

Towards a Global Unified Physical Height System

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Outline

- Motivation
- Definition of the International Height Reference System (IHR)
- International Height Reference Frame (IHRF): station selection for the reference network
- Some considerations for the determination of IHR/IHRF coordinates
- Empirical experiments for a reliable determination of potential values
- Outlook



Motivation

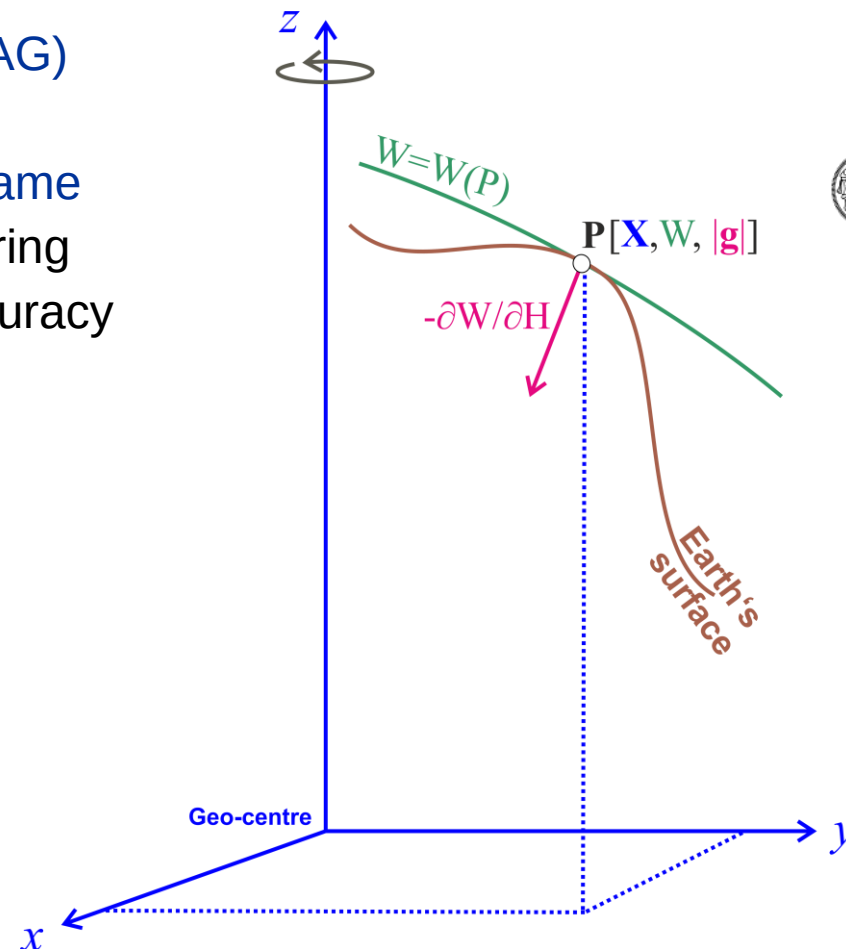
A main objective of the International Association of Geodesy (IAG) and its Global Geodetic Observing System (GGOS) is the implementation of an integrated Global Geodetic Reference Frame (GGRF) that supports the consistent determination and monitoring of the Earth's geometry, rotation, and gravity field with high accuracy worldwide.

The GGRF includes:

- Geocentric Cartesian coordinates $\mathbf{X}$, $\dot{\mathbf{X}}$
- Gravity vector $\mathbf{g}$, $\dot{\mathbf{g}}$
- Potential of the Earth's gravity field W , $\dot{W}$
- Physical height H , $\dot{H}$

Positions of points or objects on or close to the Earth's surface are to be given by:

$$W(\mathbf{X}), P\{W, \mathbf{X}\}, \text{ or } P\{\mathbf{X}, W, \mathbf{g}\} = P\{\mathbf{X}, W, -\nabla W\}$$
$$\text{and } g = -\partial W / \partial H$$



See: IAG (2017) *Description of the Global Geodetic Reference Frame*. Position paper adopted by the IAG Executive Committee in April 2016. J Geod 91,113–116, <https://doi.org/10.1007/s00190-016-0994-6>.

Geocentric Cartesian coordinates today

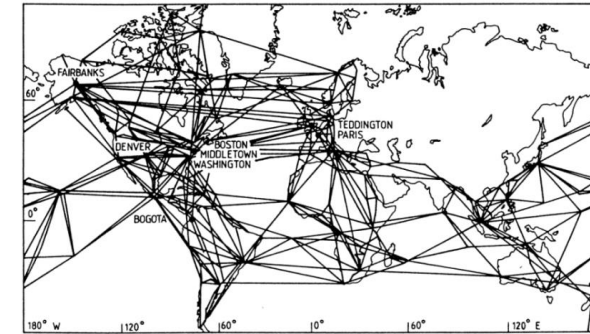
- refer to the [International Terrestrial Reference System \(ITRS\) and Frame \(ITRF\)](#)
- standardised computation through the IERS (International Earth Rotation and Reference Systems' Service);
- supported by a strong operational infrastructure (reference stations, data centres, analysis centres, combination centres, product centres, etc.) acting under the responsibility of the geometric IAG services;
- worldwide unified reference frame;
- reliability at the cm-level;
- based on an experience larger than 30 years (first realisation of the ITRS in 1988 – ITRF0).
- “automatically” accepted and used by everyone using GNSS techniques and ITRF-based satellite orbits.



References for physical coordinates today

1) Surface gravity observations refer to the [International Gravity Standardisation Net 1971 \(IGSN71\)](#)

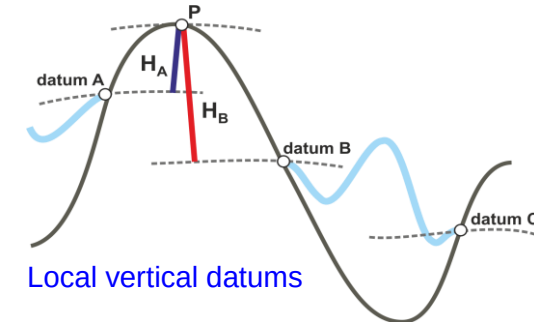
- Accuracy: $1\mu\text{ms}^{-2}$ (100 μGal)
- 10 absolute gravity stations
- 1,200 pendulum and 24,000 relative observations
- Potsdam datum correction -14mGal



IGSN71 (after Morelli et al. 1976)

2) Physical heights refer to more than [100 vertical datums](#)

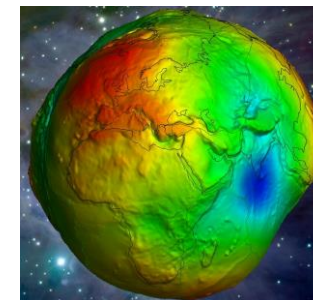
- Different reference levels (many [dm] of discrepancy);
- Different types of heights (normal, orthometric, etc.);
- Omission of (sea and land) vertical variations with time;
- Unprecise combination of h-H-N



Local vertical datums

3) (Static) geoid

- Accuracy at the cm-level at the long wavelengths (~ 100 km) thanks to the satellite gravity missions, but [more than 70 models since 2008](#)
- Accuracy at the short wavelengths depends on the availability of surface (terrestrial, airborne, marine) gravity data and terrain models
- Different geoid modelling approaches lead to different results (discrepancies up to the dm-level).



EIGEN-6C4 geoid model, ICGEM

IAG Resolutions 2015

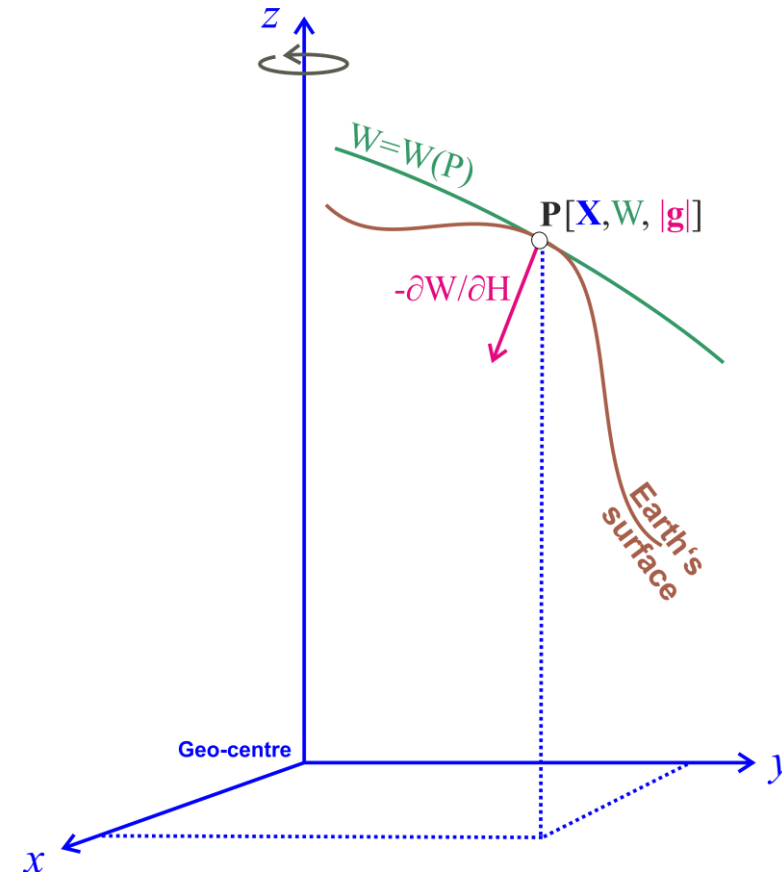
The establishment of a GGRF demands the implementation of a worldwide-unified (standardised) physical reference system.

A first concrete step oriented to this purpose was the release of two IAG resolutions during the IUGG2015 General Assembly (Prague, July 2015):

- one for the **definition and realisation of an International Height Reference System (IHR)**, and
- the second one for the establishment of an **International Gravity Reference System (IGRS)** based on absolute gravity measurements (as replacement of the IGSN71).

→ For the IAG resolutions, see Drewes et al. (2016), *The Geodesist's Handbook 2016*, J Geod, <https://doi.org/10.1007/s00190-016-0948-z>

→ This presentation concentrates on the IHR. For more details on the IGRS, please refer to Wziontek et al. (2021), *Status of the International Gravity Reference System and Frame*, J Geod 95, 7, <https://doi.org/10.1007/s00190-020-01438-9>



Definition of the International Height Reference System (IHR)

IAG Resolution No. 1, Prague, July 2015

- 1) Vertical coordinates are **potential differences** with respect to a **conventionally fixed** W_0 value:

$$C_P = C(P) = W_0 - W(P) = -\Delta W(P)$$

$$W_0 = \text{const.} = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

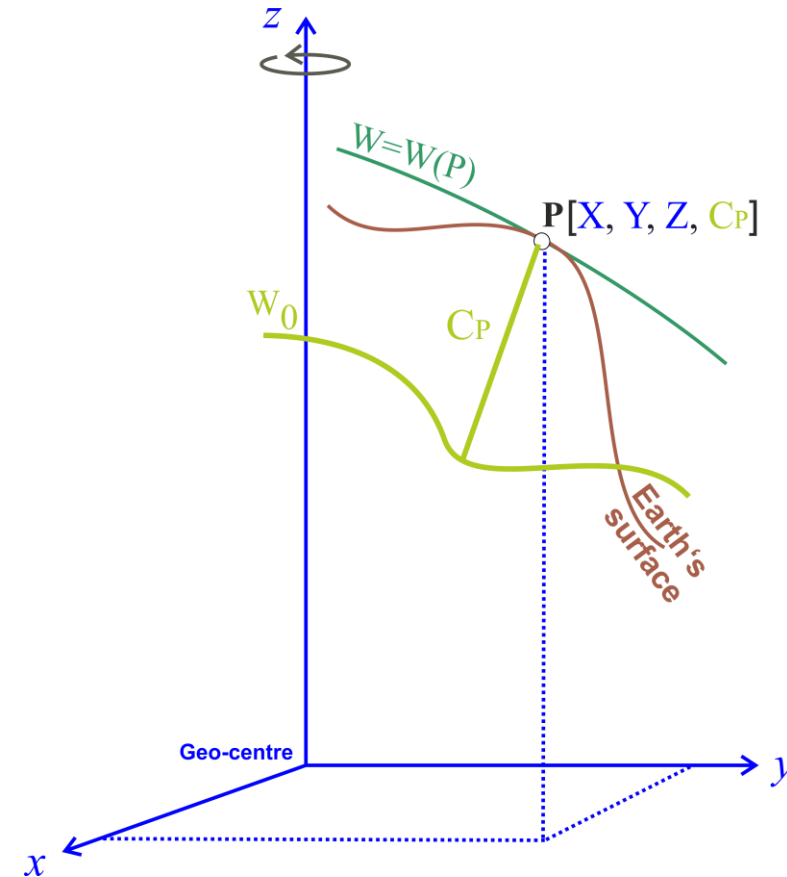
- 2) The position P is given in the ITRF

$$\mathbf{X}_P (X_P, Y_P, Z_P); \text{ i.e., } W(P) = W(\mathbf{X}_P)$$

- 3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).

- 4) Coordinates are given in **mean-tide system** / **mean (zero) crust**.

- 5) The unit of length is the **meter** and the unit of time is the **second** (SI).



→ Ihde et al. (2017), *Definition and proposed realization of the International Height Reference System (IHR)*. Surv Geophy 38(3), 549-570, <https://doi.org/10.1007/s10712-017-9409-3>

→ Sánchez et al. (2016), *A conventional value for the geoid reference potential W_0* , J Geod, 90(9): 815-835, <https://doi.org/10.1007/s00190-016-0913-x>,

Realisation of the IHRS

A reference frame realises a reference system in two ways:

- physically, by a **solid materialisation of points** (or observing instruments),
- mathematically, by the **determination of coordinates** referring to that reference system. The coordinates of the points are computed from the measurements following the definition of the reference system.

The implementation of the IHRS mainly requires:

- 1) A global **reference network** for the IHRS realisation: the International Height Reference Frame (IHRF)
- 2) **The determination of reference IHRF coordinates (W , $\dot{W}$, $\mathbf{X}$, $\dot{\mathbf{X}}$)** at the reference stations
- 3) Clear **standards, conventions and procedures** to ensure consistency between the definition (IHRS) and the realisation (IHRF)
- 4) **Operational and organisational infrastructures** (reference stations, data centres, analysis centres, combination centres, product centres, etc.) to guarantee maintenance and availability of the IHRF.



Criteria for the IHRF reference network configuration



1) Hierarchy:

- A **global network** → worldwide distribution, including
- A **core network** → to ensure sustainability and long term stability
- **Regional and national densifications** → local accessibility

2) Collocated with:

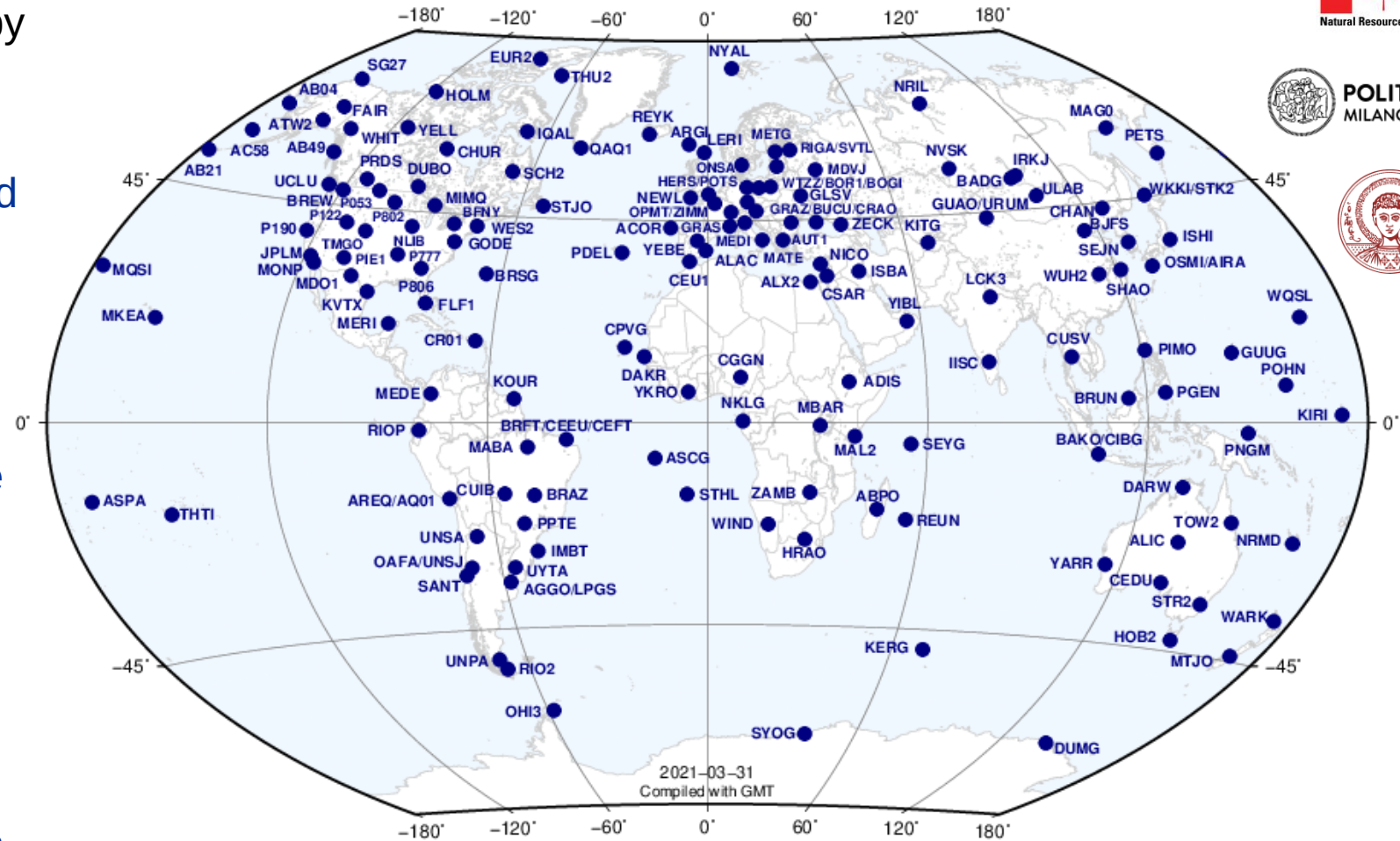
- fundamental **geodetic observatories** → connection between $\mathbf{X}$, W , g and time realisation (reference clocks);
- **continuously operating reference stations** → to detect deformations of the reference frame (preference for ITRF and regional reference stations, like SIRGAS, EPN, APREF, etc.);
- **reference tide gauges and national vertical networks** → to facilitate the vertical datum unification;
- reference stations of the new **International Gravity Reference Frame - IGRF** → to integrate the gravity and physical height reference frames.

3) Main requirement: **availability of surface gravity data around the IHRF reference stations for high-resolution gravity field modelling (i.e., precise estimation of W).**

First proposal for the IHRF reference network (~170 stations)



- Station selection coordinated by the **GGOS-FA Unified Height System** in agreement with the **GGOS Bureau of Networks and Observations**, the **Bureau Gravimétrie International** (absolute gravity stations), as well as with the **IAG regional sub-commissions** for reference frames and gravity field modelling.
- A **living network**: new stations and decommission of stations.
- To be **extended by regional/national densifications**.



Basic considerations on the IHRF/IHRF coordinates



- 1) The IHRF/IHRF is based on the combination of
 - a geometric component given by the **coordinate vector $\mathbf{X}$** in the ITRS/IHRF and
 - a physical component given by the determination of **potential values W at $\mathbf{X}$** .
- 2) The determination of **$\mathbf{X}$ follows the IERS Conventions** and will not be further considered here.
- 3) According to the **GGOS Terms of Reference**, the expected accuracy of W is
 - Positions: $\approx \pm 3 \times 10^{-2} \text{ m}^2\text{s}^{-2}$ (about **3 mm**)
 - Velocities: $\approx \pm 3 \times 10^{-3} \text{ m}^2\text{s}^{-2}/\text{a}$ (about **0.3 mm/a**)
- 4) For the moment, our goal is $\pm 1 \times 10^{-1} \text{ m}^2\text{s}^{-2}$ (about **1 cm**)
- 5) The IHRF/IHRF coordinates include the determination/modelling of time variations. For the moment, we consider **static coordinates only**.

Approaches for the determination of IHRF potential coordinates



- 1) Global gravity models of high resolution (GGM-HR), including topography-based synthetic gravity signals (like the model XGM2019e (Zingerle et al., 2019), or the combination of GGMs of degree > 2156 with topography potential models like Earth2014 (Rexer et al., 2016), ERTM2160 (Hirt et al., 2014), etc.)

- 2) Precise regional gravity field modelling (methods for the geoid/quasi-geoid determination)

$$W(P) = U(P) + \zeta(P) \cdot \gamma_Q + \Delta W_0 \quad [\text{m}^2\text{s}^{-2}] \rightarrow W(P) = W_0 - (h(P) - \zeta(P)) \cdot \bar{\gamma}_{QQ_0} \quad [\text{m}^2\text{s}^{-2}]$$

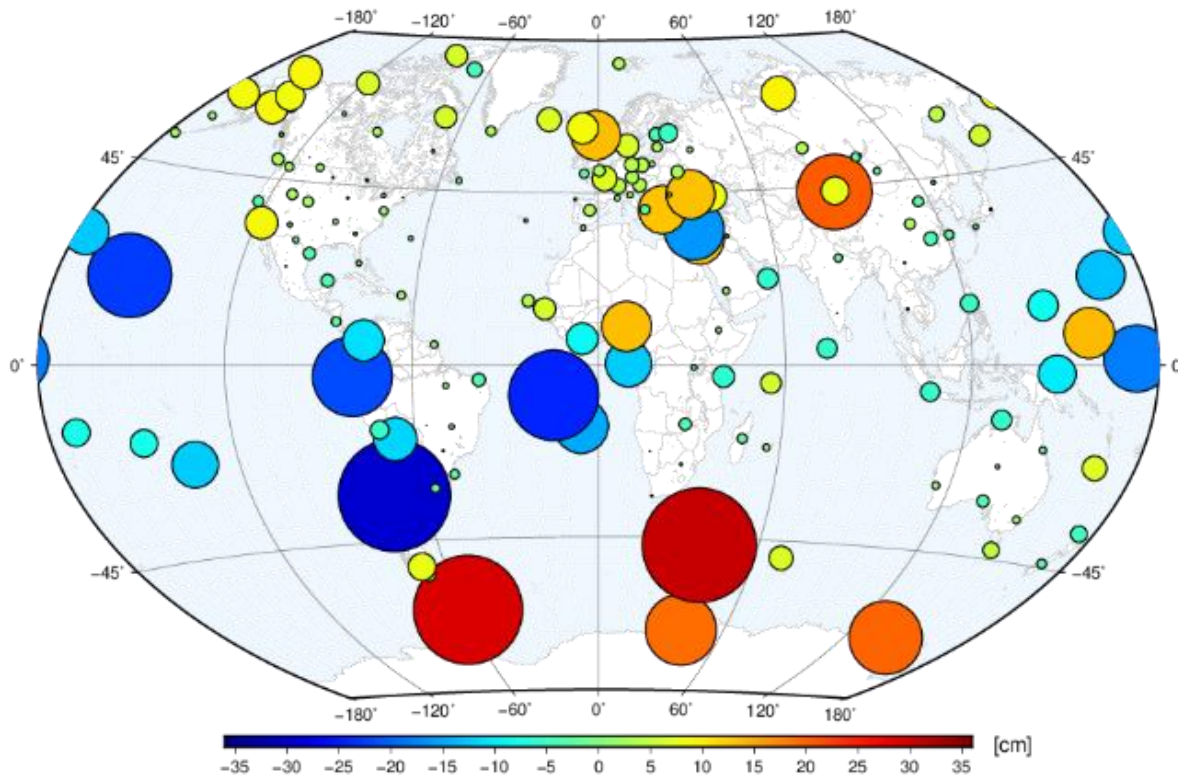
or

$$W(P) = W_0 - (h(P) - N(P)) \cdot \bar{g}(P) \quad [\text{m}^2\text{s}^{-2}] \text{ with}$$
$$\bar{g}(P) = g(P) + 0.424 \times 10^{-6} \cdot (h(P) - N(P)) + TC(P) \quad [\text{ms}^{-2}]$$

- 3) Vertical datum unification of the local height systems into the IHRF (details discussed by Sánchez and Sideris (2017), <https://doi.org/10.1093/gji/ggx025>, not further considered here)

$$W(P) = (W_0^{local} + \delta W) - C_p^{local} \quad \text{with} \quad \delta W = W_0^{IHRF} - W_0^{local}$$

IHRF potential coordinates based on global gravity models



Mean: 0.5 cm, STD: ± 8.2 cm, Min: -30.5 cm, Max: 31.0 cm

See Sánchez et al. (2021), *Strategy for the realisation of the IHRF*,
J Geod 95, 33 (2021). <https://doi.org/10.1007/s00190-021-01481-0>

- Comparison of normal heights inferred from **XGM2019e (degree 5540)** with the mean value of the normal heights obtained from **EIGEN-6C4 (degree 2190; Förste et al., 2015)**, **GECO (degree 2190; Gilardoni et al., 2016)**, and **SGG-UGM-1 (degree 2159; Liang et al., 2018)**.
- These differences are mainly attributable to the contribution of **surface gravity data posterior to EGM2008 (Pavlis et al., 2012)** and the **gravity synthetic effects inferred from the Earth2014 topography model**.
- To evaluate the reliability of these values, independent data (e.g., levelling+gravimetry) are needed.
- New and better surface gravity data distribution and quality (as in preparation for the EGM2020) will strongly improve the GGM-based estimates.

IHRF potential coordinates based on regional gravity field modelling



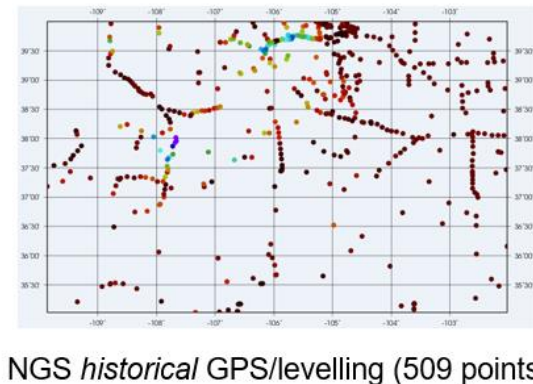
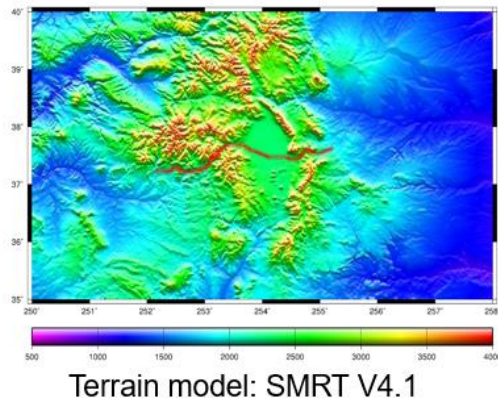
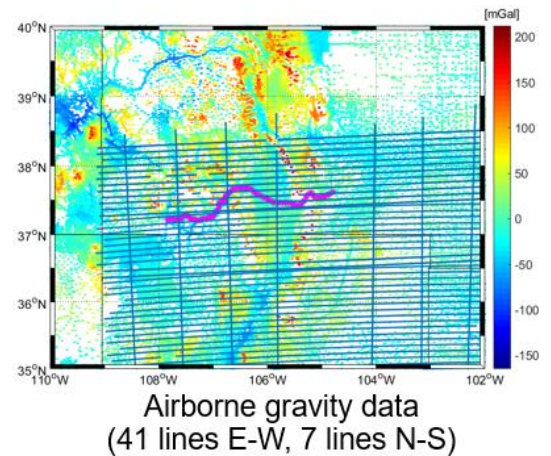
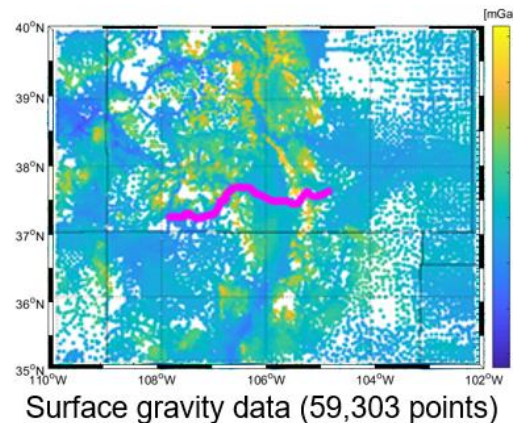
- GGMs based on SLR, GRACE and GOCE are **very precise** ($\pm 1 \dots \pm 2$ cm @ 100 km)
- Mean omission error globally: $\approx \pm 45$ cm
- Goal is to **reduce these ± 45 cm to ± 1 cm** by solving the Geodetic Boundary Value Problem (GBVP) with the combination of a GGM + surface gravity data + topography effects

$$W_P = U_P + T_P \quad T_P = T_{P, \text{satellite-only}} + T_{P, \text{residual}} + T_{P, \text{terrain}}$$

- The determination of T_P demands a series of approximations, which influence the results; i.e., **different methodologies produce different potential values**
- A “**centralised**” **computation** (like in the ITRF) is quite complicated due to the restricted accessibility to surface gravity data. So, regional/national experts have to be involved in the determination of the potential coordinates in their regions/countries
- A “**standard**” **computation procedure may be not appropriate** as
 - different data availability and different data quality exist around the world
 - regions with different characteristics require particular approaches (e.g. modification of kernel functions, size of integration caps, geophysical reductions like GIA, etc.).

Comparison of computation methods: Colorado experiment

Objective: to compute **geoid**, **quasi-geoid** and **potential values** using exactly the same input data, a **set of basic standards**, and the **own methodologies** (software) of colleagues involved in the gravity field modelling.



- Initiated in July 2017
- Data provided by US NGS
- Standards prepared by L Sánchez, J Ågren, J Huang, YM Wang, R Forsberg
- Three computations (two iterations) finished in June 2019
- Fourteen (final) contributing solutions
- Special Issue on “**Reference Systems in Physical Geodesy**” of the Journal of Geodesy with computation methods and comparison of geoid and quasi-geoid models, see <https://link.springer.com/journal/volumesAndIssues/190?tabName=topicalCollections>

Colorado experiment: contributing solutions



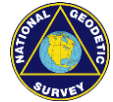
Faculty of Engineering, Minia University, [Egypt](#)



Istanbul Technical University, Istanbul, [Turkey](#)



Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki, [Greece](#)



National Geodetic Survey, [USA](#)



Natural Resources Canada, [Canada](#)



Lantmäteriet, Swedish mapping, cadastral and land registration authority, [Sweden](#)



School of Earth and Planetary Sciences and The Institute for Geoscience Research, Curtin University, [Australia](#)



Deutsches Geodätisches Forschungsinstitut, Technische Universität München, [Germany](#)



Ingenieurinstitut für Astronomische und Physikalische Geodäsie, Technische Universität München, [Germany](#)



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Faculty of Geodesy, University of Zagreb, [Croatia](#) - Research Institute of Geodesy, Topography and Cartography, [Czech Republic](#)



National Space Institute, Technical University of Denmark, [Denmark](#)



Geography and Crustal Dynamics Research Center, Geospatial Information Authority of Japan, [Japan](#)



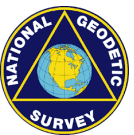
Colorado experiment: summary of approaches and models

- GGMs: GOCO05s, XGM2016, XGM2018, xGEOID17B, EIGEN-6C4
- Topographic effects based on SRTM V4.1, EARTH2014, ERTM2160

Solutions based on the **quasi-geoid** computation using **FFT integration** and a **Wong-Gore** modification of the integral kernel

(1)  Curtin University

(2) 

(3) 

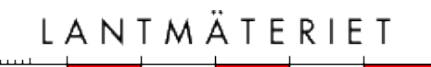
(4) 

A

Solutions based on the **quasi-geoid** computation with a **least-squares** modification of Stokes' formula with additive corrections (LSMSA)

(5) 

(6) 

(7) 

(8) 

B

Solutions based on the **quasi-geoid** computation using **spherical radial basis functions** (9) and **least-squares collocation** (10, 11)

(9)  (DGFI)

(10)  (IAPG)

(11)  POLITECNICO
MILANO 1863

C

Solutions based on the **geoid computation** using the **Helmert-Stokes (H-S)** scheme and then converted to the quasi-geoid

(12) 

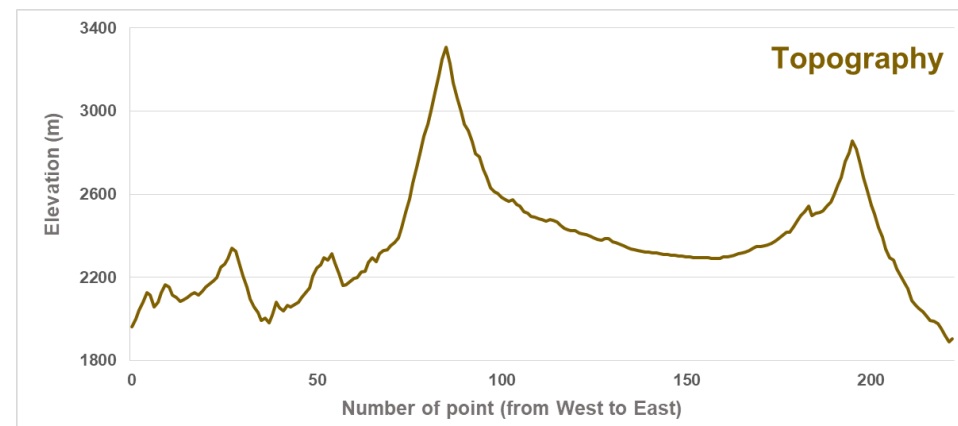
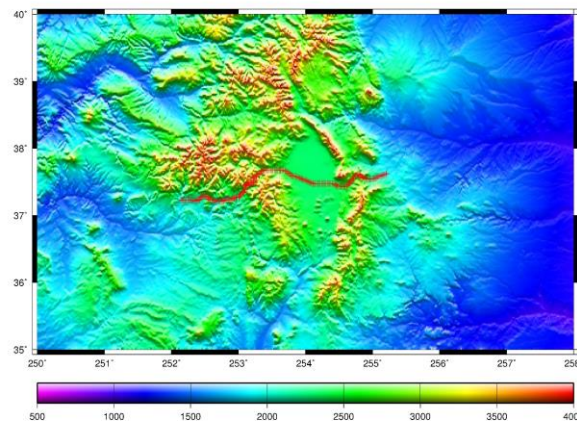
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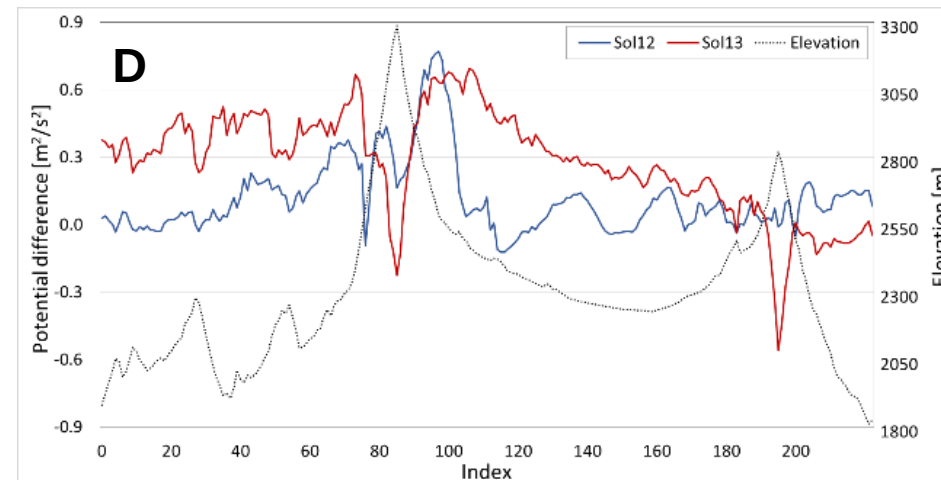
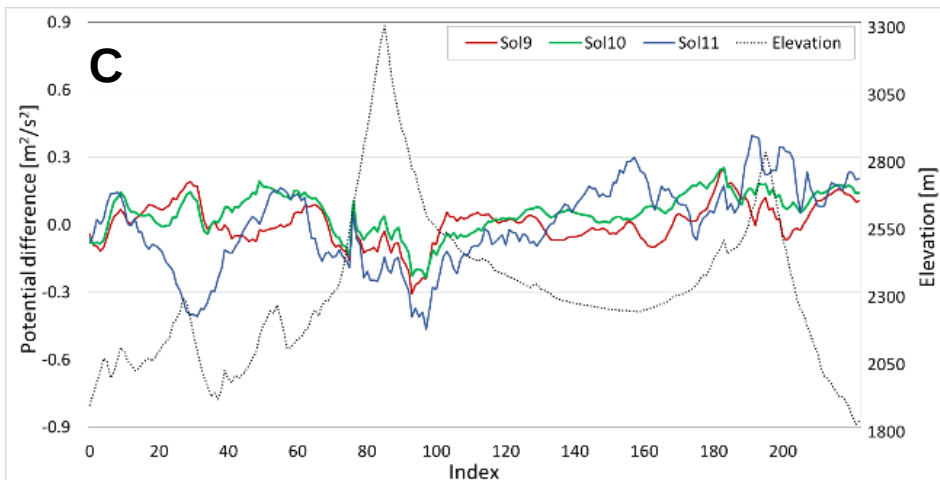
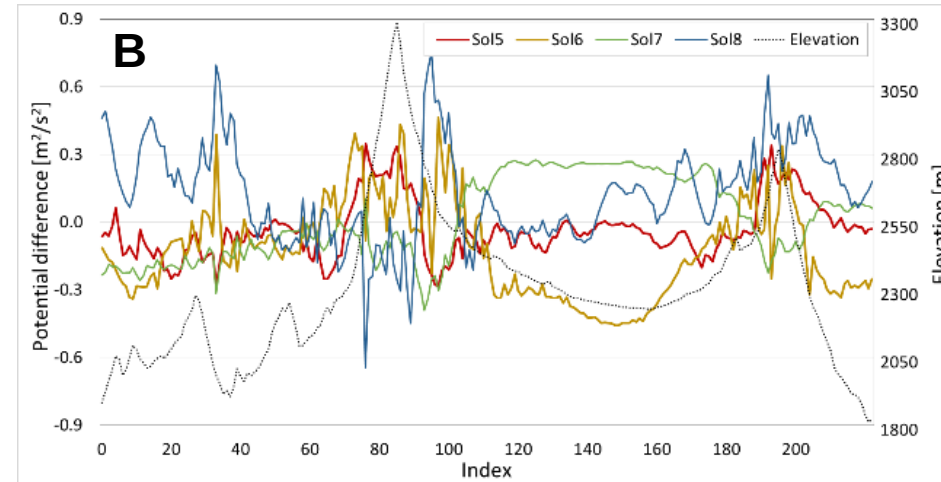
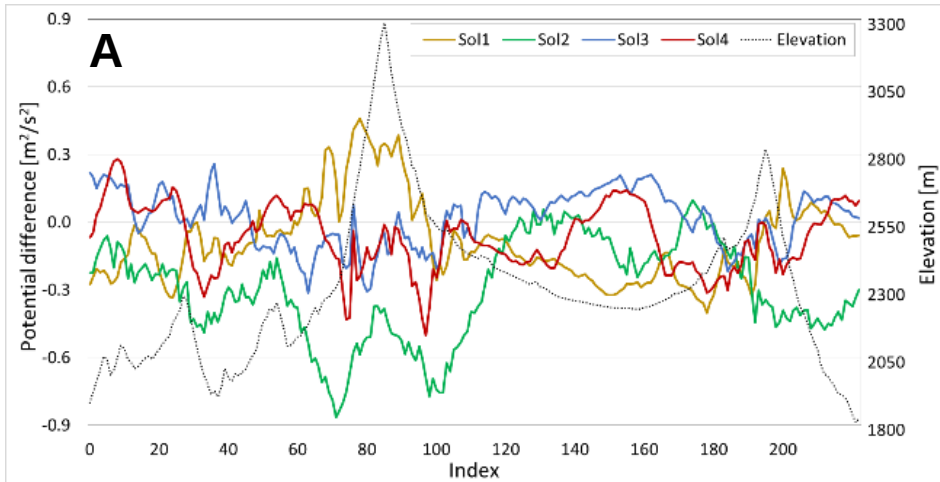
Colorado experiment: comparison of potential values



- 1) The comparison is carried out at **223 GSVS17 marks (Geoid Slope Validation Survey 2017)**, a test profile with levelling, GNSS, and gravity data of high quality.
- 2) Participants in the experiment got φ , λ , h ; levelling was not provided, but it is used for evaluation
- 3) The potential values provided by the different solutions are converted to **geopotential numbers** with respect to the IHRs W_0 value, $C(P) = W_0 - W(P)$
- 4) and are **compared with each other (to evaluate the consistency between methods)** and **with the potential differences inferred from levelling + gravimetry (to evaluate the reliability of the results)**



Colorado experiment: comparison of potential values wrt mean value (model – mean)



- Agreement within $\pm 0.09 \text{ m}^2\text{s}^{-2}$ ($\pm 0.9 \text{ cm}$) and $\pm 0.23 \text{ m}^2\text{s}^{-2}$ ($\pm 2.3 \text{ cm}$) in terms of STD wrt mean value.
- The overall differences range from $-0.86 \text{ m}^2\text{s}^{-2}$ (-8.8 cm) to $+0.77 \text{ m}^2\text{s}^{-2}$ ($+7.9 \text{ cm}$).
- Discrepancies present a strong correlation with the topography.

See Sánchez et al. (2021), *Strategy for the realisation of the IHRs*, J Geod 95, 33 (2021).
<https://doi.org/10.1007/s00190-021-01481-0>

Colorado experiment: comparison of potential differences with levelling (+ gravimetry)

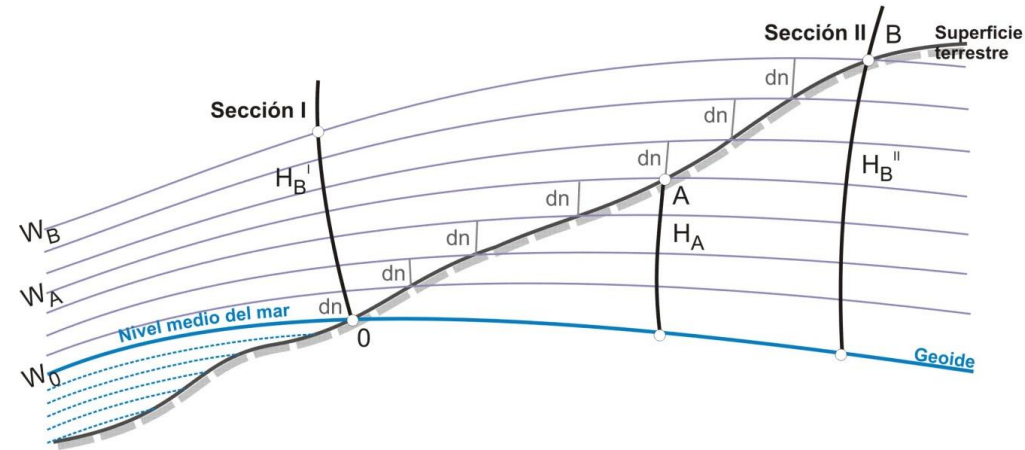
1) Between **consecutive benchmarks**:

$$\Delta C_{i,i+1} = (C_{mod,i} - C_{mod,i+1}) - (C_{lev,i} - C_{lev,i+1})$$

2) Between **all possible baseline lengths** (d_{ij}) of the GSVS17 profile:

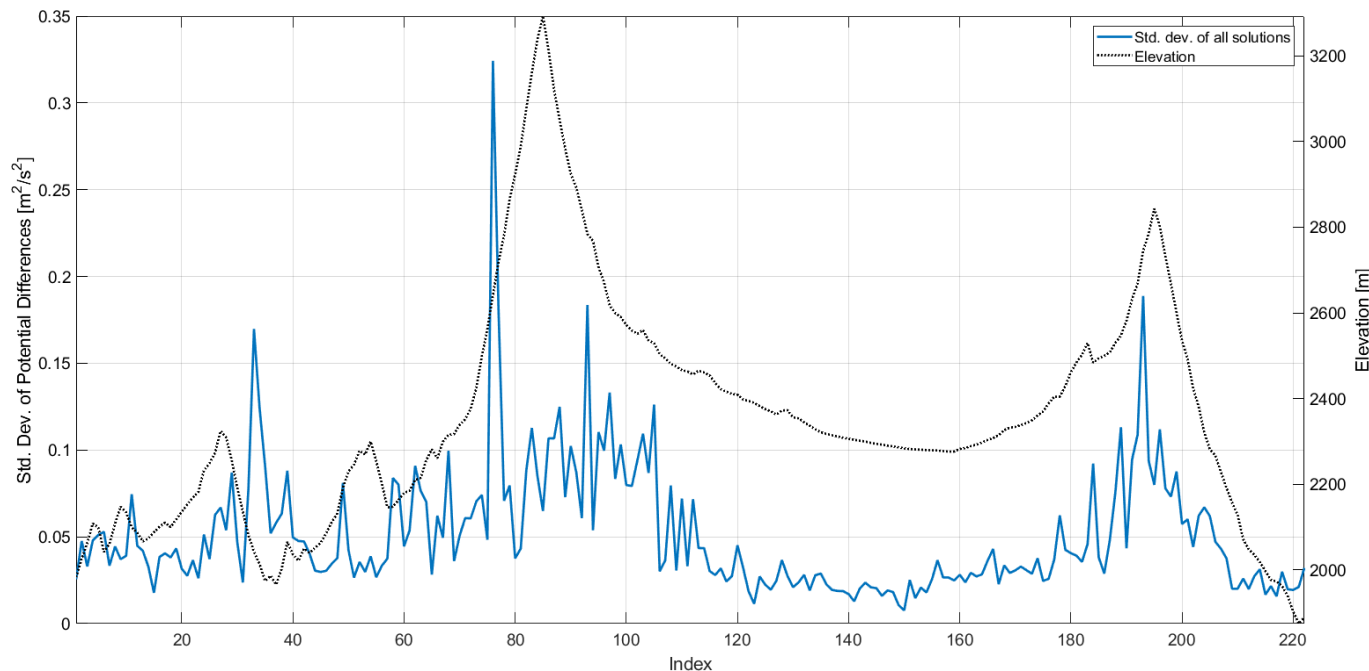
$$\Delta C_{i,j} = (C_{mod,i} - C_{mod,j}) - (C_{lev,i} - C_{lev,j})$$

- i and j are the indexes of the GSVS17 benchmarks and d_{ij} is the distance in [km] along the levelling path between each pair of points.
- The d_{ij} values are grouped in **15 class intervals varying from 15 km to 108 km**. The varying length of the class intervals is needed **to ensure that at least 2,000 benchmark pairs** are available per class.



Colorado experiment: comparison of potential values with levelling (+ gravimetry)

- 1) Between **consecutive benchmarks**: Standard deviation (STD) of all 13 regional solutions for each respective $(i, i+1)$

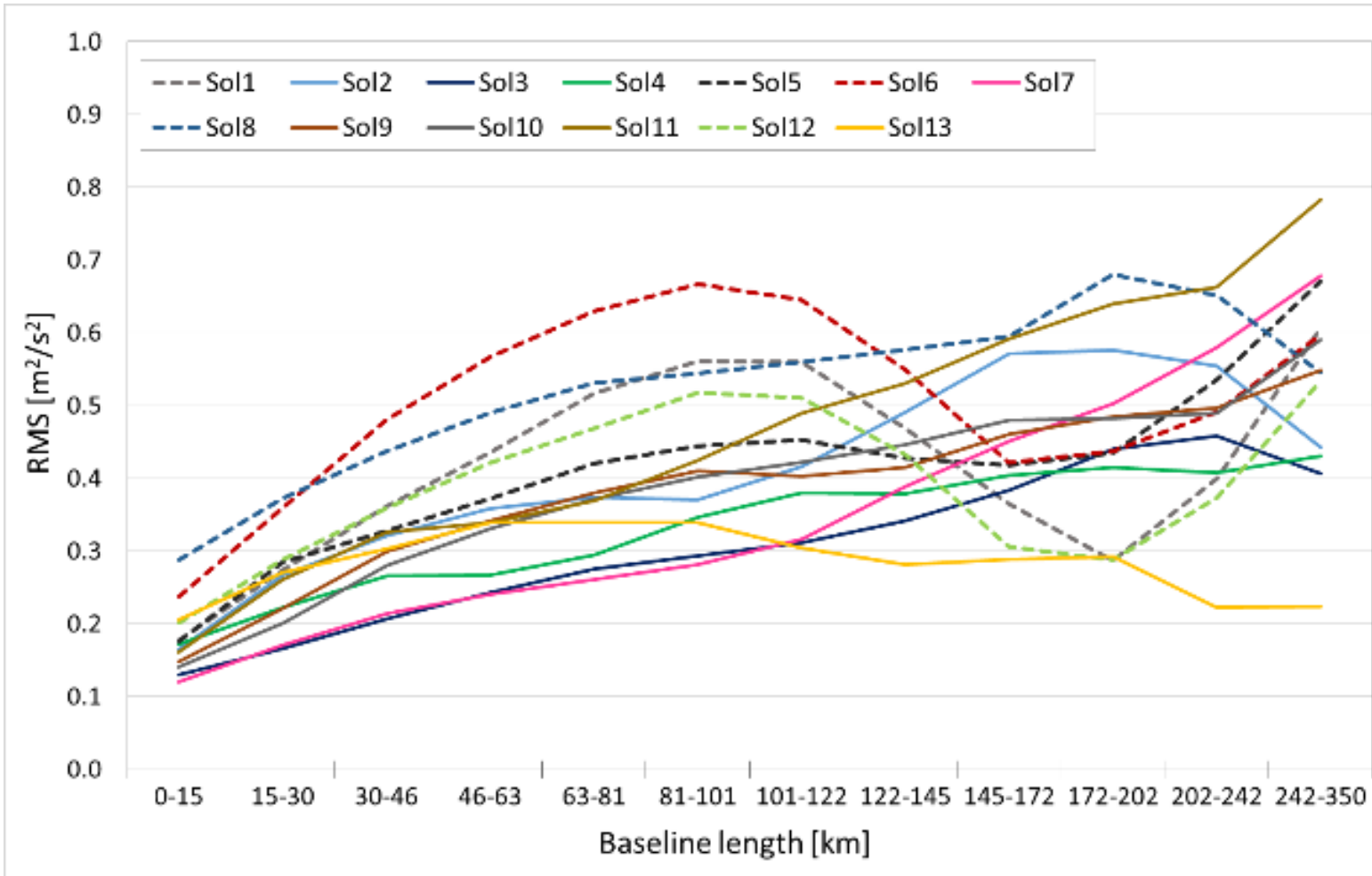


See Sánchez et al. (2021), *Strategy for the realisation of the IHRs*, J Geod 95, 33 (2021). <https://doi.org/10.1007/s00190-021-01481-0>

- The smallest STD values occur where the topography is smooth.
- STD values increase with the roughness of topography, occurring the largest values at the strongest topographic gradients.
- This shows that the large scatter is associated with the rough topography where the omission error tends to be large, and the topography effects are treated quite differently among the 13 solutions.
- Consequently, **a challenge (or limitation) in the determination of model-based geopotential numbers is a better modelling of terrain effects.**

Colorado experiment: comparison of potential values with levelling (+ gravimetry)

2) Between all possible baseline lengths (d_{ij}) of the GVSV17 profile: $\Delta C_{ij} = (C_{mod,i} - C_{mod,j}) - (C_{lev,i} - C_{lev,j})$



The RMS values of the ΔC_{ij} differences for each interval indicates the consistency between the model-based and levelling-based potential values as a function of the distance.

See Sánchez et al. (2021), *Strategy for the realisation of the IHRs*, J Geod 95, 33 (2021).
<https://doi.org/10.1007/s00190-021-01481-0>

Learnings from the Colorado experiment

The determination of potential values may be classified in three main scenarios:

- a) Regions without (or with very few) surface gravity data,
 - The only option to determine potential values is the use of GGM-HRs
 - Expected mean accuracy values around the $\pm 4.0 \text{ m}^2\text{s}^{-2}$ ($\pm 40.0 \text{ cm}$) level or even worse in regions with strong topography gradients
 - It could be improved for instance to the $\pm 1.0 \text{ m}^2\text{s}^{-2}$ ($\pm 10.0 \text{ cm}$) level if new and better surface gravity data are included in the GGMs.
 - To avoid multiple potential values provided by different GGM-HRs at the same point, it is necessary to select one GGM-HR as reference model.
- b) Regions with some surface gravity data, but with poor data coverage or unknown data quality,
 - The reliability of the existing (quasi-)geoid models is poor
 - Additional gravity surveys around the IHRF stations to increase the accuracy of the geopotential numbers computed at those specific stations.
- c) Regions with good surface gravity data coverage and quality.
 - Potential values may be inferred from precise (quasi-)geoid regional models



On going activities

Computation of a first static solution for the IHRF based on

- the existing regional/national geoid/quasi-geoid models
- GGM-HR + topographic gravity signals

Regional/national geoid/quasi-geoid models play a main role in the realisation of the IHRF because

- they include surface gravity data that are not always available for the determination of GGM-HR,
- they can assimilate new regional/local gravity surveys very quickly, and
- national/regional experts on gravity field modelling have the best possible knowledge about the local conditions (topography, data distribution, geophysical corrections, validation data, etc.) to be considered in the computation of the (quasi-)geoid, or more precisely, in the determination of the disturbing potential T in their countries/regions

A detailed guideline to standardise the recovering of potential values from regional (quasi-)geoid models was prepared and distributed to about 40 colleagues worldwide. Results are expected within the next weeks.



Outlook

- 1) To continue learning from the Colorado experiment
 - Detailed comparison of the contributing solutions at different processing stages to improve the document with the standards for the determination of potential values.
- 2) To investigate the determination of potential changes with time $\dot{W}$.
- 3) To install an 'IHRF/IHRF element' within the IGFS to ensure the maintenance and availability of the IHRF:
 - Regular updates of the IHRF to take account for new stations;
 - coordinate changes with time $\dot{\mathbf{X}}$, $\dot{W}$;
 - improvements in the estimation of $\mathbf{X}$ and W (more observations, better standards, better models, better computation algorithms, etc.).

