

Investigating Fast- and Slow-settling Phosphorus Fractions in Lakes using Steady-state Modeling

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Introduction

Phosphorus (P) is a vital nutrient in the eutrophication of freshwater resources. Lake retention of phosphorus has been studied for decades and different hypotheses and models are proposed for predicting lake P concentration. We recently have reassessed four pairs of competing hypotheses using steady-state models developed explicitly based on the hypotheses [1]. Our results showed that the hypothesis that a fraction of P input into the lake is associated with fast settling particles that are removed rapidly from the water column at the lake entrance is supported by the data. We estimated this fraction is on average about 30% of the input loading of P and applied it as a constant value. Although this hypothesis increased the R^2 of the top-performing mechanistic model by about 2.5%, the R^2 of the retention ($Ret_{TP} = 1 - (TP_{lake}/TP_{in})$) decreased by about 2.6%. In this paper, we assume that the fraction of fast- and slow-settling P is not constant among the lakes and try to investigate the relationship between this fraction and the landscape and lake characteristics using regression tree analysis.

Materials and Methods

Estimation of fast- and slow-settling fraction of P

A general mass balance model for total P (TP) in the lakes can be written as follows:

$$\frac{\Delta m_{TP}}{\Delta t} = Input - Output - Loss \quad (1)$$

where m_{TP} is the mass of TP in the lake water column. The simplifying assumptions are (i) lake is in the steady-state and water input and output are equal; (ii) lake is a continuously mixed-flow reactor ($TP_{out} = TP_{lake}$);

(iii) The mass balance terms are expressed per unit volume of the lake and TP loss is expressed as a second-order reaction function of lake TP concentration as follows:

$$\frac{1}{V} \frac{\Delta m_s}{\Delta t} = \sigma_2 TP_{lake}^2 \quad (2)$$

where V is the volume of the lake, m_s is the mass of TP incorporated into sediments and σ_2 is the second-order volumetric reaction coefficient for TP ($m^3 m g^{-1} yr^{-1}$). By replacing the loss term Eq. (2) in Eq. (1) and applying the aforementioned assumptions, we have:

$$\frac{1}{V} \frac{\Delta m_{TP}}{\Delta t} = \frac{Q \cdot TP_{in}}{V} - \frac{Q \cdot TP_{lake}}{V} - \sigma_2 \cdot TP_{lake}^2 = 0 \quad (3)$$

By assuming that a fraction of TP_{in} is associated with fast-settling particles and it removes rapidly at lake entrance, we can show that a α -fraction of TP_{in} reaches the main basin of the lake to participate in reaction; hence we have:

$$TP_{lake} = \frac{-1 + \sqrt{1 + 4\alpha\sigma_2 TP_{in}\tau_w}}{2\sigma_2\tau_w} \quad (4)$$

Database

A large database (n=738) is gathered using the published lake databases as explained by Khorasani and Zhu [1]. The majority of the lakes (about 91%) are located in Northern America and the rest are in Europe. All lakes are between latitude 25° and 60° of the northern hemisphere [See Fig. A].

Regression Tree Analysis

The TP model shown in Eq. (4) is calibrated using the database and the unknown parameters are found to be $\alpha = 0.702$ and $\sigma_2 = 0.011 m^3 m g^{-1} yr^{-1}$ by Khorasani and Zhu [1].

To find the patterns with which α varies, we let σ_2 be equal to the calibrated value and then calculate α for each lake. The hypothesis is that there are classes of lakes that α within them

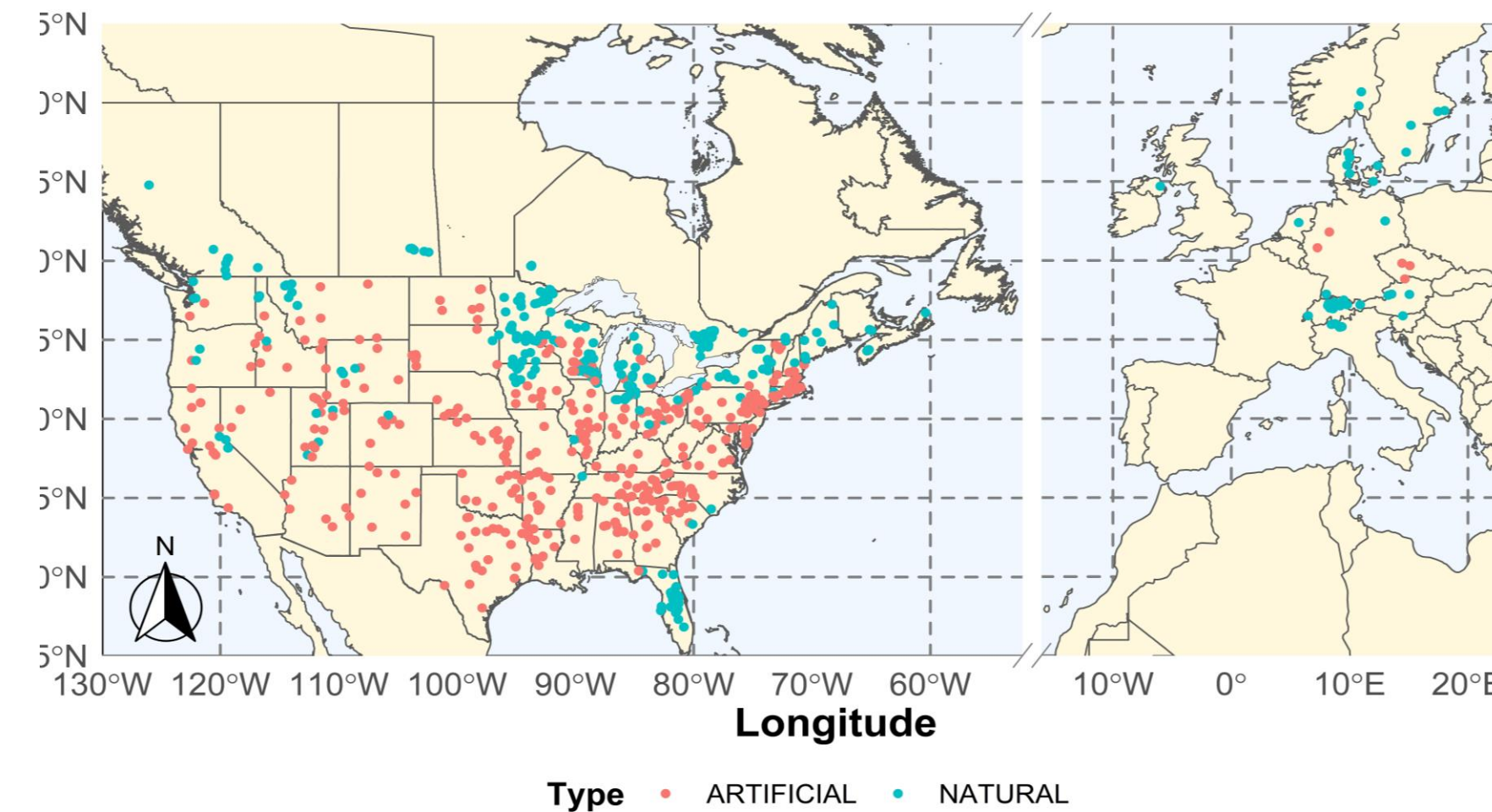


Figure A: A map of the lakes in the database [1].

can be considered constant while the prediction power of retention (Ret_{TP}) is not negatively affected in comparison to the hypothesis that $\alpha = 1$. The regression trees are trained using MATLAB programming software.

Results

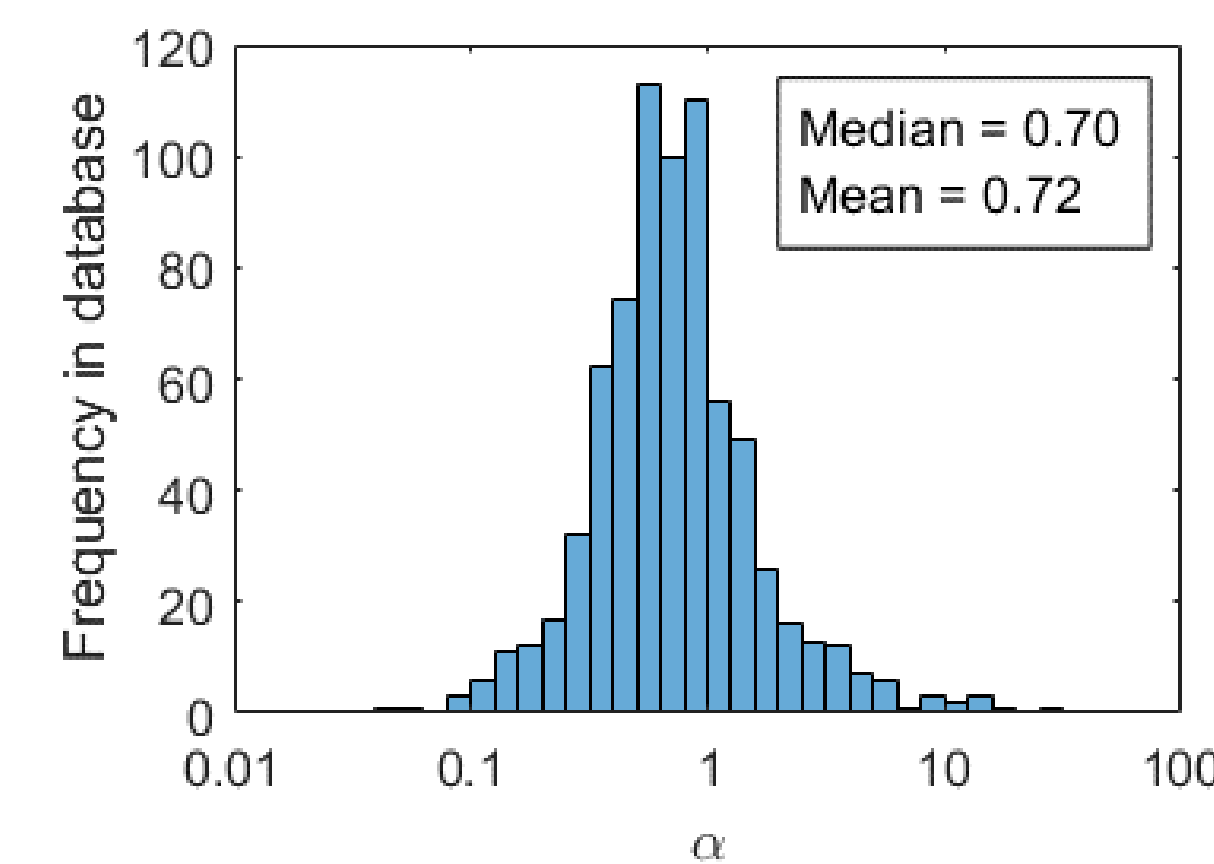


Figure B: The distribution of α fractions beside the median and the mean of log-transformed values.

As shown in [Fig. B] in 196 lakes (26% of points) the calculated α is larger than 1 which implies that more than 100% of TP_{in} is participating in loss reactions. This can be due to persistent internal loading of TP or errors in measurement of TP_{in} and TP_{lake} . We decided to keep the lakes with $\alpha > 1$ since the errors resulted in these values probably spread through the database, but we removed regression trees with a leaf with $\alpha > 1$.

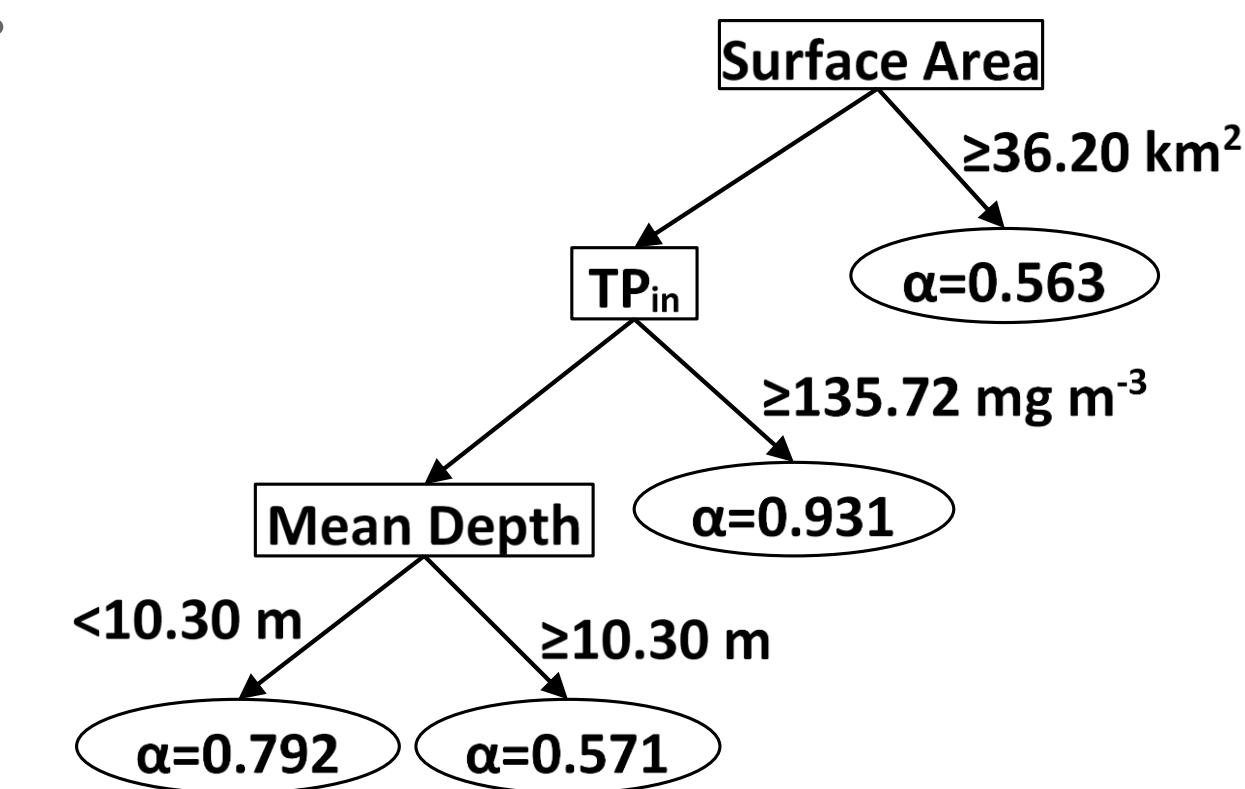


Figure C: the results of regression tree analysis for α fractions of the lakes based on lake area, TP_{in} and mean depth of the lake.

Comparison of performance of models for TP_{lake} and Ret_{TP}

Model	Description	$R^2(log(TP))$	$R^2(Ret_{TP})$
1	$\alpha=1.000, \sigma_2=0.027$	0.731	0.259
2	$\alpha=0.702, \sigma_2=0.011$	0.756	0.233
3	$\alpha=variable, \sigma_2=0.011$	0.770	0.273

Conclusion

Although considering a constant fast settling fraction ($1 - \alpha$) for the lakes increased the prediction power of TP_{lake} , but the prediction power of TP retention decreased since no lake could have a retention coefficient smaller than ($1 - \alpha$). The regression tree analysis showed that this fraction depends on area of the lake, TP_{in} and mean depth. Large lakes (Area ≥ 36.2 km²; n=183) allow about 44% of TP_{in} to settle rapidly regardless of other classifiers. Among smaller lakes, if TP_{in} is high (≥ 135.7 mg/m³; n=207) only about 7% of it rapidly settles. Among smaller lakes with lower TP_{in} , the average depth of 10.3 m decides either 43% of TP_{in} settles rapidly in deep lakes (n=235) or only 21% in shallow lakes (n=113). The mean depth of 10.3 m has been also previously found to be an important classifier in lakes [2].

References

1. Khorasani, H. & Zhu, Z. Phosphorus Retention in Lakes: A Critical Reassessment of Hypotheses and Static Models. (Under Review)
2. Cheng, V., et al. 2010, A revaluation of lake-phosphorus loading models using a Bayesian hierarchical framework. *Ecological Research*. 25. 59-76