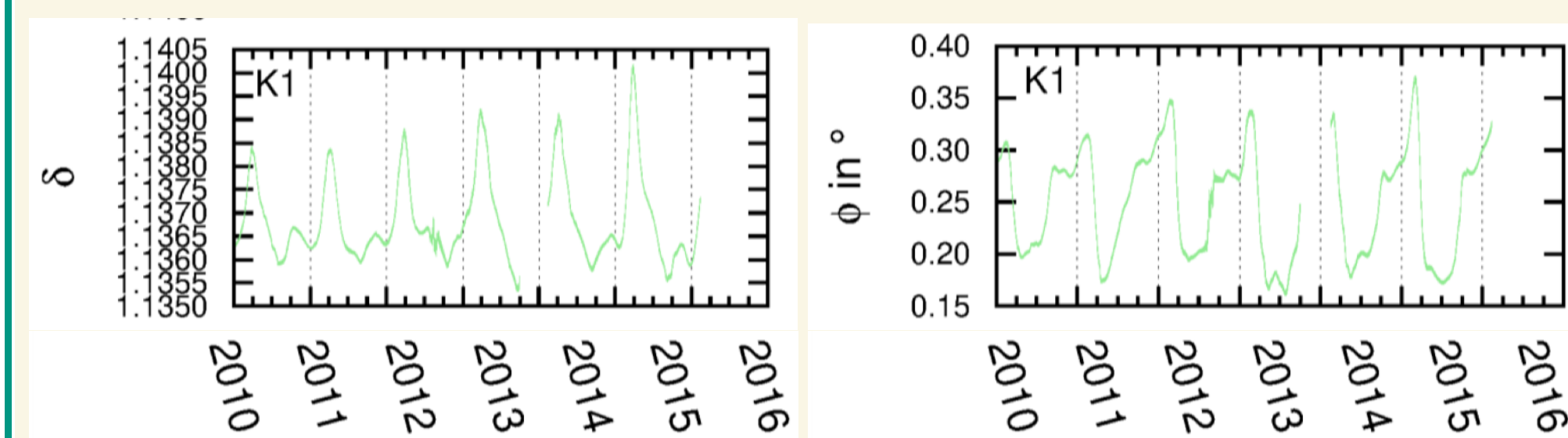


Investigation of bias in traditional grouping by means of regularization

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1. Motivation

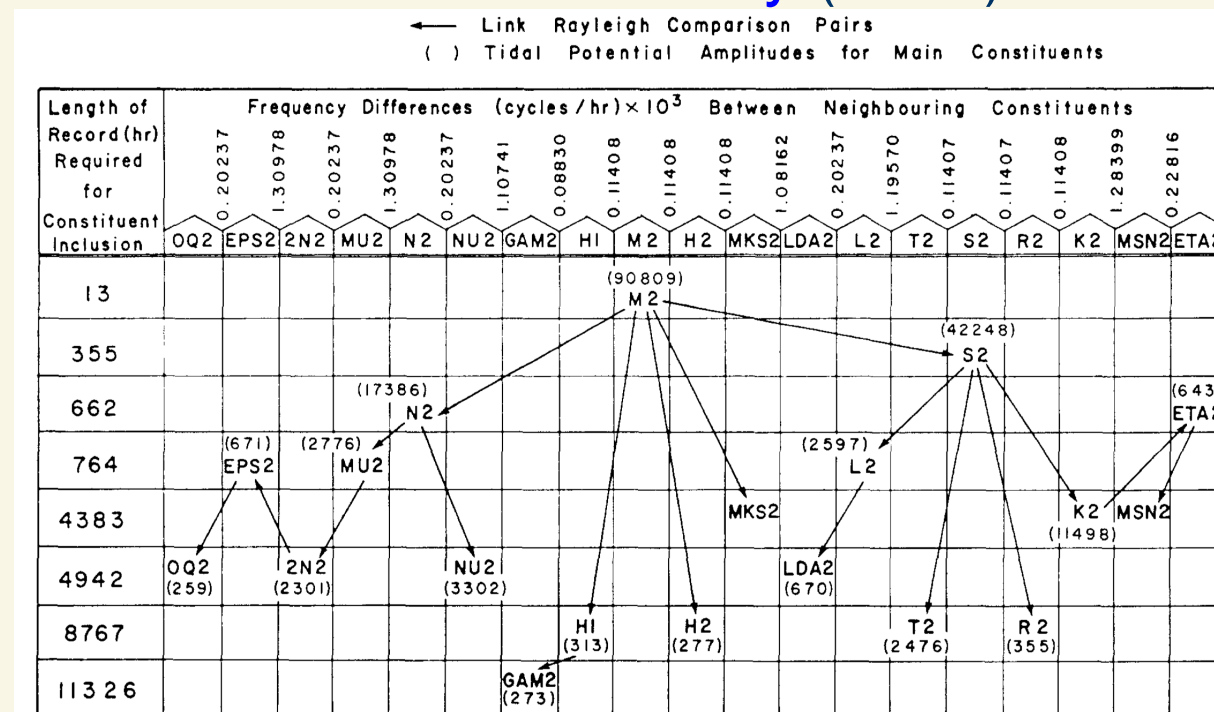
The research was initiated by reported temporal variations of tidal parameters in moving window tidal analysis of gravity recordings (Schroth et al., 2018; Meurers et al., 2016). Since gravimetric factor and phase reflect elastic and anelastic properties of the Earth, they should not change within a short time scale (a few years). We seek for an explanation of that phenomenon.



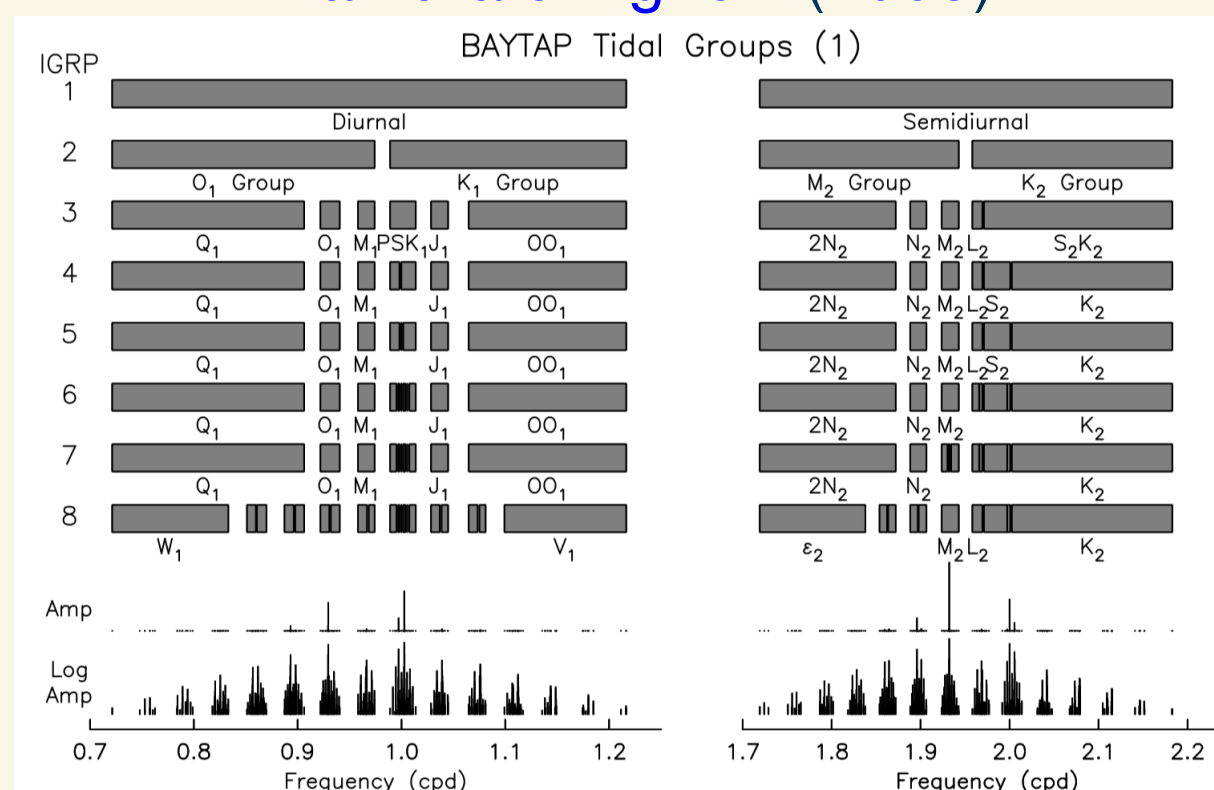
Temporal variations of tidal parameters in K1 group, windows of 90 hours of measurements (Schroth et al., 2018)

2. Wave grouping bias

Foreman & Henry (1989)



Tamura & Agnew (2008)



Wenzel (1996)

SA	SSA	MM	MTM	G1	O1	M1	P1	S1	K1	PS11	PH11	J1	OO1	2N2	N2	M2	L2	S2	K2	M3M6
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885	0.054747	0.020885
0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748	0.091348	0.054748
0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349	0.501369	0.091349
0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370	0.911390	0.501370
0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391	0.947991	0.911391
0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992	0.981854	0.947992
0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855	0.998632	0.981855
0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633	1.004107	0.998633
1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108	1.006846	1.004108
1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847	1.023623	1.006847
1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624	1.057485	1.023624
1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486	1.470243	1.057486
1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244	1.880265	1.470244
1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266	1.914128	1.880266
1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129	1.950419	1.914129
1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420	1.984283	1.950420
1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284	2.451944	1.984284
2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945	7.000000	2.451945
7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001	-	7.000001

To determine gravimetric (tidal) parameters, a least-square (LS) method for wave groups that are sums of inseparable harmonics was introduced (Venedikov, 1962; Chojnicki, 1973). The groups are defined by the Rayleigh criterion: i.e. for a given time series of length T , the difference between frequencies of major constituents Δf satisfy $\Delta f \cdot T > R$ (Godin, 1970). However, the tidal analysis problem boils down to an algebraic set of equations (Munk & Hasselmann, 1964; Ducarme & Schüller, 2018).

3. Methods

We abandoned the a priori wave grouping, so **each tidal harmonic receives its set of tidal parameters**. Because this would result in an ill-conditioned matrix equation, we introduced model constraint (Tikhonov regularisation) to the LS objective function. The new software, named **RATA** (*Regularized Approach to Tidal Analysis*), was developed for that purpose (release soon). We minimize the objective function:

$$\varphi(\vec{m}) = \|\mathbf{G}\vec{m} - \vec{g}\|_2^2 + \alpha_1^2 \|\mathbf{W}_1 \vec{m}\|_2^2 \quad (1)$$

in the Least Squares (LS) sense. We define $\mathbf{G}$ as a forcing matrix (derived from equilibrium tides), $\vec{g}$ is the observed gravity record vector, $\vec{m}$ is a difference to the reference model and α_1 is a trade-off parameter. The normal equation can be factorized to:

$$(\mathbf{A}^T \mathbf{A} + \alpha_1^2 \mathbf{I}) \vec{x} = \mathbf{A}^T \vec{d} \quad (2)$$

Singular value decomposition of $\mathbf{A} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T$ increases the efficiency of solution computation, because the estimated vector can be expressed as:

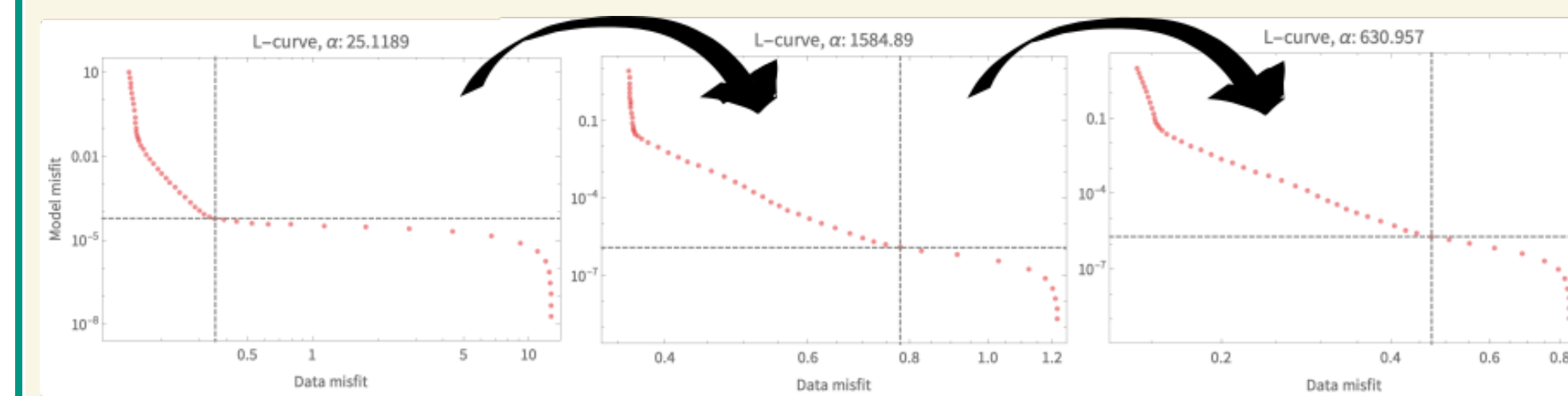
$$\vec{x} = \sum_{i=1}^K f_i \frac{\mathbf{U}_i^T \vec{d}}{\lambda_i} \mathbf{V}_i \quad (3)$$

and $K = \min(M, N)$ such that all nonzero singular values λ_i and vectors are included. The quantities

$$f_i = \frac{\lambda_i^2}{\lambda_i^2 + \alpha_1^2} \quad (4)$$

are Wiener filter factors.

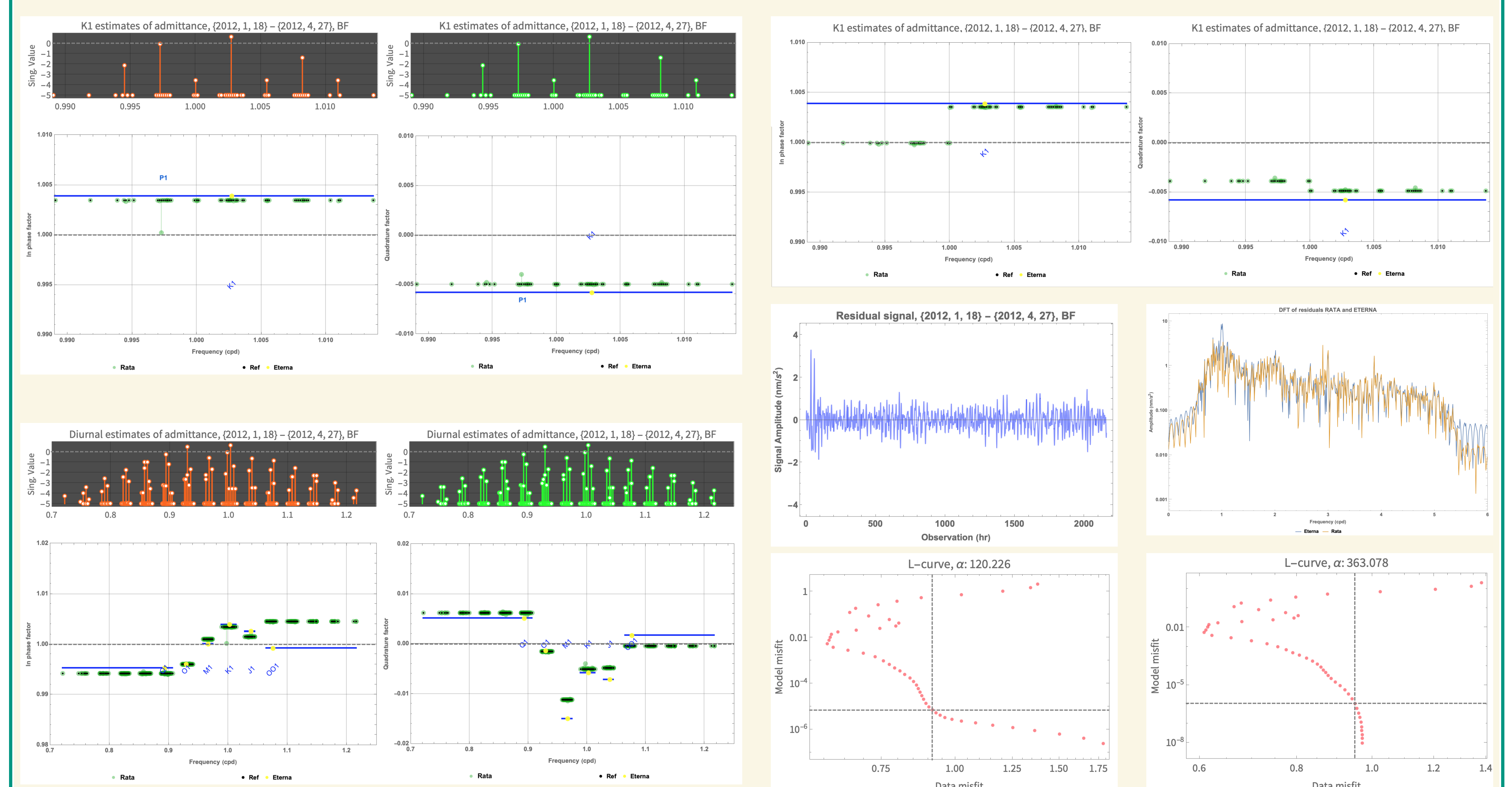
4. Iterative reference model



5. Setup

In our study, we used a harmonic development of the tide-generating potential catalogue from Tamura (1987). The analysed data were recorded with a double-sphere superconducting gravimeter GWR SG-056 at the Black Forest Observatory.

6. Results



7. Summary

- Wave groups do not need to be assumed prior to analysis, the only grouping exists in the adaptive reference model.
- Bias in the standard approach might be investigated using all tidal harmonics individually in the analysis
- There is a possibility to identify significant forcing contributions to the gravity signal.
- The method how **exactly** the final solution could be chosen is not established yet.
- Resolution (inference) in Earth tidal analysis might be increased.
- Investigations with focus on radiation tides and different degree tides will be carried out.

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