Power-law frequency scaling of surface temperature spatial degrees of freedom —estimated from instrumental data, reanalysis and climate model simulations—

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Abstract

What is the spatial scale of climate fluctuations, and how does this scale depend on the timescale under consideration? To answer this question, the spatio-temporal correlation structure of global surface temperature fields is characterized, for the period 1850-present, by estimating frequency spectra of the effective spatial degrees of freedom (ESDOF). These ESDOF spectra serve as a simple summarizing metric of the frequency-dependent spatial auto-correlation function. ESDOF spectra are estimated from: (a) the HadCRUT global gridded temperature anomaly dataset, based exclusively on instrumental measurements, including detailed error variance estimates; (b) the NOAA 20th Century Reanalysis; and (c) a large ensemble of CMIP historical climate model simulations. When comparing (i) error corrected ESDOF spectra from the instrumental data to (ii) those obtained from the reanalysis and the model simulations, with HadCRUT data gaps imposed, results are found to be highly consistent among the three data sources. When the analysis is applied to the entire globe, the ESDOF spectra exhibit an almost uniform power-law frequency scaling with about 100 ESDOFs at monthly timescales and only about 2 ESDOFs at multidecadal timescales. Second-order differences in this scaling behaviour are found when the analysis is restricted to various spatial subdomains of the globe, namely, the tropics, extra-tropics, land areas, and ocean areas. A few implications of the diagnosed ESDOF reduction towards the longer timescales are briefly discussed.

The 2 minutes PICO presentation (EGU21-13589, a contribution to session NP3.2/CL4.36):

Power-law frequency scaling of surface temperature Effective Spatial Degrees Of Freedom (= ESDOF)

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Definition **ESDOF = global mean local PSD / PSD of global mean** (= number of independent samples; coherent fields → ESDOF = 1)

Data Global 5°x5° gridded 2m-temperature fields:
- + Instrumental: **HadCRUT4** (1850-2019)
  - spatio-temporal coverage: 60%
  - including spatial error covariances
  - assuming no temporal error correlation
- + Reanalysis: **NOAA20CR.v3** (1850-2015)
- + Models: **CMIP6-historical** (1850-2014)
  - 27 models x 3 members = 81 runs

Method PSD estimation from gappy fields:
- + **Bias-free** from auto-covariance function
  - **no interpolation**, using **all data** points
- + HadCRUT-corrected = HadCRUT – Errors
- + NOAA20CR.v3, CMIP6-hist with gaps imposed

Fig. 1: Power spectral density (PSD)

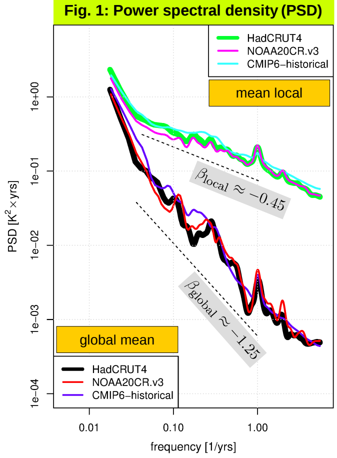
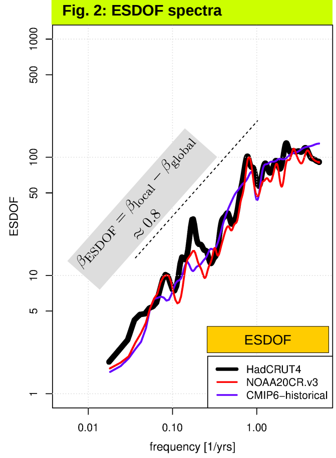


Fig. 2: ESDOF spectra



Outcome Highly consistent among the three datasets:
- + Uniform freq. scaling: **ESDOF(freq) ~ freq^0.8**
- + annual **ESDOF ≈ 100**, multi-decad. **ESDOF ≈ 2**
- + second-order differences when computed from land/ocean/extra-tropics/tropics
- + for additional information / details: see associated **display material**

1. Definition

A simple measure of the effective spatial degrees of freedom (ESDOF) of a time-varying spatial field (e.g., a global near-surface temperature field) is given by $D = \sigma^2_{\text{loc}}/\sigma^2_{\text{glb}}$, where σ^2_{loc} is the spatial average of the local variance and σ^2_{glb} is the variance of the spatial average (see, for example, Smith et al. 1994; Jones et al. 1997). For fields covering the entire globe, the spatial averages are global averages. The interpretation is as follows: If the field is constant in space (globally coherent fluctuations), then $D = 1$. For smaller scale fluctuations, we have $D > 1$ and the value of D specifies the effective number of independent spatial samples. Thus, the larger the value of D, the smaller the average spatial scale of the fluctuations. To obtain a frequency-dependent ESDOF measure $D(f)$, we replace the mean local variance σ^2_{loc} by the mean local power spectral density (PSD) $S_{\text{loc}}(f)$, and the variance of the global mean σ^2_{glb} by the PSD of the global mean $S_{\text{glb}}(f)$. Accordingly, we define

$$D(f) = \frac{S_{\text{loc}}(f)}{S_{\text{glb}}(f)}. \tag{1}$$

As in the frequency-independent case, this measure characterizes the spatial scale of the fluctuations, but now separately at each frequency.

2. Data

The above frequency-dependent ESDOF measure is estimated from monthly mean global gridded $5°{\times}5°$ near-surface temperature deseasonalized anomaly fields, obtained from the following three data sources.

a. Instrumental: HadCRUT4 (1850–2019)

The HadCRUT4 dataset is based exclusively on instrumental data, combining temperature measurements from land weather stations with sea surface temperature measurements from ocean platforms (Morice et al. 2012). Grid boxes without any data in a given month are represented as data gaps. The average spatio-temporal coverage of this global dataset is $\sim 60\%$. The dataset is complemented by detailed error variance estimates, including spatial error covariances for ocean grid boxes (due to moving ocean platforms), and land station measurements are assumed to be uncorrelated in space. For our analysis we further assume errors for both land and ocean grid boxes to be uncorrelated in time. Finally, in our analysis we ignore the error component from the bias correction uncertainty as it is found to be very small (unless, perhaps, at the very lowest frequencies). Thus, we use only the ensemble mean of the HadCRUT4 dataset.

b. Reanalysis: NOAA20CR version 3 (1850–2015)

We use the ensemble mean of the NOAA 20th Century Reanalysis version 3 dataset (NOAA20CR.v3 hereafter; see Slivinski et al. 2019), remapped to a global 5°×5°-grid to match the grid of the HadCRUT4 dataset. This allows to optionally impose the HadCRUT4 data gaps onto the reanalysis.

c. Climate models: CMIP6-historical (1850–2014)

We use an ensemble of 81 (= 27 models with 3 members each) CMIP6-historical climate model simulations (Eyring et al. 2016), from each of which the PSDs and ESDOFs are estimated. Data are remapped to a global $5°{\times}5°$-grid to match the grid of the HadCRUT4 dataset, which allows to optionally impose the HadCRUT4 data gaps onto the model data.

3. Method

For the HadCRUT4 dataset, as well as for the reanalysis and model simulations with HadCRUT4 gaps imposed, we need to estimate the mean local PSD, $S_{\mathrm{loc}}(f)$, and the PSD of the global mean, $S_{\mathrm{glb}}(f)$, from gappy data fields. Thus, the mean local PSDs cannot be simply computed from the Fourier transform at each gridpoint due to the data gaps. Likewise the PSD of the global mean cannot be simply computed from the time series of the global mean (although there are at least some grid boxes at each time step), because of the incomplete spatial coverage affecting the variance (or PSD) estimates. Often this issue is addressed by interpolating across the gaps in time and/or space. However, interpolation always introduces some artificial spatial/temporal correlation structure to the data which, in the present context, should be avoided as it would contaminate the spatio-temporal statistics to be characterized by the ESDOF measure. Furthermore, interpolation usually requires to discard some fraction of the data between which the gaps are too large for meaningfully applying interpolation.

Therefore, we apply an alternative approach to estimate the PSD from gappy data fields, which avoids any interpolation and, at the same time, uses all data points available in the dataset. The approach relies on computing the PSD as the Fourier transform of the temporal auto-covariance function. Details of the approach will be presented in a forthcoming paper by the authors that is currently in preparation. The approach yields unbiased PSD estimates, and the additional uncertainty (due to the data gaps) is still relatively small given the HadCRUT4 spatio-temporal coverage.

To obtain error-corrected PSDs for the HadCRUT4 dataset, we estimate the PSDs from the raw HadCRUT4 data fields and subtract from them the PSDs of the error components. These error PSDs are computed from the error covariance matrices (provided with the HadCRUT4 dataset) in a way that is fully consistent with our approach to estimate PSDs from gappy data fields, although the errors are specified as variances.

From the reanalysis and the model simulations with HadCRUT4 gaps imposed we compute the PSDs in exactly the same way as described above for HadCRUT4. We also compute the PSDs from the complete fields to investigate spatial non-stationarity effects of the data gaps. For the model simulations, we compute the PSDs separately for each ensemble member and then compute the ensemble mean PSD. From these ensemble mean PSDs we finally compute the frequency-dependent ESDOF measure.

When the analysis is restricted to land or ocean areas, mixed grid boxes (covering partly land and partly ocean) are excluded. Thus, the sum of the land- and ocean-only areas is less than the surface area of the entire globe.

4. Results

a. HadCRUT4 error-corrected vs. NOAA20CR.v3 and CMIP6-historical with gaps imposed **(Figures 1 and 2)**

- The PSD and ESDOF estimates are highly consistent among the three data sources.
- The largest discrepancy in terms of PSDs appears to be the stronger ENSO variability over tropical land areas in the CMIP6-historical simulations, compared to HadCRUT4 and NOAA20CR.v3.
- Since both the local and the global PSD over-estimate this variability by the same factor, the corresponding ESDOF spectra do not exhibit a discrepancy at ENSO timescales compared to the other data sources.
- The PSDs and ESDOFs exhibit a power-law frequency scaling. The scaling exponent of the ESDOF spectra is near 0.8. Only at subannual timescales the scaling is somewhat flatter (i.e., it has a smaller exponent).
- There is a pronounced reduction of ESDOFs towards lower frequencies. Specifically, there are about 100 ESDOFs at annual and only about 2 ESDOFs at multi-decadal timescales.

b. HadCRUT4 error-corrected vs. raw **(Figures 3 and 4)**

- From the PSDs it is obvious that the effect of the error correction is largest at the high frequencies because we assume temporally uncorrelated errors.
- The effect is much larger over ocean areas than over land areas, because measurement and sampling errors are larger for sea surface temperatures.
- Since errors from ocean grid boxes are spatially correlated, they also have an effect on the PSD of the global mean because they do not average out as much as the errors from land grid boxes.
- In terms of the ESDOF spectra, however, the error correction effect appears to be restricted mainly to tropical ocean areas.

c. NOAA20CR.v3 with vs. without gaps **(Figures 5 and 6)**

- Over tropical areas the data gaps do not have a notable impact on the PSDs and ESDOFs.
- Over extra-tropical ocean areas, the unobserved regions (i.e., the gaps) appear to have above-average variance at subannual timescales.
- But the mean local PSD changes by the same factor as does the PSD of the mean, indicating that the estimated spatial correlation structure is not altered by the data gaps. Accordingly, the ESDOF spectrum over extra-tropical ocean areas is not affected much by the data gaps.
- Over extra-tropical land areas, however, from subannual to decadal timescales the PSD of the mean is smaller without gaps whereas the mean local PSD is hardly affected, indicating that the variability in the unobserved regions is less correlated (in itself and/or with the observed regions), compared to the average spatial correlation in the observed regions.
- Accordingly, the ESDOFs are underestimated over extra-tropical land areas when the analysis is restricted to the observed regions.

d. CMIP6-historical with vs. without gaps **(Figures 7 and 8)**

- PSDs and ESDOF spectra are much smoother because they are based on ensemble means, compared to the HadCRUT4 and NOAA20CR.v3 spectra.
- The effect of the data gaps on the model simulations is similar to the effect on the reanalysis.
- However, the above-average variance in the unobserved regions over extra-tropical ocean areas, already found for the reanalysis at subannual timescales, extends across all timescales in the model simulations, but only in terms of the mean local PSD. This indicates that the variability in the unobserved regions is less correlated (in itself and/or with the observed regions), compared to the average spatial correlation in the observed regions.
- Accordingly, the ESDOFs are underestimated over extra-tropical ocean areas from annual to multi-decadal timescales when the analysis is restricted to the observed regions.

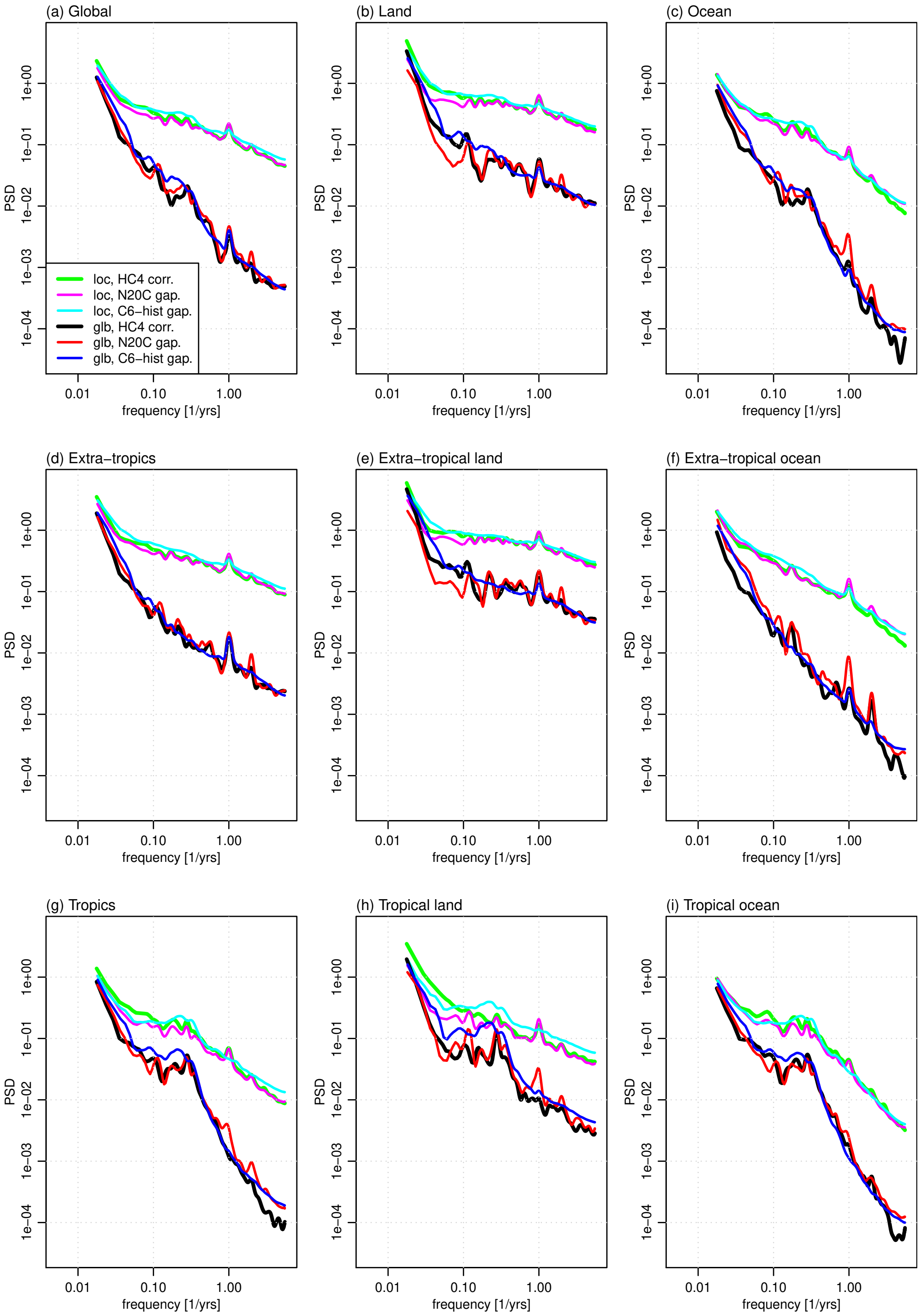


FIG. 1. Mean local PSD and PSD of the mean, for the entire globe and for various spatial subdomains; for HadCRUT4 error-corrected, and for NOAA20CR.v3 and CMIP6-historical with HadCRUT4 gaps imposed.

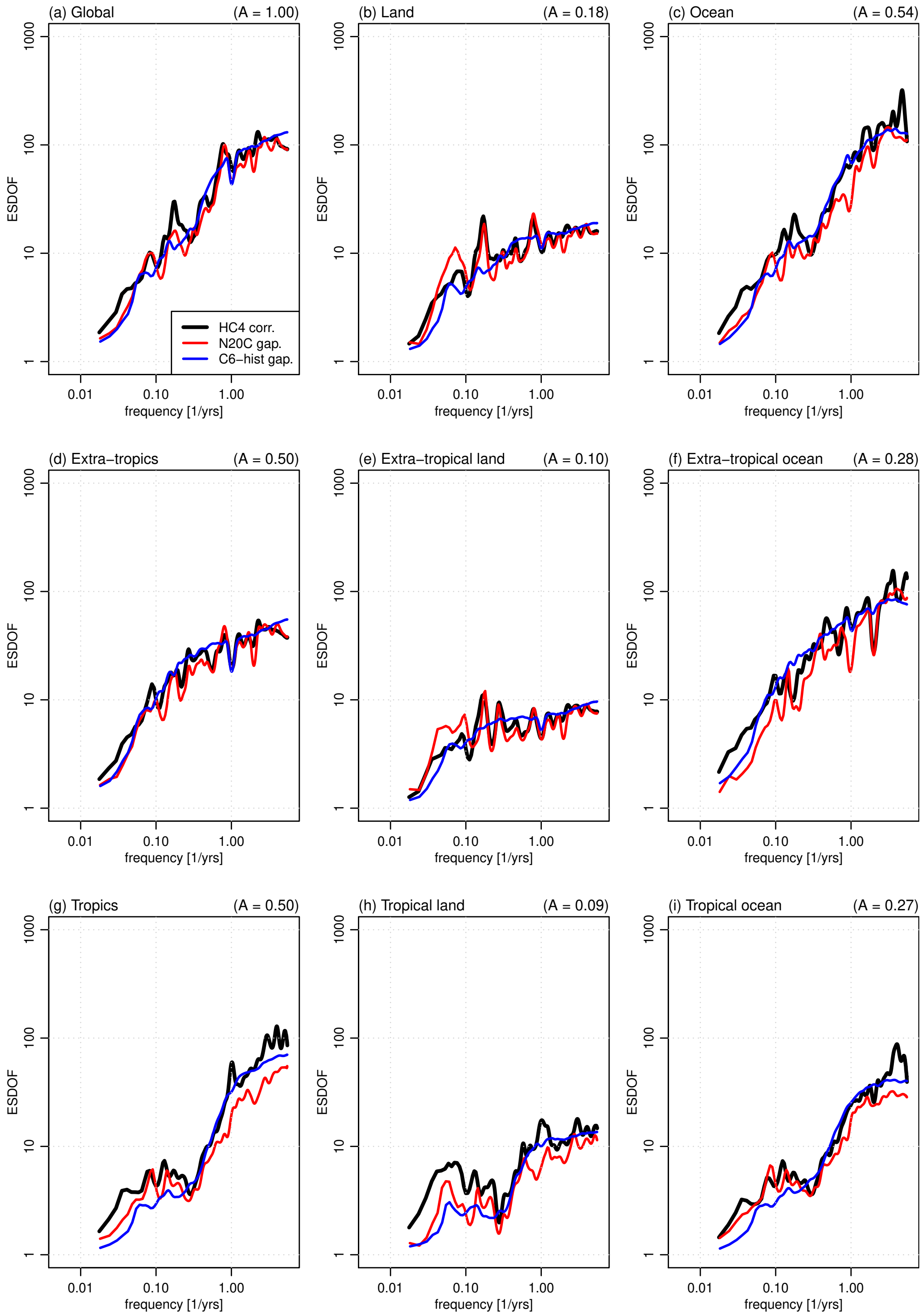


FIG. 2. ESDOF spectra for the entire globe and for various spatial subdomains; for HadCRUT4 error-corrected, and for NOAA20CR.v3 and CMIP6-historical with HadCRUT4 gaps imposed. The area fraction A of each domain, relative to the entire globe, is indicated at the top of each panel. Note: The smaller the area fraction A, the smaller the ESDOF value given the same spatial scale of the fluctuations.

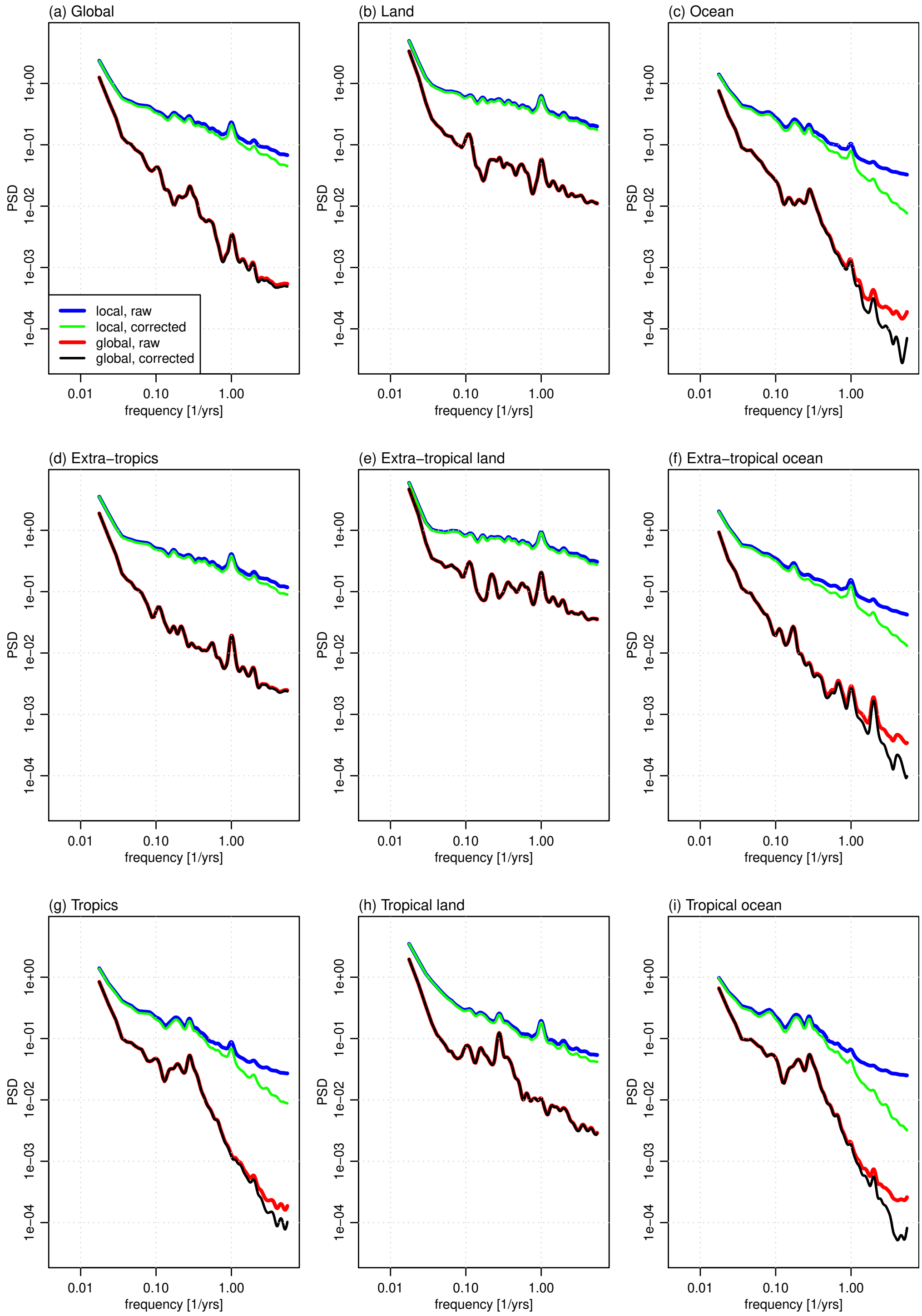


FIG. 3. As Fig. 1, but for HadCRUT4 with and without error correction.

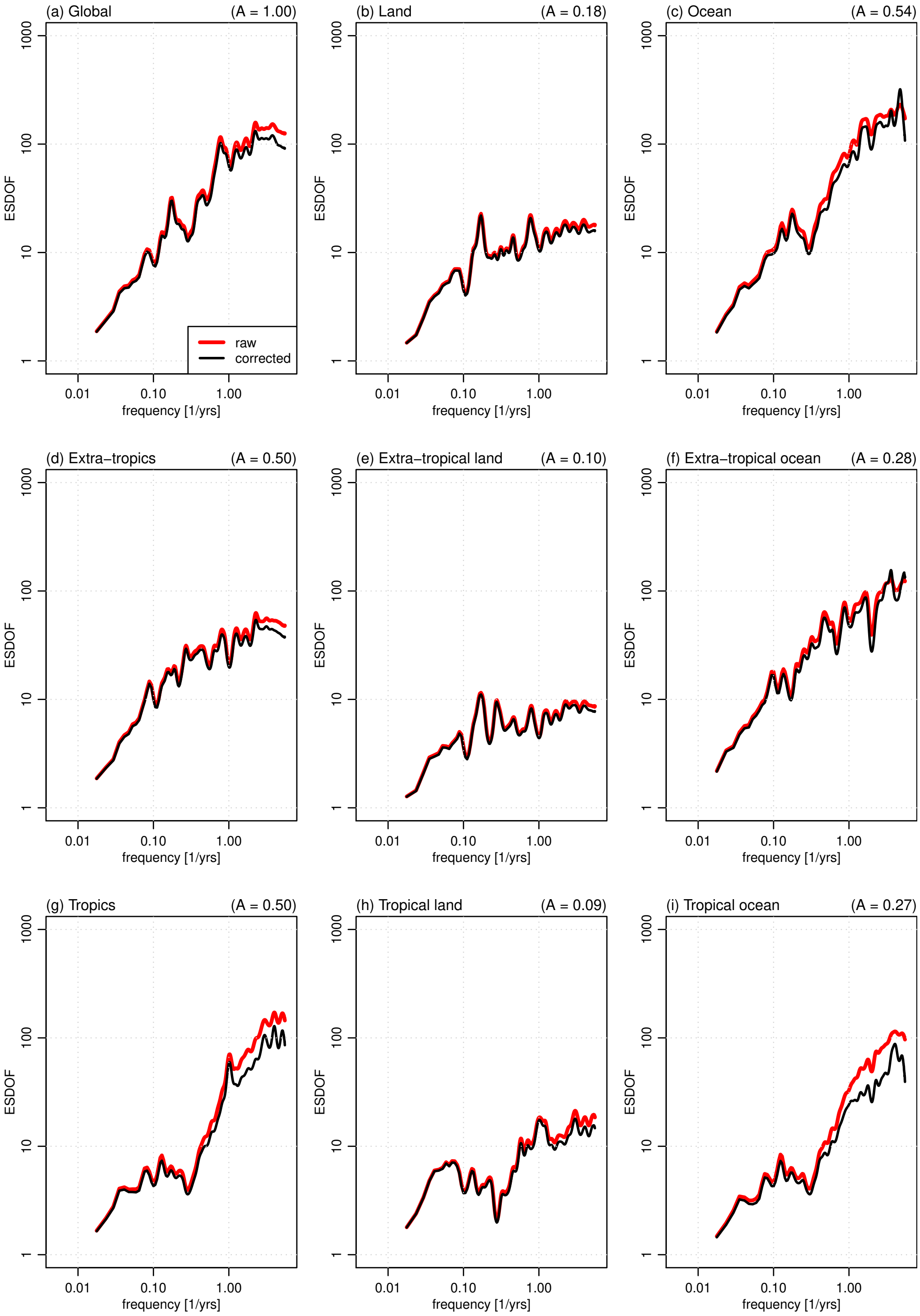


FIG. 4. As Fig. 2, but for HadCRUT4 with and without error correction.

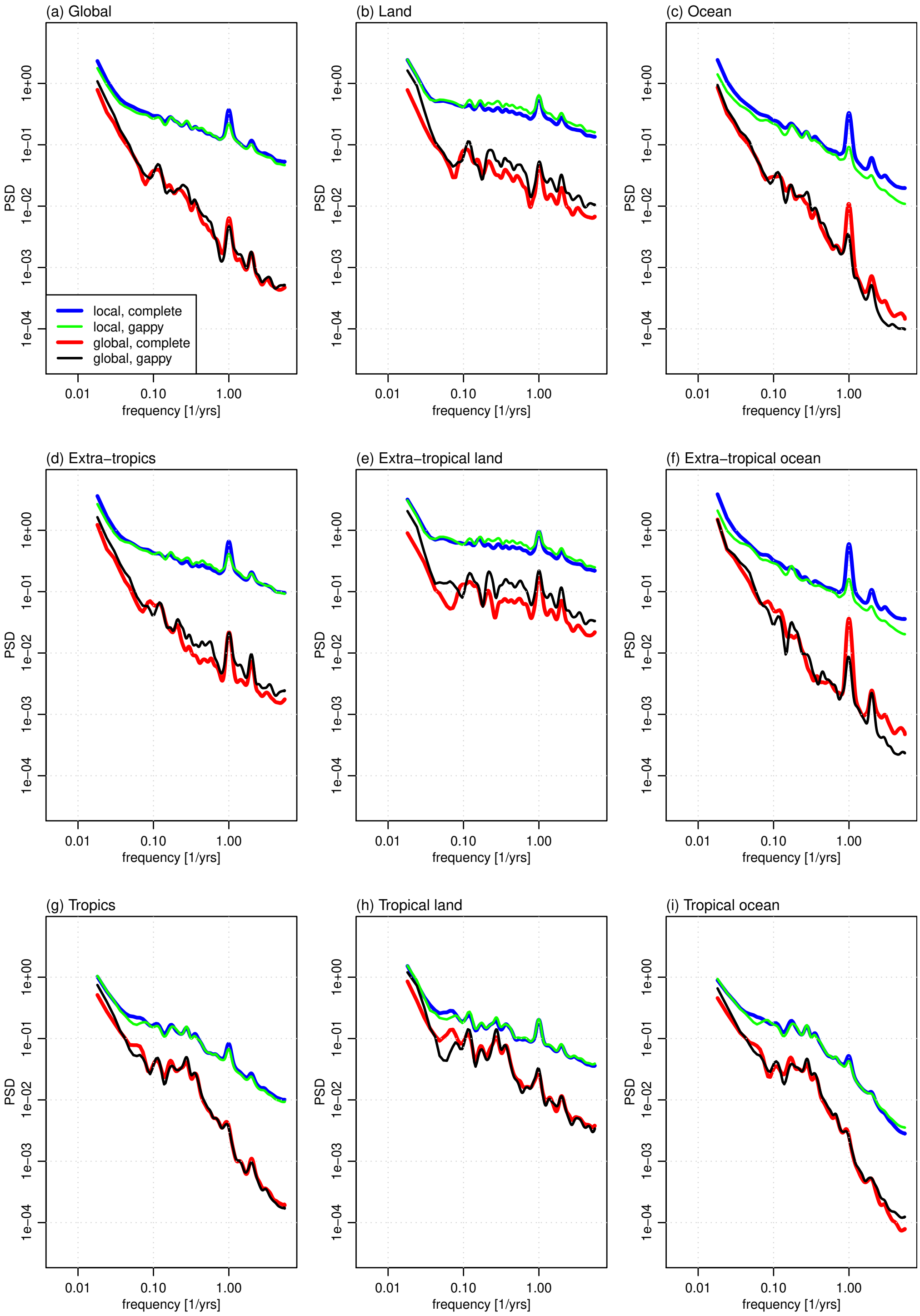


FIG. 5. As Fig. 1, but for NOAA20CR.v3 with and without HadCRUT4 gaps imposed.

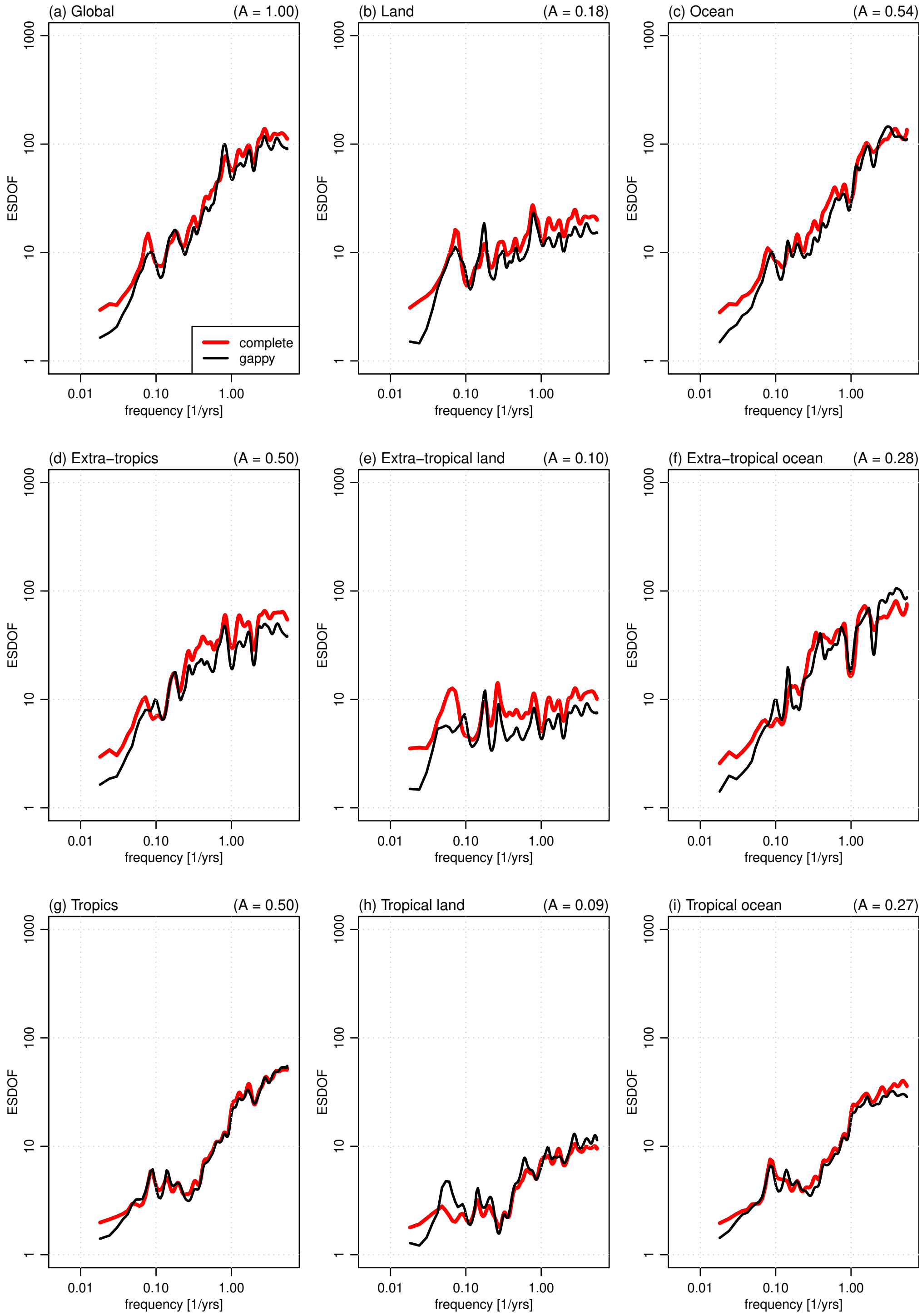


FIG. 6. As Fig. 2, but for NOAA20CR.v3 with and without HadCRUT4 gaps imposed.

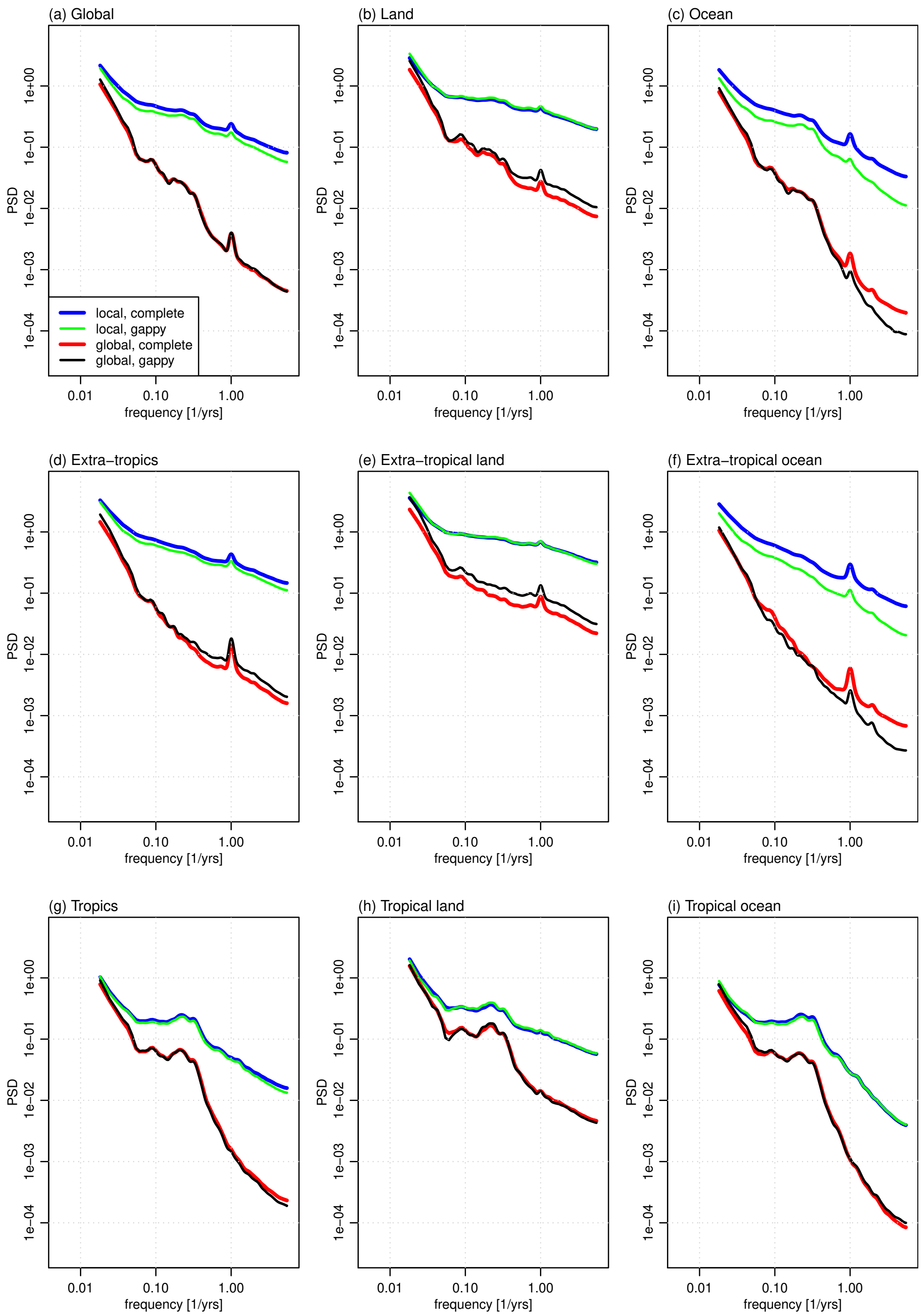


FIG. 7. As Fig. 1, but for CMIP6-historical with and without HadCRUT4 gaps imposed.

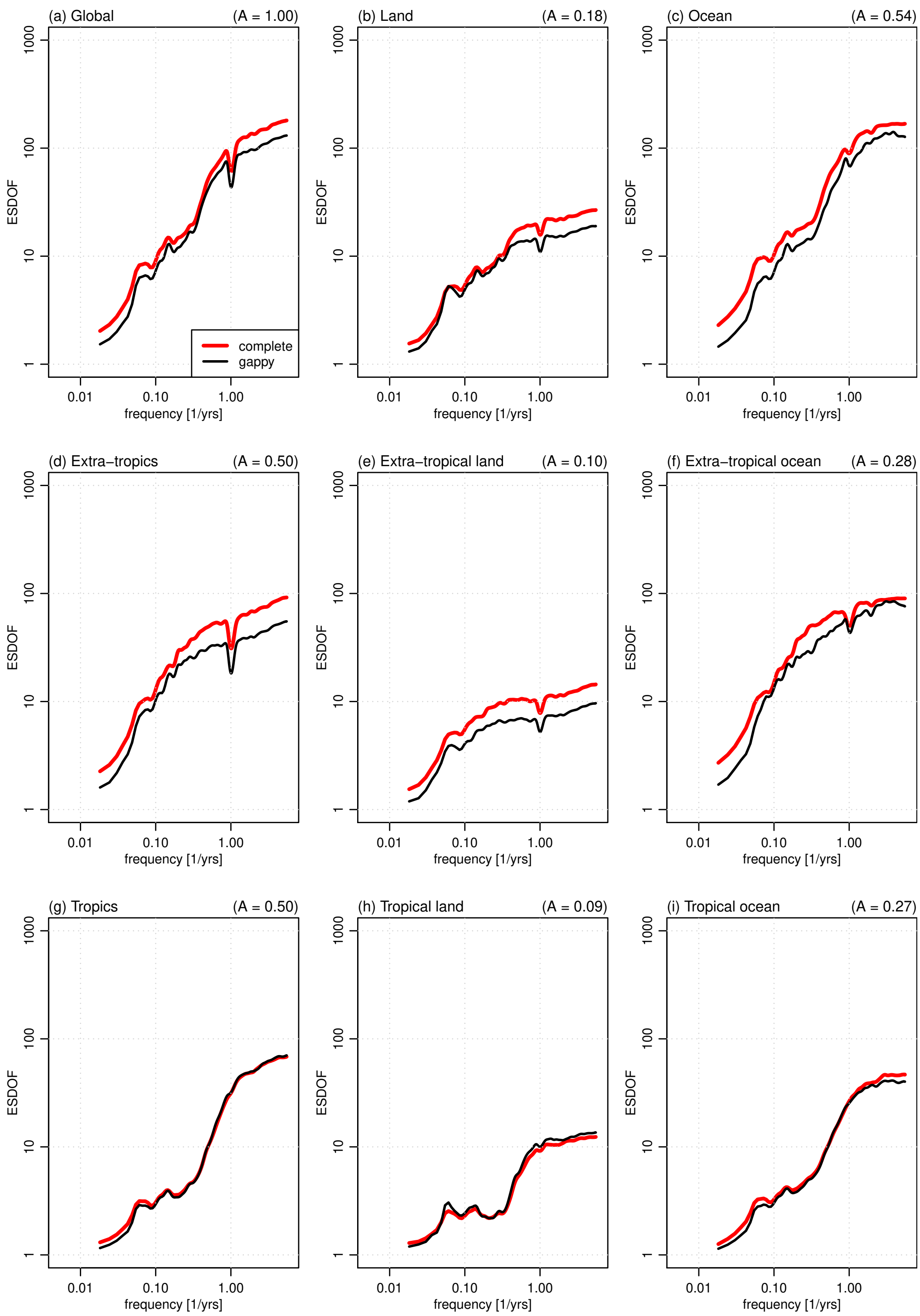


FIG. 8. As Fig. 2, but for CMIP6-historical with and without HadCRUT4 gaps imposed.

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