
On the multiscale fractal features of a low-order coupled ocean-atmosphere model in comparison with reanalysis data

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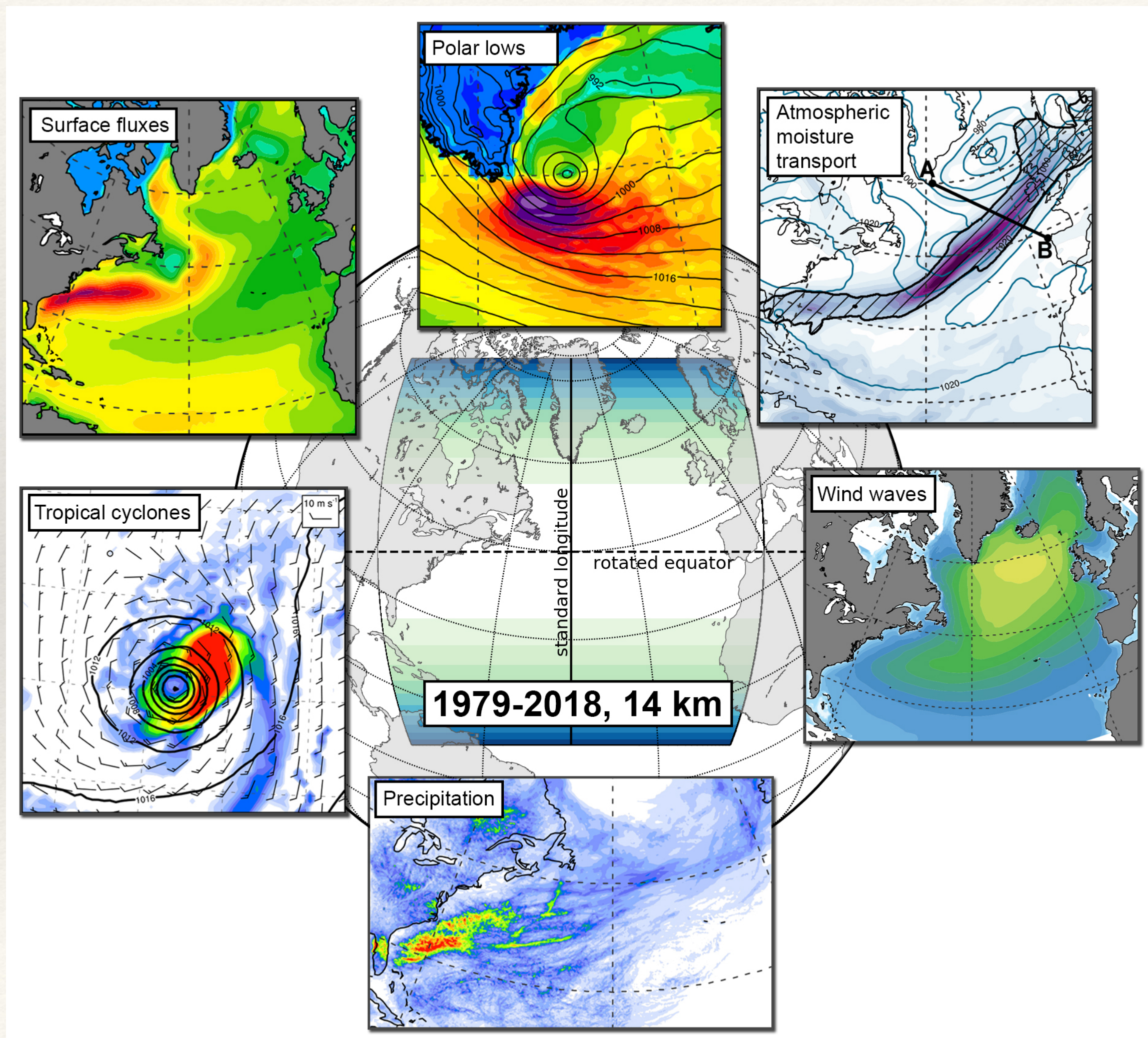
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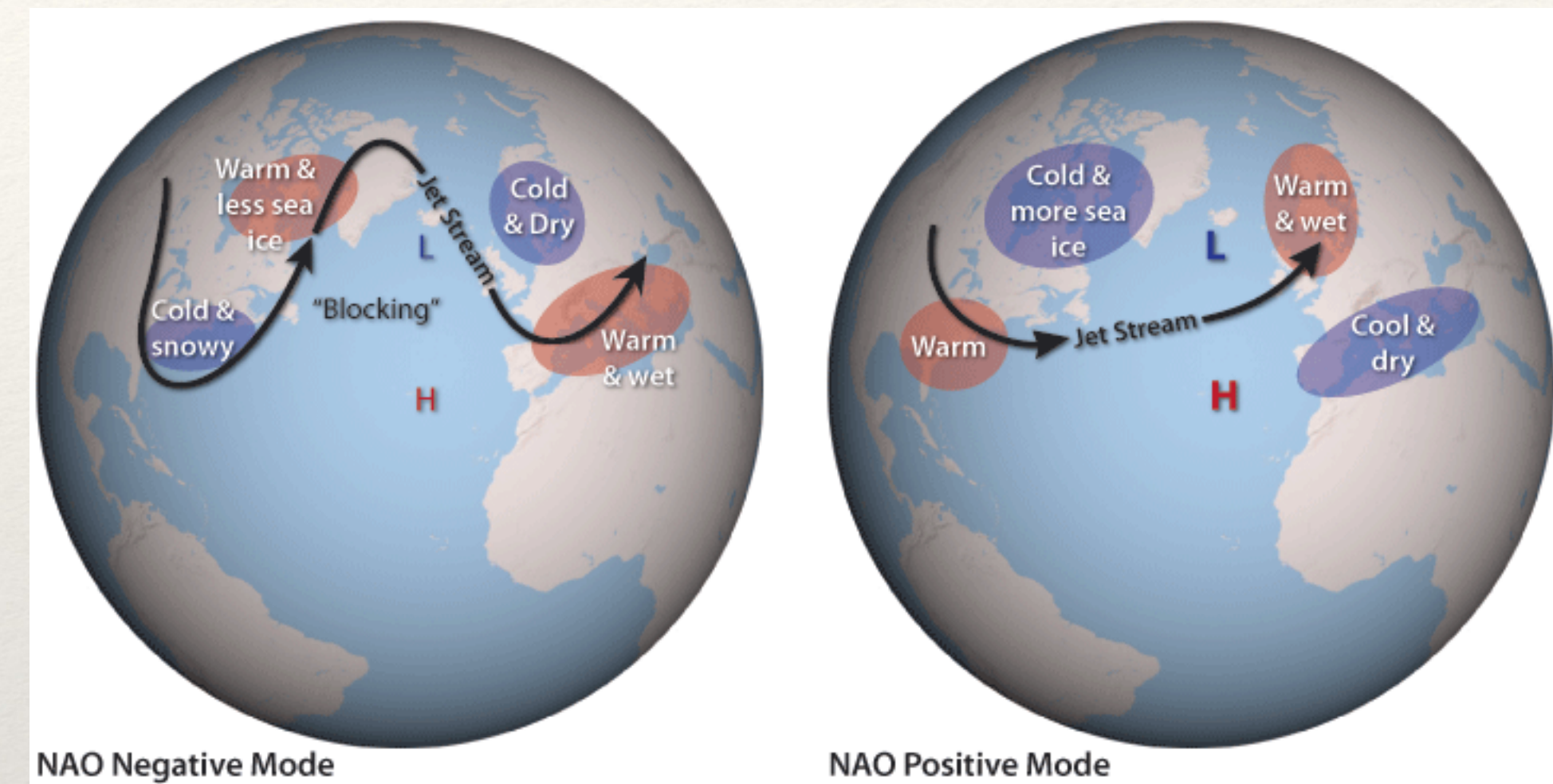


The ocean-atmosphere complex system

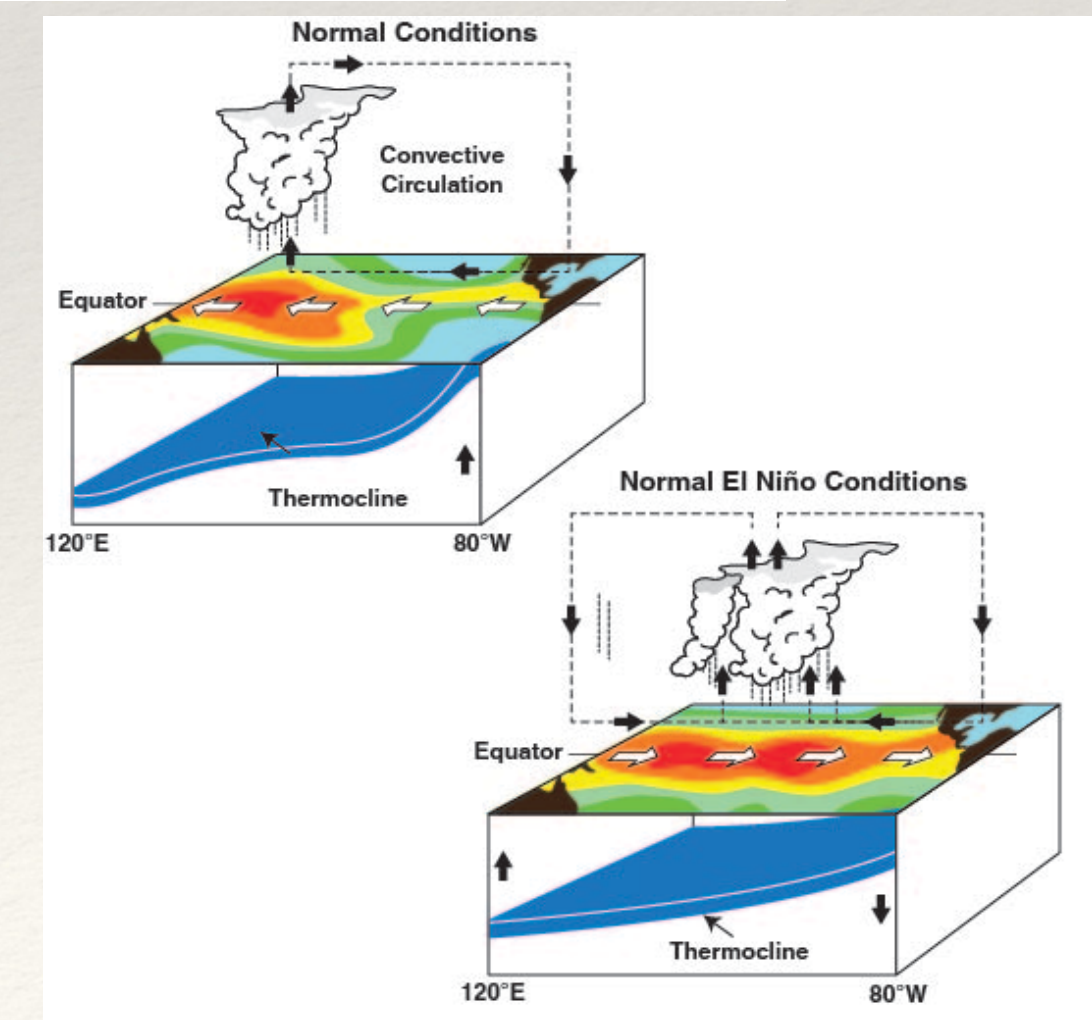
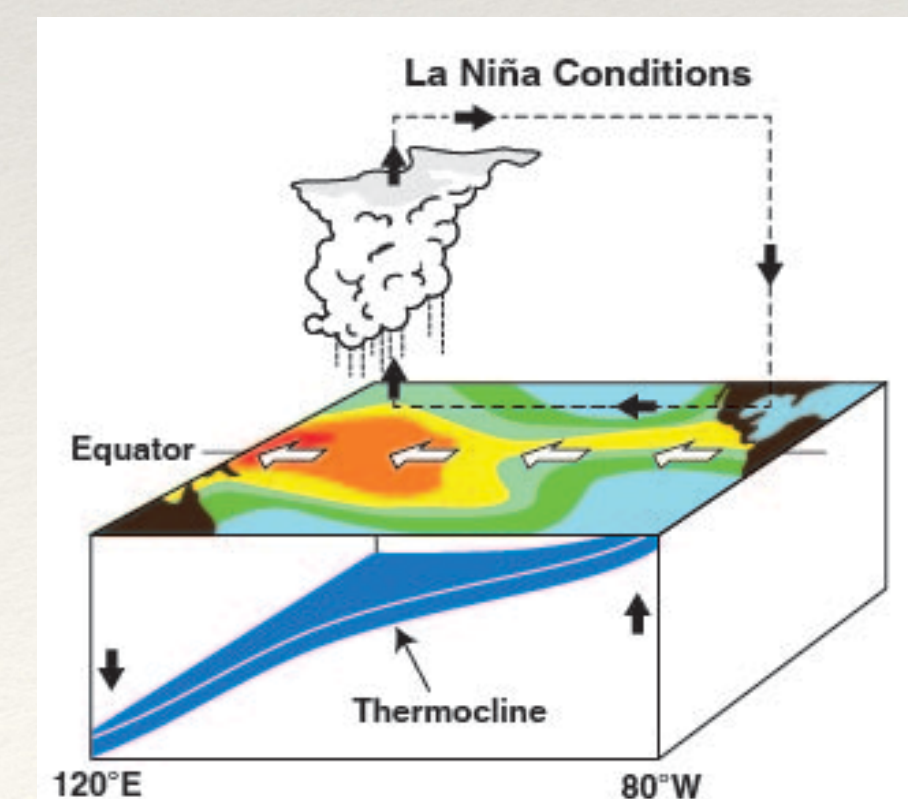


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The reduced order ocean-atmosphere model

- ❖ Reduced order coupled ocean-atmosphere models are key tools in the hierarchy of climate models
- ❖ a suitable reduced order model of the coupled ocean-atmosphere system has been developed starting from the quasi-geostrophic equations describing the interaction between a two-layer atmosphere and a one-layer ocean over an infinitely deep quiescent ocean layer
- ❖ By expanding the solutions of these equations into Fourier series, and by truncating them at low wavenumbers, and by projecting onto the Fourier modes retained, a set of ordinary differential equations is derived.
- ❖ Four relevant fields, i.e., the barotropic and baroclinic atmospheric streamfunctions, the ocean streamfunction and the ocean temperature, are given by $\psi_a = \sum_{i=1}^{10} \psi_{a,i} F_i$, $\theta_a = \sum_{i=1}^{10} \theta_{a,i} F_i$, $\Psi_o = \sum_{i=1}^8 \Psi_{o,i} \phi_i$, $T_o = \sum_{i=1}^8 T_{o,i} \phi_i$ where F_i and ϕ_i are simplified notations for the sets of modes (see Vannitsem, 2017).
- ❖ Depending on the choice of the surface friction coefficient C , different solutions are found with a highly chaotic dynamics without marked LFV in the atmosphere for small values of C , but a more moderately chaotic dynamics with stronger LFV in both the ocean and the atmosphere (related to the development of a coupled mode) for larger values of C .

Scale-dependent dimensions $D_{q,\tau}$

- ❖ A complex system (like a turbulent flow) is formed by many interacting components at different scales
- ❖ Assume for the sake of simplicity to have a time dependent signal

$$x(t) = \langle x(t) \rangle + \delta x_{\tau_1}(t) + \dots + \delta x_{\tau_k}(t) = x_0 + \sum_{\tau} \delta x_{\tau}(t)$$

- ❖ let be $\mathcal{M}_{\tau}$ the phase-space representative of the system for all scales $\tau' < \tau$ and let $\mathcal{S}_{\tau}$ a partition of $\mathcal{M}_{\tau}$ into $\mathcal{B}_{i,\tau}$ hypercubes of size r
- ❖ let now define a normalized measure

$$\mathcal{P}_{i,\tau} = \frac{\mu(\mathcal{B}_{i,\tau})}{\mu(\mathcal{M}_{\tau})}$$

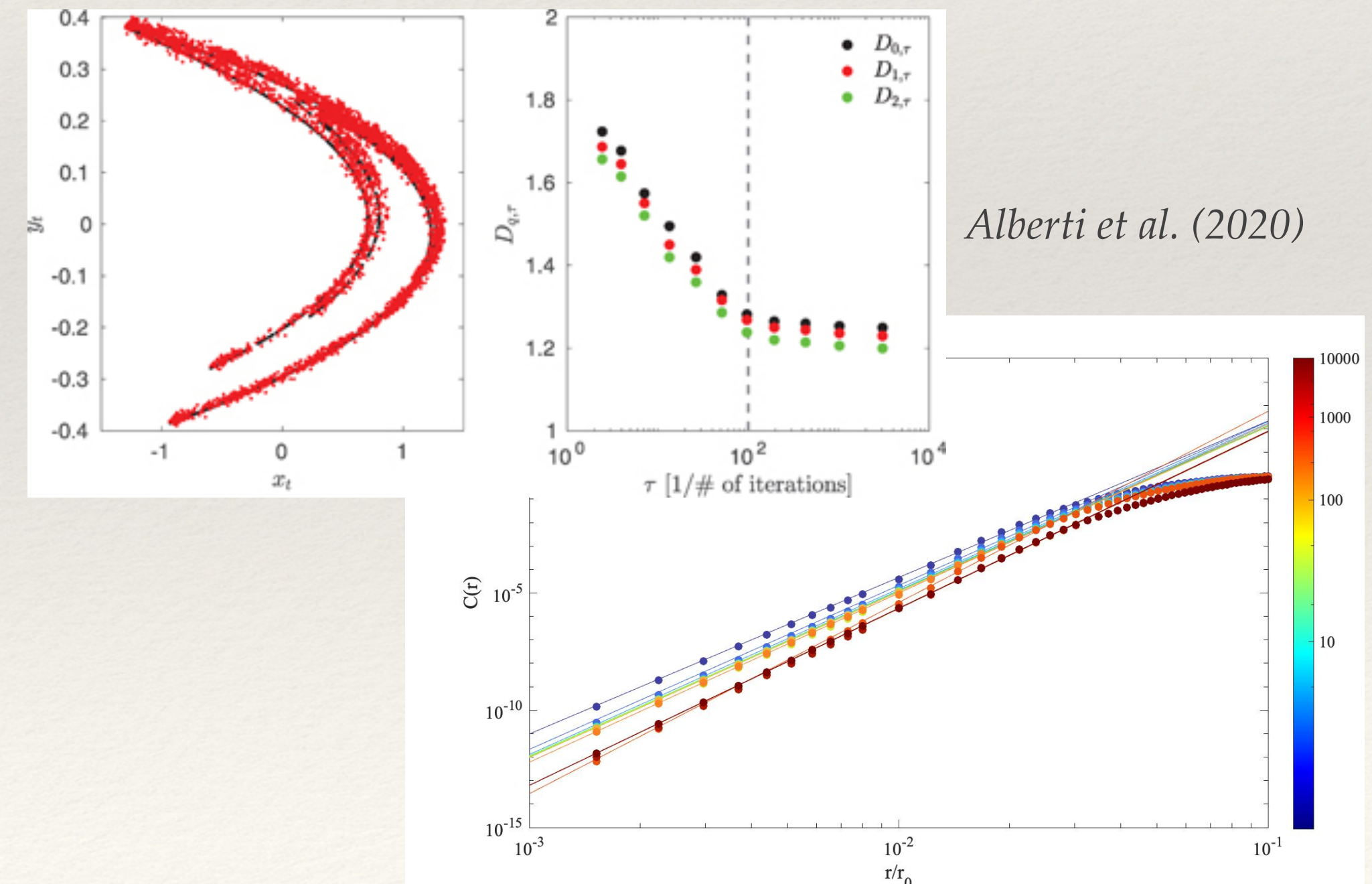
- ❖ if we define a partition function

$$\chi(r, q, \tau) = \sum_i \mathcal{P}_{i,\tau}^q$$

- ❖ it will scale as

$$\chi(r, q, \tau) \propto r^{(q-1)D_{q,\tau}}$$

- ❖ where $D_{q,\tau}$ are measures of the scale-dependent complexity of a system



Alberti et al. (2020)

The Multivariate Empirical Mode Decomposition (MEMD)

- ❖ MEMD directly works on the data domain with the aim of being as adaptive as possible to minimize mathematical assumptions and definitions (Huang et al., 1998) in extracting scale-dependent structures (Rehman and Mandic, 2010)
- ❖ Given a multivariate signal $\mathbf{s}(t) = \{s_1(t), s_2(t), \dots, s_n(t)\}$ via the MEMD we can write

$$\mathbf{s}(t) = \sum_j \mathbf{c}_j(t) + \mathbf{r}(t) \quad (1)$$

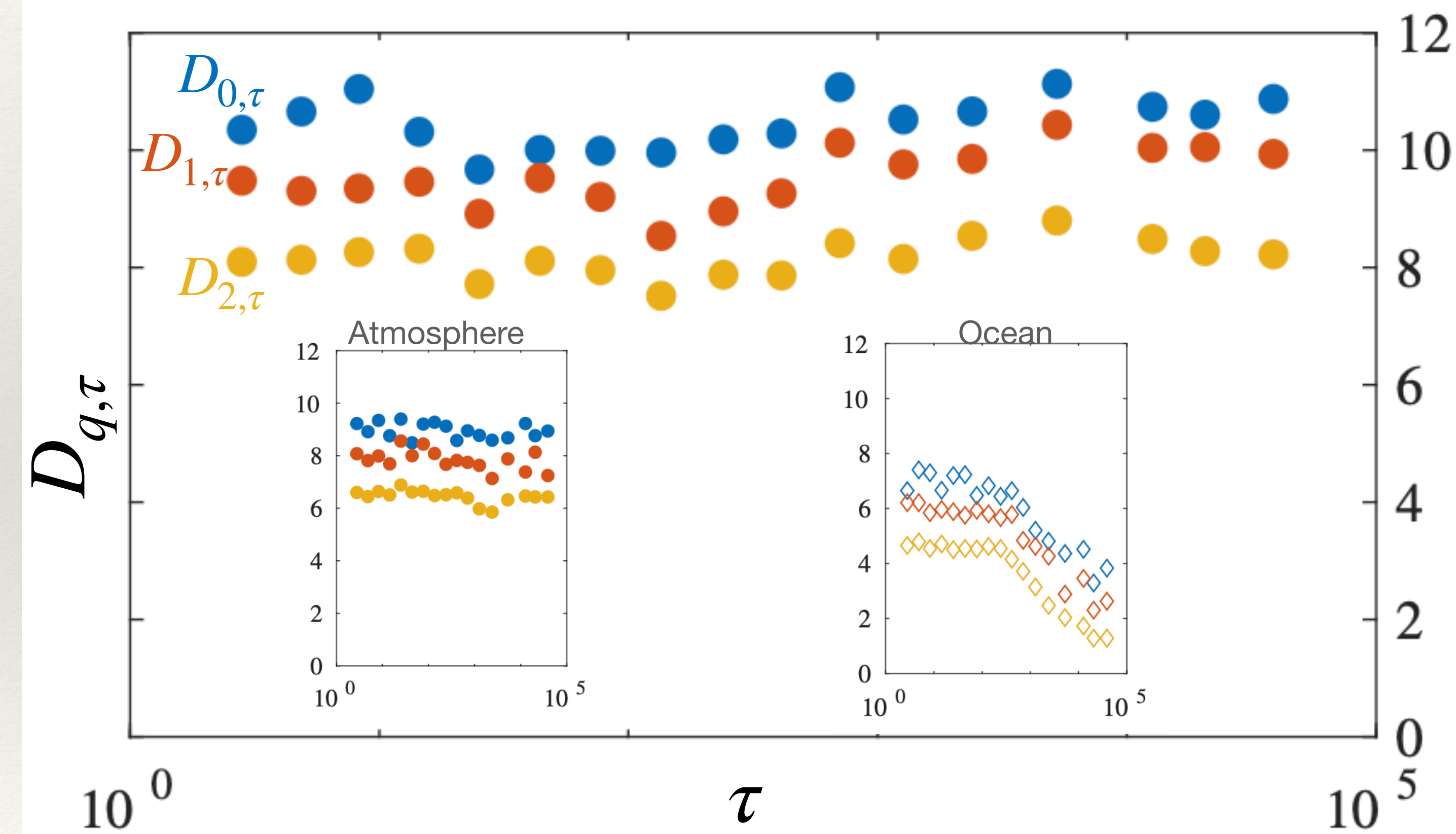
- ❖ where $\mathbf{c}_j(t)$ are the so-called Multivariate Intrinsic Mode Functions (MIMFs) and $\mathbf{r}(t)$ is a non-oscillating pattern

- ❖ For each MIMF we can define a multivariate mean timescale as $\tau_j = \frac{1}{T} \int_T t' \langle \mathbf{c}_j(t') \rangle dt'$

- ❖ Since MIMFs form a complete and orthonormal basis the dynamics at scale $\tau' < \tau$ can be easily exploited by partial sum of Eq. (1) as

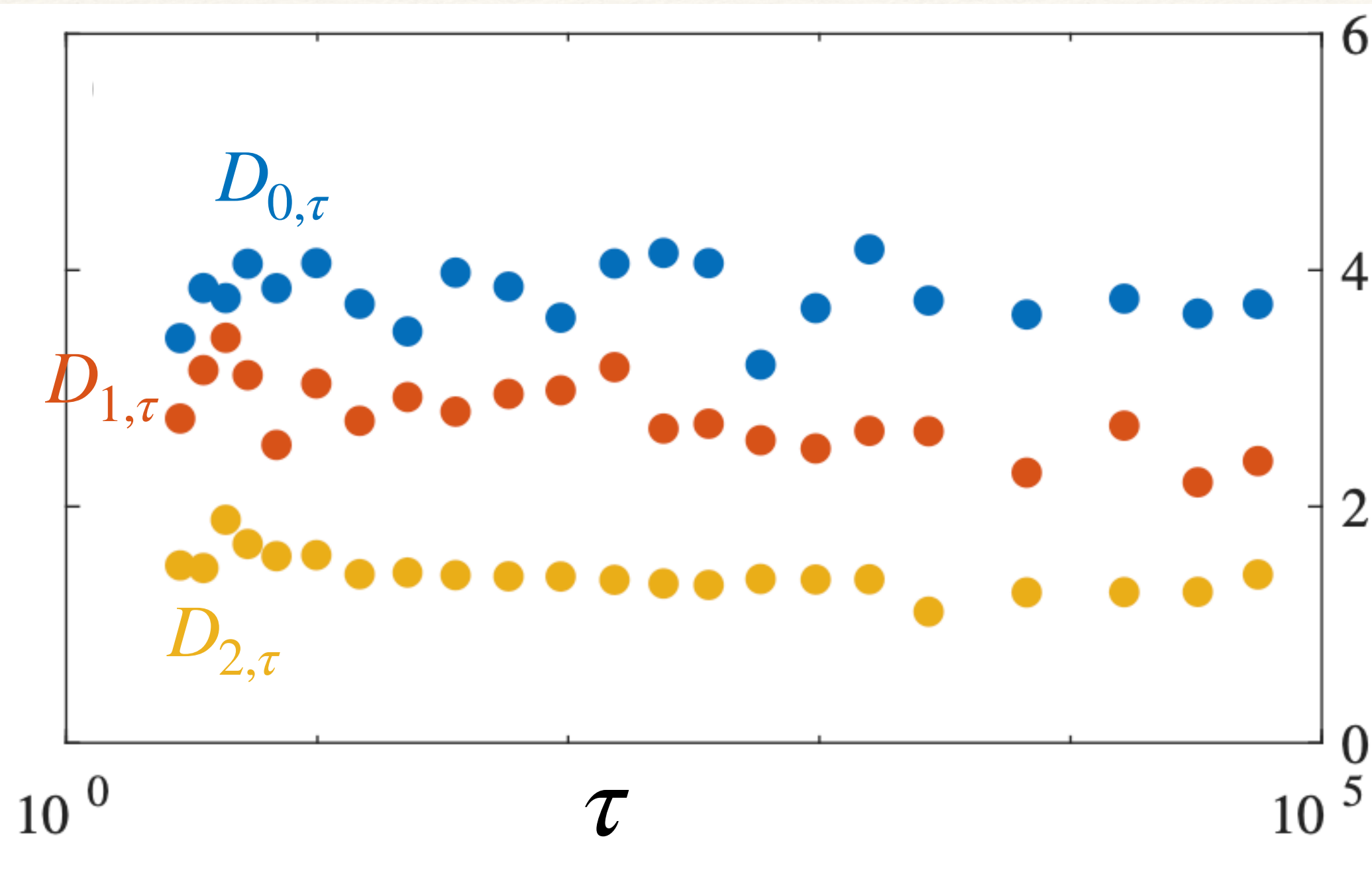
$$\delta \mathbf{s}_\tau(t) = \sum_{j' < j} \mathbf{c}_{j'}(t) \quad \text{s.t. } \tau_{j'} < \tau$$

Scale-dependent dimensions $D_{q,\tau}$ for $C=0.008$



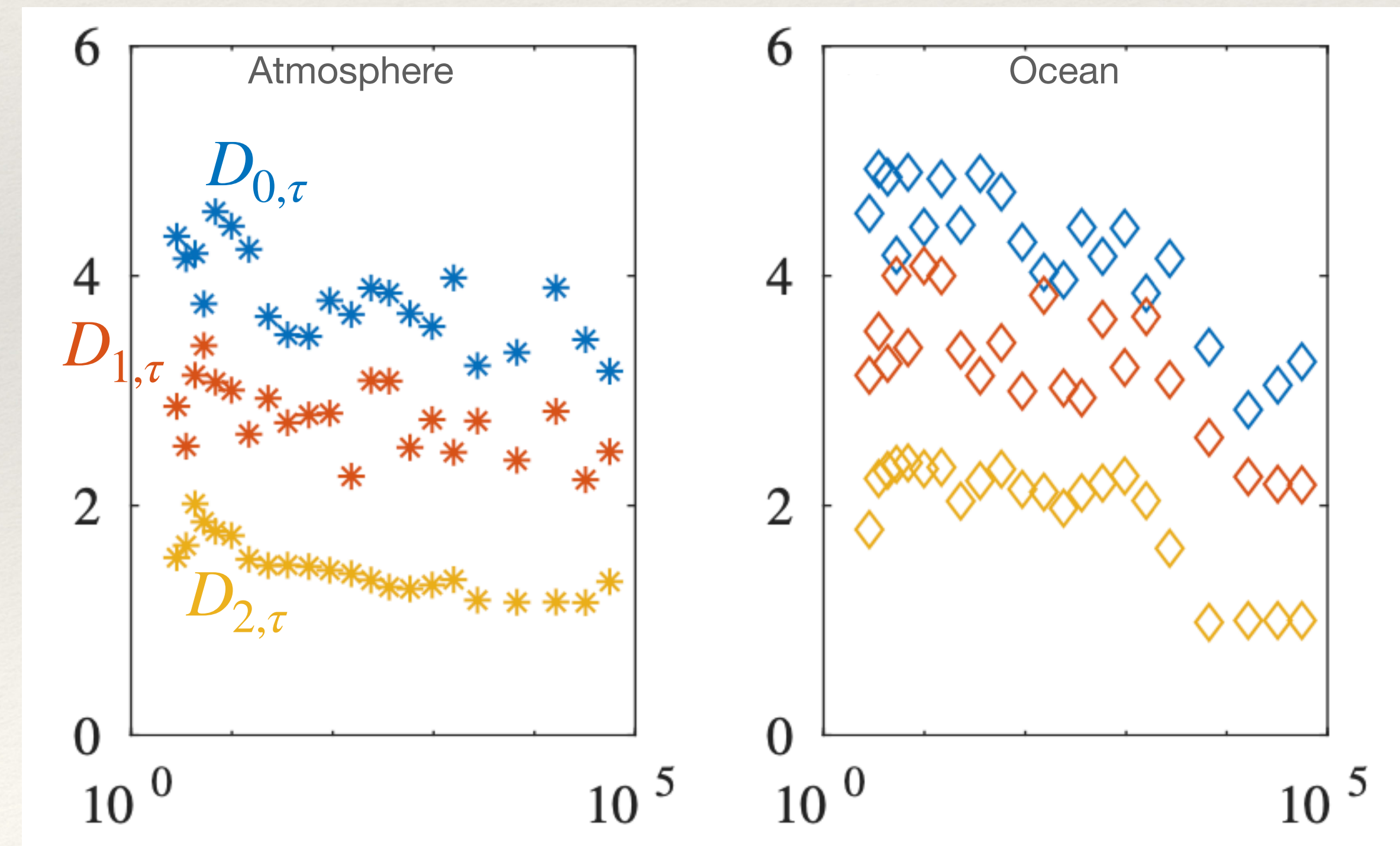
- ❖ The overall dynamics of the system is characterized by a scale-independent behavior of $D_{q,\tau}$ with typical values of the order of 8-11
- ❖ The atmosphere is characterized by scale-independent $D_{q,\tau}$ being representative of a high-dimensional system whose prime dynamics occurs at short timescales and with little effects of large-scale processes on the collective dynamics of the atmosphere
- ❖ by contrast, a clearly scale-dependent behavior is found for the oceanic variables, with the multiscale generalized dimensions decreasing at larger timescales, reflecting the effects of large-scale dynamics dominating with respect to the short-term one for the ocean variability.

Scale-dependent dimensions $D_{q,\tau}$ for $C=0.015$



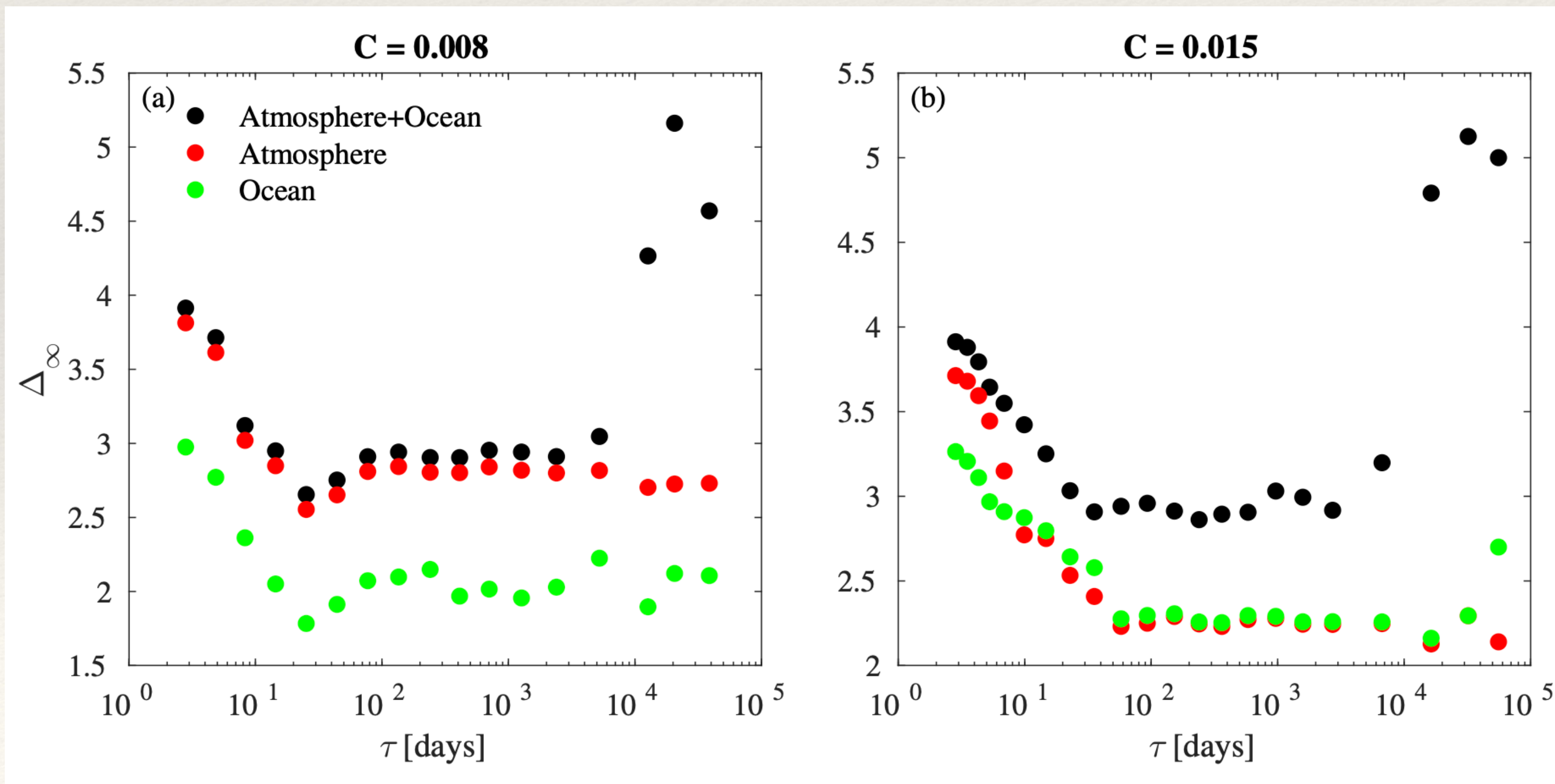
- ❖ The overall dynamics of the system is characterized by a scale-independent behavior of $D_{q,\tau}$ with typical values of the order of 2-4
- ❖ The reduced values of $D_{q,\tau}$ reflect the role of the friction coefficient C

- ❖ The atmosphere is characterized by scale-independent $D_{q,\tau}$ while they are scale-dependent for the ocean
- ❖ However, both the atmosphere and the ocean are now characterized by similar values of $D_{q,\tau}$ reflecting the different topological properties of the attractor as C is varied



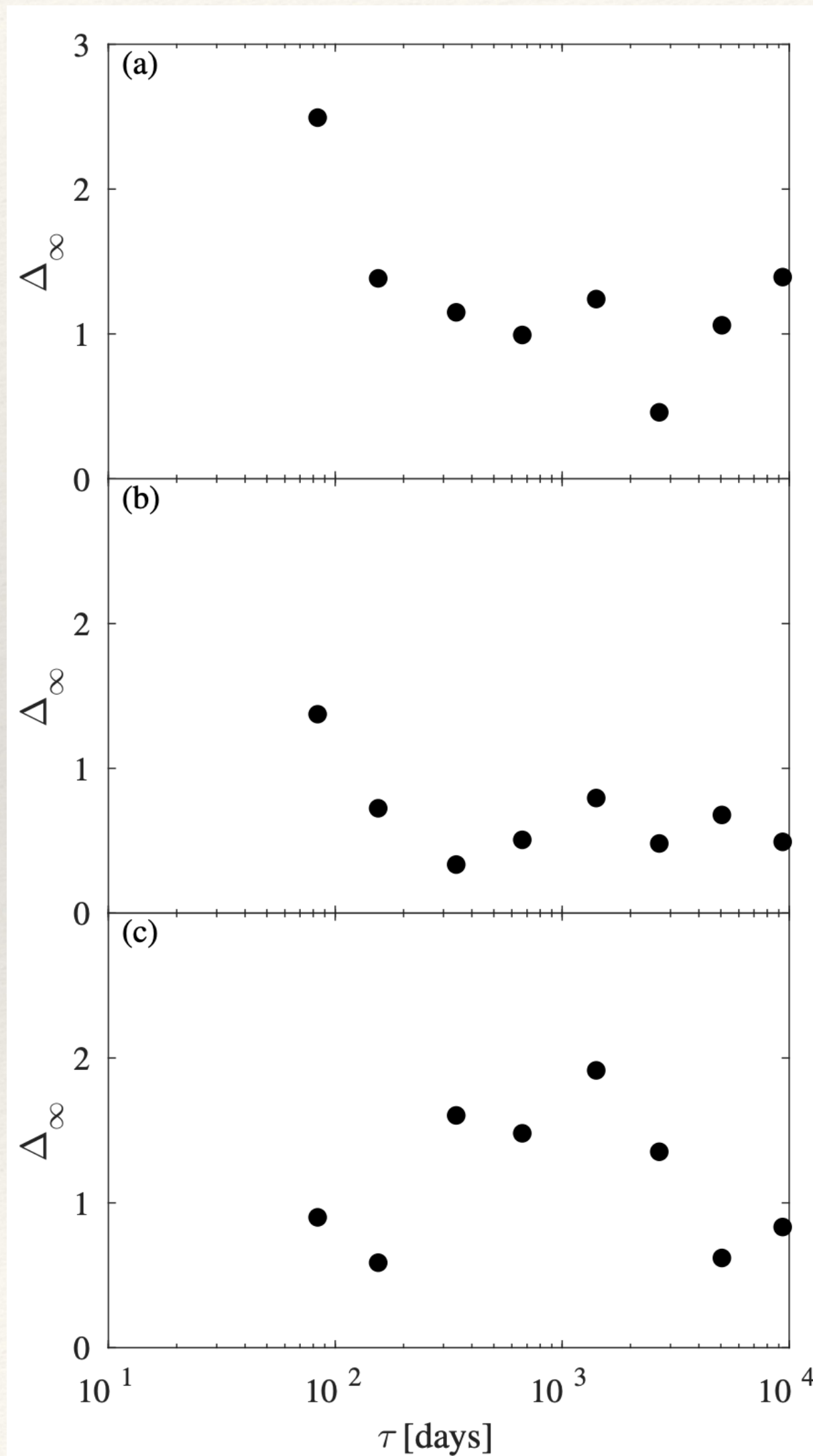
Multifractal features

- ❖ We can characterize how (multi)fractal properties of the system evolve with the timescale τ by evaluating the full spectrum of generalized dimensions and provide an estimate of $D_{\pm\infty,\tau}$
- ❖ Thus, we define the multifractal width as $\Delta_\infty \doteq D_{-\infty,\tau} - D_{\infty,\tau}$



- ❖ $\Delta_\infty = 3$ for $\tau \in [\tau_S, \tau_L]$ days, while $\Delta_\infty > 3$ for both $\tau < \tau_S$ and $\tau > \tau_L$, with $\tau_S \sim 20$ days and $\tau_L \sim 1$ year
- ❖ by evaluating the difference Δ_∞ separately for the atmosphere and the ocean we see that larger values, of the order of 4, are found for the atmosphere, at almost all timescales (and especially at shorter timescales), for both values of C
- ❖ This clearly suggests that the presence of strong multifractality in the full system can be essentially attributed to the atmosphere, with only a marginal role of the ocean variability in determining the fractal structure of the full system.

Comparison with reanalysis data



- ❖ we have compared our results for the reduced order coupled ocean-atmosphere model with those derived from reanalysis data (Poli, 2015) by using three sets of different regional time series from the ORA-20C project
- ❖ (a) North Atlantic, (b) North Pacific, and (c) Tropical Pacific
- ❖ both the North Atlantic and the North Pacific regions (a, b) are characterized by a decreasing Δ_∞ as τ increases, being representative of a transition from a short-term multifractal nature to long-term monofractal one
- ❖ Conversely, by looking at the Tropical Pacific region (c) we clearly see an enhancement of Δ_∞ , i.e., the emergence of multifractal features at annual/multi-annual timescales (i.e., $\tau \sim 1 - 8$ years)
- ❖ This could be related to the role of the El Niño–Southern Oscillation (ENSO) cycle manifesting at these timescales (between 2 and 7 years), which is likely responsible for the different scale-dependent behavior as compared to the two other extratropical regions.

Conclusions

- ❖ We have provided a first time systematic investigation of the multiscale dynamics of a reduced order coupled ocean-atmosphere model (Vannitsem et al., 2015) as described by means of the topological and geometric features (Alberti et al., 2020).
- ❖ For weak ocean-atmosphere coupling, the resulting dimensions of the two model components are very different, while for strong coupling the scaling properties are more similar especially at longer time scales.
- ❖ For both considered values of the friction coefficient C , the full system exhibits signatures of multifractality at all timescales, especially pronounced at short and long as compared to intermediate timescales.
- ❖ We have clearly evidenced that for the atmosphere, there is a scale-independent behavior for all q , rendering the multifractal spectra almost invariant with respect to the timescale. By contrast, a scale-dependent behavior emerges for the ocean for the lower value of C . This clearly suggests that the presence of strong multifractality in the full system can be attributed to the atmosphere, with only a marginal role of the ocean variability in determining the fractal structure of the full system.
- ❖ By using reanalysis data we observed that the multiscale multifractal features of different regions show different scale-dependent behaviors: both the North Atlantic and the North Pacific regions are characterized by a scale-dependent behavior, with a transition from a short-term multifractal nature to long-term monofractal one, while the Tropical Pacific region is characterized by the emergence of multifractal features at annual / multi-annual timescales (i.e., $\tau \sim 1 - 8$ years), as a manifestation of the El Niño–Southern Oscillation (ENSO) cycle.

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