A unified first order hyperbolic model for nonlinear dynamic rupture processes in diffuse fracture zones

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Dynamic rupture modeling in diffuse fracture zones

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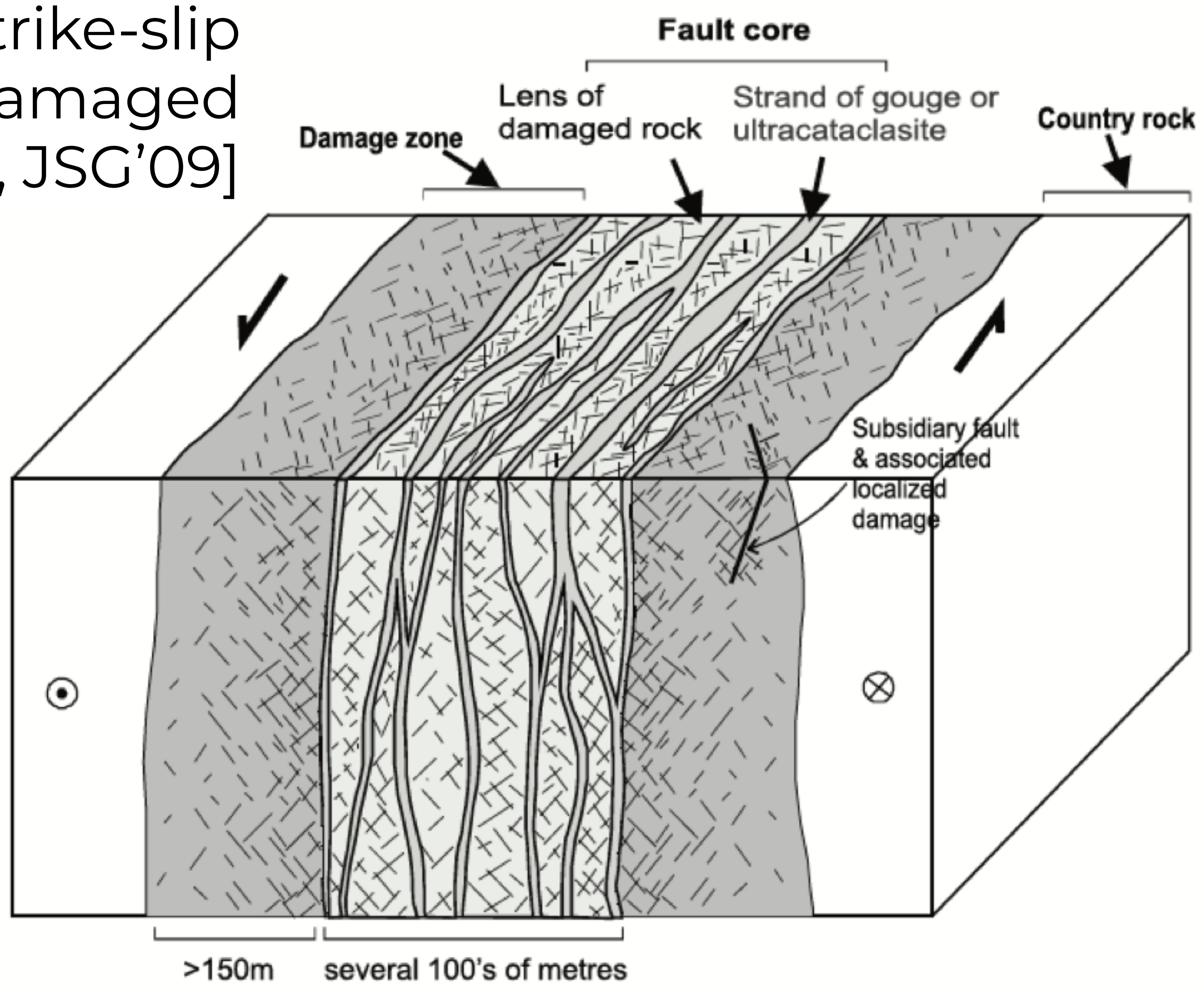
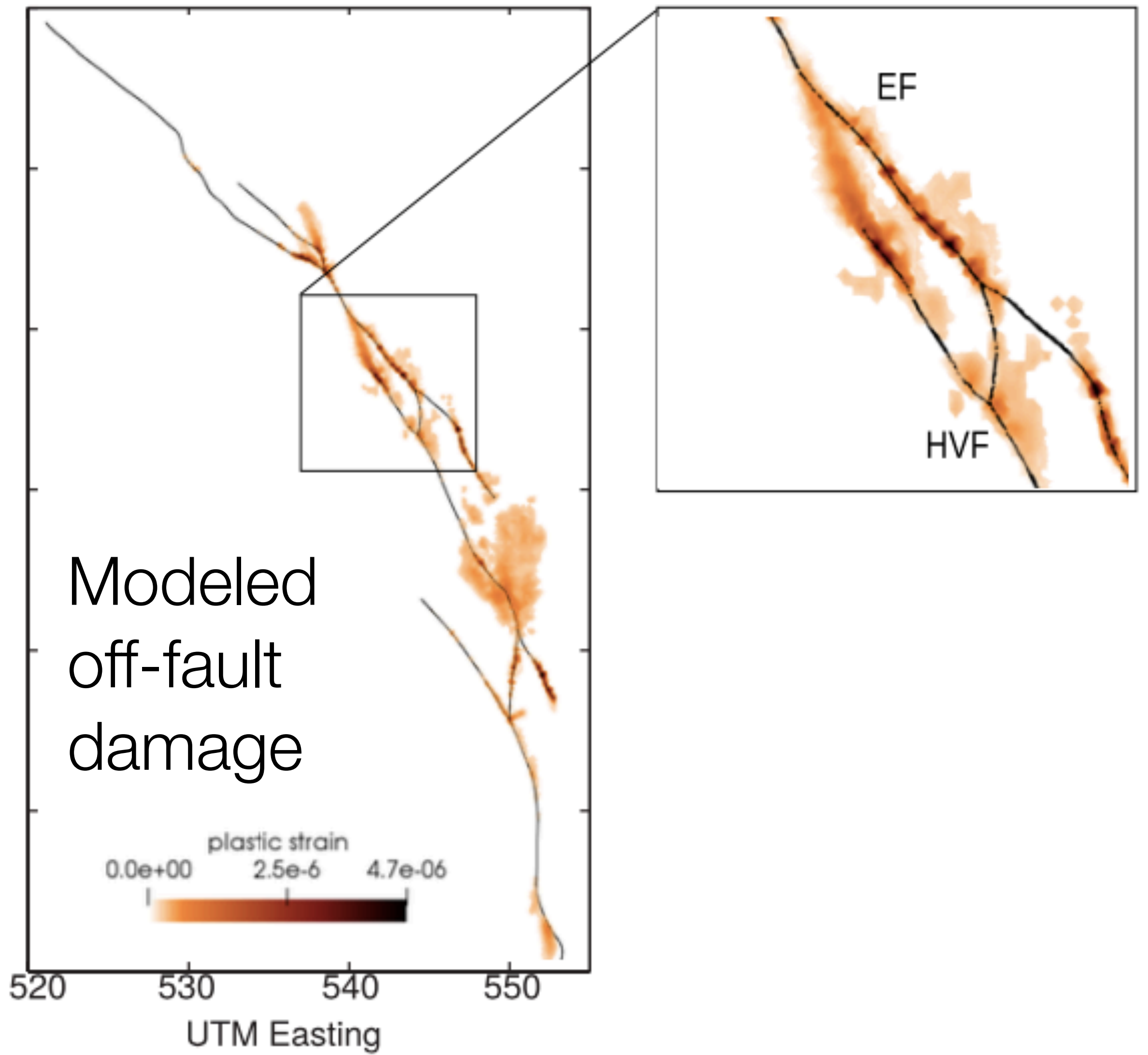
Conceptual view of a strike-slip fault zone including a damaged zone [Mitchell, Faulkner, JSG'09]

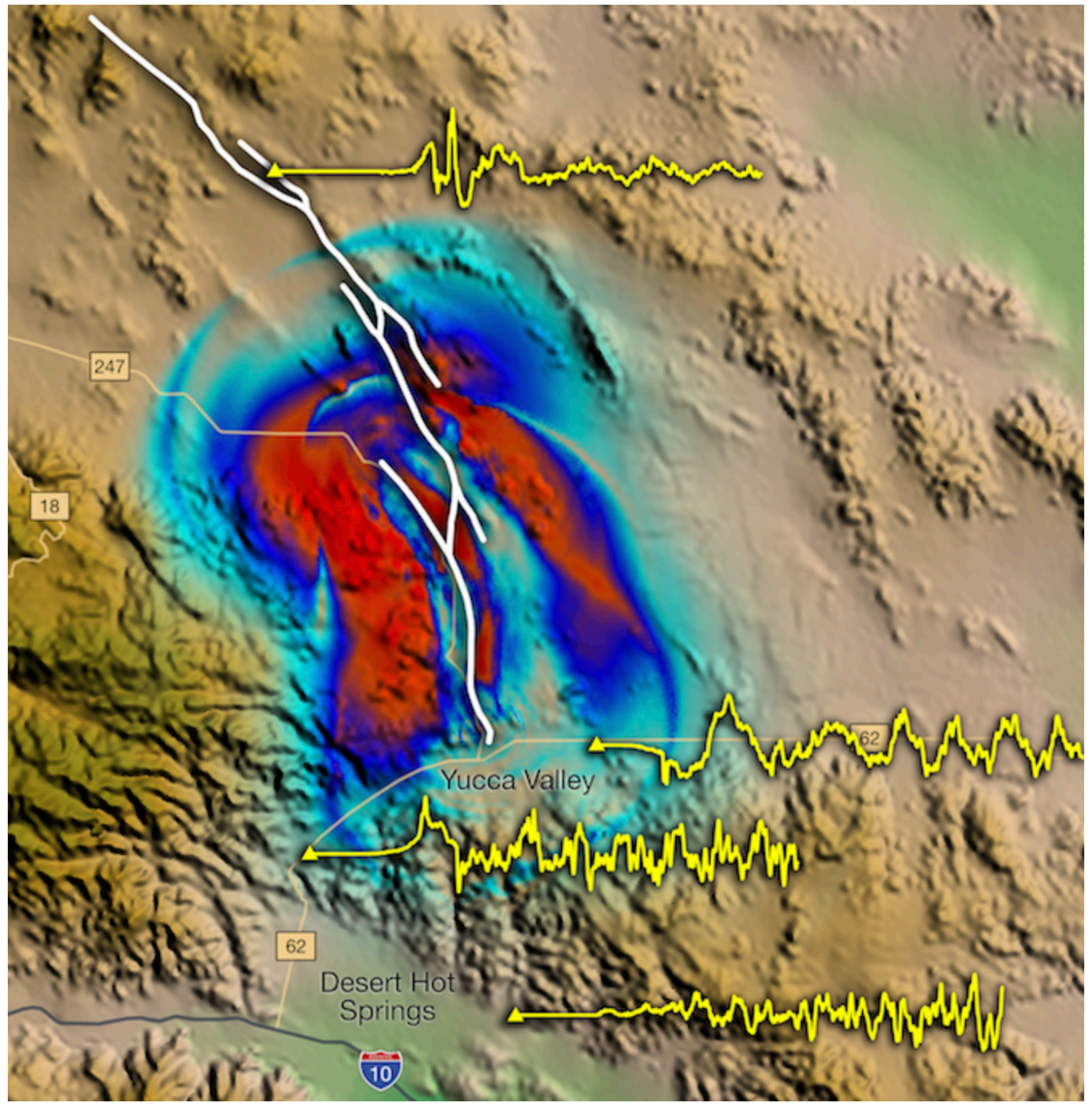


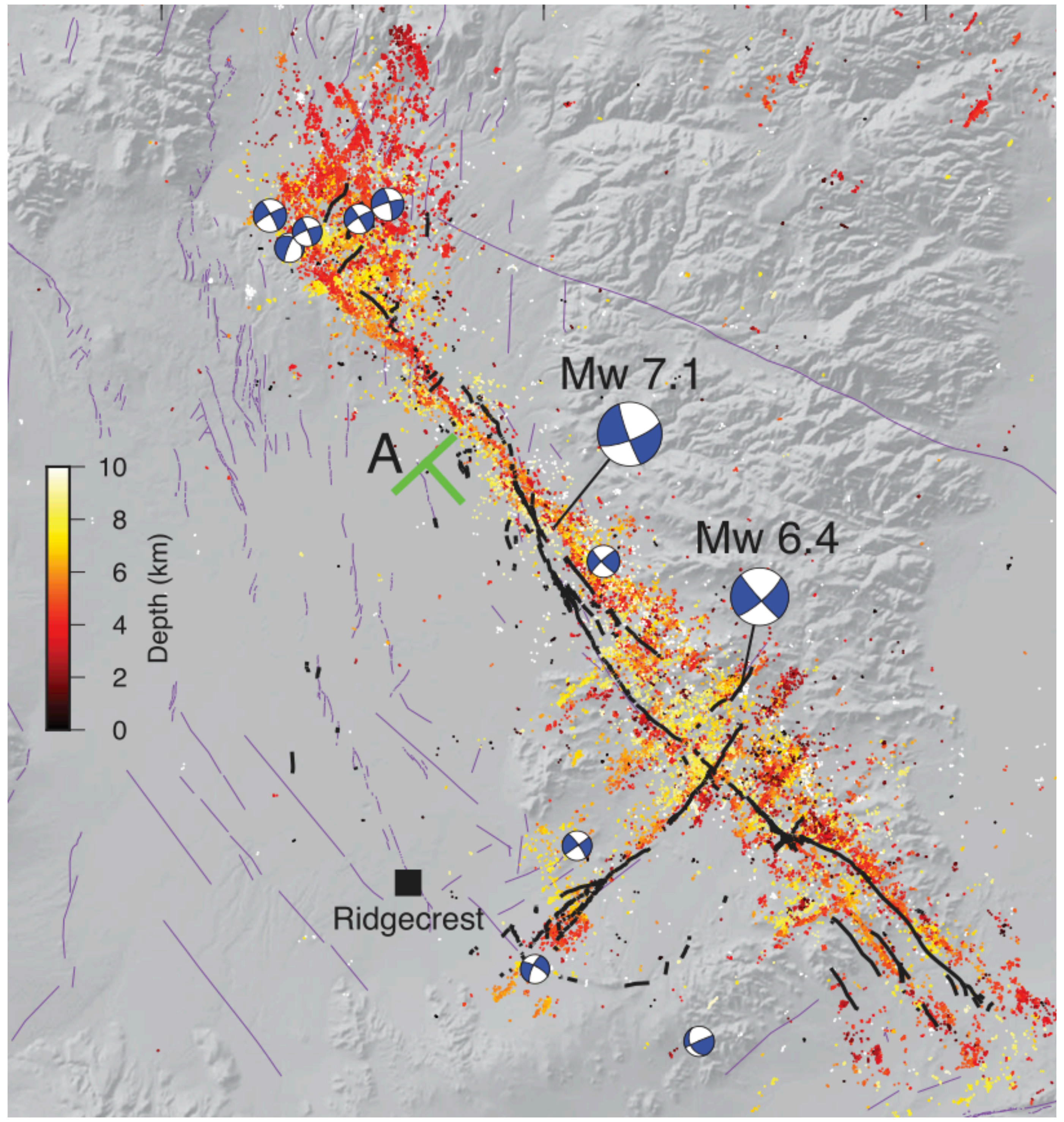
- Earthquake fault zones are **more complex, both geometrical and rheological, than an idealised infinitely thin plane embedded in linear elastic material**

- **TEAR ERC StG** aims to develop open-source methods for volumetric earthquake modelling using high performance computing (HPC) to understand the mechanics of slip in deforming fault zones from short-term rupture dynamics to long-term seismic sequences.

Dynamic rupture model of the 1992 Landers earthquake [Wollherr et al., 2019]

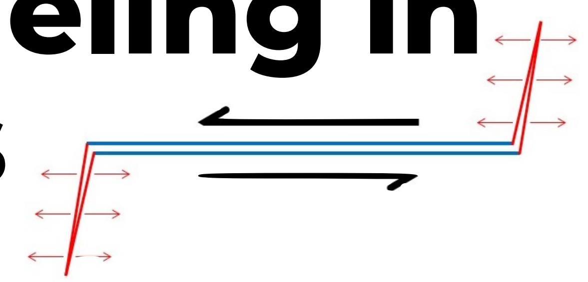






Hierarchical interlocked rupture during the 2019 Ridgecrest, CA, earthquake sequence [Ross et al., 2019]

Dynamic rupture modeling in diffuse fracture zones



- **Godunov-Peshkov-Romenski (GPR)** model incorporates finite strain elasto-visco-plasticity and viscous fluids in a **single PDE system** ➠

- **GPR mathematical formulation:** Eulerian thermodynamically compatible, unified description of nonlinear elasto-plasticity, material damage and viscous Newtonian flows

[Godunov and Romenski, J. Appl. Mech. Tech. Phys., 1972; Peshkov and Romenski, Continuum Mech. Thermodyn. 2016; Tavelli et al., JCP 2019]

Conservations of mass, momentum and energy in diffuse interface approach

$$\partial_t \alpha + v_k \partial_k \alpha = 0, \quad \partial_t \bar{\rho} + \partial_k(\bar{\rho} v_k) = 0,$$

$$\partial_t(\bar{\rho} v_i) + \partial_k\left(\bar{\rho} v_i v_k + \alpha p \delta_{ik} - \alpha \sigma_{ik}\right) = \bar{\rho} g_i,$$

$$\partial_t A_{ik} + \partial_k(A_{im} v_m) + v_m\left(\partial_m A_{ik} - \partial_k A_{im}\right) = -\theta_1^{-1}(\tau_1) E_{A_{ik}},$$

$$\partial_t J_k + \partial_k\left(v_m J_m + T\right) + v_m\left(\partial_m J_k - \partial_k J_m\right) = -\theta_2^{-1}(\tau_2) E_{J_k},$$

$$\partial_t \xi + v_k \partial_k \xi = -\theta E_\xi,$$

$$\partial_t(\bar{\rho} S) + \partial_k\left(\bar{\rho} S v_k + \bar{\rho} E_{J_k}\right) = \bar{\rho}\,(\alpha T)^{-1}\left(\theta_1^{-1} E_{A_{ik}} E_{A_{ik}} + \theta_2^{-1} E_{J_k} E_{J_k} + \theta E_\xi E_\xi\right) \geq 0,$$

$$\partial_t(\bar{\rho} E) + \partial_k\left(v_k \bar{\rho} E + v_i(\alpha p \delta_{ik} - \alpha \sigma_{ik}) + q_k\right) = \bar{\rho} g_i v_i,$$

$$\theta_1(\tau_1) = \tau_1 c_s(\xi)^2/3|\mathbf{A}|^{-5/3}, \qquad \theta_2(\tau_2) = \tau_2 \frac{c_h^2}{\rho T}$$

α: colour function, =1 inside solid body
$\bar{\rho} = \alpha\rho$: mass conservation
v_k: velocity
σ_{ik}: stress tensor (tangential + thermal)
$p = \rho^2 E_\rho$: thermodynamic pressure
g_i: gravitational acceleration
$E = E(\rho, S, \mathbf{v}, \mathbf{A}, \mathbf{J}, \xi)$: total energy potential

A_{ik}: distortion (local basis triads)
J_k: thermal impulse (heat conduction)
S: entropy
T=E_S: temperature
$\theta, \theta_1, \theta_2$: scalar functions controlling dissipation (strain localization, heat flux, chemical kinetics)
ξ: damage coefficient [0,1]
τ_1: strain relaxation time
τ_2: heat flux relaxation time

Dynamic rupture modeling in diffuse fracture zones

- **Godunov-Peshkov-Romenski (GPR)** model incorporates finite strain elasto-visco-plasticity and viscous fluids in a **single PDE system**
- **GPR mathematical formulation:** Eulerian thermodynamically compatible, unified description of nonlinear elasto-plasticity, material damage and viscous Newtonian flows
- Coupled with a hyperbolic model for **continuous modeling of damage** ➠ including brittle and ductile fracture as well as viscous Newtonian flows, which results in a diffuse representation of material damage (as per **phase-field**)

[Godunov and Romenski, J. Appl. Mech. Tech. Phys., 1972; Peshkov and Romenski, Continuum Mech. Thermodyn. 2016; Tavelli et al., JCP 2019]

Evolution of damage coefficient ξ:

$$\theta = \theta_0(1-\xi)(\xi+\xi_\epsilon)\left[(1-\xi)\left(\frac{Y_{eq}}{Y_0}\right)^a + \xi\left(\frac{Y_{eq}}{Y_1}\right)\right]$$

shear relaxation time of intact rock τ_I
shear relaxation time of damaged rock τ_D
initial shear relaxation time τ_0

von Mise stress Y_{eq}
material constants $\alpha_I, \beta_I\, \alpha_D, \beta_D, Y_0, Y_1$
damage rate degradation constants $\xi_\epsilon, \theta_0, a$

Constitutive relations of damage:

$$\tau_1 = \left(\frac{1-\xi}{\tau_I} + \frac{\xi}{\tau_D}\right)^{-1},$$

$$\tau_I = \tau_0 \exp(\alpha_I - \beta_I(1-\xi)Y_{eq}),$$

$$\tau_D = \tau_0 \exp(\alpha_D - \beta_D \xi Y_{eq}),$$

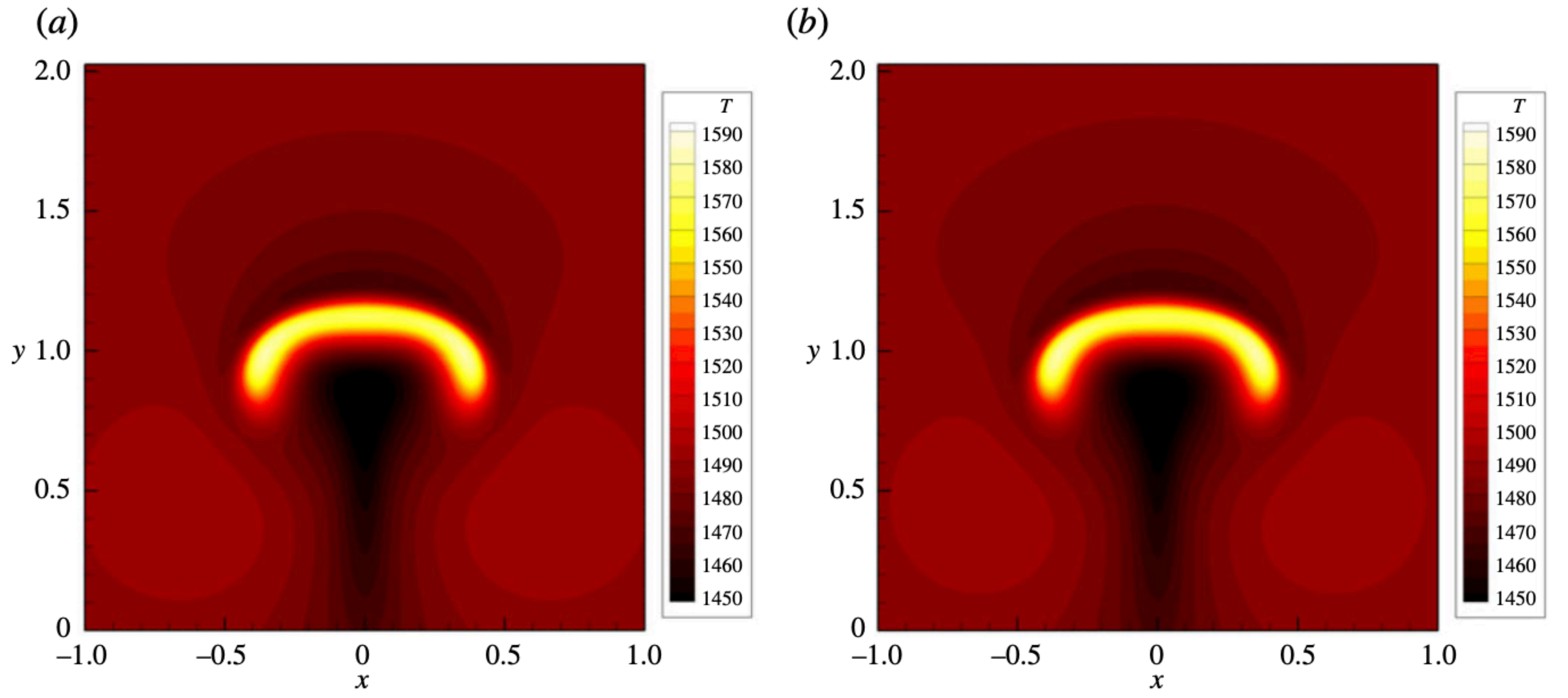


Figure 6. Temperature contours for the rising bubble problem in molten rock-like material at time $t = 4$. Solution obtained with the GPR model (*a*) and Navier–Stokes reference solution (*b*). The melting temperature is set to $T_c = 1000$. (Online version in colour.) The damaged phase represents here a Newtonian fluid.

Dynamic rupture modeling in diffuse fracture zones

- **Diffuse interface approach:** each material element as a mixture of an intact and a fully damaged phase
- **Arbitrary geometries** of fractures and material boundaries without the necessity of generating interface-aligned meshes (Eulerian formulation)
- Non-trivial **constitutive parameter selection**
- **ExaHyPE** (www.exahype.org) PDE engine supporting dynamic adaptive mesh refinement and based on an ADER-DG scheme combined with a Finite Volume sub-cell limiter

[Godunov and Romenski, J. Appl. Mech. Tech. Phys., 1972; Peshkov and Romenski, Continuum Mech. Thermodyn. 2016; Tavelli et al., JCP 2019]

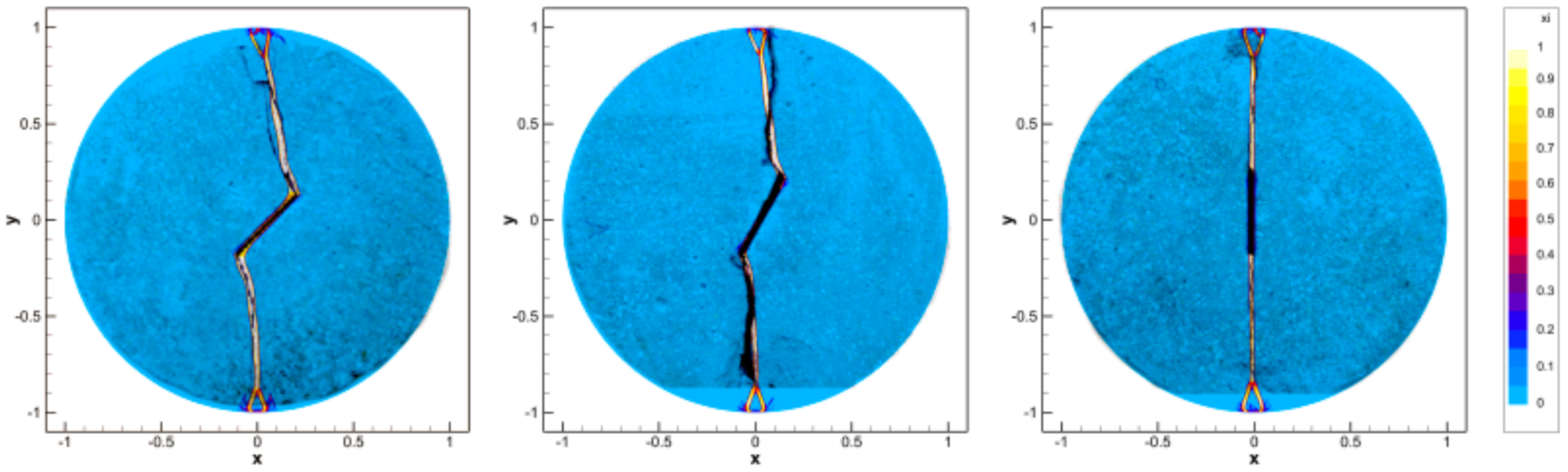
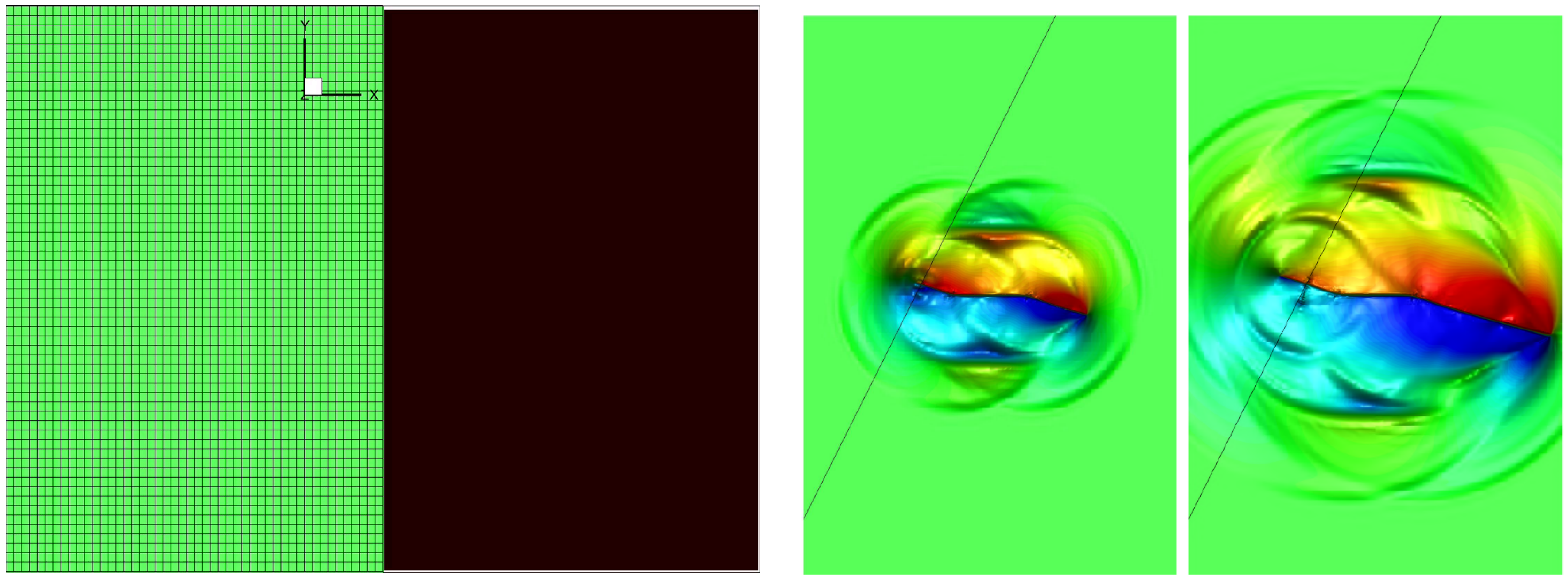


Figure 5. Crack formation in a rock-like disc under vertical load (Brazilian test) for different angles of the pre-damaged area. Comparison of the contour colours of the damage variable ξ obtained in the numerical simulations of the GPR model with the cracks observed in experiments. The simulation results are overlaid on top of the photographs from [86]. From left to right: 45°, 60° and 90°. Only the regions of the disc where $\alpha > 0.5$ are shown. (Online version in colour.)

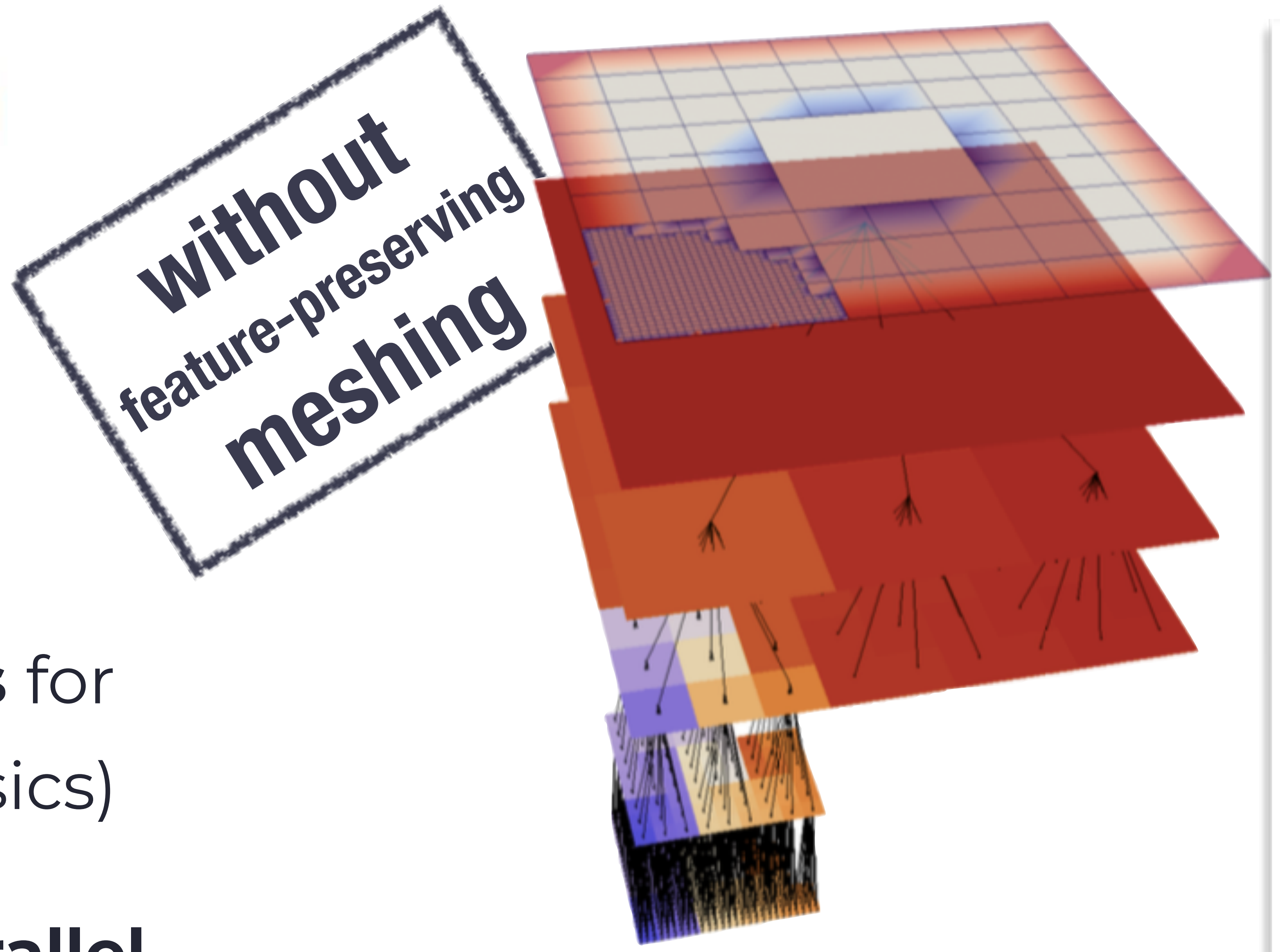


Propagation of an out-of-plane brittle crack using the GPR model [Tavelli et al., JCP'20].

ExaHyPE
An exascale hyperbolic PDE engine

Reinarz et al., CPC'20

https://gitlab.lrz.de/exahype/ExaHyPE-Engine/-/tree/TEAR-ERC

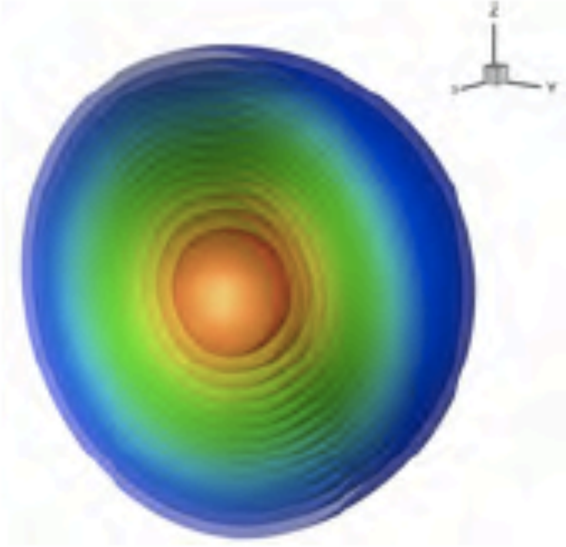


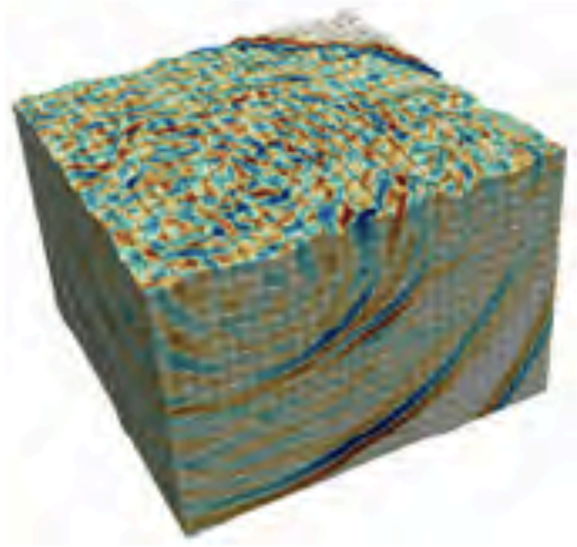
Assumptions on Exascale HPC:
(selected; ExaHyPE project proposal, 2014)

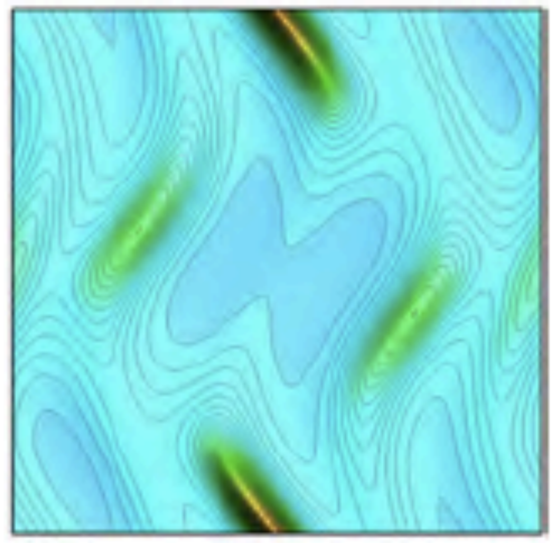
- Equal work load will no longer lead to balanced computation time.
- Moving data is the thriving constraint for performance and energy consumption.
- Cores will fail and cause whole nodes to crash.

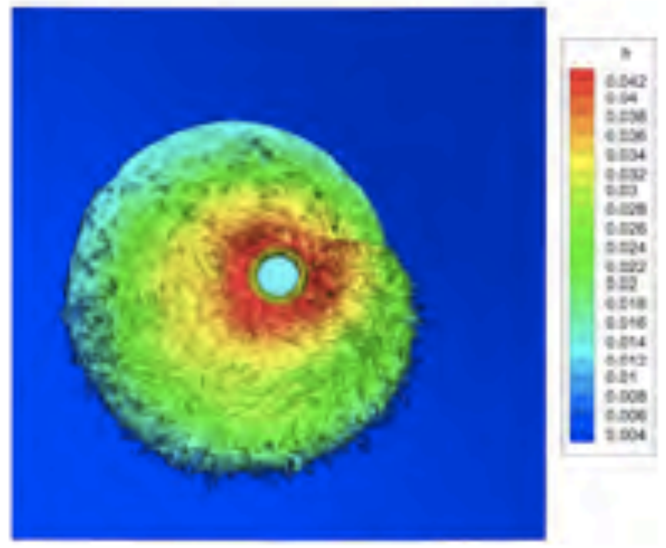
→ Grand challenge applications require tailoring of existing codes to the specific challenge and cannot rely on general-purpose solutions.

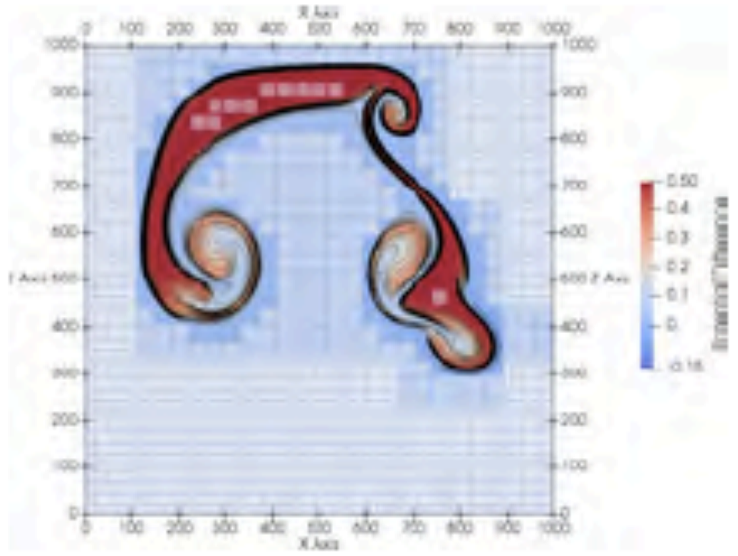
- A Hyperbolic PDE Engine for **exascale supercomputers** for hyperbolic conservation laws (e.g. seismology, astrophysics)
- User is provided with abstraction layer of: ADER-DG, **parallel AMR** (Peano), **cartesian** space-tree grids, FV limiters

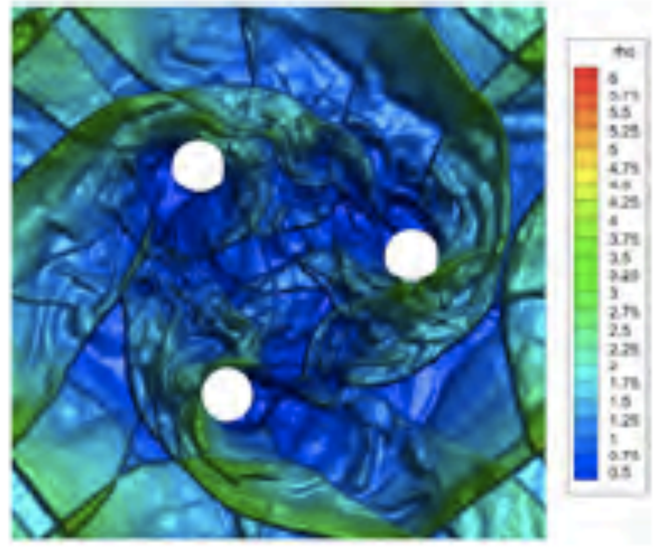

neutron stars


seismic waves


magneto-hydrodynamics

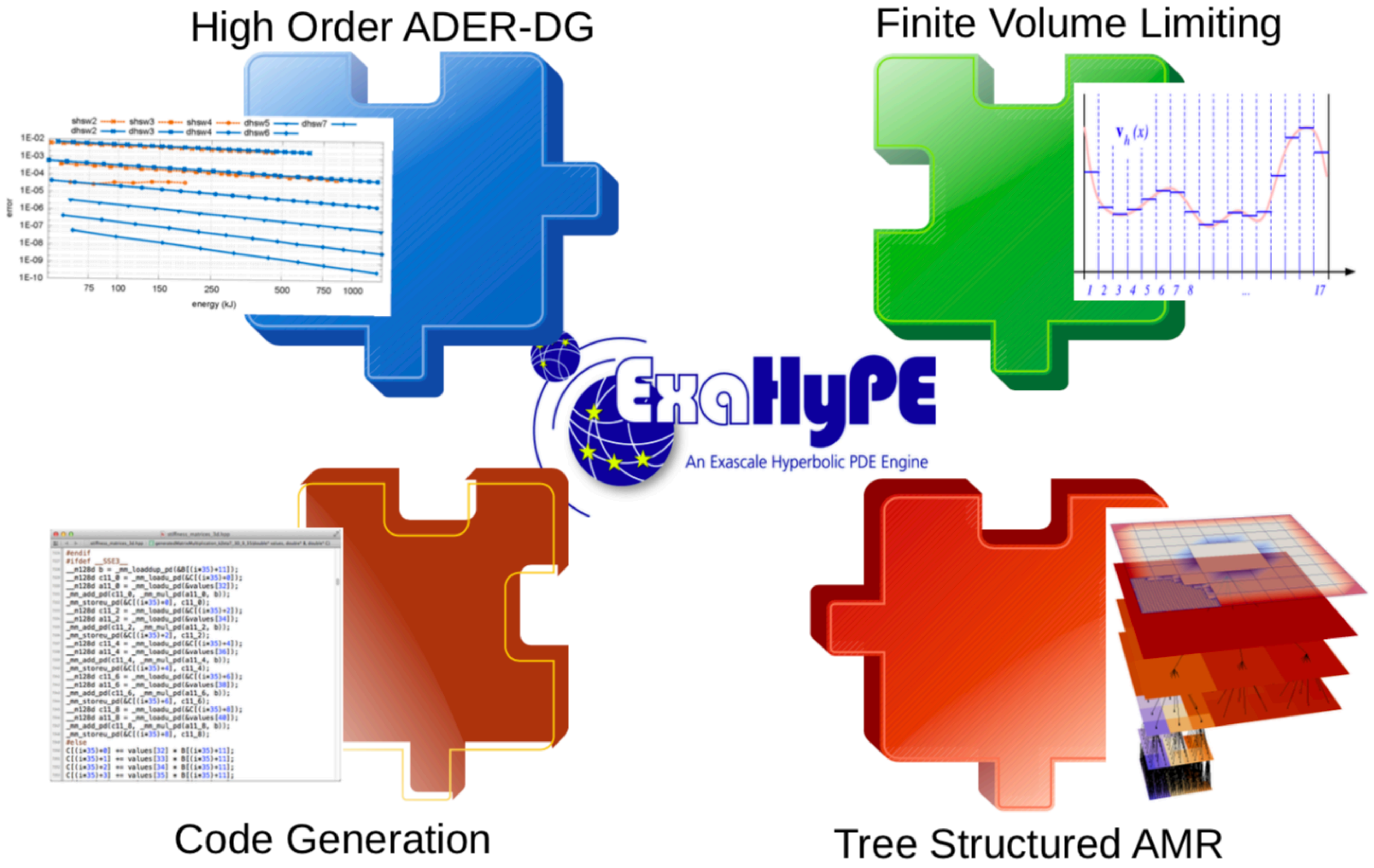

shallow water turbulence


cloud formation

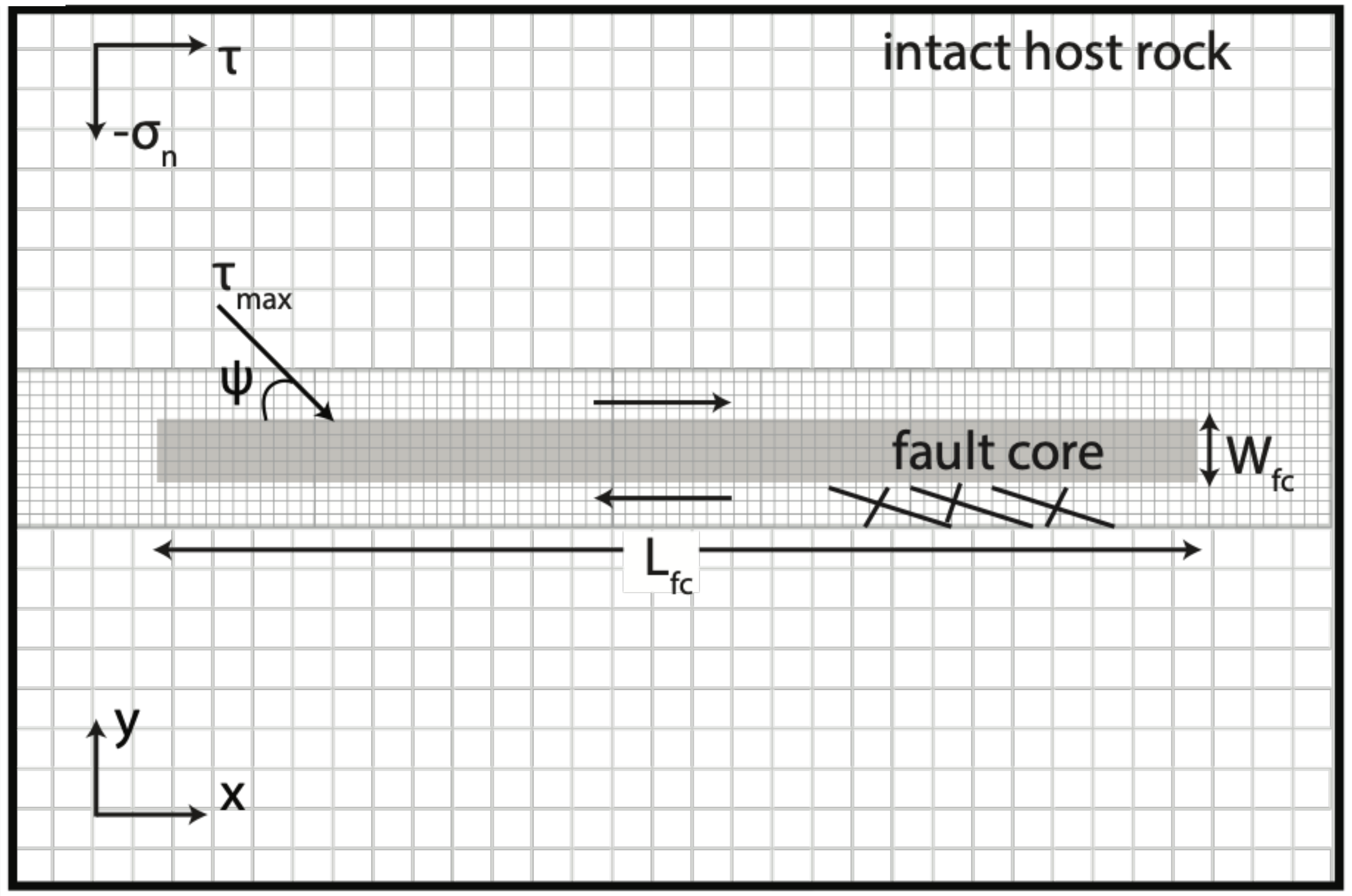

fluid structure interaction

: Engine
/perbolic
with abs

Components of the ExaHyPE Engine

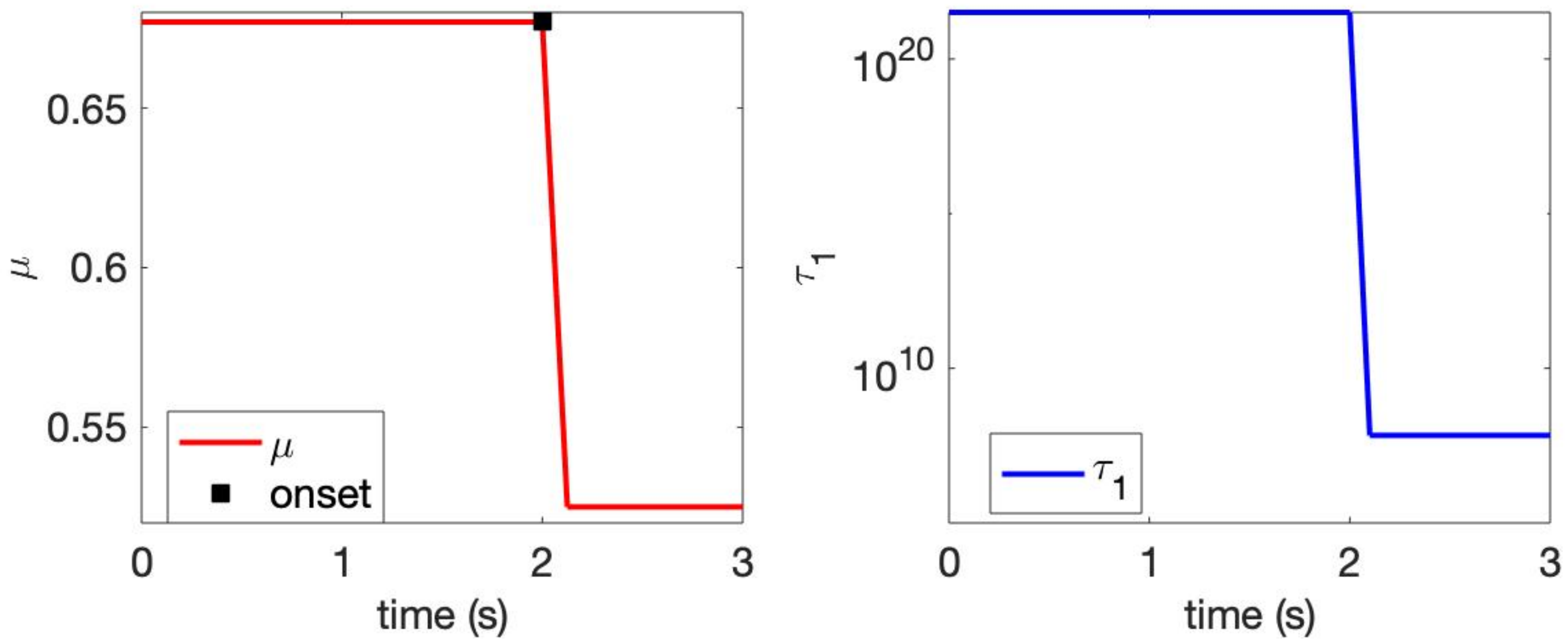


Dynamic rupture modeling in diffuse fracture zones

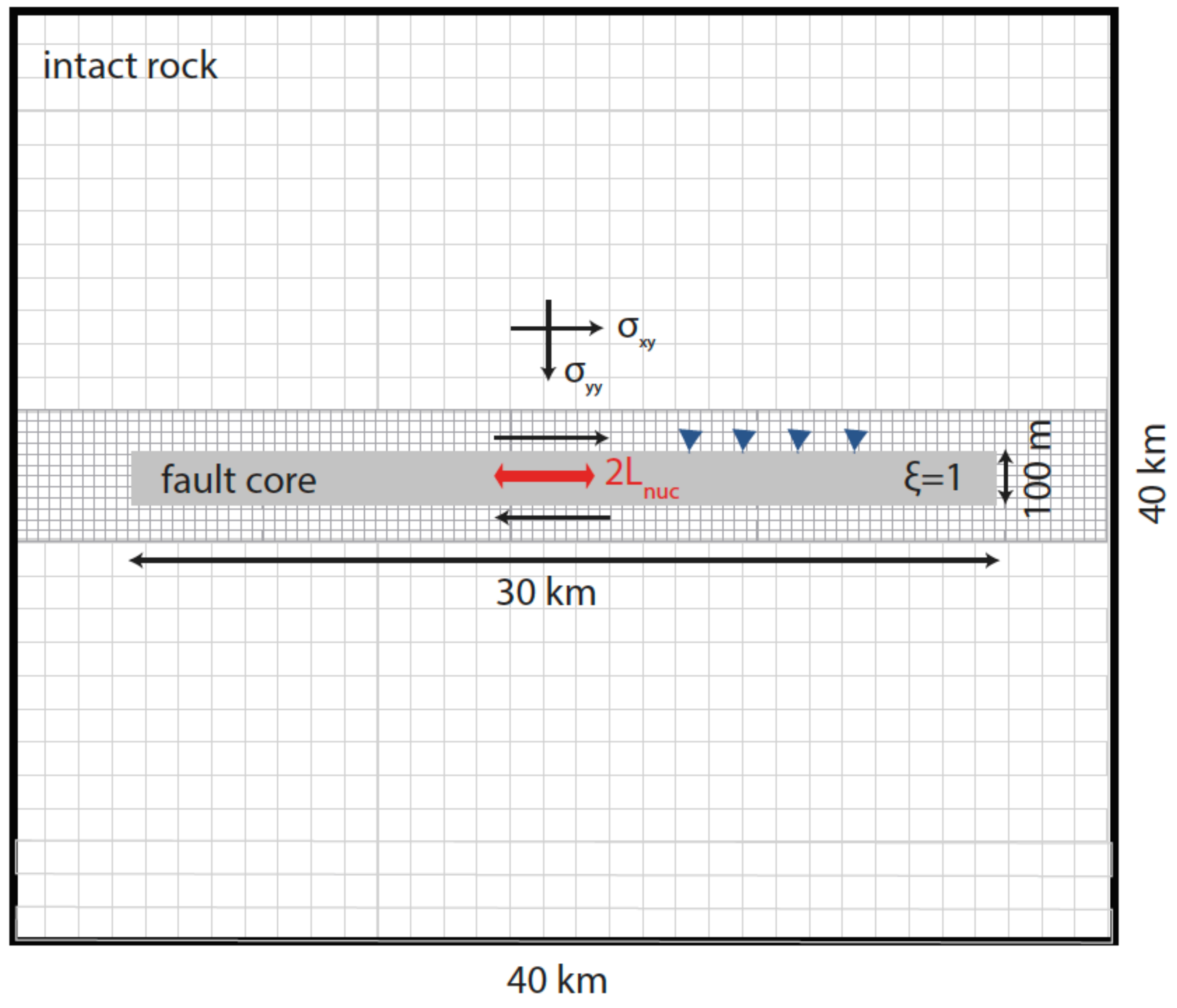


- **Fault core:** τ_1 calibrated constitutive relation of **damage resembling linear-slip weakening friction**
- Using a **single distortion field** representing the local deformation of the mixture element
- Individual rheological properties of the phases are taken into account via the dependence of a **relaxation time on the damage variable** (e.g. no friction = low relaxation time, as if fault is filled with an as if it were filled with an inviscid fluid)

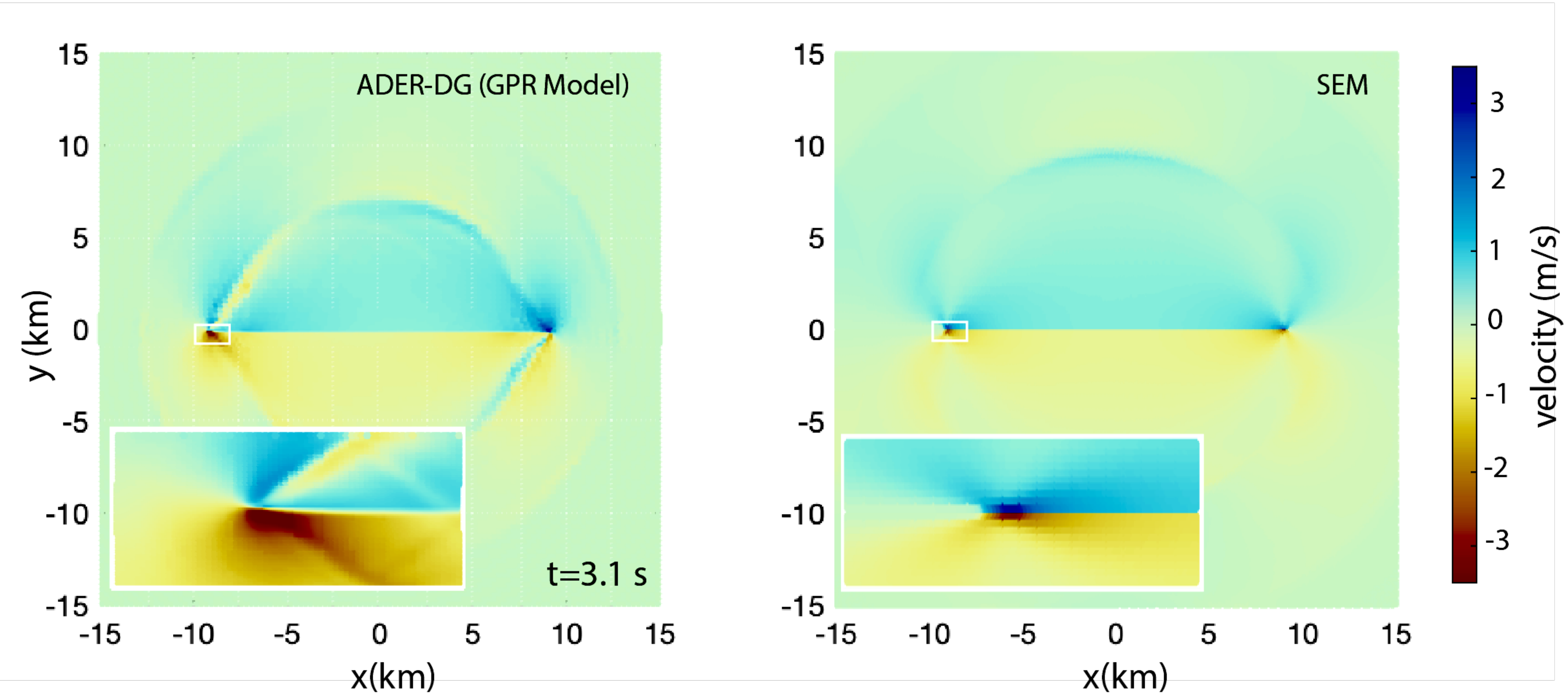
analogous linear slip weakening via τ_1



Dynamic rupture modeling in diffuse fracture zones

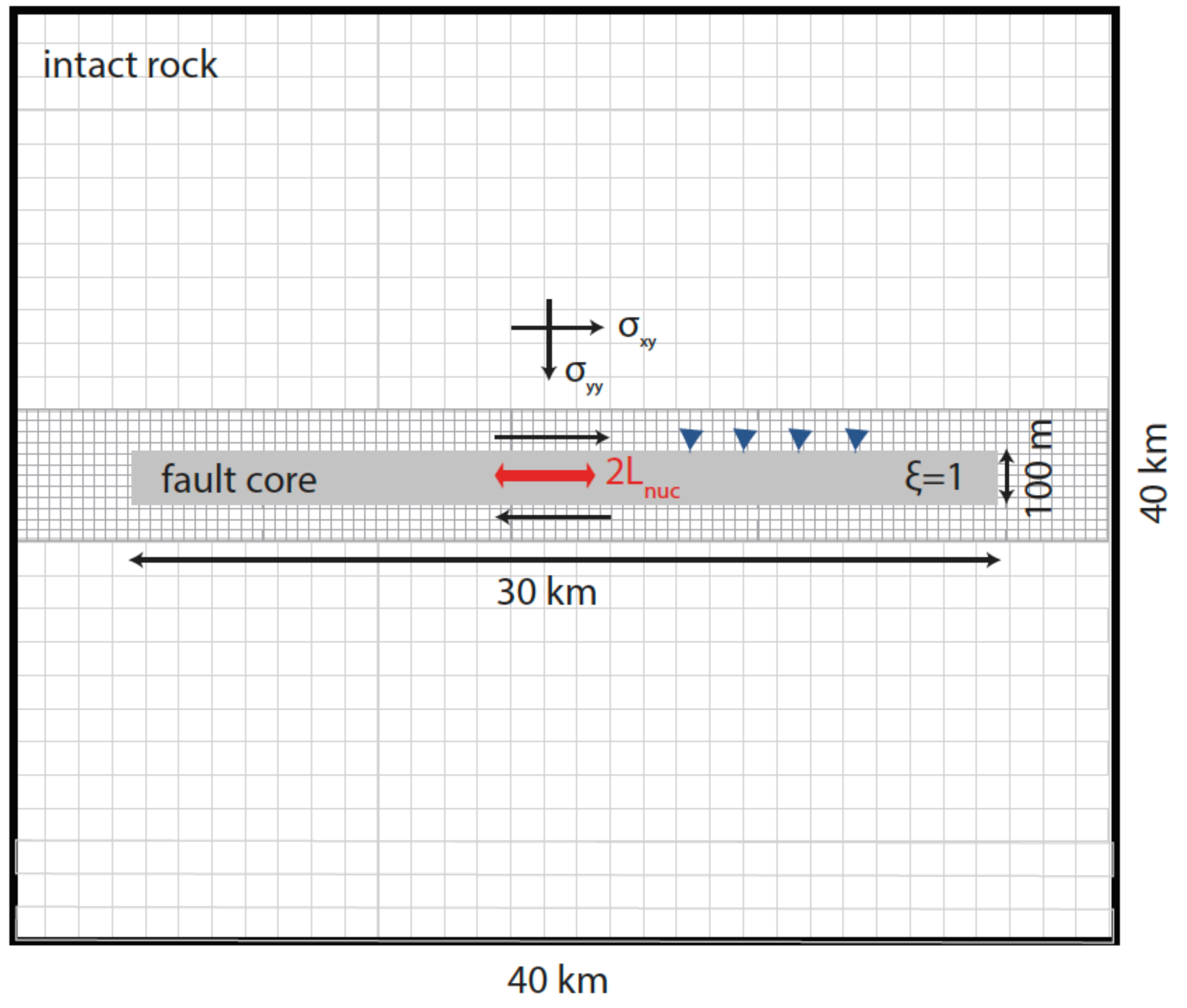


- amplified velocity pulses in the emitted wave field
- out-of-plane rotation at the rupture tip
- non-frictional control of rupture speed

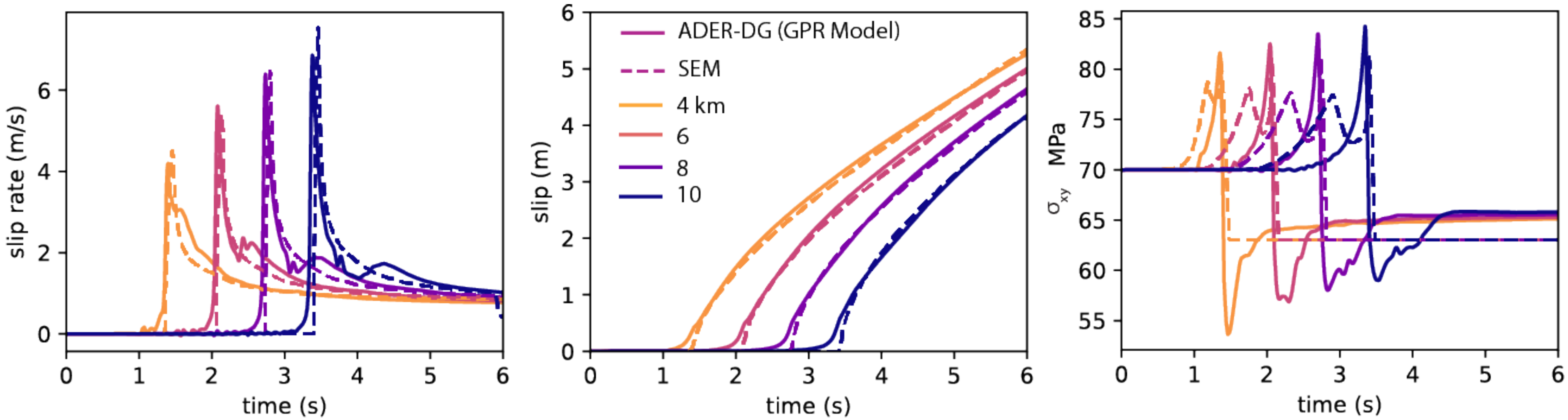


Comparison wavefield for 2D SCEC TPV3 *[Harris et al., SRL'18]* linear slip weakening dynamic rupture benchmark (ADER-DG, *p*=6, static AMR). T**he GPR model fault core is 100 m wide and 10 km long. P-wave and S-wave velocity in the fault core 30% reduced cf. off-fault linear elastic material.** SEM solution uses SEM2DPACK: https://github.com/jpampuero/sem2dpack

Dynamic rupture modeling in diffuse fracture zones



- **Fault slip rates limited in peak, modulated by dynamic complexity**
- **Fault slip asymptotically resembles the discontinuous, elastic solution**
- **Differences in shear stress may reflect the oblique shear developing and/or viscous deformation within the diffuse fault core**



Comparison on-fault time series for 2D SCEC TPV3 *[Harris et al., SRL'18]* linear slip weakening dynamic rupture benchmark (ADER-DG, *p*=6, static AMR). **The GPR model fault core is 100 m wide and 10 km long. P-wave and S-wave velocity in the fault core 30% reduced cf. off-fault linear elastic material.** SEM solution uses SEM2DPACK: https://github.com/jpampuero/sem2dpack

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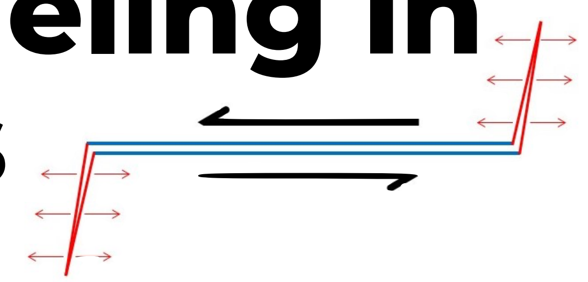
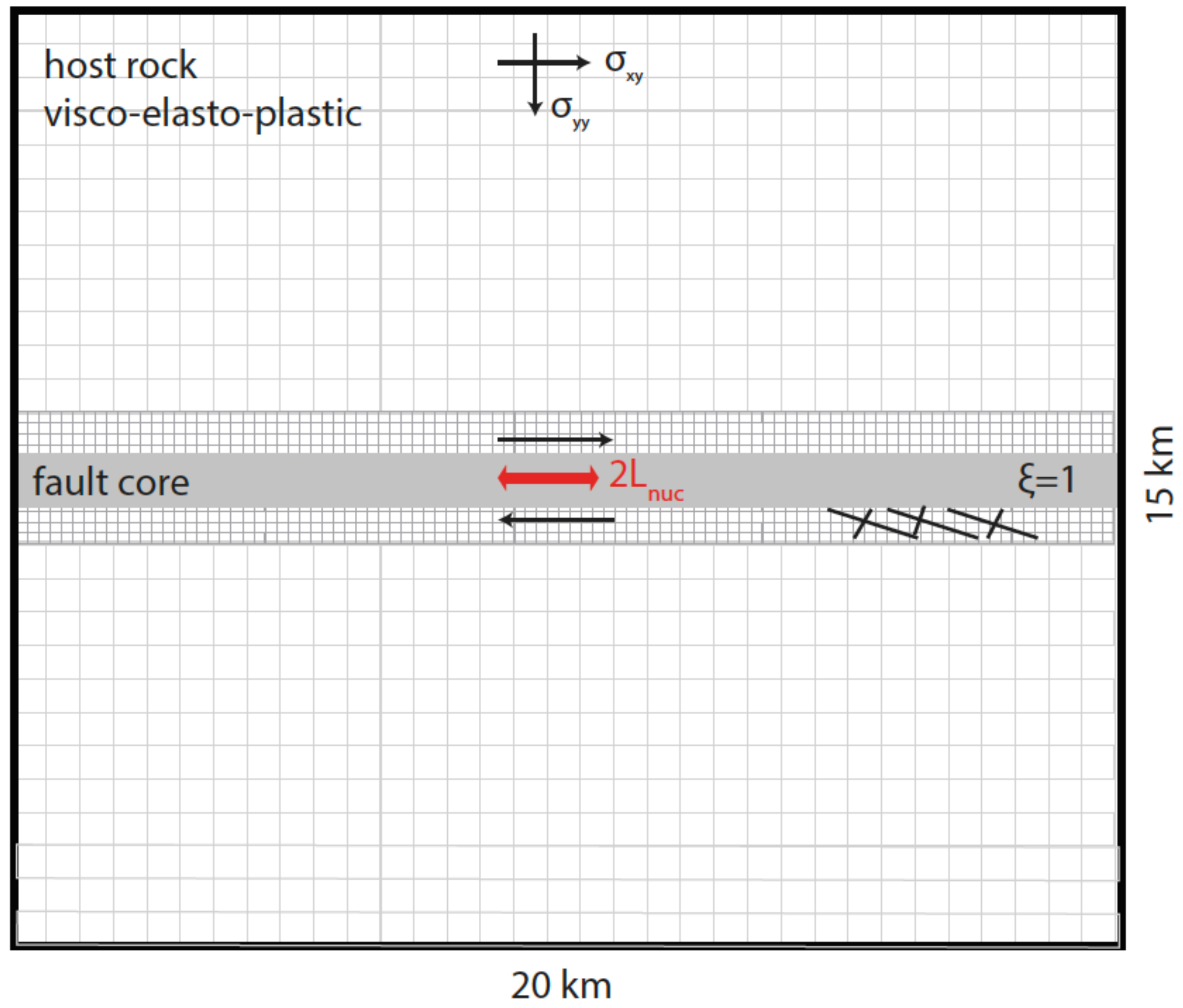
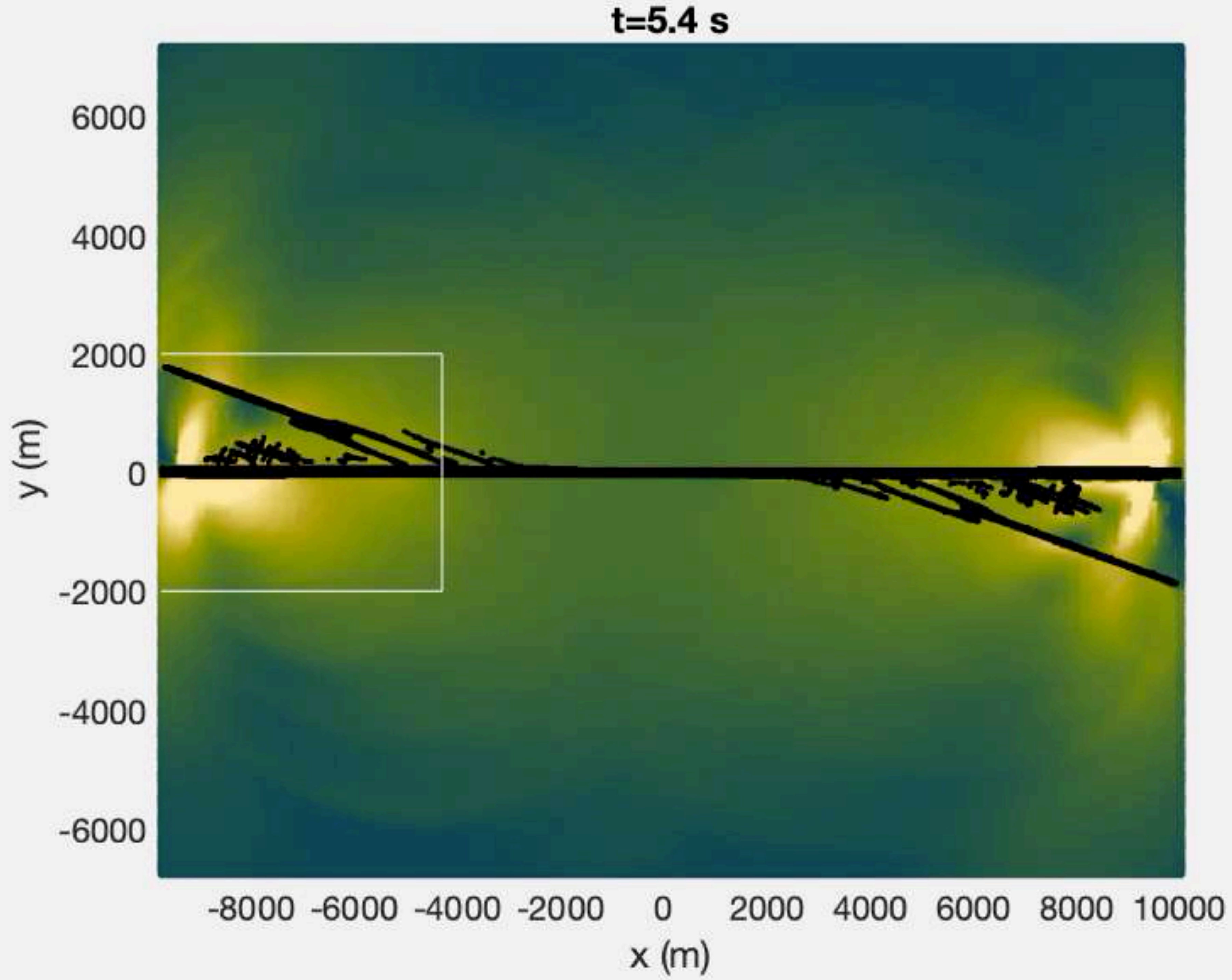
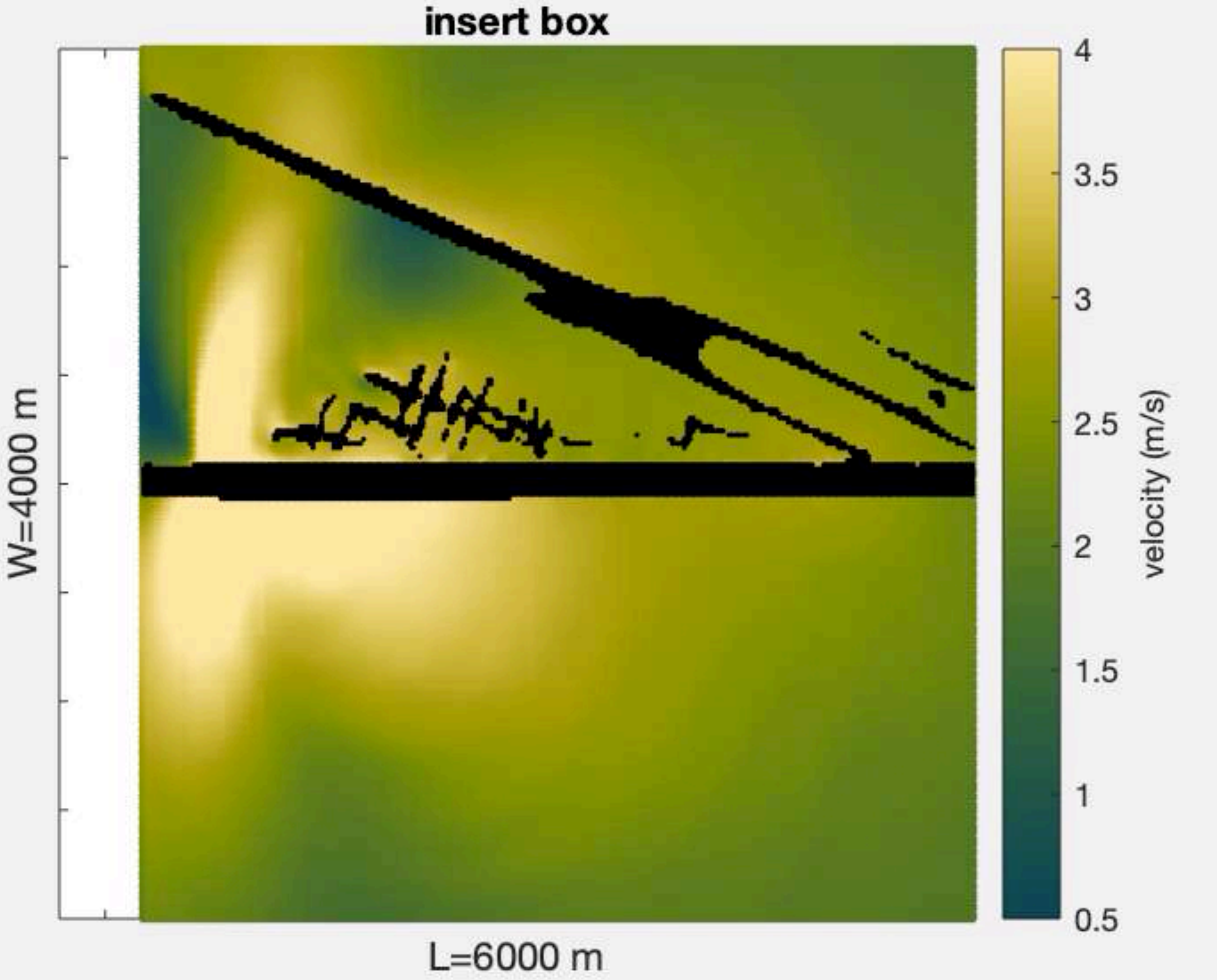
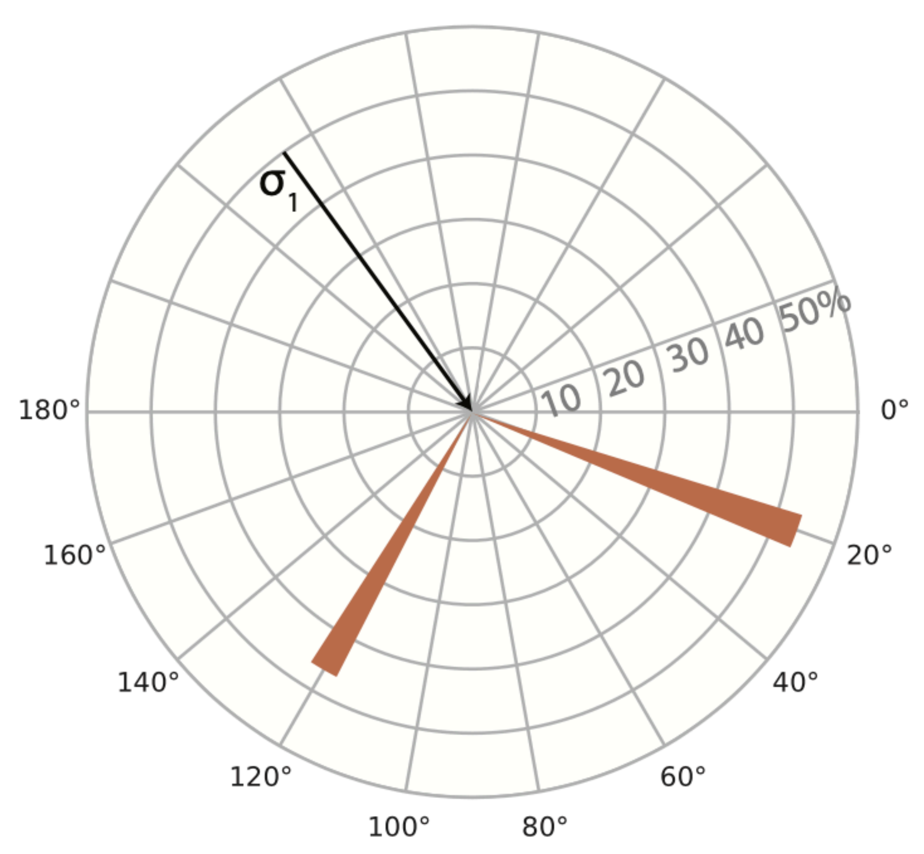


Drucker-Prager visco-elasto-plastic host rock (using ADER-DG, p=6 and dynamic AMR)









- **Mesoscopic off-fault shear cracks spontaneously generated** in the tensile lobes behind the diffuse dynamic rupture front
- Dynamic development of **interlaced conjugate shear faulting around two favourable orientations** resembling recent high-resolution imaging of earthquakes
- Off-fault cracks introduce **larger stress drop within the fault zone and suppress supershear** transition

Summary

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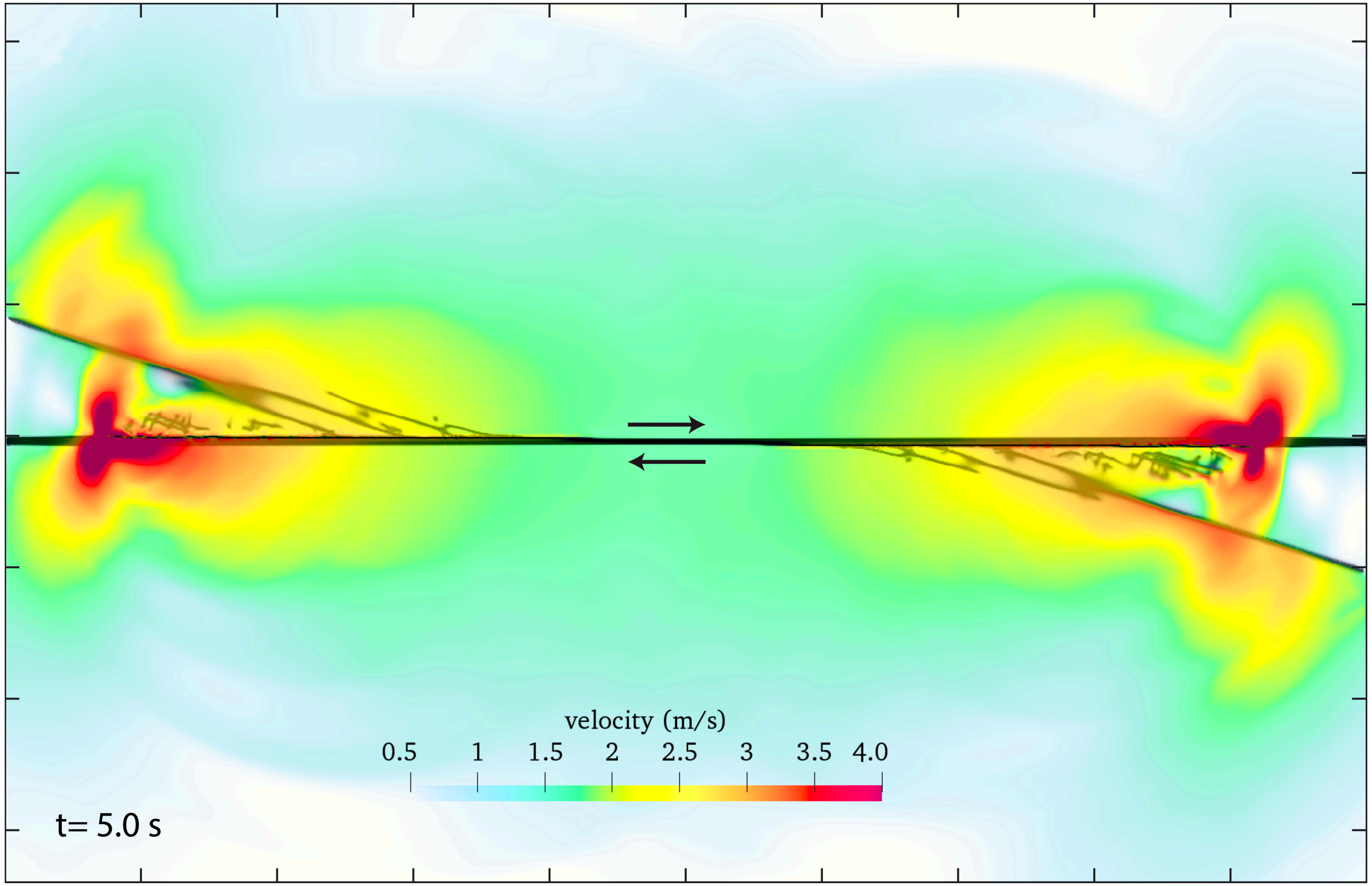
A diffuse interface unified first-order hyperbolic model for nonlinear dynamic rupture processes in diffuse fracture zones potentially allows to model volumetric fault zone shearing during earthquake rupture including spontaneous partition of fault slip into intensely localized shear deformation within weak fault core and more distributed damage within fault rocks and foliated gouges (and frictional melting)



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