

Dissemination of High-Resolution Ionosphere Information from VTEC B-spline Expansions for Single-Frequency Positioning

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Motivation

Methodology

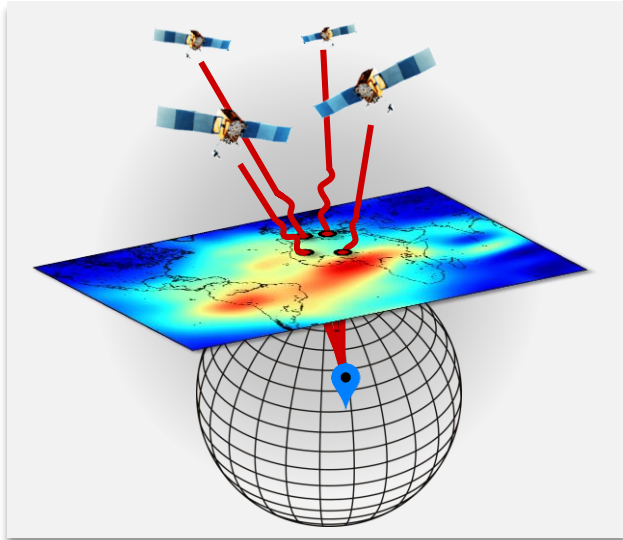
Testing

Validation

Summary

Motivation

- Among the various errors in GNSS-based positioning and navigation, **the ionospheric delay is one of the largest.**
- 
- Modern applications, such as autonomous driving or precision agriculture require **real-time (RT) information about the ionosphere** with **high spectral and spatial resolution.**
- The Ionosphere Associate Analysis Centers (IAACs) of the International GNSS Service (IGS) provide so-called **global ionosphere maps (GIM)** representing the Vertical Total Electron Content (VTEC) mostly as spherical harmonic (SH) expansions with a maximum degree value $n_{max} = 15$.
 - The DGFI-TUM uses series expansions in terms of **B-splines**, which provide **a large number of advantages** compared with SHs, e.g. adaptation of the model to both the data distribution and the signal structure, and allows for a **high-resolution modelling of the VTEC**.



Problem Definition:

- Currently, the only data format used for the dissemination of these ionospheric corrections in real time is based on the State Space Representation (SSR) message and the RTCM standards and allows for the provision of coefficients of SHs.

Research Question:

- How can the high-resolution B-spline models be transformed into SHs to meet RTCM requirements?

Results and Interpretation:

- Improved single-frequency positioning by applying the transformed B-spline model for ionosphere correction on the GNSS observations.

Motivation

- Positioning and navigation using GNSS is subject to errors of several error sources.
- Among the different error sources, the influence of the ionosphere on the pseudorange observations is one of the largest.
- The Ionosphere Associate Analysis Centers (IAACs) of the International GNSS Service (IGS) provide so-called **global ionosphere maps (GIM)** representing the Vertical Total Electron Content (VTEC).

Error Source	Error in [m]
Receiver noise	± 0.3 m
Troposphere	± 0.5 m
Multipath Effect	± 1.0 m
Satellite Clocks	± 2.0 m
Orbit	± 2.5 m
Ionosphere	± 5.0 m

Numbers taken from: <https://novatel.com/an-introduction-to-gnss/chapter-4-gnsserror-sources>

A distinction has to be undertaken related to the use of the GIM information, namely

- for precise positioning or research in post-processing mode:
 - ➡ As long as the data is not immediately used, the VTEC information can be prepared with highest resolution and quality and be disseminated to a user by means of any appropriate data format (IONEX) and platform (File Transfer Protocol (FTP))
- for precise navigation and positioning in real-time (RT):
 - ➡ Thereby the selection of possible dissemination formats is limited, because a user must receive the ionospheric information within seconds and with high precision, in order to correct the GNSS measurements used, e.g. for positioning.



Modern applications, such as autonomous driving or precision agriculture require real-time (RT) information about the ionosphere with high spectral and spatial resolution.

Motivation – Problem Definition

- In terms of GIM we distinguish between different **Product Types** depending on the input data which is used for modelling.

Product Types	„Final“	„Rapid“	„Ultra-rapid“	„Near Real-Time“	„Real-Time“
Orbit information	Final GNSS orbits	Rapid GNSS orbits	Ultra rapid GNSS orbits	Ultra rapid GNSS orbits	Ultra rapid GNSS orbits
Station information	All available GNSS stations	Stations providing daily data	Stations providing hourly data	Real-time stations data	Real-time stations data
Data Latency	> 1 week	> 24 hours	1,5h – 2,5h	~ 1 sec	~ 1 sec
Product Latency	> 2 weeks	> 24 hours	< 3 hours	< 15 min	ca. 1 min

IONEX

Product
Formats

Grid based format:

- typically provided with a grid resolution of $2.5^\circ \times 5^\circ$ for snapshot maps with temporal sampling intervals of $\Delta T = 2$ hours, as daily IONEX files.
- shorter sampling intervals are possible and frequently applied.

RTCM

- Streaming format for dissemination of SH coefficients up to degree 16

Motivation – Problem Definition

Most of these products are based on series expansions in terms of spherical harmonics.

IAAC	Model	Spectral Degree	Product Types 	RTCM
CODE	SH	$n_{max} = 15$	Final ('codg'), rapid	YES
UPC	voxel	divers	Final, rapid ('uqrg'), RT	NO
ESOC/ESA	SH	$n_{max} = 15$	Final, rapid	YES
JPL	splines	divers	Final, rapid	NO
CAS	SH	$n_{max} = 15$	Final, RT	YES
EMRG	SH	$n_{max} = 15$	Final, rapid	YES
WHU	SH	$n_{max} = 15$	Final, rapid, RT	YES
DGFI-TUM	B-splines 	$J_1 = 5 \approx n_{max} = 34$ $J_2 = 3 \approx n_{max} = 12$	ultra-rapid ('othg'), RT	NO

The RTCM/SSR VTEC message is not applicable for several GIMs of the IAACs.

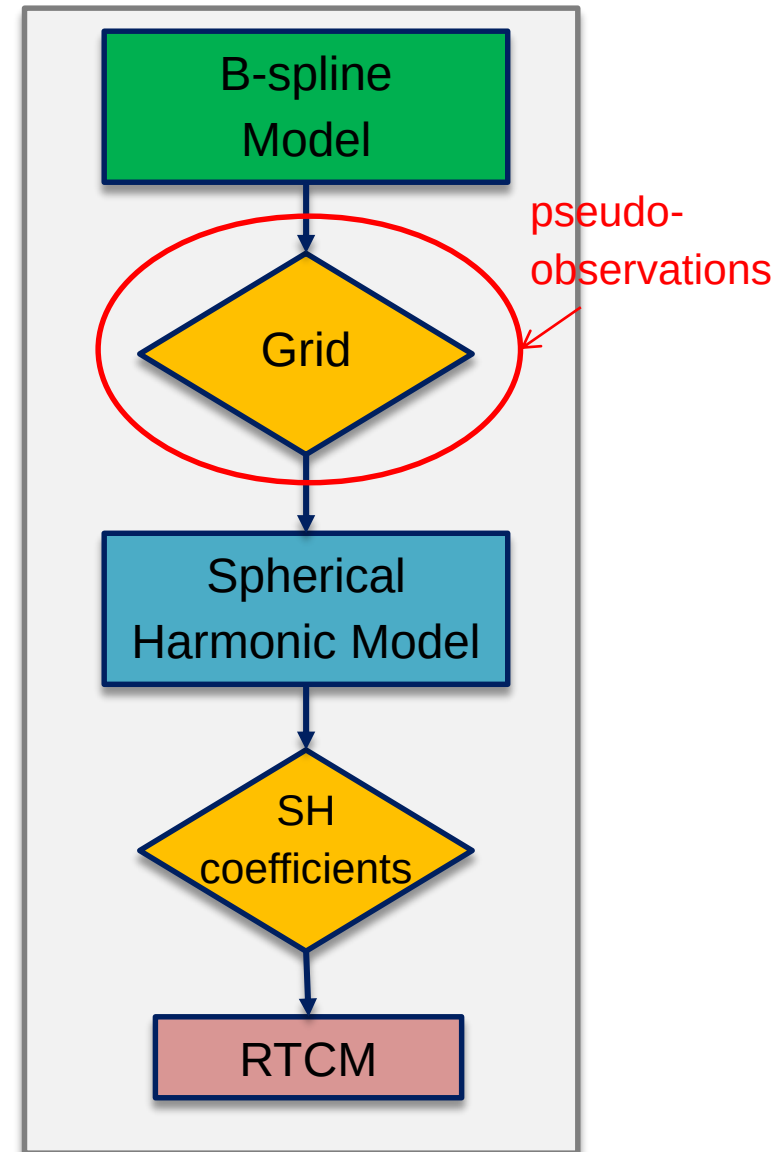


Previous investigations and validations (dSTEC analysis, comparison with VTEC from altimetry) have shown that the **GIMs based on B-splines are more precise** than maps based on SHs with degree 15 due to their **high spectral resolution**.
(Goss et al., 2019; Goss et al., 2020a; Erdogan et al., 2017; Erdogan et al., 2020)

Methodology – Flowchart

The transformation of the B-spline GIM to a SH GIM comprises the following steps:

1. **B-spline Model:** 
Generation of the B-spline Model
2. **Pseudo-observations:** 
Generation of pseudo-observations for VTEC on a grid using the B-spline model.
3. **Transformation:** 
Estimation of the SH coefficients by means of the generated pseudo-observations from 1.
4. **Dissemination:** 
writing SH coefficients into the RTCM message format and provision of the coefficients to users of GNSS.

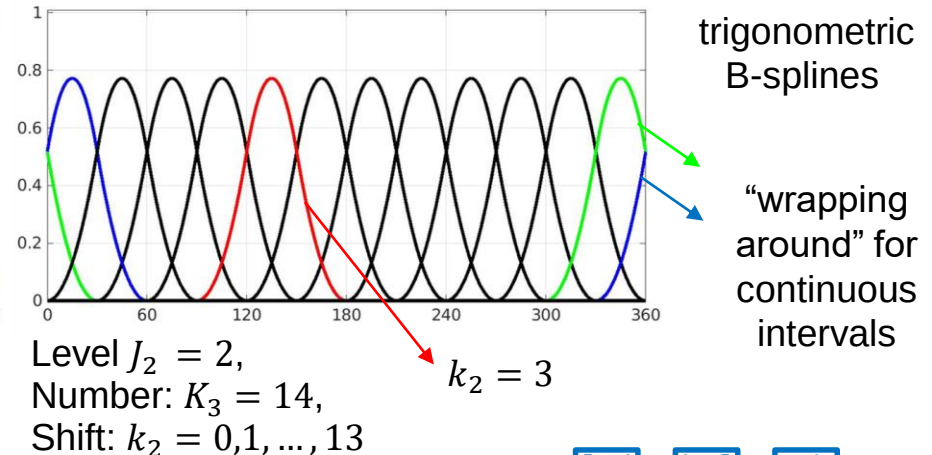
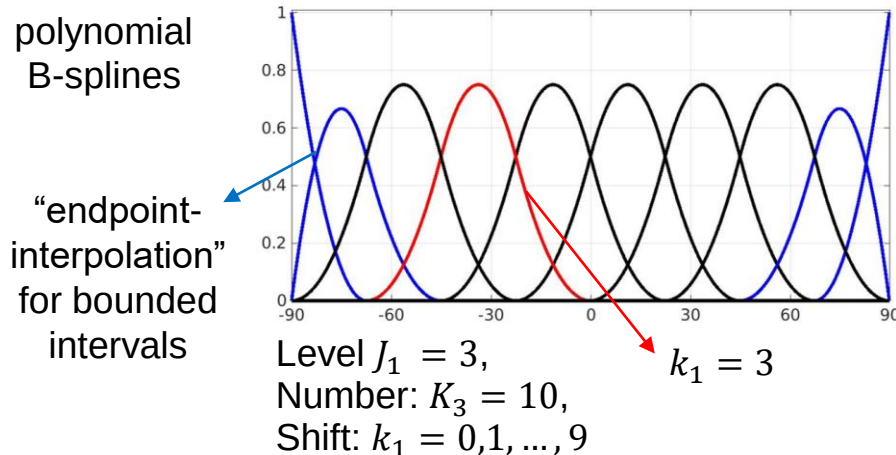


Methodology – B-spline Model

VTEC is represented by a series expansion in terms of **B-spline functions** defined separately for **longitude** λ and **latitude** φ and combined as tensor products (Schmidt et al., 2011).

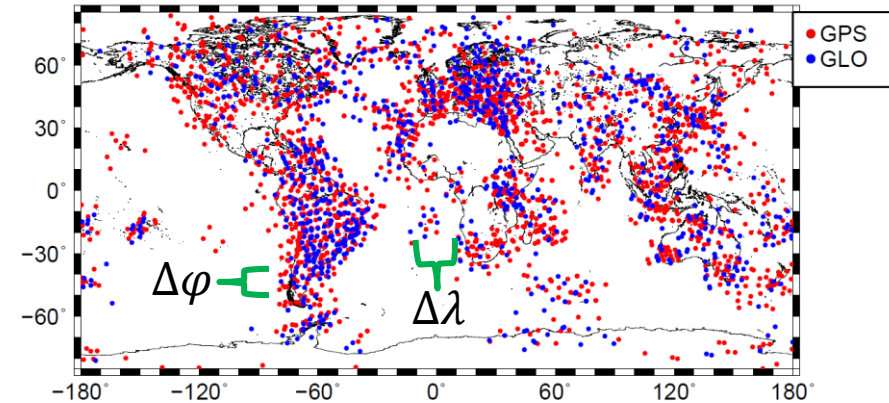
$$VTEC(\varphi, \lambda, t_s) = \sum_{k_1=0}^{K_{J_1}-1} \sum_{k_2=0}^{K_{J_2}-1} d_{k_1, k_2}^{J_1, J_2}(t_s) \phi_{k_1}^{J_1}(\varphi) \tilde{\phi}_{k_2}^{J_2}(\lambda)$$

- Therein $\phi_{k_1}^{J_1}(\varphi)$ are polynomial B-splines of shift k_1 and level J_1 and $\tilde{\phi}_{k_2}^{J_2}(\lambda)$ the trigonometric B-splines of shift k_2 and level J_2 .
- The level values define the number $K_{J_1} = 2^{J_1} + 2$ of polynomial B-splines functions distributed along the latitudes and the number $K_{J_2} = 3 \cdot 2^{J_2}$ of trigonometric B-splines functions along the longitude.
- The B-spline (BS) functions are different from zero only in a local environment, i.e. they are characterized by a **compact support**. Due to the **localizing feature** of the B-spline functions **data gaps** (oceans, deserts, etc.) can be handled appropriately.



Methodology – B-spline Model Resolution

- In general, the measure to define the spectral content of the estimated model on a sphere is the SH degree $n \in \mathbb{N}_0$.
- Analogously, the level values J_1 and J_2 define the spectral resolution of the B-spline model.
- Typically, the B-spline levels are chosen according to the given **mean sampling intervals** $\Delta\varphi$ and $\Delta\lambda$ of observations in latitude and longitude respectively.
- The following table provides the relation between the levels J_1 and J_2 and the maximum SH degree n_{max} (Goss et al., 2019).

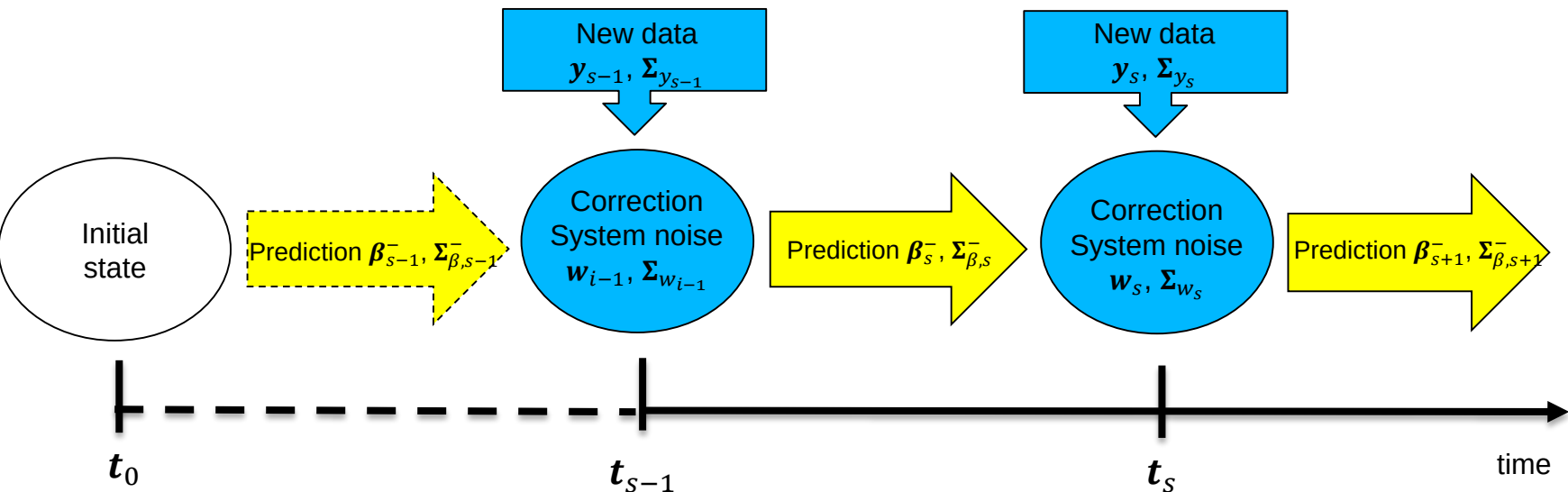


Distribution of IPPs from GPS and GLONASS observed by the IGS station Network. (Goss et al., 2020a)

Latitude						Longitude					
$J_1 \leq \log_2 \left(\frac{180^\circ}{\Delta\varphi} \right) \leq \log_2 (n_{max} - 1)$						$J_2 \leq \log_2 \left(\frac{120^\circ}{\Delta\lambda} \right) \leq \log_2 \left(\frac{2n_{max}}{3} \right)$					
Level	1	2	3	4	5	Level	1	2	3	4	5
n_{max}	3	5	9	17	33	n_{max}	3	6	12	24	48
$\Delta\varphi$	60	36	20	10.5	5.5	$\Delta\lambda$	60	30	15	7.5	3.75

Methodology – B-spline Model Estimation

- Estimation of the unknown B-spline coefficients $d_{k_1, k_2}^{J_1, J_2}(t_s)$ for consecutive time moments t_s with temporal sampling $\Delta T \geq t_s - t_{s-1}$ by means of a Kalman filter.
- The Kalman filter consists of a
 - **state equation** $\beta_s^- = F_s \hat{\beta}_{s-1} + w_{s-1}$ and an
 - **observation equation** $y_s + e_s = A_s \beta_s^-$.
- The first equation describes the **prediction step** of the state vector β_s at time **moment** t_s .
- The second equation is the **time update**, e.g. the **correction** of the predicted state vector β_s^- using new observations y_s .
- The Kalman filter predicts and updates the vector β_s sequentially.



Methodology – Pseudo-observations

- For the estimation of SH coefficients, appropriate number of pseudo-observations need to be distributed equally on the sphere, with a mean sampling of

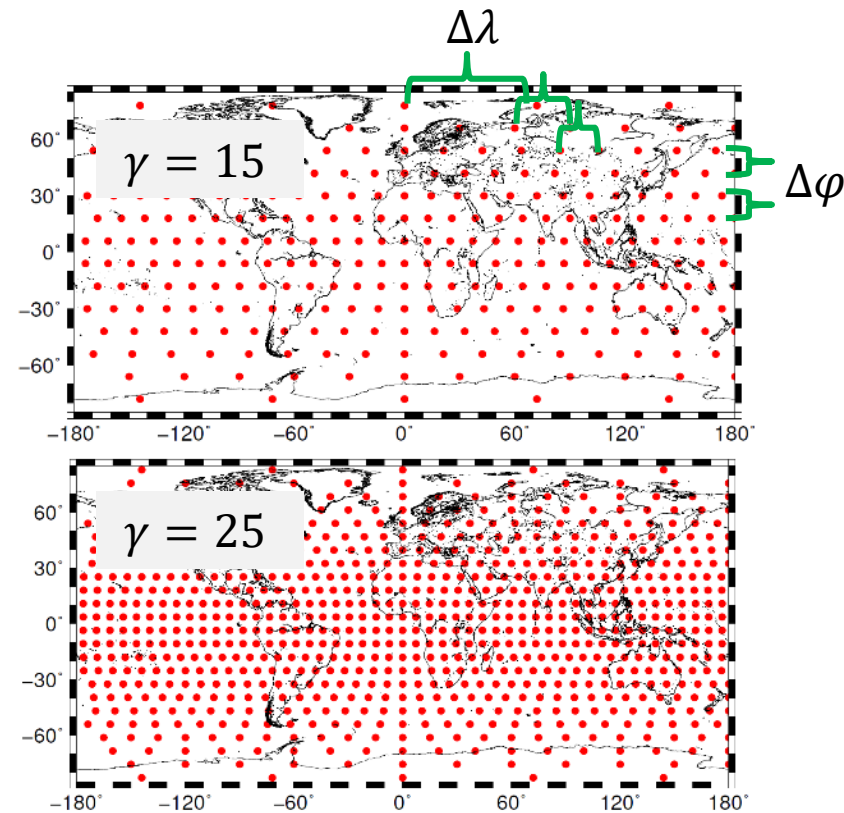
$$\Delta\varphi < \frac{180^\circ}{n_{max}} \text{ and } \Delta\lambda < \frac{180^\circ}{n_{max}},$$

depending on the degree n_{max} .



The Reuter grid is optimally adapted to a sphere and provides a homogeneous distribution of grid points.

- The sampling distance $\Delta\varphi$ in latitude is determined by the control parameter γ as $\Delta\varphi = \frac{180^\circ}{\gamma}$.
 - The total number of points along the meridian is given with $\gamma + 1$.
- Along the circles of latitudes, the number of points decreases with increasing latitude. The sampling $\Delta\lambda$ along the equator is equal to $\Delta\varphi$.



Methodology – Transformation

Step 1:

Given the $u \times 1$ vector $\hat{\beta}_s$ of the $u = K_{J_1} \cdot K_{J_2}$ estimated B-spline coefficients for a time $\hat{d}_{k_1, k_2}^{J_1, J_2}(t_s)$ moment t_s , the $V \times 1$ vector $\hat{f}_s$ of the VTEC pseudo observations $\widehat{VTEC}(\varphi_v, \lambda_v, t_s)$ at the Reuter grid with $v = 1, 2, \dots, V$ points can be calculated as

$$\hat{f}_s = \mathbf{A} \hat{\beta}_s,$$

where the elements of the $V \times u$ matrix $\mathbf{A}$ are defined as the B-spline tensor products.

Step 2:

The $N = (n_{max} + 1)^2$ SH coefficients $c_{n,m}$ of degree $n = 0, 1, \dots, n_{max}$ and order $m = -n, \dots, n$ can be estimated from the linear equation system

$$\hat{f}_s + \mathbf{e}_s = \mathbf{X} \mathbf{c}_s$$

where $\mathbf{e}_s$ is a $V \times 1$ consistency vector. The $N \times 1$ vector $\mathbf{c}_s = (c_{n,m})$ comprises the unknown SH coefficients and the $V \times N$ matrix $\mathbf{X}$ contains the SHs itself.

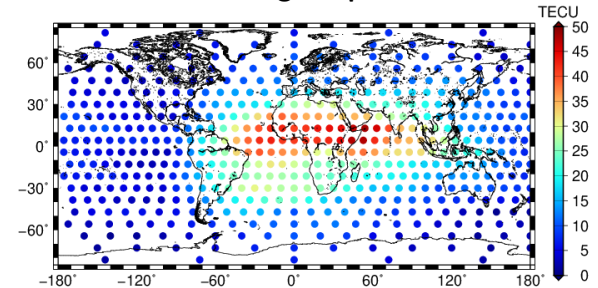
Step 3:

In the case that the matrix $\mathbf{X}$ is of full column rank the least-squares estimation yields with a given $V \times V$ positive definite weight matrix $\mathbf{P}$ the solution reads

$$\hat{c}_s = (\mathbf{X}^T \mathbf{P} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{P} \mathbf{A} \hat{\beta}_s = \mathbf{T} \hat{\beta}_s,$$

with $N \times u$ matrix $\mathbf{T}$.

Pseudo-observations
Reuter grid $\gamma = 15$



The complete derivation of the transformation can be found in:
Goss et al. (2020b), 'High-Resolution Ionosphere Corrections for Single-Frequency Positioning'

Test of the Methodology

- Following the steps of the methodology, a B-spline model with the level values $J_1 = 5$ and $J_2 = 3$ was used for ten consecutive days in September 2017 to generate pseudo-observations on a Reuter grid.
- The VTEC signal generally features finer structures in latitude than in longitude direction, thus, the B-spline level in latitude direction is chosen with a higher level.
- The B-spline level $J_1 = 5$ in latitude direction corresponds to a degree SHs of $n_{max} = 33$, while the level $J_2 = 3$ in longitude direction can be relate to a degree of $n_{max} = 12$. 
- In order to find an appropriate degree for the transformation of the B-spline model, five different cases with different control parameter γ for the Reuter grid and different degrees n_{max} for the SH expansion are considered. 
- The following table provides the values γ for the corresponding sampling intervals $\Delta\varphi_l$ and $\Delta\lambda_l$ of the pseudo-observations, as well as the degree n_{max} and the corresponding required minimum sampling interval $\Delta\varphi$ and $\Delta\lambda$.

	Reuter Grid						Spherical Harmonics				
γ	16	21	25	31	35	n_{max}	15	20	24	30	34
$\Delta\varphi_l, \Delta\lambda_l$	11.25°	8.57°	7.2°	5.8°	5.14°	$\Delta\varphi, \Delta\lambda$	12	9	7.5	6	5.3
V	317	563	797	1225	1561	N	256	441	625	961	1224

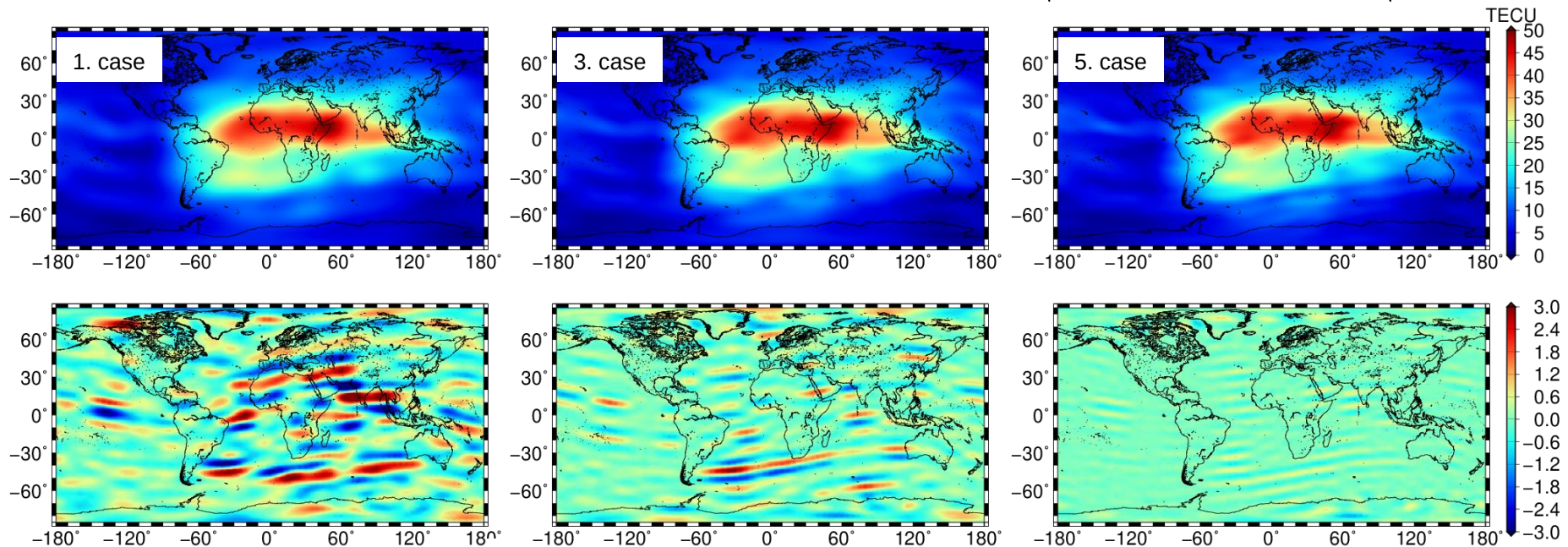
Test of the Methodology

	1. Case	2. Case	3. Case	4. Case	5. Case
γ	16	21	25	31	35
V	317	563	797	1225	1561
n_{max}	15	20	24	30	34
N	256	441	625	961	1224
$\widetilde{\Delta t}$	1.43 s	1.85 s	3.12 s	4.99 s	8.23 s
rel. RMS [%]	9.23	5.83	4.19	2.54	1.83
RMS [TECU]	1.31	0.83	0.60	0.36	0.26
δ_{max} [TECU]	8.22	5.91	5.23	2.22	1.79
δ_{min} [TECU]	-8.06	-6.29	-4.21	-2.23	-1.9
δ_{mean} [TECU]	0.016	0.0012	0.014	0.0033	0.003

For the comparison of the different cases the quality characteristics

- computational time $\widetilde{\Delta t}$
- RMS in [%] and in [TECU]
- Minimum δ_{min} , maximum δ_{max} and mean δ_{mean} values of the differences $\delta = VTEC_{BS} - VTEC_{SH}$ are considered.

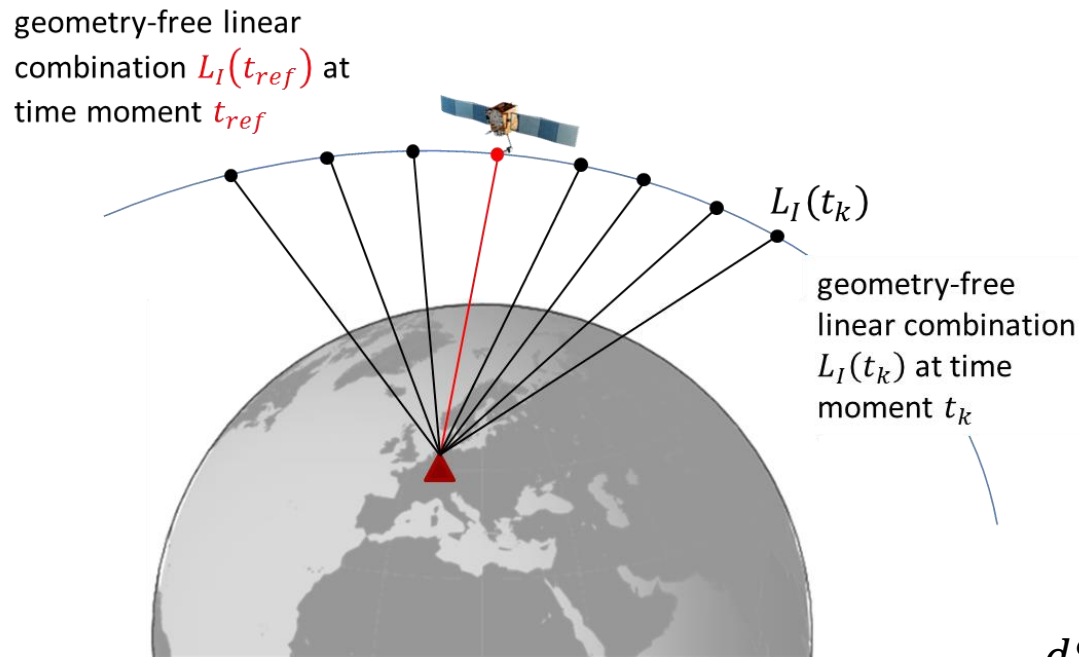
$$RMS[\%] = 100 \cdot \sqrt{\sum (VTEC_{BS} - VTEC_{SH})^2} / \sqrt{\sum VTEC_{BS}^2}$$



Validation – dSTEC

- The validation with external GIMs is based on the **self-consistency analysis (dSTEC)**, which provides an accuracy of 0.1 TECU.

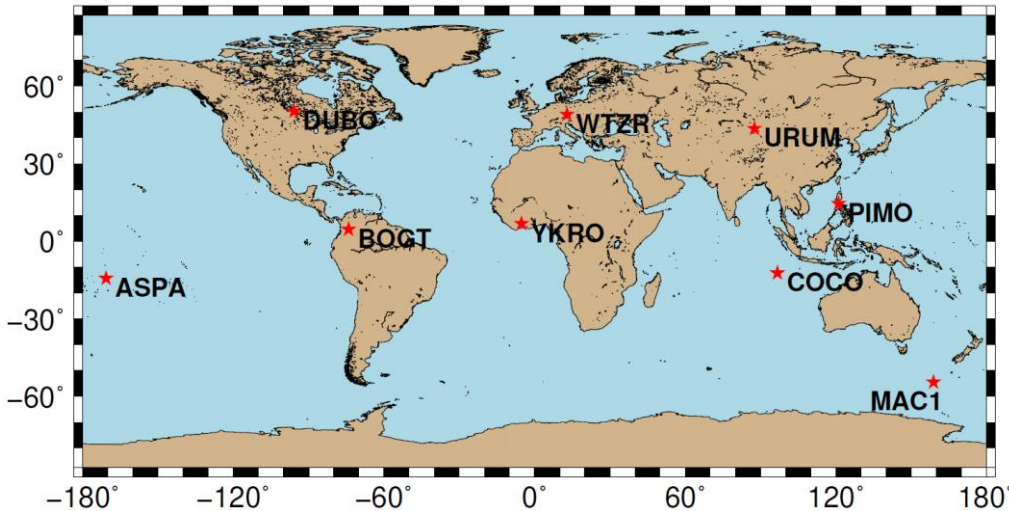
- The self consistency analysis is based on the comparison of ...



- differenced STEC values computed from the GNSS geometry-free linear combination of carrier-phase observables (along a phase-continuous arc):**
 $dSTEC_{obs,k}$
- and differenced STEC values computed from the VTEC maps:**
 $dSTEC_{map,k}$

$$dSTEC_k = dSTEC_{obs,k} - dSTEC_{map,k}$$

Validation – dSTEC



- We compare the B-spline product denoted as `othg` with its transformations based on SHs of different degrees n_{max} = 15, 20, 24, 30, 34 ; i.e. (`o15g`, `o20g`, `o24g`, `o30g`, `o34g`)

Similar results as the `othg` can be seen for the reconstructions with degree $n_{max} \geq 30$. These reconstructions contain nearly all spectral frequencies which have originally been represented by `othg`.

GIM	othg	o15g	o20g	o24g	o30g	o34g	codg	uqrg
Model	B-splines	SHs	SHs	SHs	SHs	SHs	SHs	Voxel & Kriging
n_{max}		15	20	24	30	34	15	n.a.
J_1/J_1	5/3							
ΔT	10 min	10 min	10 min	10 min	10 min	10 min	60 min	15 min
Latency	<3 h	<3 h	<3 h	<3 h	<3 h	<3 h	>1 week	>1 day
Reference	[15]	Section 3	Section 3	Section 3	Section 3	Section 3	[31]	[13]
RMS [TECU]	0.91	1.18	1.05	1.00	0.93	0.92	1.01	0.85

Table taken from Goss et al., (2020b), references can be found therein.

Validation – Positioning

- For the validation in the positioning domain we use the **open source software RTKLIB**.
- RTKLIB enables various options, such as PPP (static, kinematic, fixed), single frequency (L1, L2, L5), dual/multi frequency.
- The ionospheric and tropospheric information, as well as the satellite orbits and clocks can be used differently in RTKLIB.

Specifications used in the study:

- PPP kinematic
- Single frequency (L1) positioning
- Ionosphere: from IONEX
- Troposphere: Estimated ZTD
- Satellite orbits (precise) – ultra-rapid orbits (.sp3)
- Clock information every 5s (from CODE)

Options ✕

Setting1 Setting2 Output Stats Positions Files Misc

Positioning Mode PPP Kinematic ▼

Frequencies / Filter Type L1+2 ▼ Combinec ▼

Elevation Mask (°) / SNR Mask (dBHz) 10 ▼ ...

Rec Dynamics / Earth Tides Correction ON ▼ Solid ▼

Ionosphere Correction Iono-Free LC ▼

Troposphere Correction Estimate ZTD ▼

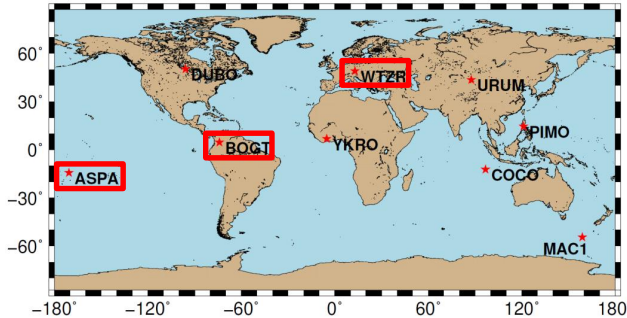
Satellite Ephemeris/Clock Precise ▼

☐ Sat PCV ☐ Rec PCV ☐ PhWindup ☐ Reject Ed ☐ RAIM FDE

Excluded Satellites (+PRN: Included)

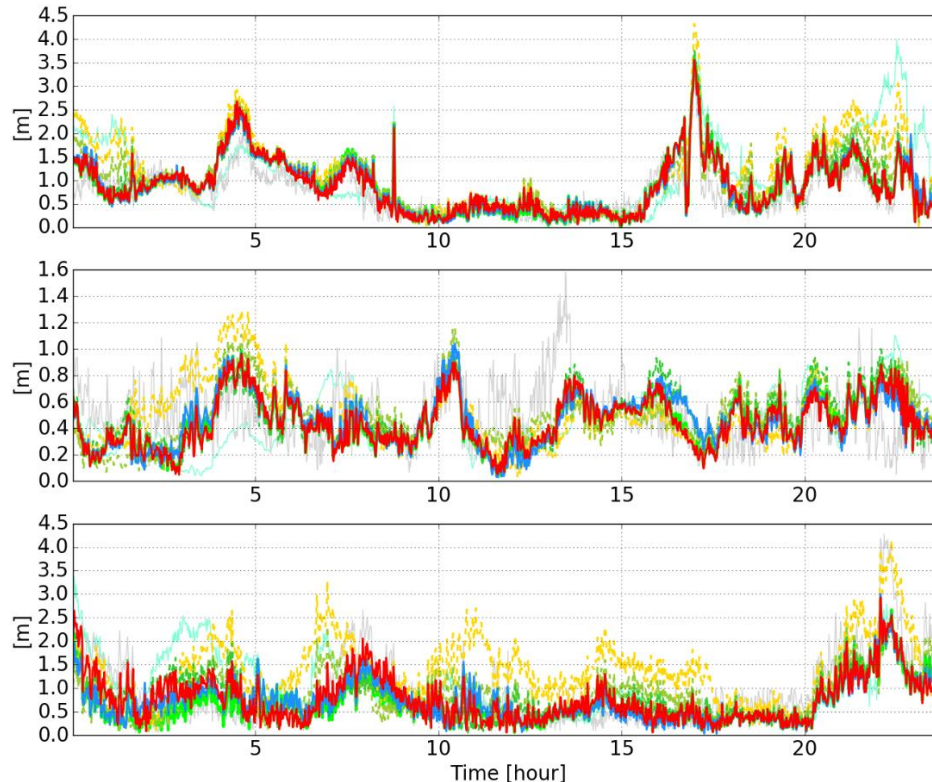
☒ GPS ☐ GLO ☐ Galileo ☐ QZSS ☐ SBAS ☐ BeiDou

Validation – Positioning



The selection of stations to be used for the single frequency positioning test is based on their geographical location:

- **BOGT**: close to geomagnetic equator with geomagnetic disturbances
- **WTZR**: generally quiet ionospheric conditions, with dense observation distribution
- **ASPA**: close to geomagnetic equator and oceanic region

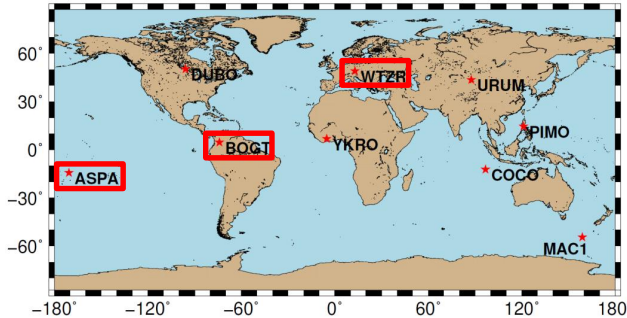


Difference between the precise position (accuracy in cm level) of the stations and the position determined by single frequency PPP throughout the day.

2nd of September 2017:
low solar activity

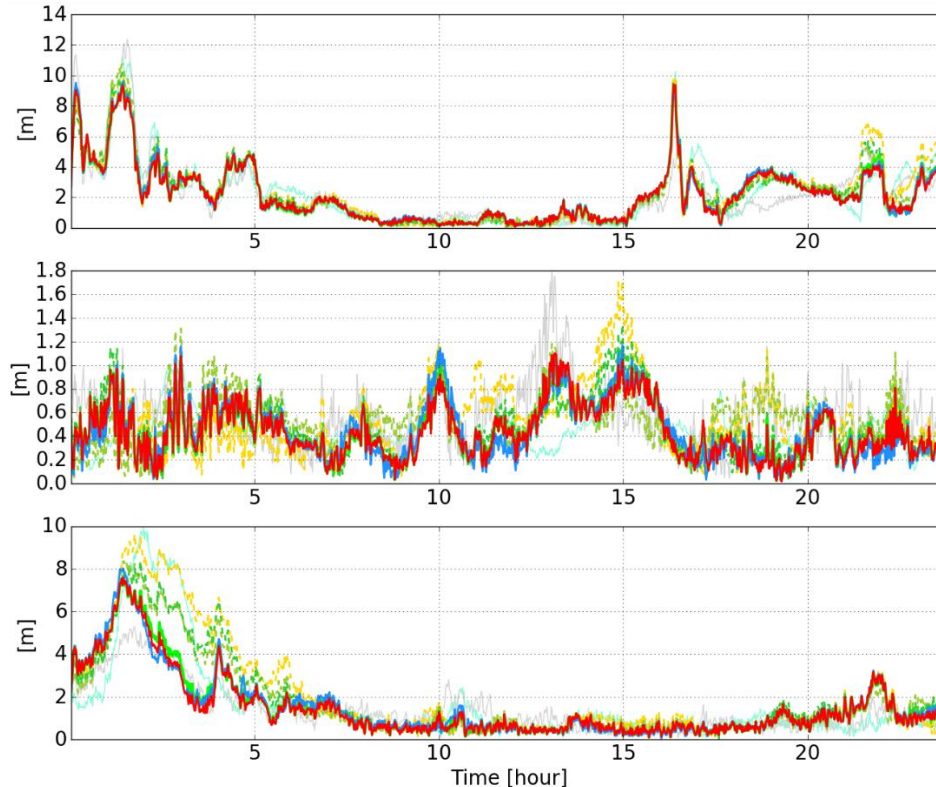


Validation – Positioning



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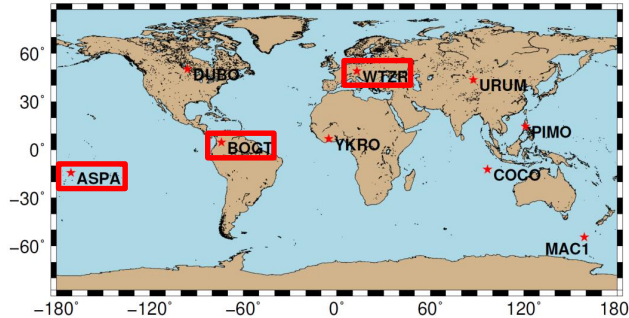


Difference between the precise position (accuracy in cm level) of the stations and the position determined by single frequency PPP throughout the day.

8th of September 2017:
geomagnetic storm



Validation – Positioning



The selection of stations to be used for the single frequency positioning test is based on their geographical location:

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- **WTZR**: generally quiet ionospheric conditions, with dense observation distribution
- **ASPA**: close to geomagnetic equator and oceanic region

$\bar{S}$ is the mean value of the differences between the actual position and the position resulting from the position with a single frequency throughout the 2nd (DOY 245) and 8th (DOY 251) of September 2017.

	Value	DOY	othg	o15g	o20g	o24g	o30g	o34g	codg	uqrg
BOGT	RMS [TECU]	245	1.07	1.35	1.17	1.09	1.05	1.05	1.29	0.90
		251	2.91	3.19	3.12	3.03	2.91	2.86	3.14	2.98
	$\bar{S}$ [m]	245	0.90	1.13	1.00	0.92	0.89	0.89	1.05	0.77
		251	2.22	2.44	2.34	2.31	2.23	2.20	2.35	2.18
WTZR	RMS [TECU]	245	0.49	0.55	0.53	0.52	0.50	0.48	0.51	0.55
		251	0.49	0.90	0.57	0.54	0.50	0.49	0.51	0.62
	$\bar{S}$ [m]	245	0.45	0.50	0.49	0.48	0.46	0.44	0.46	0.50
		251	0.44	0.53	0.52	0.47	0.44	0.44	0.47	0.57
ASPA	RMS [TECU]	245	0.81	2.53	0.98	0.92	0.85	0.92	1.13	1.19
		251	2.18	3.12	2.59	2.67	2.17	2.11	2.65	1.80
	$\bar{S}$ [m]	245	0.69	1.37	0.86	0.79	0.73	0.77	0.95	0.94
		251	1.53	2.10	1.76	1.84	1.54	1.48	1.64	1.40

Table taken from Goss et al., (2020b).

Summary and Conclusions

- For precise positioning in modern applications, high-precision information about the ionospheric state is required for correcting the ionospheric delay on the GNSS measurements.
- Typically, ionospheric information is transmitted to users in real-time via data streams and the RTCM-based message 1264, which only allows the **dissemination of coefficients of SHs up to degree 16**.
- The distribution of the high-resolution B-spline GIM using data streams requires ...
 - ... an adaptation of the RTCM message for B-splines or
 - ... an **appropriate transformation of the B-splines model using SHs**.
- The shown study pursued the second possibility. In this case, both the **computation time required for the transformation** and the **sufficient spectral degree for the transformation** of the original B-spline model ``othg'` have been considered.
 - The computation time increases with increasing degree n_{max} , since larger values γ for the Reuter grid have to be chosen and more grid points/pseudo-observations are generated.
 - The transformation quality increases with the degree n_{max} . The B-spline product with level values $J_1 = 5$ and $J_2 = 3$ can be represented almost exactly with a highest degree of $n_{max} > 30$.
 - The original B-spline model ``othg'` provides a RMS value of 0.91 TECU in the dSTEC analysis and performs better than the SHs product ``codg'` with a RMS value of 1.01 TECU.
 - The transformation ``o30g'` and ``o34g'`, with highest degree values $n_{max} = 30$ and $n_{max} = 34$ provide with RMS values of 0.93 TECU and 0.92 TECU both a similar quality as the original product ``othg'`.

Summary and Conclusions

- A test in positioning domain provides a final conclusion, if the transformation of high-resolution GIMs allow for a higher positioning accuracy.
 - Among all GIMs, ``othg`` performs best, for all three stations and for the selected days.
 - The SHs product ``codg`` ($n_{max} = 15$) performs better than ``o15g`` with same spectral degree.
 - The transformations ``o30g`` and ``o34g`` perform similar to the original ``othg`` for all considered cases.
- **The dissemination of coefficients with high spectral resolution would allow an improvement in positioning for single frequency users.**
- **For future modern GNSS applications it is necessary to adapt the product formats accordingly.**

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References

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