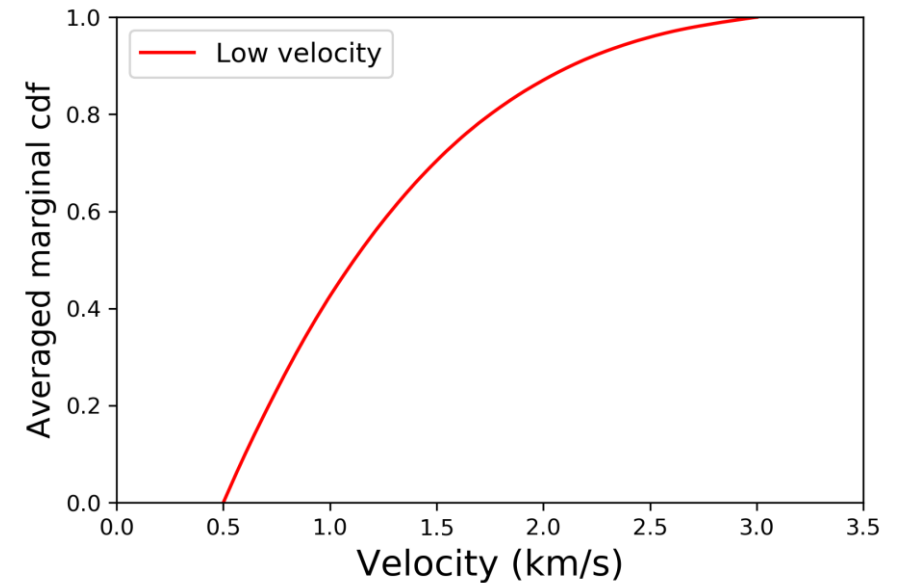
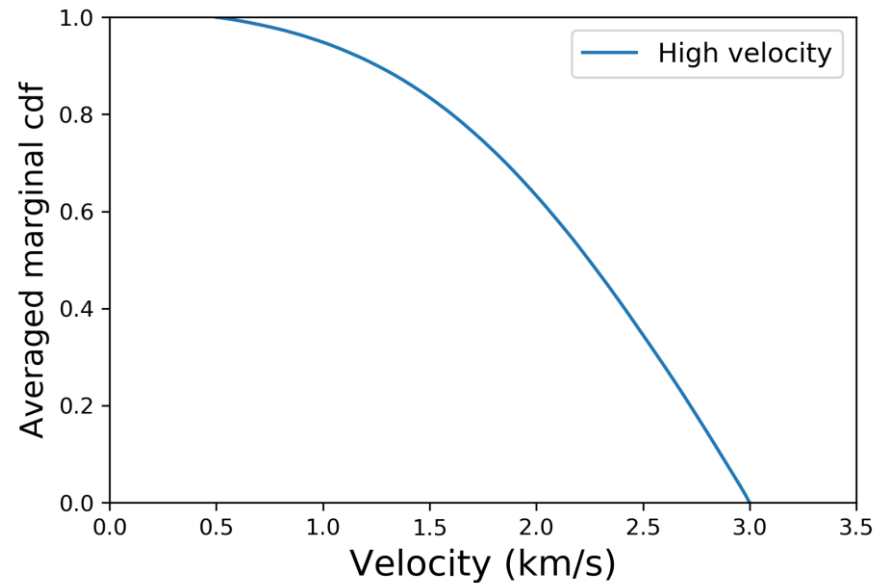
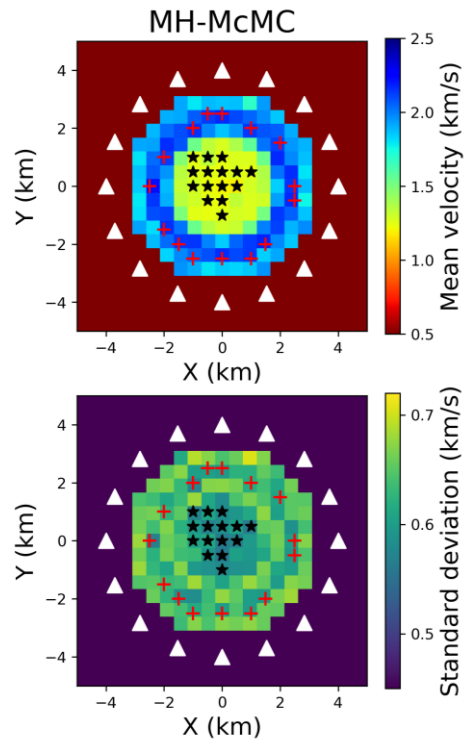
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Application of Interrogation Theory

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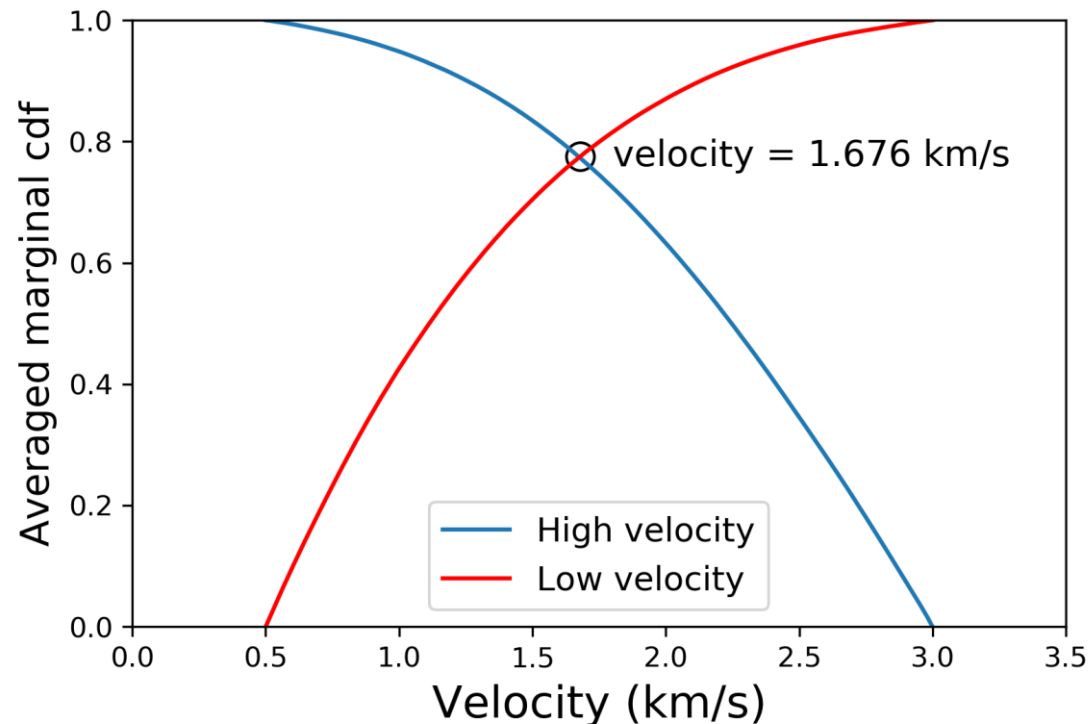
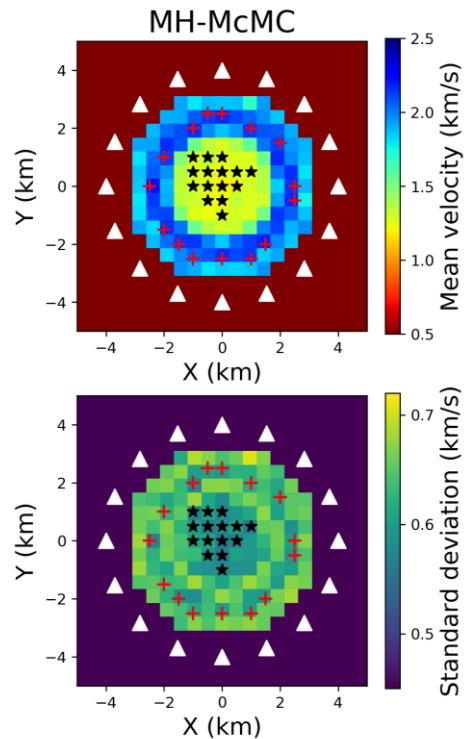
1. Find grid cells that are most likely to be low and high velocities
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Application of Interrogation Theory

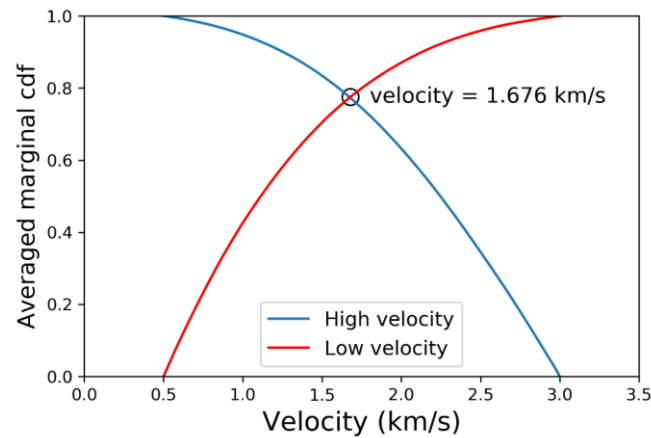
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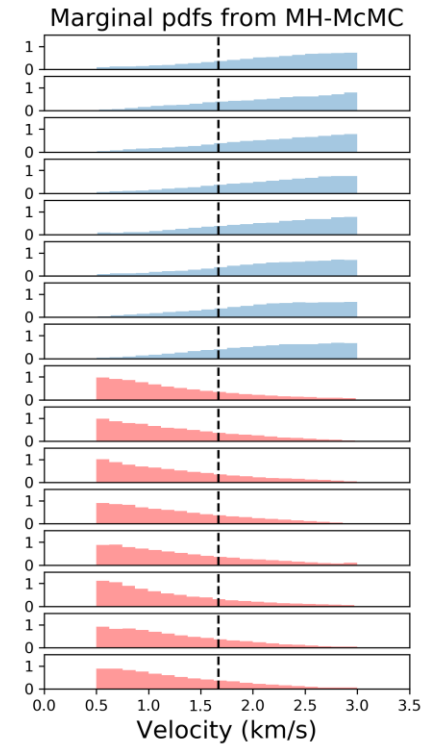
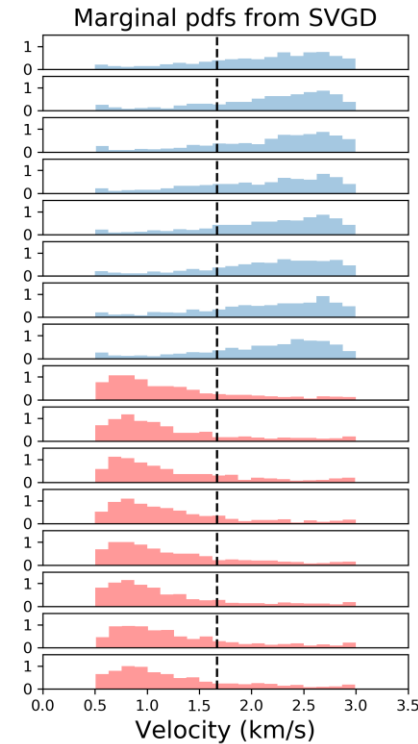
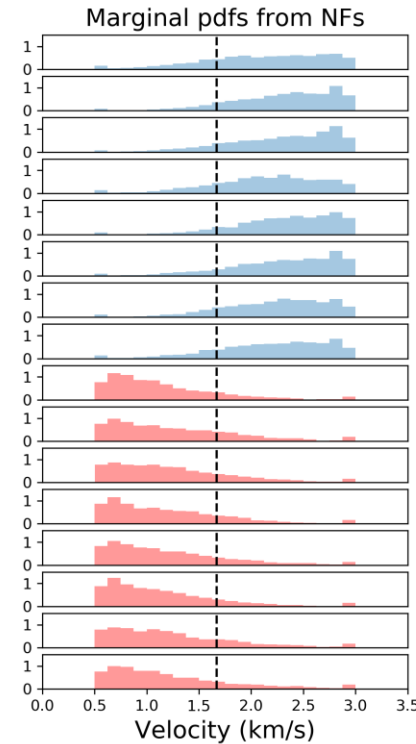
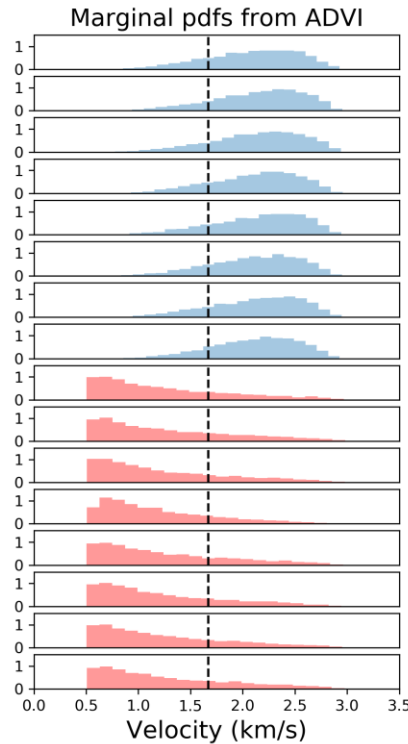


Application of Interrogation Theory

Sub-question: what is the best threshold to discriminate low from high velocities with minimal bias?



Optimal answer:
vel = 1.676 km/s

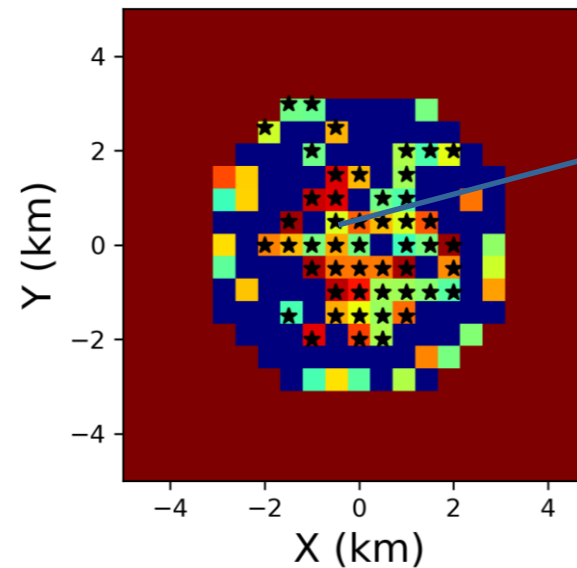
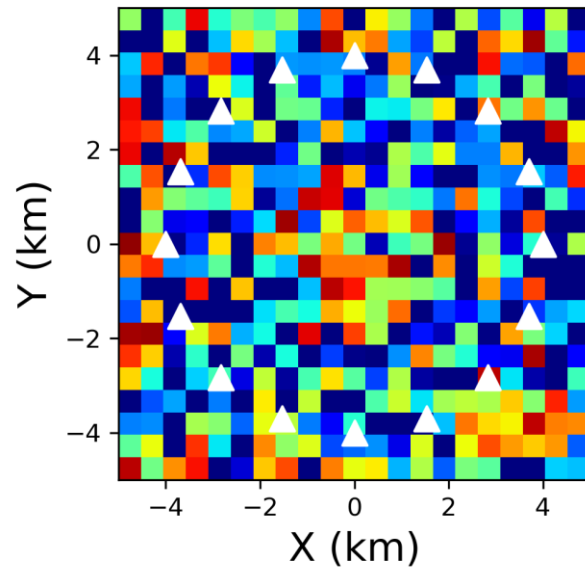


Application of Interrogation Theory

Question: what is the area of the low velocity anomaly?

Continuous low velocity body!

Optimal Answer: $a^* = E[T|E_d, d]$ $\longrightarrow$ $T(m|f(m)) = \text{Area of low velocity body}$



Low or high velocity?

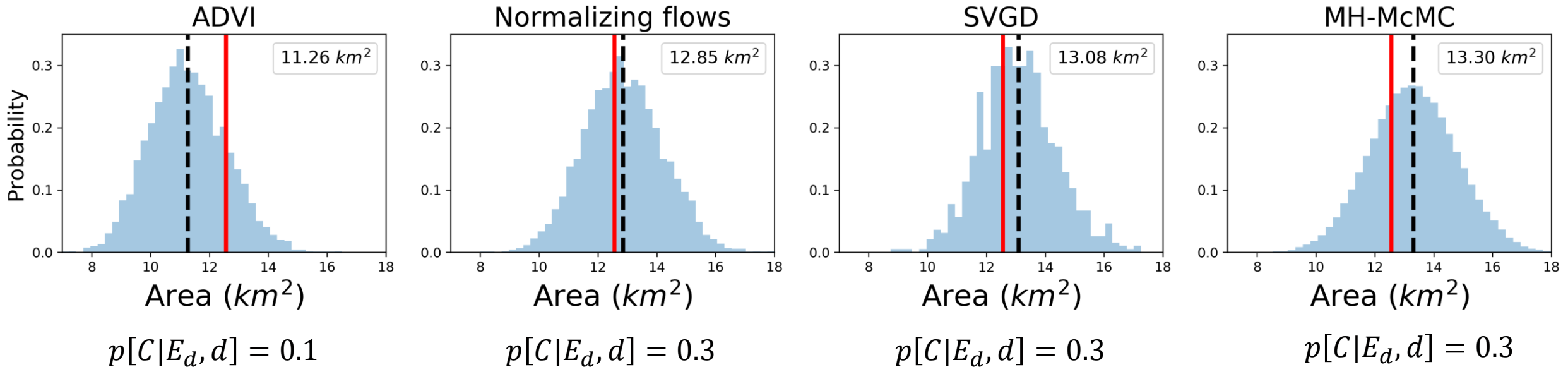
$vel < 1.676 \text{ km/s}$ or not ?

Application of Interrogation Theory

Question: what is the area of the low velocity anomaly?

Optimal Answer: $a^* = E[T|E_d, d]$

$T(m|f(m)) = \text{Area}(\text{vel} < 1.676 \text{ km/s})$



$p[C|E_d, d]$: The probability that Algorithm C would give **TRUE** posterior pdf

Optimal Answer: $a^* = E[T|E_d, d]$

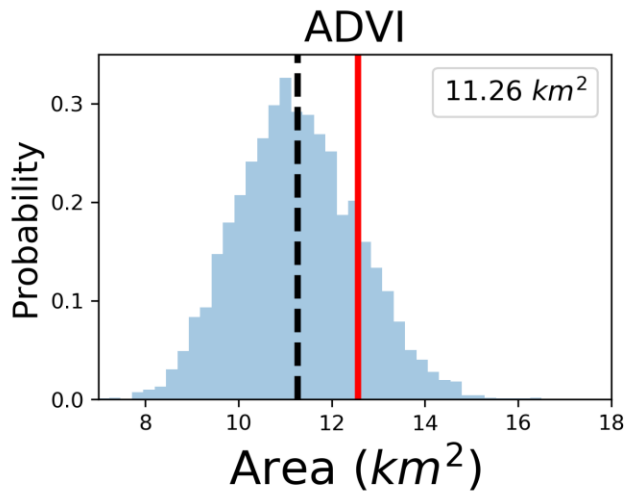
$$= \sum_{C, f(m)} p(C|E_d, d) p(f(m)|E_d, d) \int_m T(m|C, f(m)) p(m|C, f(m), E_d, d) dm$$

Application of Interrogation Theory

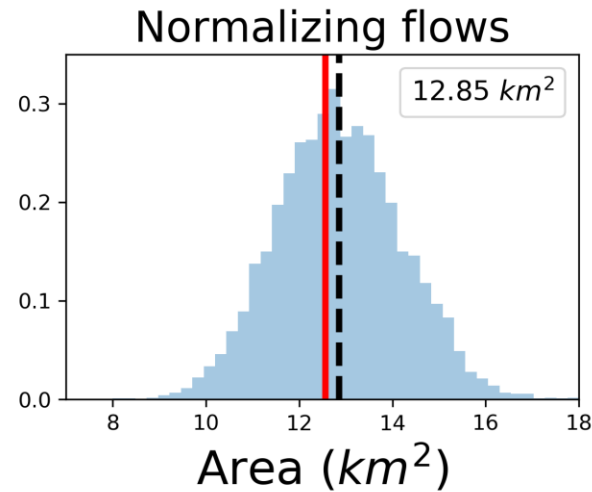
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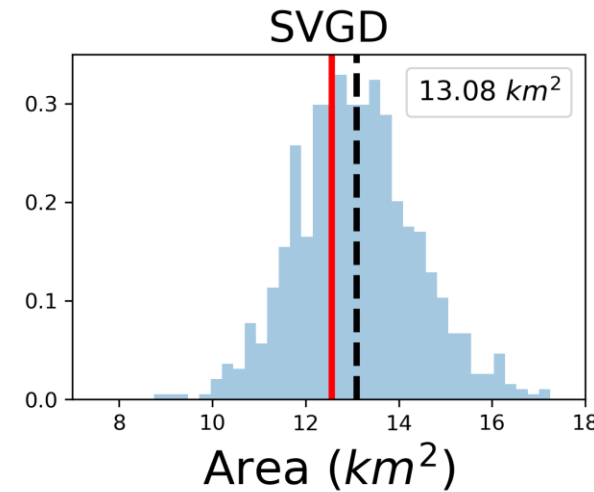
$T(m|f(m)) = \text{Area}(\text{vel} < 1.676 \text{ km/s})$



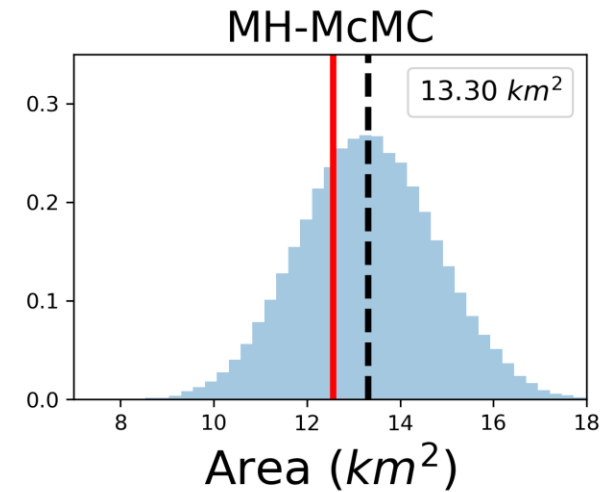
$$p[C|E_d, d] = 0.1$$



$$p[C|E_d, d] = 0.3$$



$$p[C|E_d, d] = 0.3$$



$$p[C|E_d, d] = 0.3$$

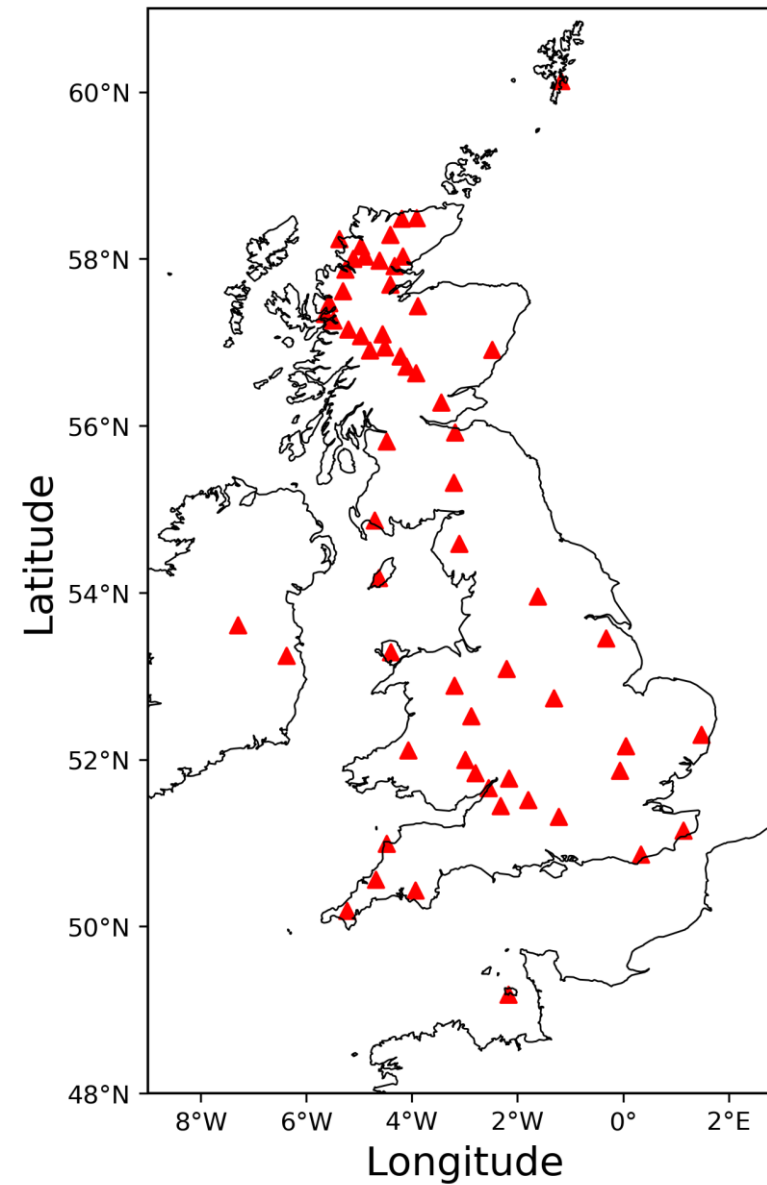
$p[C|E_d, d]$: The probability that Algorithm C would give **TRUE** posterior pdf

Optimal Answer: $a^* = 12.89 \text{ km}^2$

True answer: 12.56 km²

Interrogating the area of the East Irish Sea Basins

Receiver stations



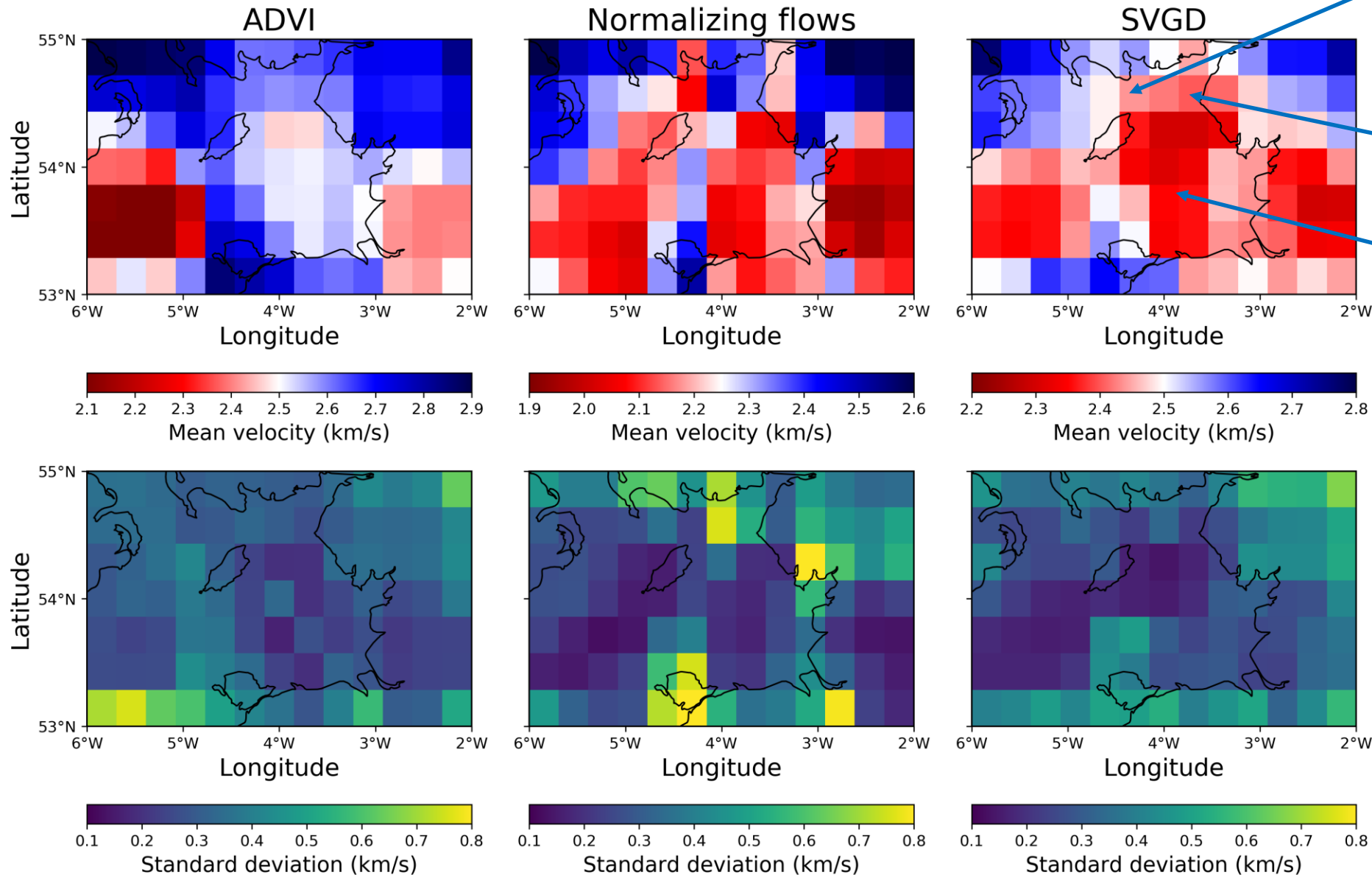
S wave velocity at Z = 200 m

What is the area of these three Sedimentary Basins?

Stranraer Basin

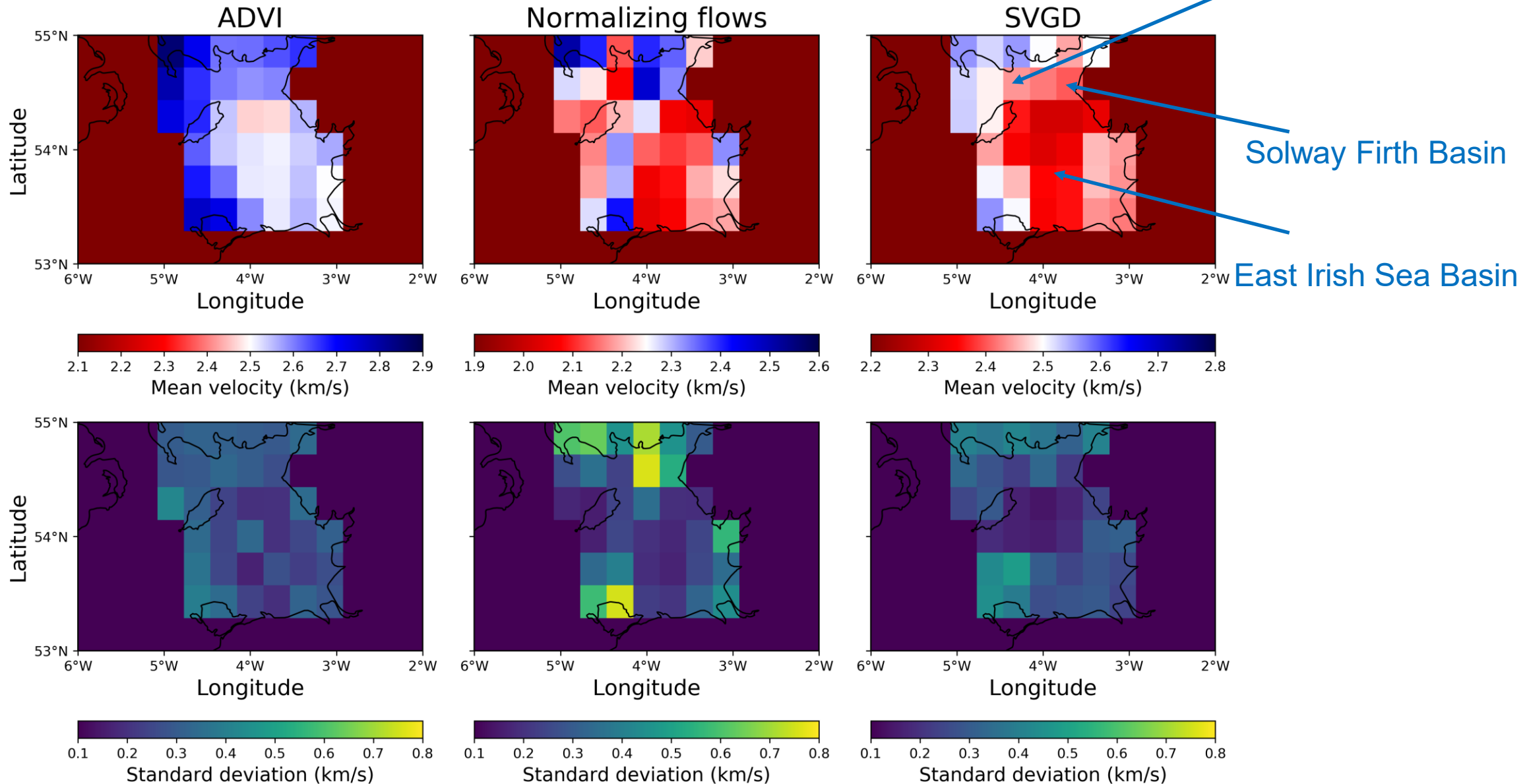
Solway Firth Basin

East Irish Sea Basin



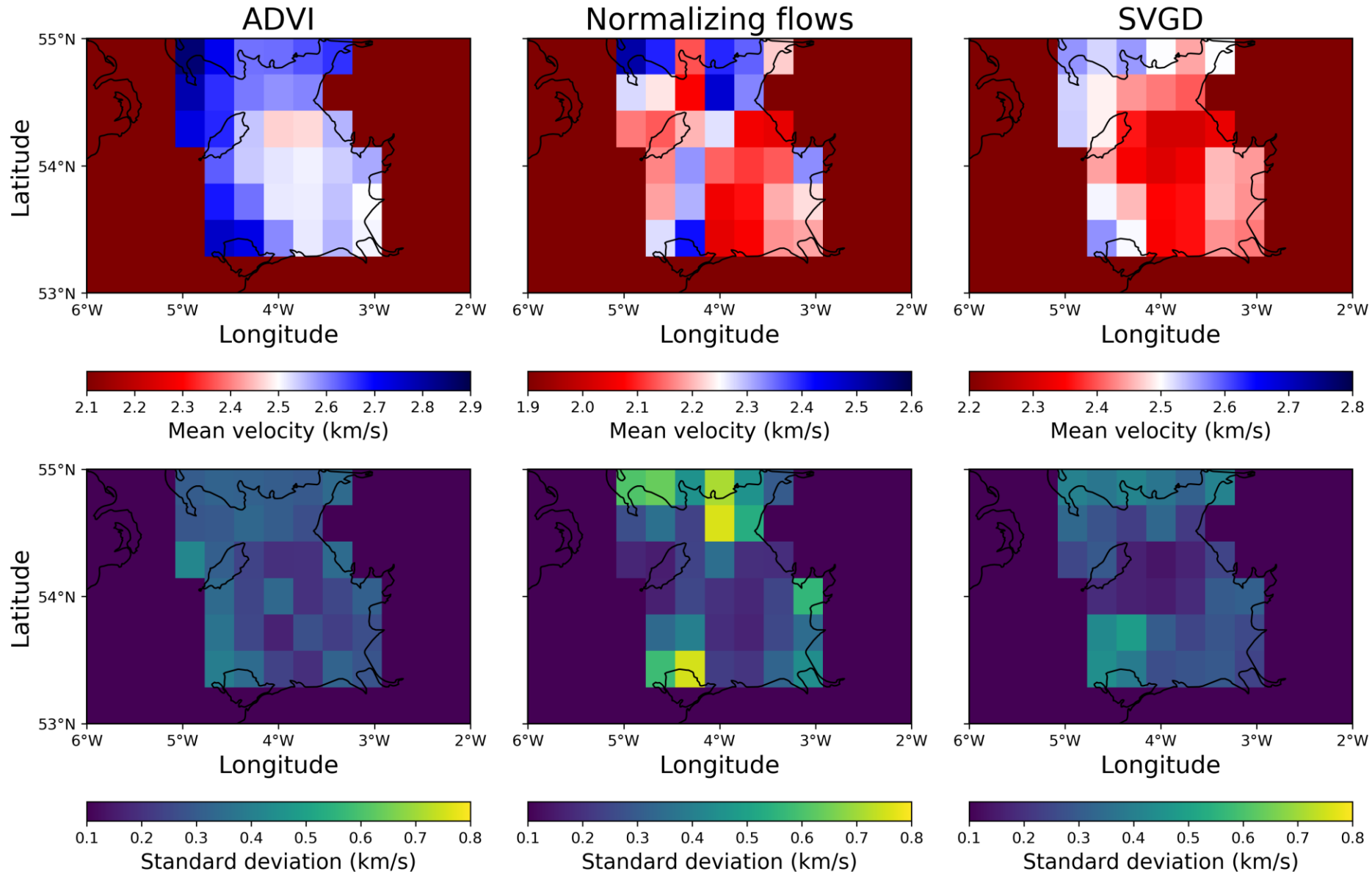
S wave velocity at $Z = 200$ m

What is the area of these three Sedimentary Basins?



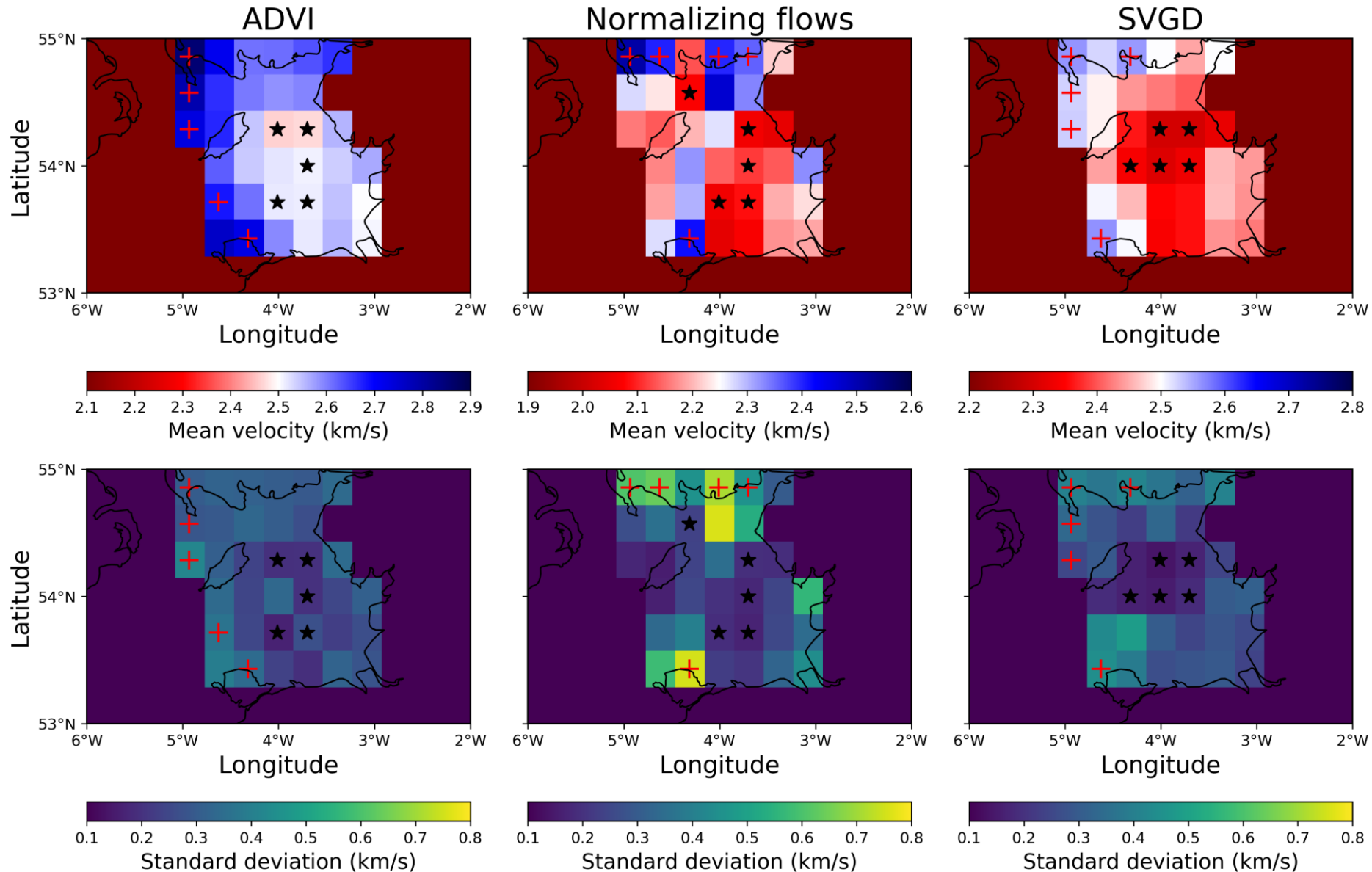
S wave velocity at $Z = 200$ m

What is the area of the continuous low velocity body?



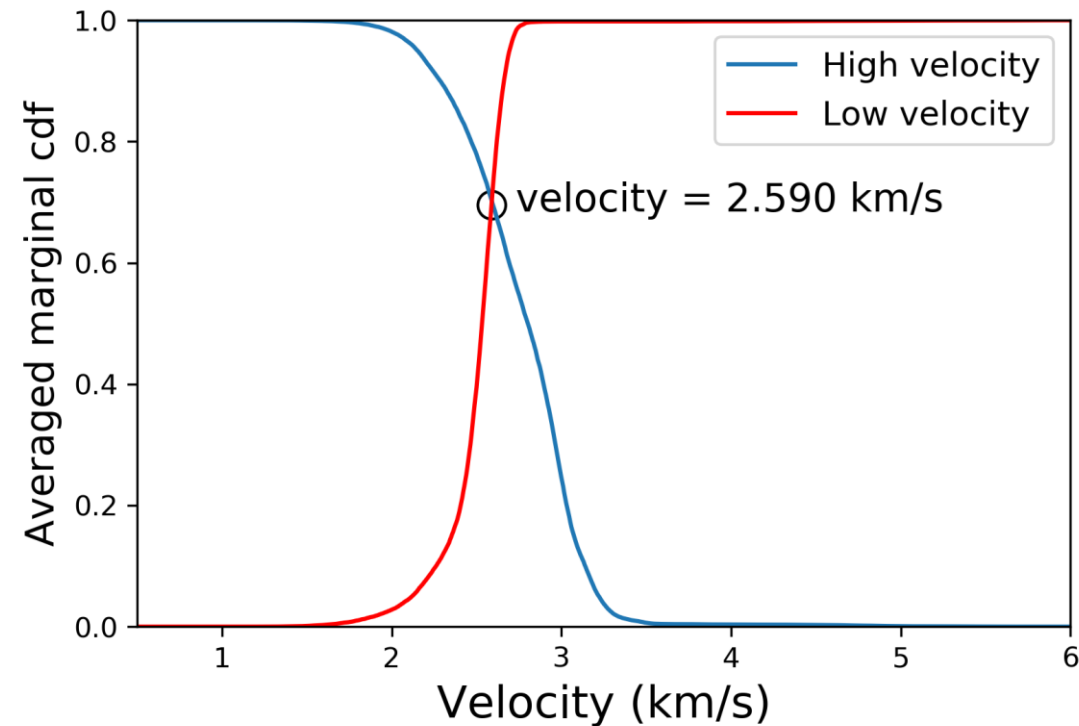
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What is the area of the continuous low velocity body?



Application of Interrogation Theory

Sub-question: what is the best threshold to discriminate low from high velocities with minimal bias?

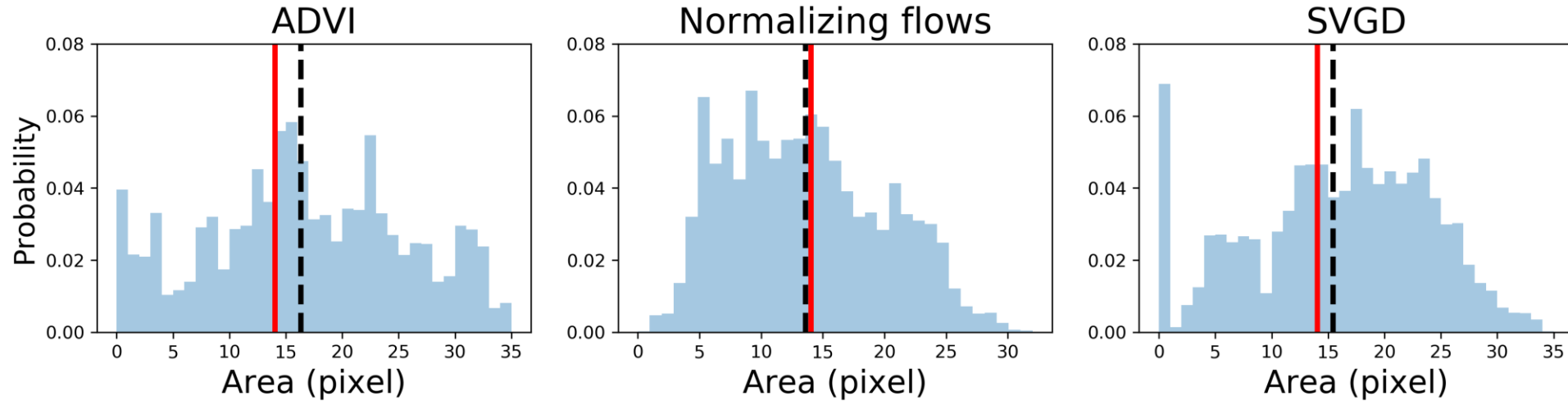


Best threshold obtained from ADVI

Application of Interrogation Theory

Question: what is the area of the three Sedimentary Basins?

Optimal Answer: $a^* = E[T|E_d, d]$



Optimal Answer: $a^* = 14.93$

True value: 14.02
(Estimated from BGS report CR/15/057)

Conclusion

1. *Interrogation theory* answers **specific** questions about the Earth
2. Synthetic example proved its effectiveness and accuracy
3. Application to field data estimates our answer to a realistic question
4. Theory is quite general – also can be applied in many other contexts



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Thank you



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