New continuity-based velocity interpolation scheme for staggered grids
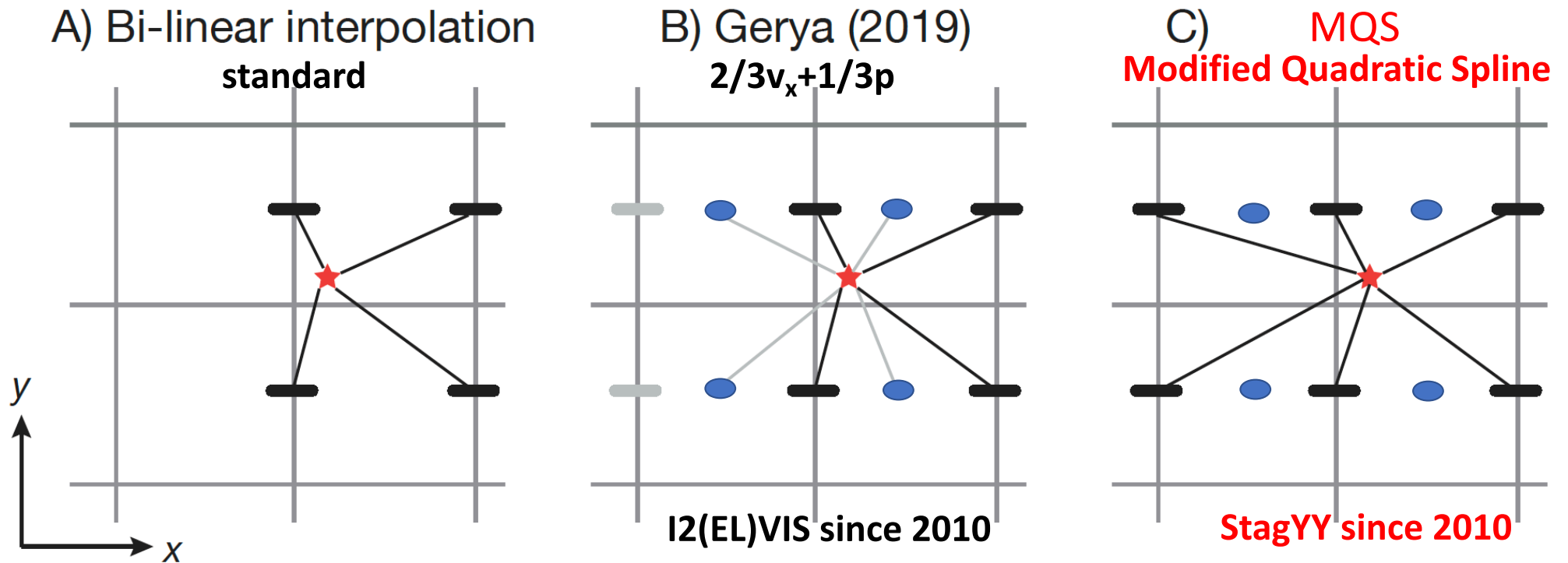
Taras Gerya[1], Paul Tackley[1], Thibault Duretz[2], Ludovic Räss[3]

[1]ETH-Zurich, Institute of Geophysics, ETH Zurich, Switzerland (taras.gerya@erdw.ethz.ch)
[2]Géosciences Rennes UMR 6118, Univ Rennes, CNRS, Rennes, France
[3]ETH-Zurich, Dept. of Civil, Environmental and Geomatic Engineering, ETH Zurich, Switzerland
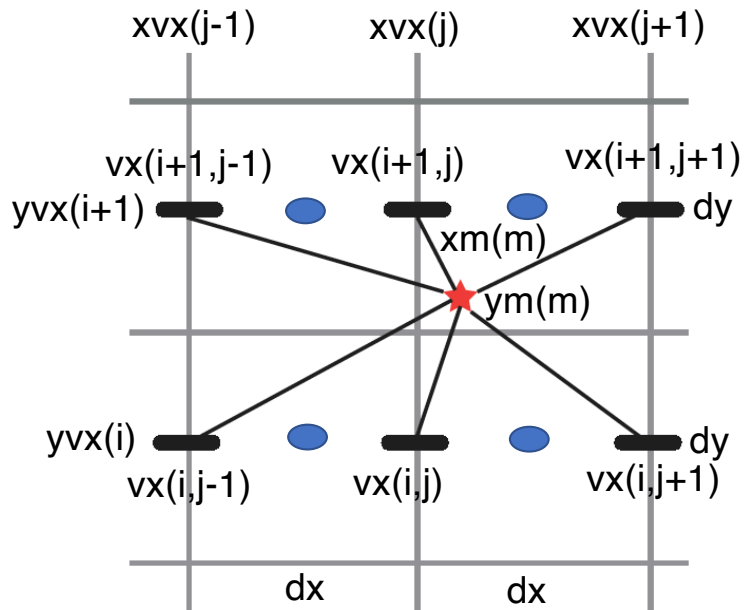
The scheme guarantee bi-linear interpolation of dvx/dx and dvy/dy from pressure nodes ⬤
where they are defined by solving (in)compressible continuity equation
Modified Quadratic Spline

reproduces average velocity in pressure nodes ⬤ **but not in original velocity nodes** ▬

Example of vx interpolation for the regular staggered grid


```
% Compute distances
dxmj=xm(m)-xvx(j);
dymi=ym(m)-yvx(i);
% Compute vx velocity with bi-linear scheme for the bottom and top
vxm13=vx(i,j)*(1-dxmj/dx)+vx(i,j+1)*dxmj/dx;
vxm24=vx(i+1,j)*(1-dxmj/dx)+vx(i+1,j+1)*dxmj/dx;
% Compute second order vx correction for the bottom and top
vxm13=vxm13+1/2*((dxmj/dx-0.5)^2)*(vx(i,j-1)-2*vx(i,j)+vx(i,j+1));
vxm24=vxm24+1/2*((dxmj/dx-0.5)^2)*(vx(i+1,j-1)-2*vx(i+1,j)+vx(i+1,j+1));
% Compute vx of the marker with bi-linear scheme in vertical direction
vxm=(1-dymi/dy)*vxm13+(dymi/dy)*vxm24;
```
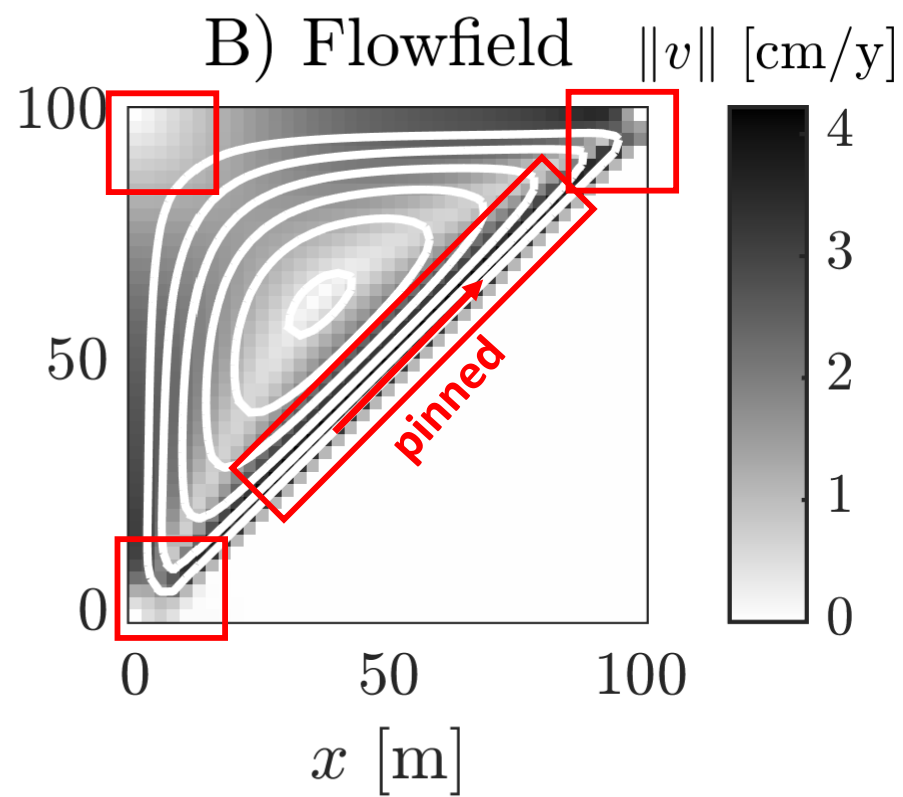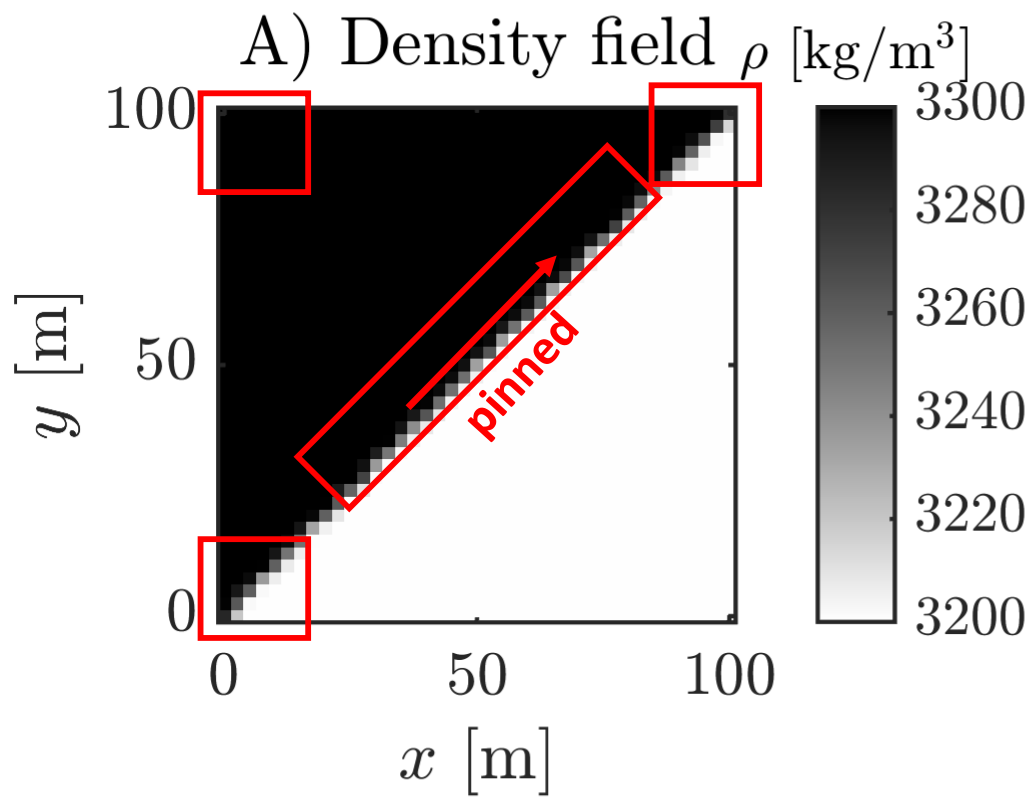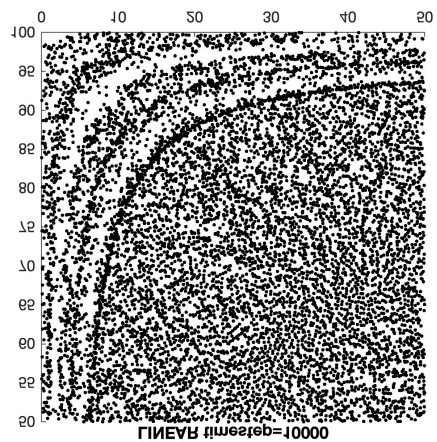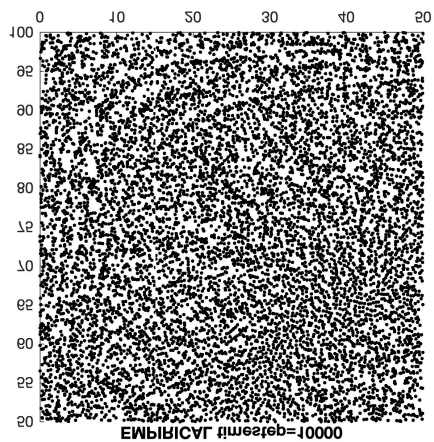
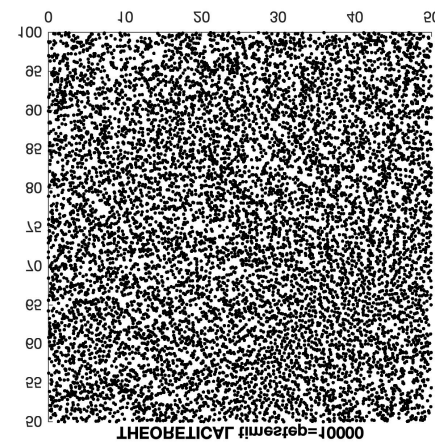**Test: circulation with diagonal shear**
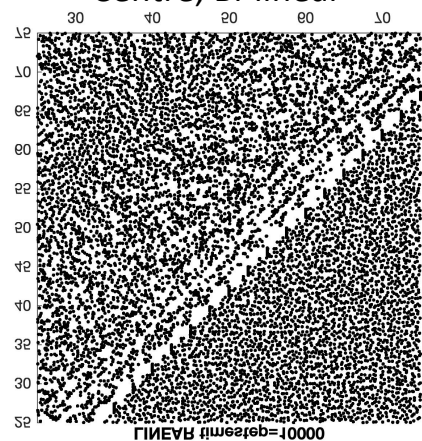
problematic regions are shown in red

Top-Left, LINEAR

Top-Left, Gerya 2019
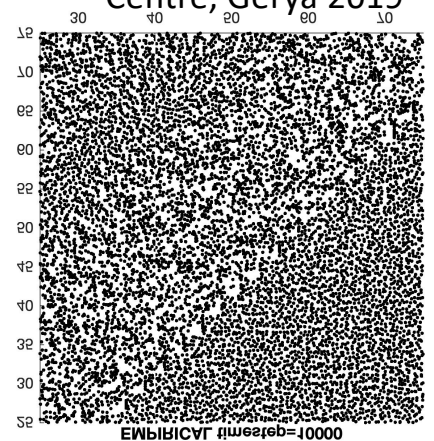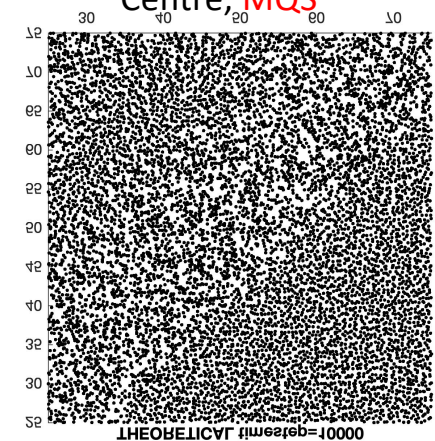
Top-Left, MQS

Centre, Bi-linear

Centre, Gerya 2019

Centre, MQS

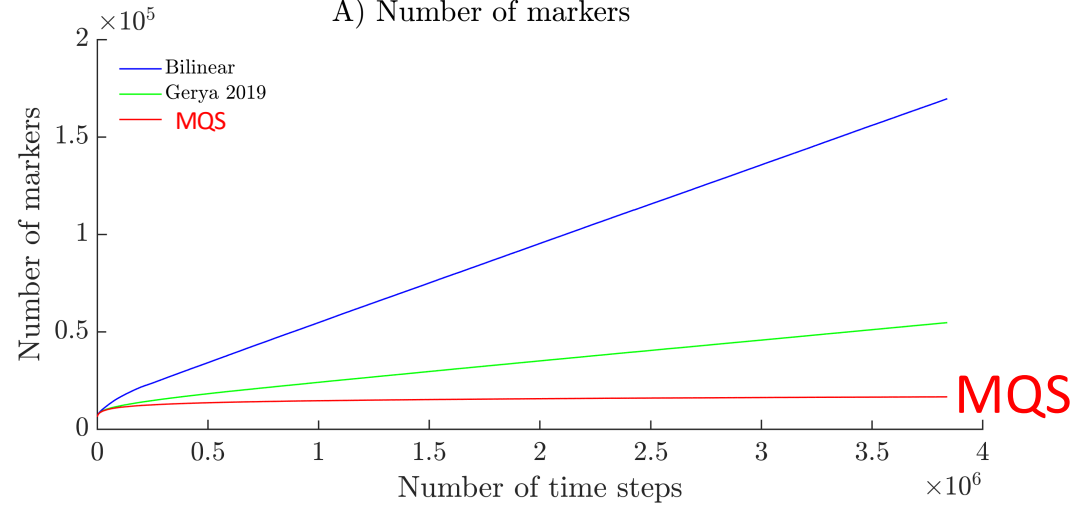# Markers injection

Bilinear at step = 590000   Bilinear at step = 2070000   Bilinear at step = 3840000

injected markers   **markers density**

Gerya 2019 at step = 590000   Gerya 2019 at step = 2070000   Gerya 2019 at step = 3840000

MQS at step = 590000   MQS at step = 2070000   MQS at step = 3840000
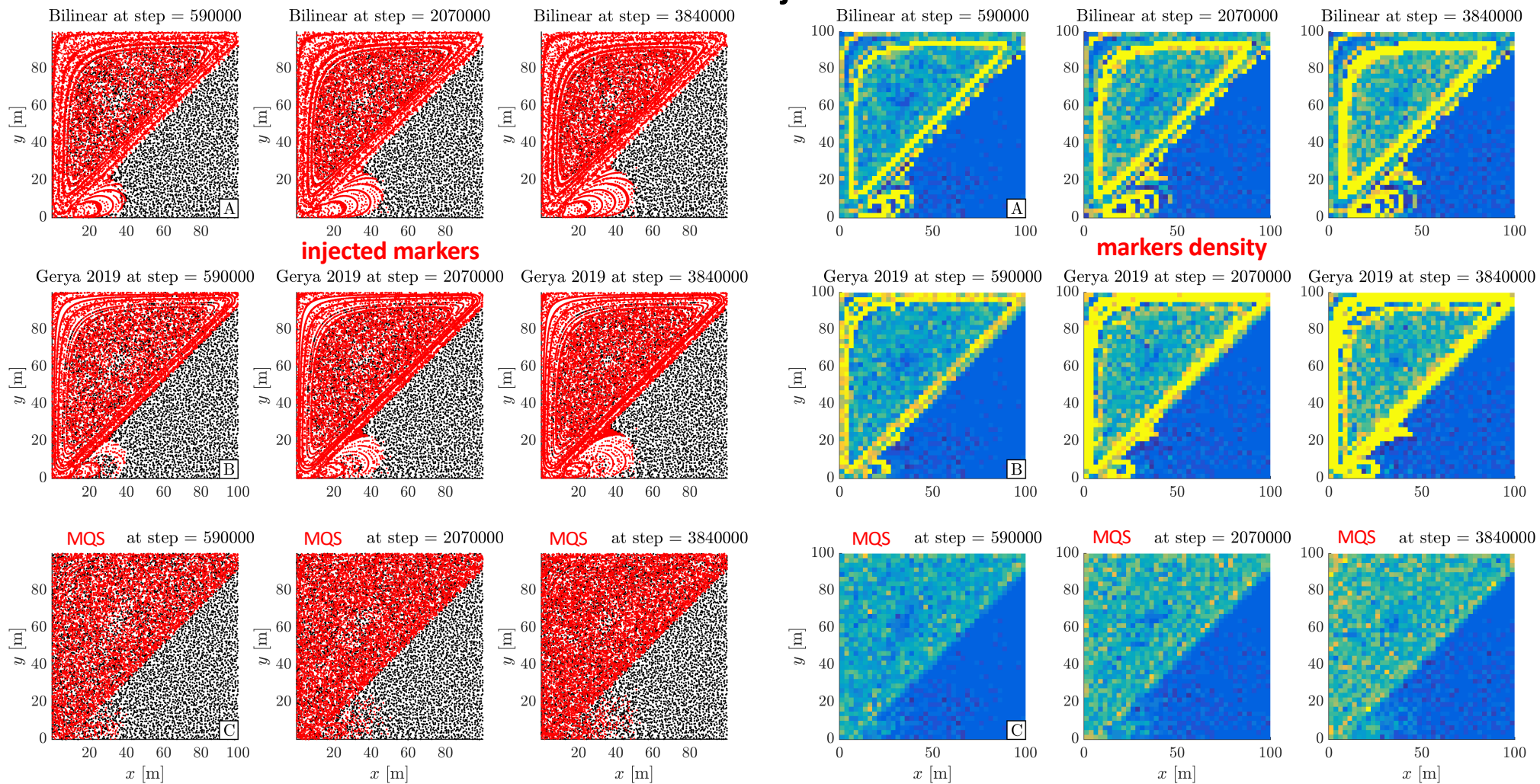
Markers injection

A) Number of markers

Legend:
- Bilinear (blue)
- Gerya 2019 (green)
- MQS (red)

Y-axis: Number of markers ($\times 10^5$)
X-axis: Number of time steps ($\times 10^6$)

MQS

B) Markers injection rate

Y-axis: Marker injection rate ($\log_{10}$)
X-axis: Number of time steps ($\times 10^6$)

MQS