Upscaling root water uptake equations despite heterogeneous soil moisture at the soil-root interface

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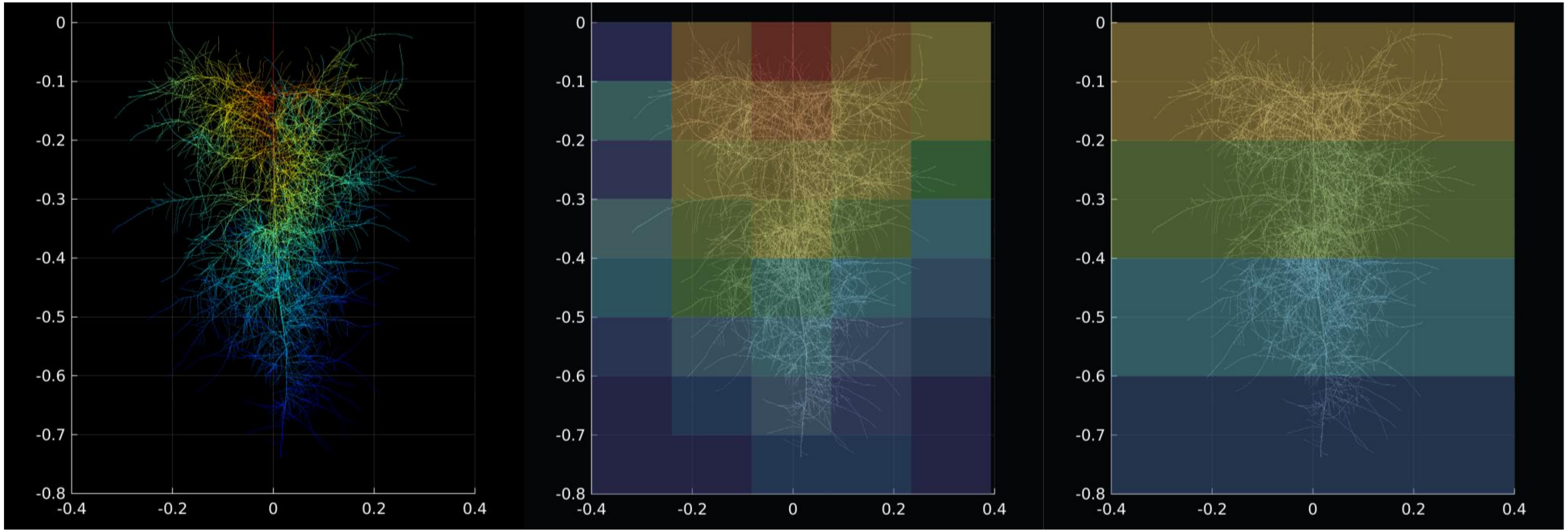
OP Výzkum a vývoj pro inovace

Upscaling root water uptake: what is it & why do it?



Full 3D solution of root water flow equations:

- high accuracy
- high computational cost

Pictured: root water potentials in *Pisum sativum* (green pea) generated with CPlantBox.

Upscaling: finding water flow equations valid over discrete blocks of soil.

- reduced computational cost ✅
- reduced accuracy ⊗

(how can we avoid this ?)

Goal: equations valid for discrete soil layers at resolutions commonly used in Earth System Models while minimising prediction error.

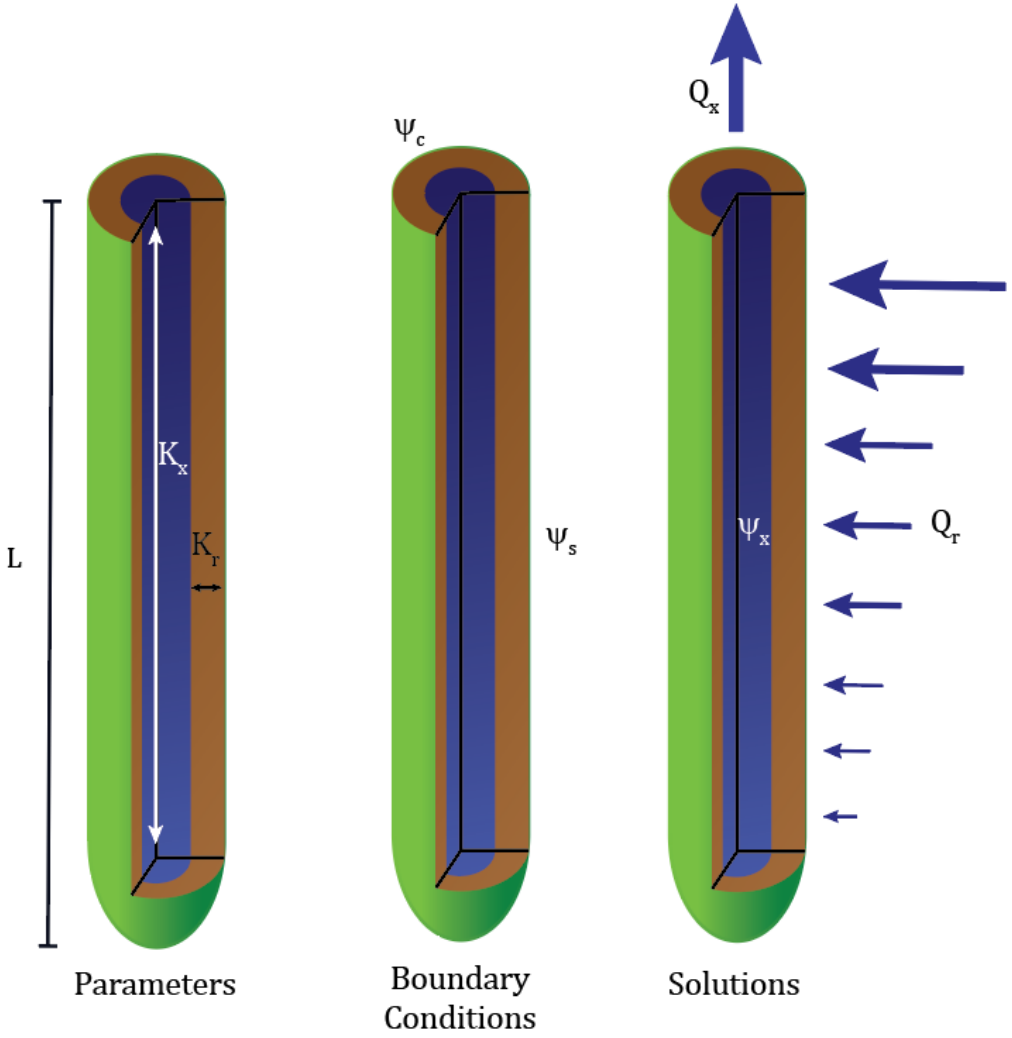
Motivation:
Effect of soil moisture on plants explains:

90% of annual variation in the terrestrial carbon sink
Humphrey et al. (2021) *Nature* **592**(7852):65.

80% uncertainty of the terrestrial carbon sink across current Earth System Models
Trugman et al. (2018) *Geophysical Research Letters* **45**(13):6495

Plant water uptake is a major player in terrestrial climate system.

Approach (1): porous pipe equation



$$\frac{\partial^2 \psi_x}{\partial s^2} = -\frac{K_r}{K_x}(\psi_x - \psi_s)$$

At root scale:
Parameters:
Root conductances K_r & K_x
Root length L; $0 \leq s \leq L$

Boundary conditions:
Soil water potential ψ_s
Root collar potential ψ_c or flow rate Q_c

Solve for:
root water potential ψ_x
then calculate flows (Q_r, Q_x)

Classically, this equation is linearised before solving by assuming discrete segments of roots, obtaining Ohm's law analogue.

But **continuous solutions** known on single root

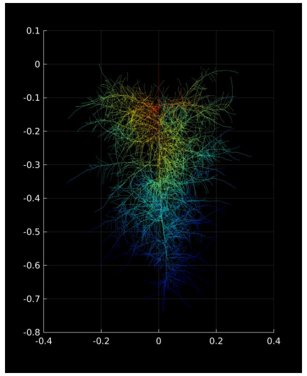
Landsberg & Fowkes (1978) *Annals of Botany* **42**:493–508

and **on full root system**

Bouda & Saiers (2017) *Advances inWater Resources* **110**: 319–334

Meunier et al. (2017) *Applied Mathematical Modelling* **52**: 648–663

For example:



Approach (2): integrals of porous pipe eq^n solutions

1) On any root segment, use continuous solutions $\psi_x = f(s)$ to define **mean segment potential**:

$$\bar{\psi}_x = \frac{\int_0^L \psi_x(s)\, ds}{L}$$

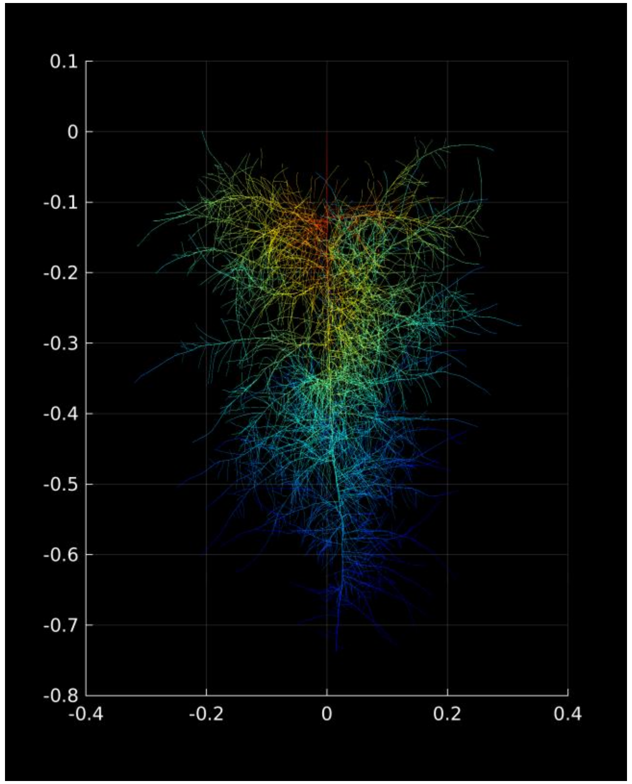
2) A **weighted average** for any soil block $\hat{\psi}_x$ can be found:

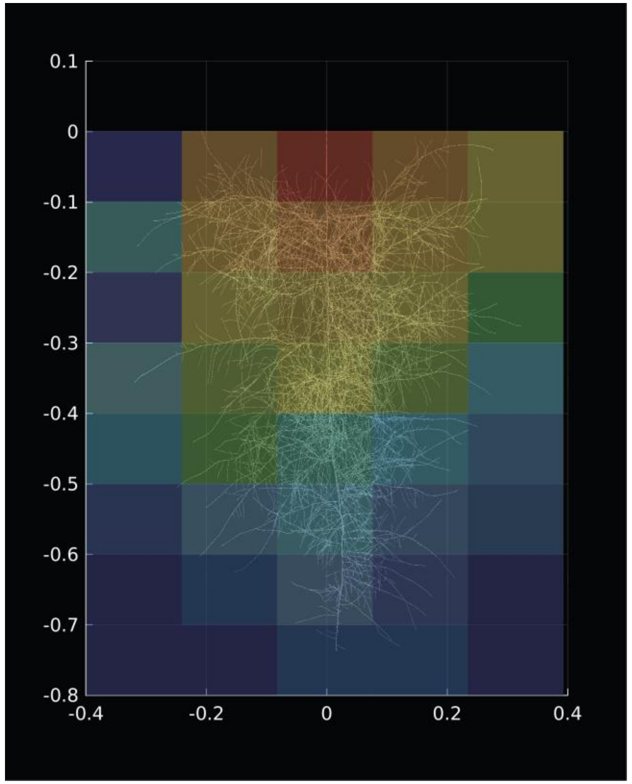
$$\hat{\psi}_x = \frac{\sum_1^n K_r L\, \bar{\psi}_x}{\sum_1^n K_r L}$$ (for *n* root segments in each soil block)

which **yield exact solutions** for total uptake in soil bloc Q_R in simple Darcian:

$$Q_R = -\sum_1^n K_r L\ (\hat{\psi}_x - \psi_s)$$

when soil water potential ψ_s is uniform in each block.





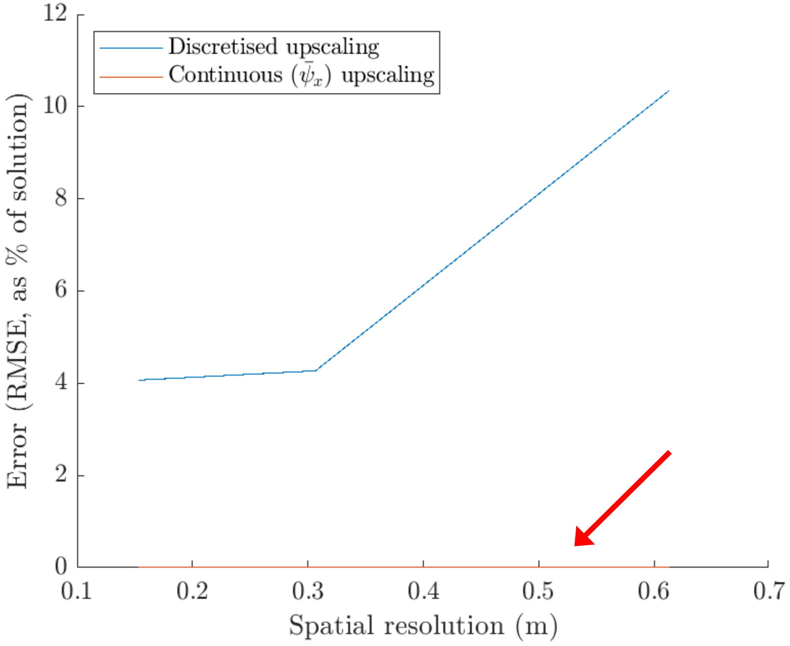
3) The full 3D problem in continuous form can be formulated as a linear system in terms of $\bar{\psi}_x$.

Bouda & Saiers (2017) *Advances in Water Resources* **110**: 319–334

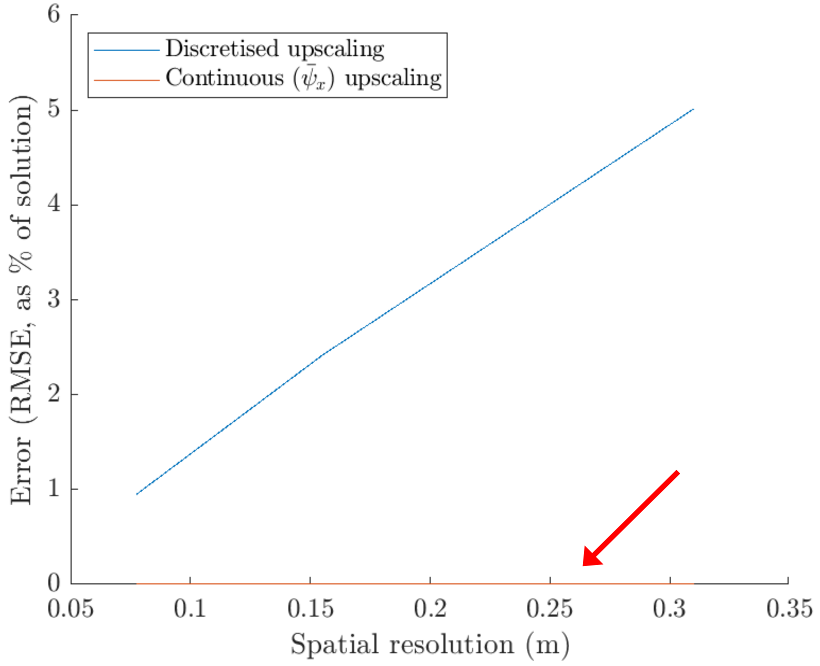
4) Which means an **exact upscaling can be achieved** by partial Gaussian elimination, yielding $\hat{\psi}_x$ for each soil block of uniform ψ_s.

Continuous upscaling eliminates discretisation error

Green pea (*Pisum sativum*)



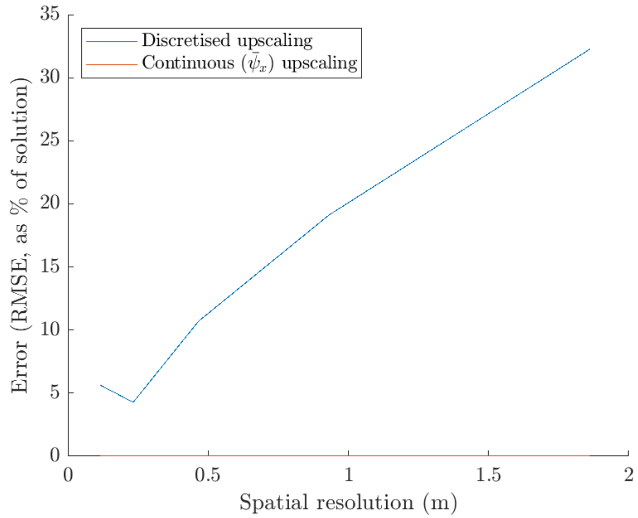
Winter wheat (*Triticum aestivum*)



- Assume soil moisture is uniform in layers of increasing thickness ('spatial resolution' in plots).
- Compare simulated hydrostatic flows between full 3D continuous model and upscaled models:
- Linearised upscaling:
  discretisation error increases with layer thickness.
- Continuous upscaling:
  No discretisation error.

This direct upscaling by Gaussian elimination also yields lowest computational cost: Single *N*x*N* matrix multiplication per solution step, where *N* is number of soil blocks.

Right: For large shrubby root system at meter resolution, discretisation error can be significant.



Code for upscaling at https://github.com/mbouda/genUpscAlg (in beta-testing)

Considering soil blocks with non-uniform ψ_s...

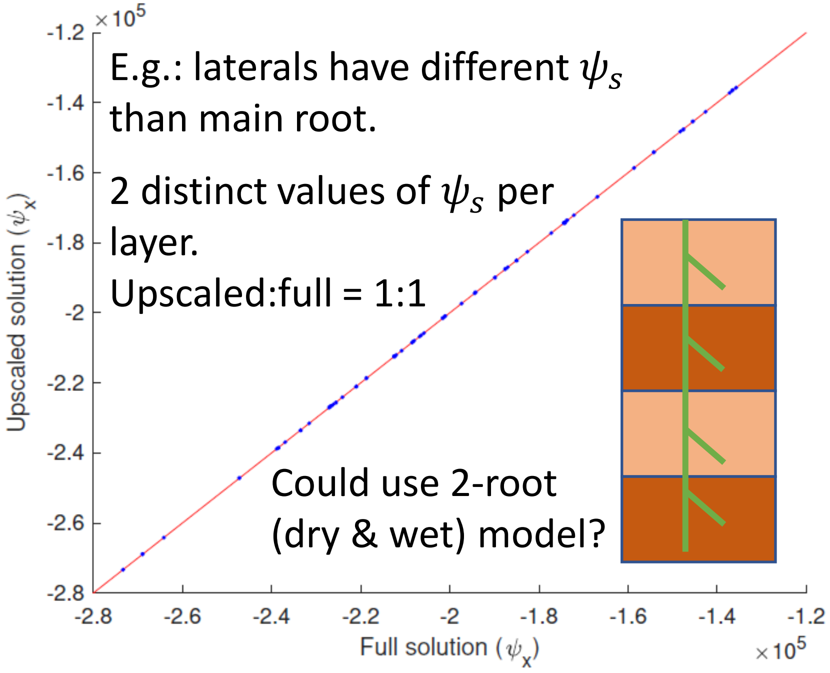
Breaking the assumption of uniform ψ_s in soil blocks yields prediction errors of comparable magnitude across upscaling approaches.

Taking the next step, into hydrodynamic situations, requires dealing with non-uniformity within the soil blocks, which arises during drying.

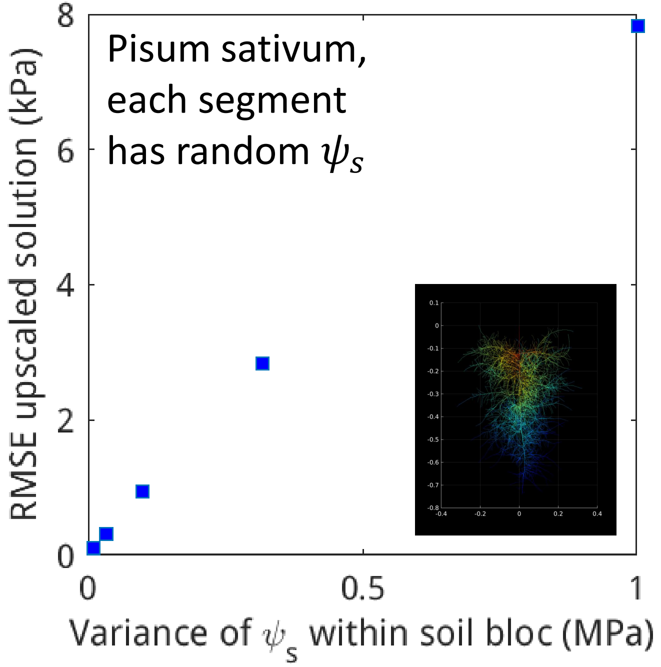
Given a weighted mean soil water potential

$$\hat{\psi}_s = \frac{\sum_1^n K_r L \psi_s}{\sum_1^n K_r L}$$

Exact upscaled solutions exist only for trivial cases.



For more distinct ψ_s using $\hat{\psi}_s$ and $\hat{\psi}_x$ solutions to fit upscaled parameters by inversion, error remains quite low: ~ kPa when variance of ψ_s is ~ MPa.



Summary & next steps on non-uniform ψ_s

- A general approach to upscaling the flow equations on root networks in continuous form
  - eliminates discretisation error,
  - minimises computational cost.
- The approach can be partially extended to non-uniform soil moisture in soil blocks
  - analytically, in trivial situations that might serve as a dry-root / wet-root model
  - by simply fitting model parameters at bulk scale, which yields low-error solutions.
- Further work in heterogeneously drying soils:
  - Merge with complementary approach of Vanderborght et al. (EGU21-8151)
  - Or try find non-iterative solution by
    - using a matric flux potential formulation to combine ψ_s with K_{soil}, and
    - examining statistics of water potential in drying soil $\hat{\psi}_s = f\left(\psi_s^{bulk}\right)$?

Acknowledgments

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Generating root system structures and general support

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