

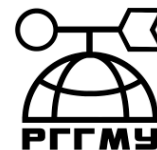
ON SELF-SIMILARITY OF WAVES IN TROPICAL CYCLONES

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- ▶ Combined TC wind-wave observations (satellites, buoy, aircraft) are critical for forecasting purposes and advance in air-sea interaction fundamental studies
- ▶ Full sophisticated models (e.g. WAM, WAWEWATCH III) are certainly capable to provide detailed wave information, but needs to use precise well-resolved surface winds and to consider large ensembles of solutions invite developments of more simplified and rapid solutions
- ▶ The simplified 1D parametric model (Kudryavtsev et al., 2015) has recently been extended for 2D case (Kudryavtsev et al., 2021a), to model wave field evolution forced by space-time varying wind field
- ▶ Equations are solved by the method of characteristics to provide efficient visualization on how the different wave systems develop under a moving TC and outrun it as long wave swell systems
- ▶ **The aim of this work** is to analytically reduce the solutions using self-similar functions, for rapid first guess evaluation for azimuthal-radial distributions of the primary wave system parameters (SWH, wavelength, direction) given the TC characteristics (maximum wind speed, radius, translation velocity)

Energy and momentum conservation
(Hasselmann et al., 1976; Phillips, 1977):

$$\partial E / \partial t + c_{gj} \partial E / \partial x_j = S^E$$

$$\partial M_i / \partial t + c_{gj} \partial M_i / \partial x_j = S_i^M$$

$$\int \int d\varphi d\omega$$

and some
algebra...

$E(\omega, \varphi) = A(\varphi - \varphi_p)F(\omega)$ - energy spectral density

$M_i = k_i E / \omega = \kappa_i \omega E / g$ - momentum spectral density

$$\kappa_i = [\cos \varphi, \sin \varphi]$$

$S^E = S_W - S_D + S_N$ - energy source

$S_i^M = \kappa_i \omega S^E / g$ - momentum source

C_{gj} - group velocity

▣ Wind input:

$$S_W = \beta \omega A(\varphi - \varphi_p)F(\omega),$$

(Miles, 1957)

$$\beta = c_\beta (u_*/c)^2 \cos^2(\varphi - \varphi_W) \text{ - growth rate}$$

(Plant, 1982; Meirink et al. 2003)

▣ Dissipation:

Wave breaking
(Longuet-Higgins, 1969)

$$D = \omega_p e (k_p^2 e / \varepsilon_T^2)^n$$

$$D = \int S_D d\varphi d\omega$$

$$e = \int E d\varphi d\omega$$

▣ Non-linear interactions:

Four-wave interactions
(Hasselmann, 1962)

Energy transfer towards low frequencies:

$$\langle S_N \rangle \sim E^3$$

(Zakharov, 2010;
Badulin et al., 2007)

$$\frac{d}{dt} x_j = \kappa_j^p \bar{c}_g - V_j \quad - \text{wave train } \underline{\text{position}}$$

$$\frac{d}{dt} \ln(\bar{c}_g e) = -\bar{c}_g G_n + \omega_p (\tilde{I}_w - \tilde{D}) \quad - \text{modified } \underline{\text{energy}}$$

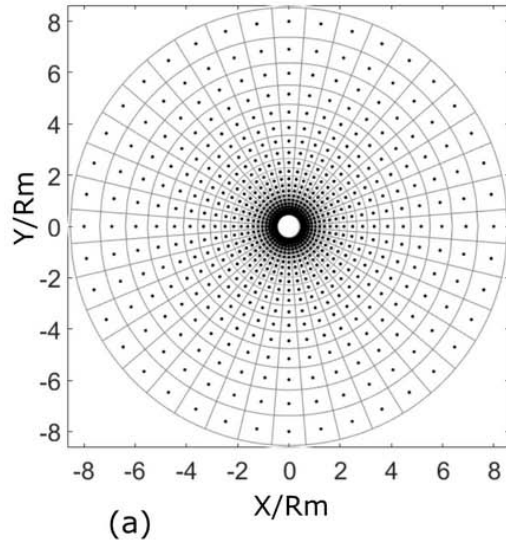
$$\frac{d}{dt} c_{gp} = -\frac{r_g C_\alpha}{2} \Delta_p g (k_p^2 e)^2 \quad - \text{spectral peak } \underline{\text{group velocity}} \text{ (from eq. for frequency)}$$

$$\frac{d}{dt} \varphi_p = C_\varphi \alpha^2 \omega_p H_p \sin[2(\varphi_p - \varphi_w)] \quad - \text{spectral peak } \underline{\text{direction}}$$

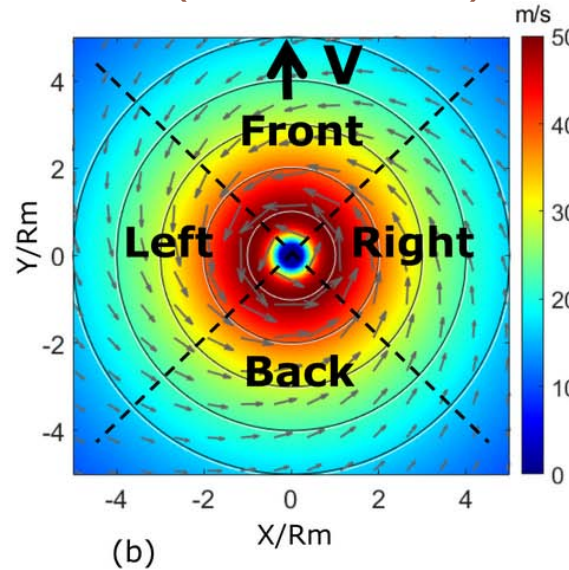
$$dG_n/dt + G_n/T + \bar{c}_g G_n^2 = G_w^n/T \quad - \text{peak direction gradient (} \underline{\text{focusing}} \text{ term), } G_n = \Delta\varphi_p/\Delta n \text{ (or two eq. instead: for } \Delta\varphi_p \text{ and } \Delta n \text{)}$$

The system describes the development of surface waves under a varying wind field in both space and time, and the evolution of swell propagation in the absence of wind forcing

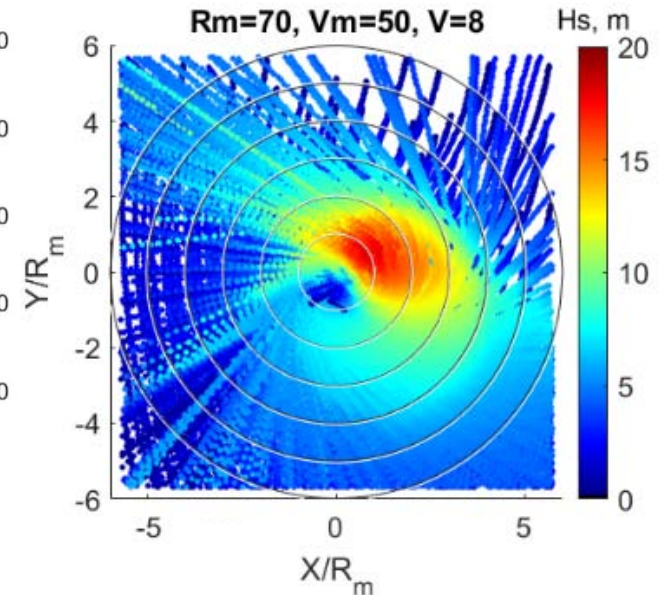
Initial wave train locations



Wind field (Holland 1980)

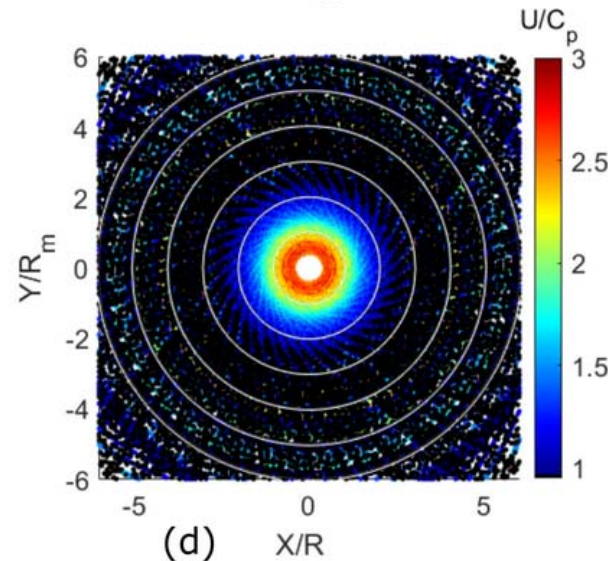
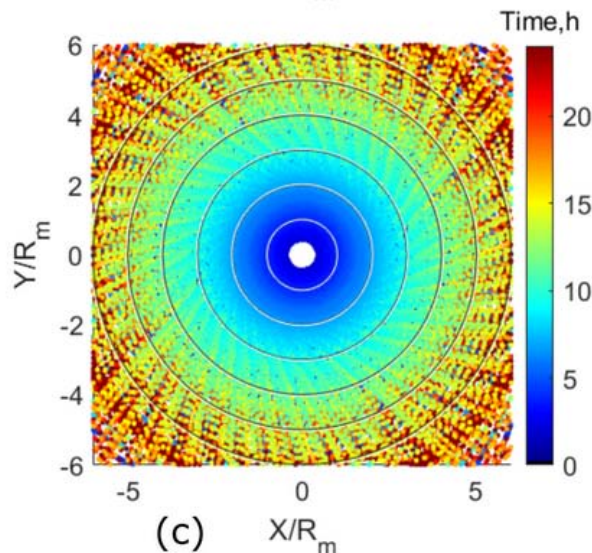
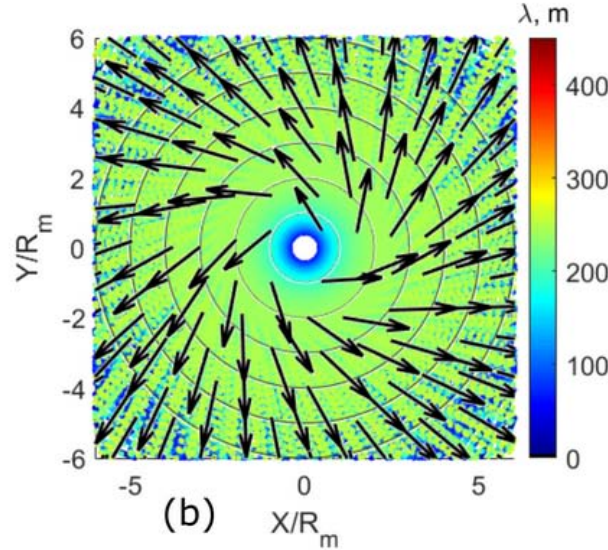
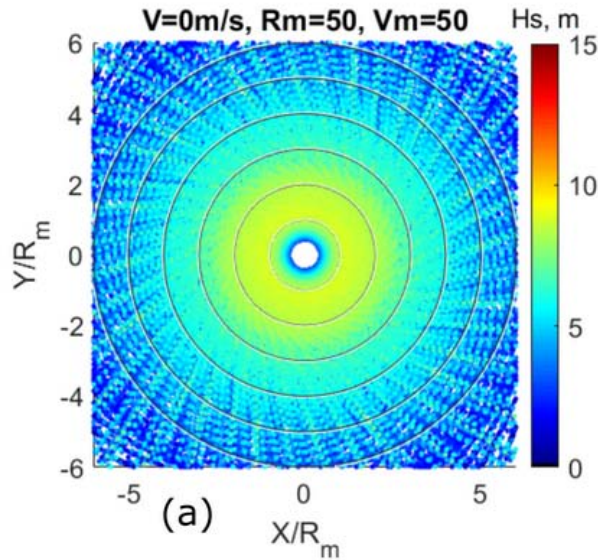


Wave rays after 48 hours



- ▶ The model is solved in the storm frame of reference with Holland (1980) wind input
- ▶ Each wave train location and wave parameters (peak frequency, energy and direction) at each moment of time are obtained with the use of 4th order Runge-Kutta scheme
- ▶ Wave-rays visualize how wave trains develop and travel through the TC varying wind field, and how they leave the storm area as swell systems.
- ▶ The wave train with maximal wave length/energy in a given grid cell can be treated as the primary wave system. Each grid cell can be considered to analyze multiple wave systems and their time evolution.

Wave Characteristics in Stationary TC

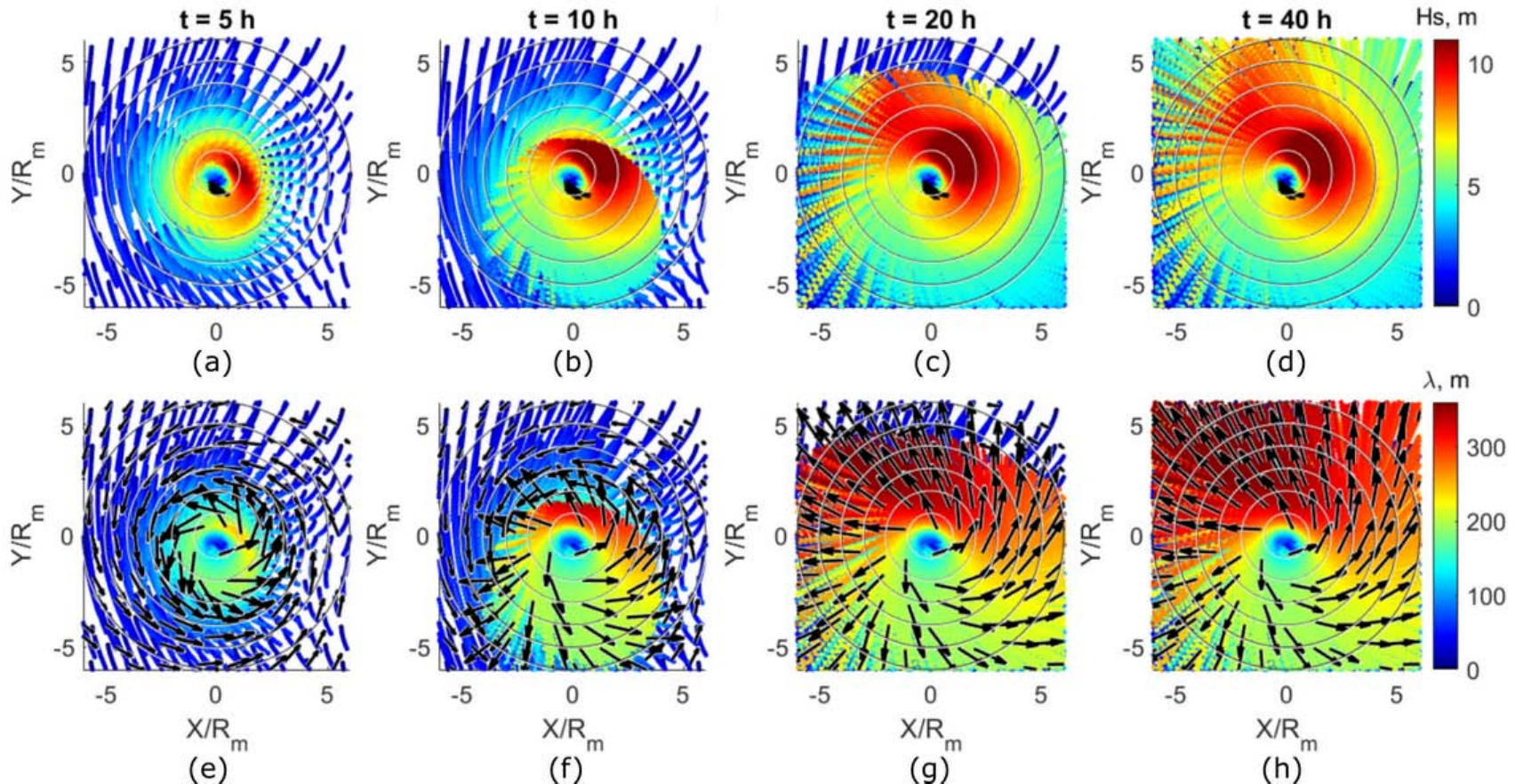


- ▶ Wave field develops from TC center toward its periphery (c)
- ▶ The spatial distribution of wave-train parameters exhibits an apparent azimuthal isotropy (a-d)
- ▶ The maximum of energy is located at radial distance close to R_m (a)
- ▶ Wave trains become locally fully developed at $r \approx 2R_m$ (d)
- ▶ Swell travels outwards almost in radial direction (b)
- ▶ Swells energy decreases, but the wavelength is conserved (a, b)

The rays with the largest local wavelength are on top of all others (hereinafter)

Wave Field Evolution in Moving TC

$R_m = 50 \text{ km}$, $u_m = 50 \text{ m/s}$, $V = 5 \text{ m/s}$



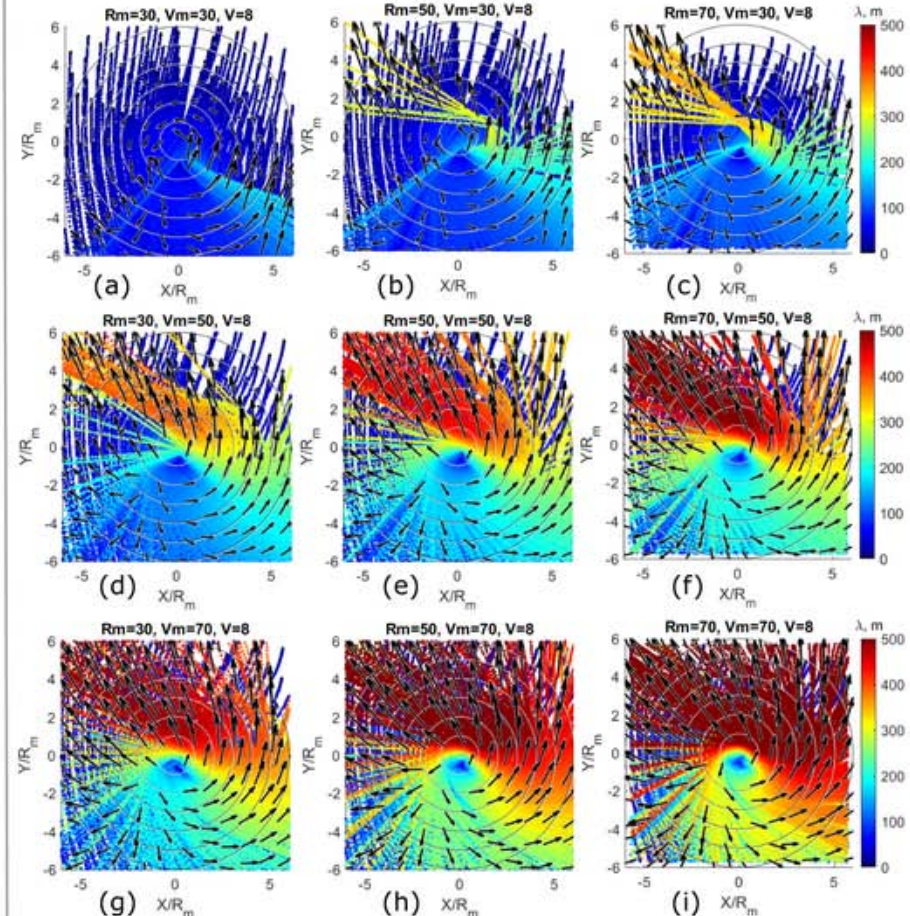
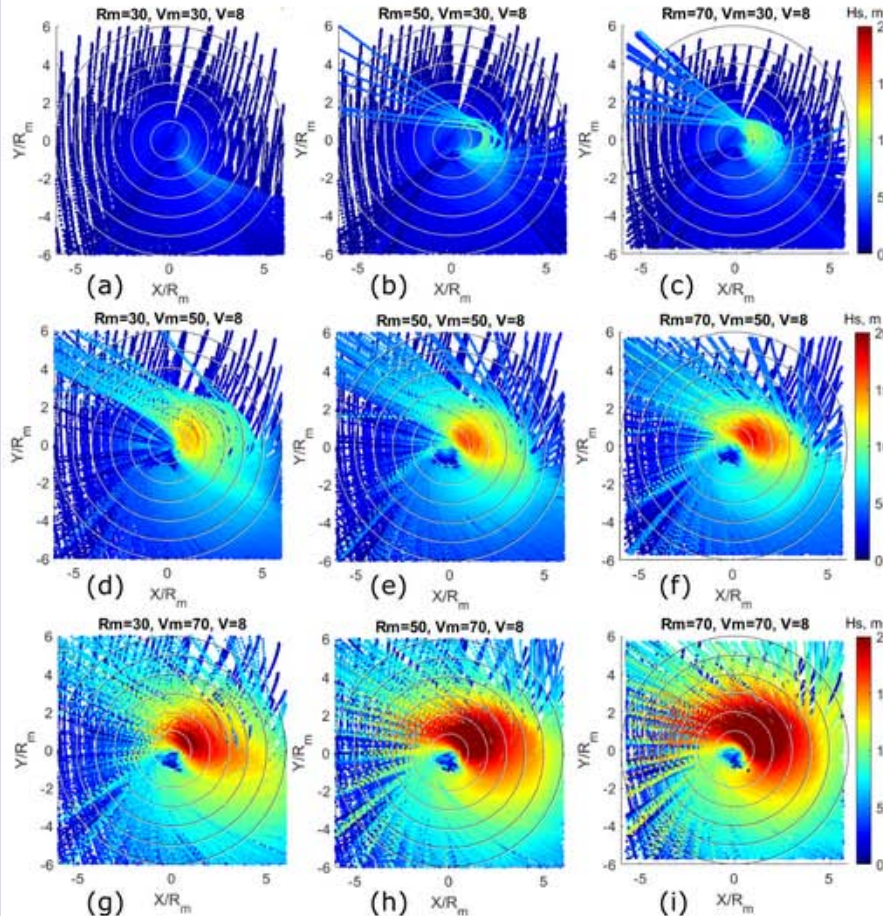
- ▶ The most developed waves appear in the vicinity of the maximum wind speed. A wave front is well detectable
- ▶ Asymmetry appears with the alignment of the group velocity of wave-trains and TC heading in the right-half, and their opposite direction in the left-half
- ▶ At time $t > 10 \text{ h}$, wave-trains subject to group velocity resonance move forward
- ▶ The time of full development ($\sim 24 \text{ h}$) can be considered as the time-scale the wavefield reaches stationarity

SWH and Wavelength for different combinations of R_m , u_m

SWH

Wavelength

$R_m = [30, 50, 70] \text{ km}$, $u_m = [30, 50, 70] \text{ m/s}$, $V = 8 \text{ m/s}$



Wave field properties:

- ▶ Large variability of SWH and peak wavelength fields depending on the TC parameters
- ▶ Similar spatial distributions for some particular combinations of TC parameters (e.g. 30km-70m/s and 70km- 50m/s)



- ▶ The spatial distribution of wave characteristics can be anticipated to be **self-similar**, using proper scaling involving the main TC parameters: Rm , um , and V .
- ▶ Classical self-similarity theory of wave growth in terms of 1D fetch law has already demonstrated its efficiency to describe TC-generated waves using the **equivalent fetch concept** (e.g., *Bowyer & MacAfee, 2005; Young, 2017* and others)
- ▶ *Kudryavtsev et al., 2015*, for wave age and wave energy in 1D case:

$$\begin{aligned} \alpha_p / \alpha_{p0} &= \varphi_\alpha (\tilde{r} / \tilde{L}_{cr}), & \alpha_{p0} &\propto \tilde{r}^q & \tilde{L}_{cr} &= c_{cr} (u_r / 2V)^{1/q} \\ \tilde{e} / \tilde{e}_0 &= \varphi_e (\tilde{r} / \tilde{L}_{cr}), & \tilde{e}_0 &\propto \tilde{r}^p & c_{cr} &= -c_\alpha^{-1/q} q / (1 + q) \end{aligned}$$

- ▶ 2D self-similar solutions in moving TC are anticipated to take the following form:

$$\begin{aligned} e / e_0 &= \Phi_e (r / L_{cr}, \theta), \\ \lambda_p / \lambda_{p0} &= \Phi_\lambda (r / L_{cr}, \theta), \\ \varphi_p - \varphi_{p0} &= \Phi_\varphi (r / L_{cr}, \theta), \end{aligned}$$

Hereinafter zeros indicate wave characteristics in stationary TC

Self-Similarity of Stationary Cyclone

- ▶ Different combinations of TC radii and maximum wind speeds:

$$R_m = 30, 50, 70 \text{ km},$$

$$U_m = 30, 50, 70 \text{ m/s}$$

- ▶ For the **same radius**:

the larger the wind speed, the larger the wavelength and H_s

- ▶ For the **same maximum wind speed**:

the larger the radius, the larger the wavelength and H_s



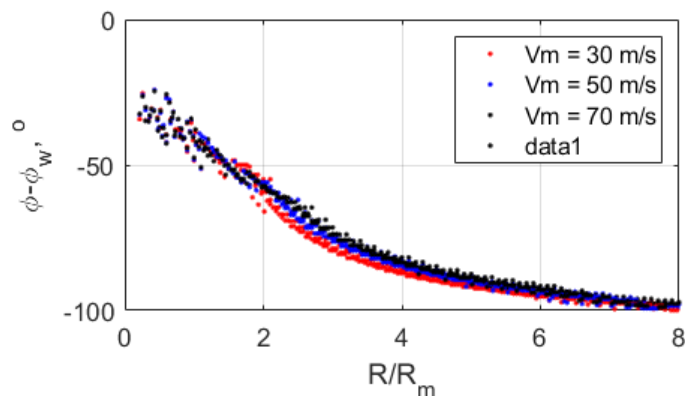
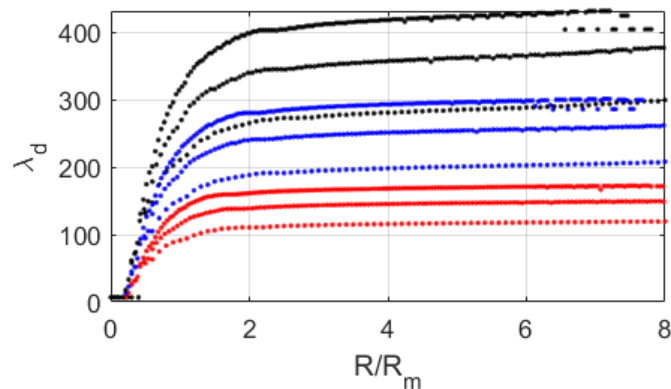
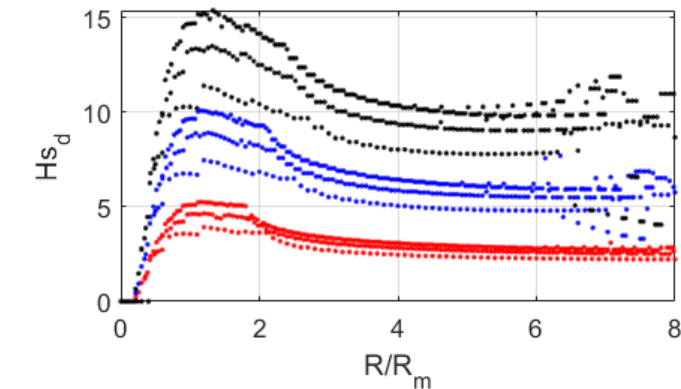
Scale by the values associated with fetch laws:

$$\tilde{H}s_d = \tilde{H}s / \tilde{R}_m^{p/2}$$

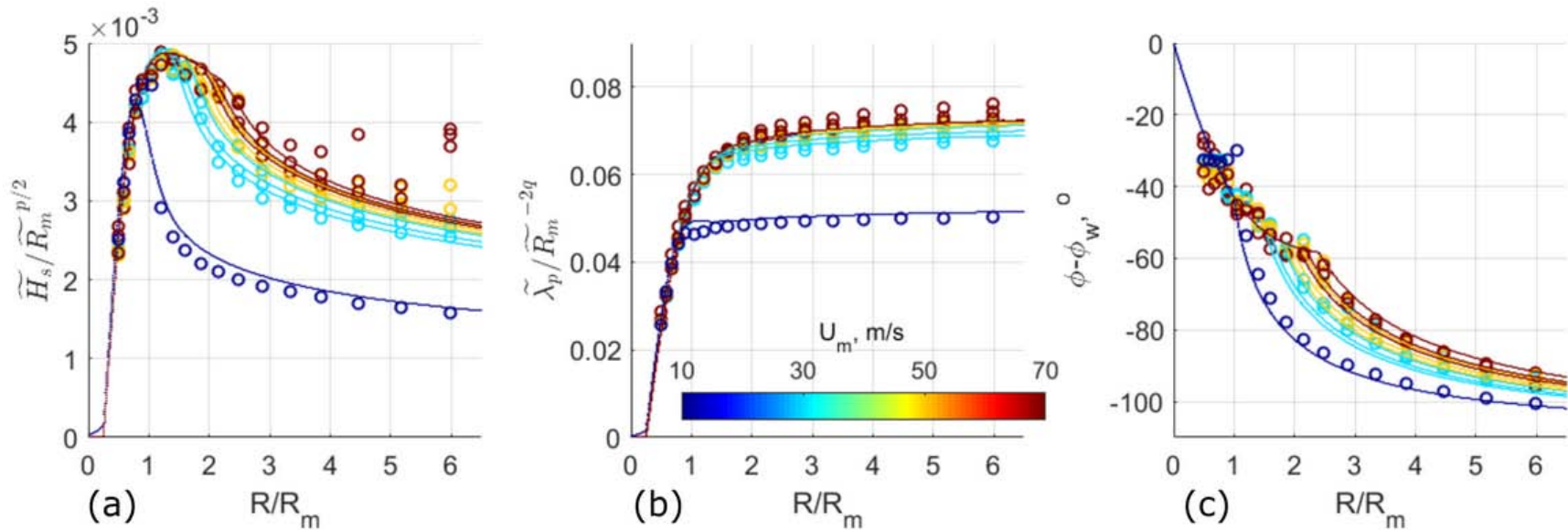
$$\tilde{\lambda}_d = \tilde{\lambda} / \tilde{R}^{-2q}$$

$$\tilde{R}_m = R_m g / u_m^2, \tilde{\lambda} = \lambda g / u_m^2, \dots$$

$$p = 3/4, q = -1/4$$



Self-Similarity of Stationary Cyclone



- ▶ Profiles collapse, exhibiting self-similar shapes for radial distances $\rho = R/R_m < 1.5$
- ▶ Swell regime after $\rho_0(\tilde{R}_m) = 5\tilde{R}_m^{-0.2}$
- ▶ Parameterization:

$$\begin{aligned}\tilde{H}_{s0} &= \tilde{R}_m^{p/2} f_h(\rho, \tilde{R}_m) \\ \tilde{\lambda}_{p0} &= \tilde{R}_m^{-2q} f_\lambda(\rho, \tilde{R}_m) \\ \varphi_{p0} - \varphi_w &= f_\varphi(\rho, \tilde{R}_m)\end{aligned}$$

$$f_y = [f_y^w - (f_y^w - f_y^s)H_y]$$

$$H_{h,\lambda} = H(\rho - \rho_0) \text{ - Heaviside function}$$

$$H_\varphi = H[\rho - (\rho_0 + 0.2)]$$

Developing waves:

$$\begin{cases} f_h^w(\rho) = 5.6 \times 10^{-3} \tanh[2(\rho - 0.25)][1 - 0.1(\rho - 0.25)] \\ f_\lambda^w(\rho) = 6.8 \times 10^{-2} m_\lambda \tanh[1.5(\rho - 0.25)] \\ f_\varphi^w(\rho) = -(\pi/3) \tanh(0.9\rho) \end{cases}$$

Swell:

$$\begin{cases} f_h^s(\rho) = f_{h0}^w \left[1 + 5.3 \times 10^3 (f_{h0}^w)^4 (f_{\lambda 0}^w)^{-5} (\rho - \rho_0) \right]^{-1/4} \\ f_\lambda^s(\rho) = f_{\lambda 0}^w \left[1 + 0.2 \ln \left(1 + 5.3 \times 10^3 (f_{h0}^w)^4 (f_{\lambda 0}^w)^{-5} (\rho - \rho_0) \right) \right]^{1/5} \\ f_\varphi^s(\rho) = -\arccos \left[((\rho_0 + 0.2)/\rho) \cos(f_\varphi^w + \varphi_{in}) \right] - \varphi_{in} \end{cases}$$

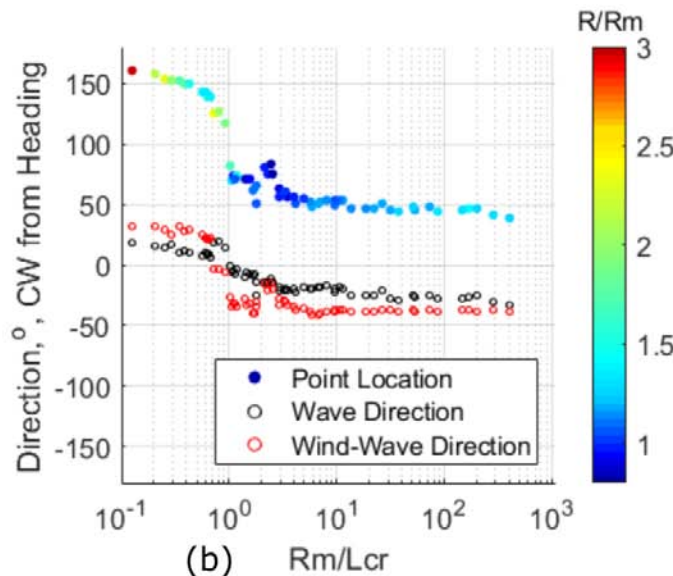
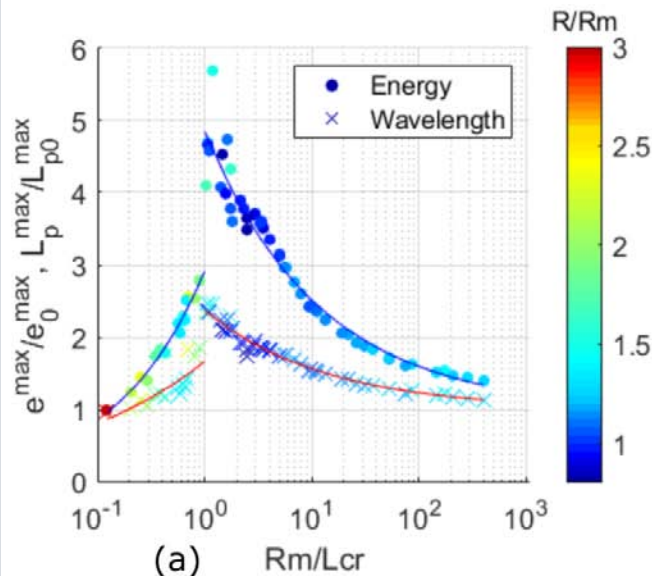
Moving TCs: Maxima of SWH and Wavelength

- 63 combinations of R_m , u_m , V with Holland (1980) wind profile
 - Maximal energy and wavelength are scaled by those of stationary TC:
- and are plotted as function of $\tilde{R}_m / \tilde{L}_{cr}^m$ ($\tilde{L}_{cr}^m = c_{cr} (u_m / 2V)^{1/q}$)

$$e_0^{\max} = a_e (u_m^4 / g^2) \tilde{R}_m^p$$

$$\lambda_{p0}^{\max} = a_\lambda (u_m^2 / g) \tilde{R}_m^{-2q}$$

- “Slow” TCs:** $\tilde{R}_m / \tilde{L}_{cr}^m > 1$; **“fast” TCs:** $\tilde{R}_m / \tilde{L}_{cr}^m < 1$
- $\tilde{R}_m / \tilde{L}_{cr}^m \approx 1$: the group velocity resonance and the largest possible waves
- The location of maximum for “slow” TCs is between front and right sectors
- The location of maximum for “fast” TCs is between right and back sectors
- Deviation of wave direction from the wind is about 40 deg to the right for “slow” TCs), and about 40deg to the left for “fast” TCs



For practical applications:

$$\tilde{e}^{\max} / \tilde{e}_0^{\max} = l_e + m_e \left(\tilde{R}_m / \tilde{L}_{cr}^m \right)^{n_e}$$

$$\tilde{\lambda}_p^{\max} / \tilde{\lambda}_{p0}^{\max} = l_\lambda + m_\lambda \left(\tilde{R}_m / \tilde{L}_{cr}^m \right)^{n_\lambda}$$

“Slow” TCs:

$$[l_e, m_e, n_e] = [1, 3.84, -0.4]$$

$$[l_\lambda, m_\lambda, n_\lambda] = [1, 1.37, -0.38]$$

“Fast” TCs:

$$[l_e, m_e, n_e] = [0, 2.92, 0.53]$$

$$[l_\lambda, m_\lambda, n_\lambda] = [0, 1.67, 0.31]$$

Equivalent fetch is substituted into the fetch laws to provide fast estimates of maximal values of energy and wavelength of waves generated by a moving TC:

$$\begin{aligned}\tilde{e}^{\max} &= c_e \tilde{X}_e^p \\ \tilde{\lambda}_p^{\max} &= c_\lambda \tilde{X}_\lambda^{-2q}\end{aligned}\quad \text{with fetch law constants:} \quad \begin{aligned}p &= 3/4 & c_e &= 1.3 \times 10^{-6} & c_\alpha &= 11.8 \\ q &= -1/4 & c_\lambda &= 2\pi c_\alpha^{-2} = 4.51 \times 10^{-2}\end{aligned}$$

$$\begin{aligned}\tilde{X}_e &= 1.1 \times \tilde{R}_m \left[l_e + m_e \left(\tilde{R}_m / \tilde{L}_{cr}^m \right)^{n_e} \right]^{1/p} \\ \tilde{X}_\lambda &= 1.8 \times \tilde{R}_m \left[l_\lambda + m_\lambda \left(\tilde{R}_m / \tilde{L}_{cr}^m \right)^{n_\lambda} \right]^{-1/(2q)}\end{aligned}$$

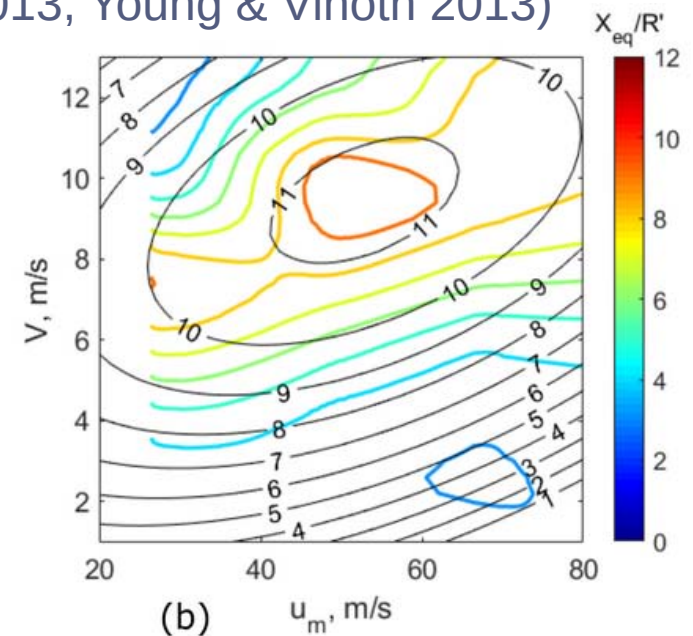
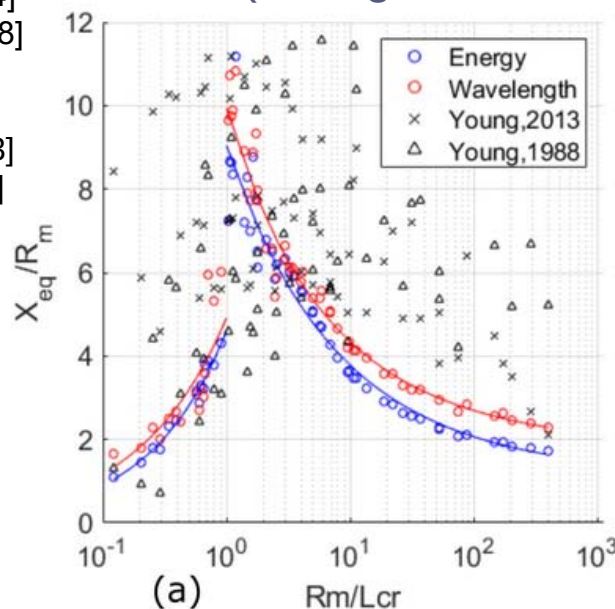
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“Fast” TCs:

$$\begin{aligned}[l_e, m_e, n_e] &= [0, 2.92, 0.53] \\ [l_\lambda, m_\lambda, n_\lambda] &= [0, 1.67, 0.31]\end{aligned}$$

Comparison with empirical relationships
(Young 1998, Young 2013, Young & Vinoth 2013)



- ▶ For 63 combinations of $[R_m, U_m, V]$:

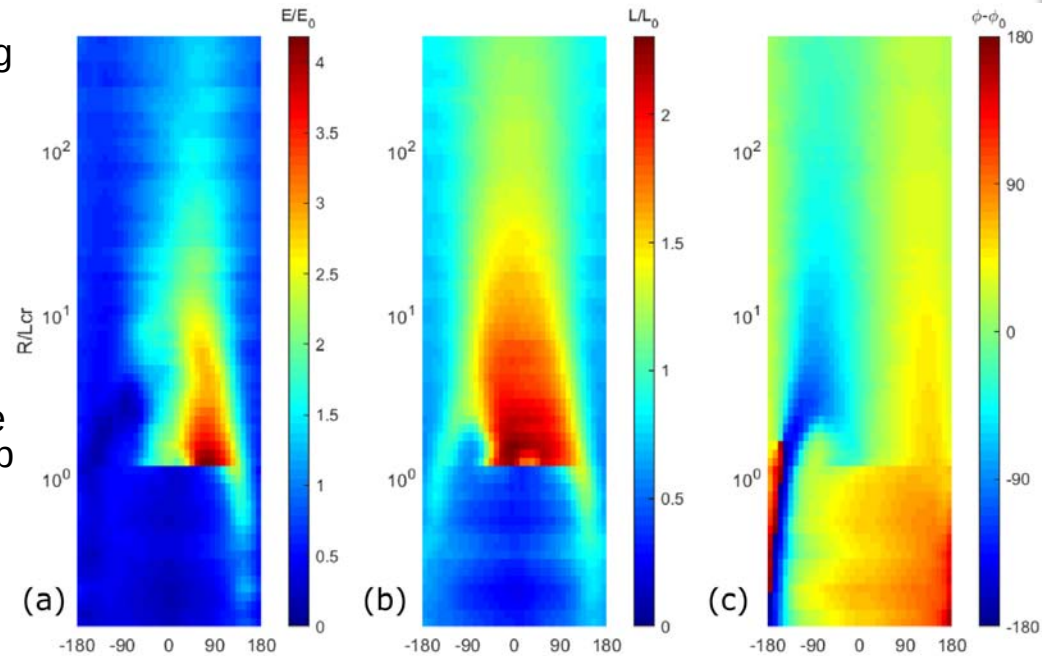
the energy and wavelength of primary wave trains are normalized by their corresponding stationary values and represented as function of azimuth and r/L_{cr} ,

- r is the distance from TC center
- L_{cr} is a critical fetch:

$$\tilde{L}_{cr} = c_{cr} (u_r/2V)^{1/q}$$

$$c_{cr} = -c_{\alpha}^{-1/q} q/(1+q)$$

- ▶ These fields were further averaged over the ensemble of 63 considered TCs in a look-up table (matrices).
- ▶ Transects of these functions over the TC azimuth at fixed $x=L_{cr}$ provide azimuth distributions of the primary wave system parameters relative to a stationary TC condition.
- ▶ Primary wave systems predicted by the TC-wave GMF on the outer radius, $r/R_m = 3$, can be further used as the boundary conditions for swell propagation



The universal functions are freely available:

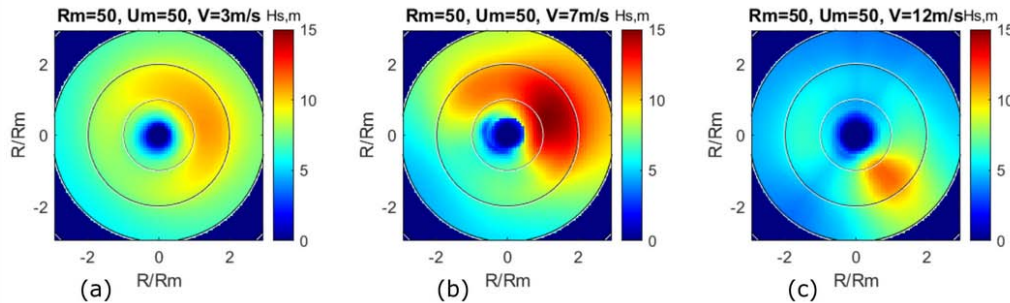
<https://zenodo.org/record/4609996#.YGmoDD9n2UI>

Self-similar solutions (TC-wave GMF) generalize the 1D fetch laws for 2D TC-generated waves

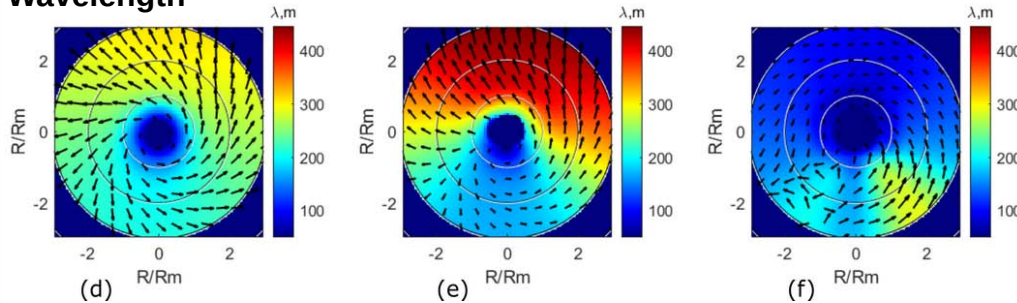
TC-wave GMF is rapid to quantitatively assess the significant wave height, wavelength and direction fields under a moving TC prescribed by R_m , u_m and V

Example of TC-Wave GMF Implementation

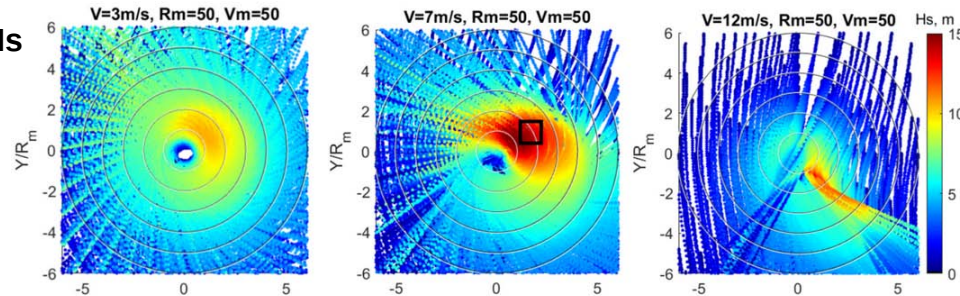
Hs



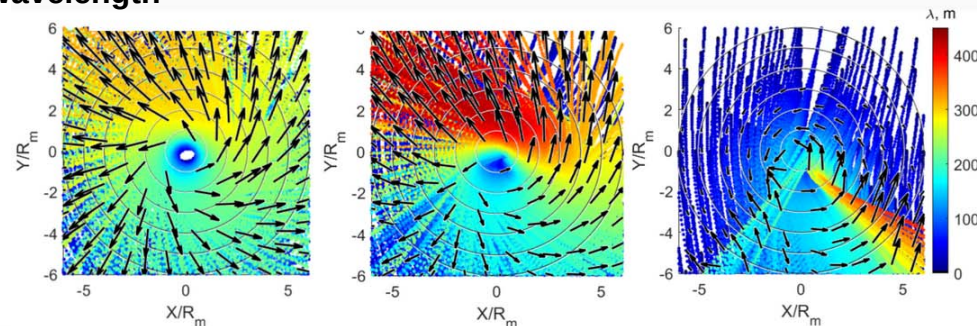
Wavelength



Hs



Wavelength



Hs and wavelength fields
from self-similar functions

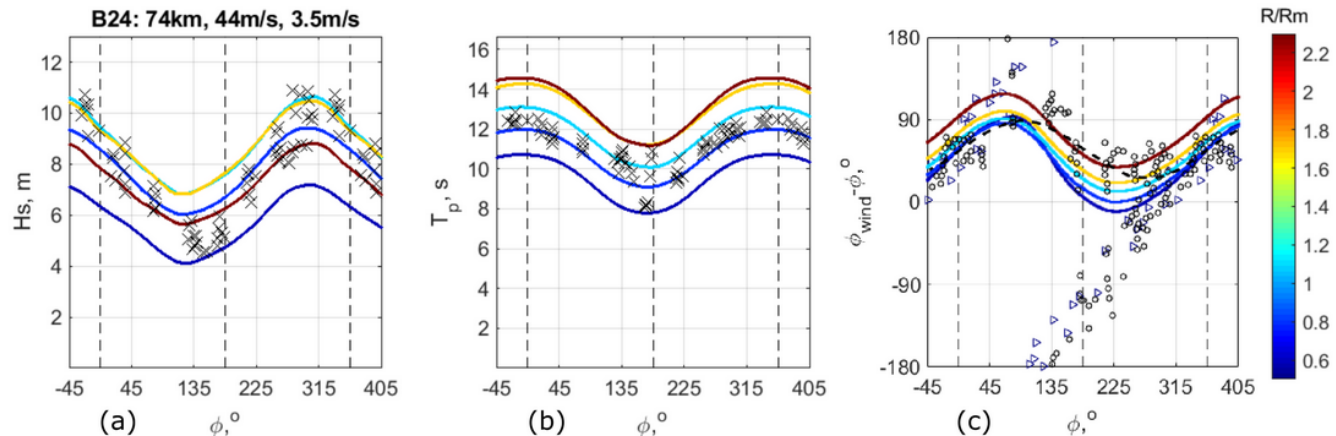
$R = 50 \text{ km}$,
 $um = 50 \text{ m/s}$
 $V = 3, 7, 12 \text{ m/s}$,

Direct model calculations

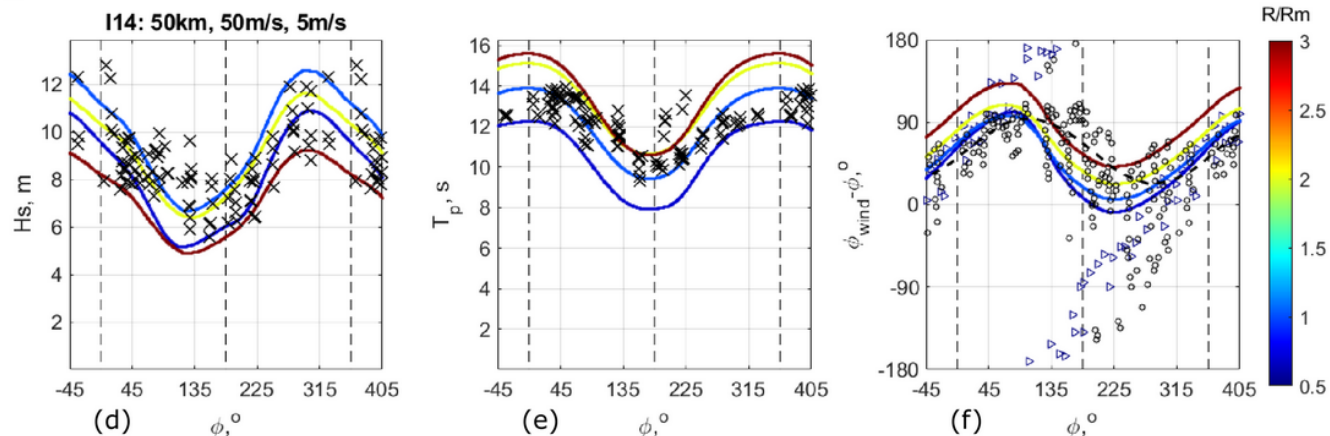
Self-similar solutions provide less detailed and smoother fields than ray composition, but stay quantitatively close

Azimuthal variations of SWH, peak wave period and wave direction relative to wind one of primary wave system generated by TCs

TC Bonnie



TC Ivan



- ▶ Solid lines are obtained from the transects of dimensionless universal functions
- ▶ Crosses and circles are the SRA measurements (Hwang and Fan 2017) Triangles are direct model calculations in the vicinity of TC center
- ▶ The model reproduces well the magnitudes and the azimuthal modulations of observed SWH, wave period and wave direction

- ▶ A new 2D-parametric model for wave field evolution forced by space-time varying wind field (Kudryavtsev et al., 2021a) has been applied to predict wave field developments under moving TCs
- ▶ Equations are solved by the method of characteristics, Lagrangian ray-paths, to provide efficient visualization on how the different wave systems develop under a moving TC, and outrun it as swell systems
- ▶ Compared with stationary TC conditions, TC motions introduce remarkable azimuthal anisotropy in the wave-ray patterns and wave-trains dynamics
- ▶ Group velocity resonance leads to the appearance of waves with abnormally high energy, further outrunning as long swell systems through the TC front sector. When the TC translation velocity exceeds a threshold value, waves cannot reach group velocity resonance, and travel backwards in TC reference system
- ▶ Solutions are analytically reduced using self-similar functions, for rapid first guess evaluation for azimuthal-radial distributions of the primary wave system parameters (SWH, wavelength, direction) given the TC characteristics, maximum wind speed, radius, translation velocity
- ▶ Self-similar solutions clearly divide TCs between slow TCs, fulfilling conditions $Rm/Lcr > 1$, and fast TCs corresponding to $Rm/Lcr < 1$, where Lcr is a critical fetch. Around the region $Rm/Lc = 1$, group velocity resonance occurs, leading to the largest possible waves generated by a TC
- ▶ Comparisons between self-similar solutions and measurements, demonstrate a reasonable agreement to warrant scientific and practical applications

More details can be found in

- Kudryavtsev, V., Yurovskaya, M., & Chapron, B. (2021a). 2D parametric model for surface wave development under varying wind field in space and time. *Journal of Geophysical Research: Oceans*, 126, e2020JC016915. doi.org/10.1029/2020JC016915
- Kudryavtsev, V., Yurovskaya, M., & Chapron, B. (2021b). Self-Similarity of Surface Wave Developments under Tropical Cyclones. *Journal of Geophysical Research: Oceans*, 126, e2020JC016916. doi.org/10.1029/2020JC016916
- <https://zenodo.org/record/4609996#.YGmoDD9n2UI> (TC-wave GMF with Matlab example)

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