On the Constitutive Equations for Coupled Reaction and Deformation

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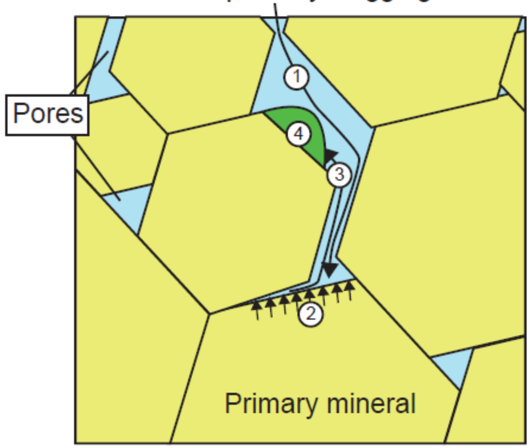
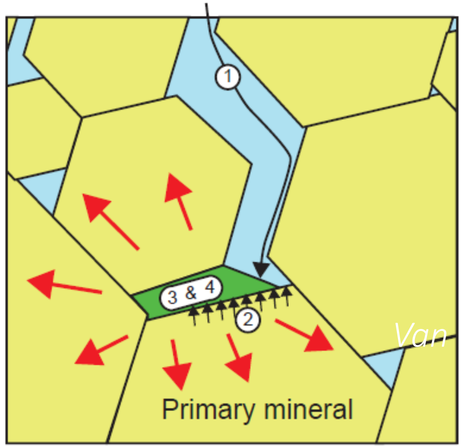
A: Classical approach:
no reaction-induced deformation
and porosity clogging



$$\nabla \cdot V_s = -\frac{1}{K_d}\frac{dp_e}{dt} - \frac{p_e}{\eta_d}$$

B: New approach:
reaction-induced swelling
and porosity preservation



$$\nabla \cdot V_s = -\frac{1}{K_d}\frac{dp_e}{dt} - \frac{p_e}{\eta_d} + \beta_d \frac{d\xi}{dt}$$

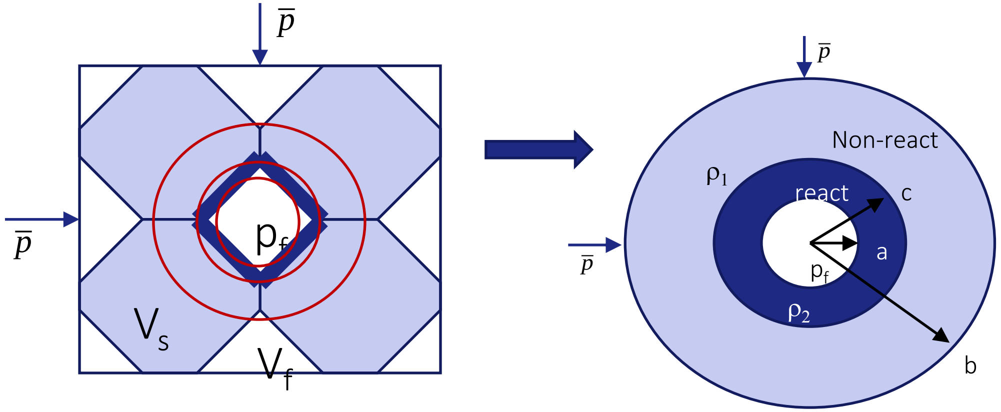
Governing equations for coupled fluid flow, reaction and deformation:

$$\begin{cases} \frac{dX_s}{dt} = \left(\frac{\phi}{\phi_p}\right)^{2/3} \frac{X_{seq}-X_s}{\tau} \\ \nabla_j \frac{k_\phi}{\eta_w}\left(\nabla_j P_w + \rho_w g e_z^j\right) = -\beta_{e_t}\frac{d\overline{P}}{dt} + \beta_{e_w}\frac{dP_w}{dt} - \frac{\overline{P}-P_w}{(1-\phi)\eta_\phi} \\ \qquad + \left(\frac{\rho_s}{\rho_w} - (1+\beta_{sr})\right)\frac{1-\phi}{1-X_s}\frac{dX_s}{dt} \\ \frac{1}{(1-\phi)}\frac{d\phi}{dt} = \left(\beta_{e_t} - \frac{\beta_s'\phi}{1-\phi}\right)\left(-\frac{d\overline{P}}{dt} + \frac{dP_w}{dt}\right) - \frac{\overline{P}-P_w}{(1-\phi)\eta_\phi} - \frac{1+\beta_{sr}}{1-X_s}\frac{dX_s}{dt} \end{cases}$$

Incomplete reaction

$$\begin{cases} \frac{dX_s}{dt} = \left(\frac{\phi}{\phi_p}\right)^{2/3} \frac{X_{seq}-X_s}{\tau} \\ \nabla_j \frac{k_\phi}{\eta_w}\left(\nabla_j P_w + \rho_w g e_z^j\right) = -\beta_{e_t}\frac{d\overline{P}}{dt} + \beta_{e_w}\frac{dP_w}{dt} - \frac{\overline{P}-P_w}{(1-\phi)\eta_\phi} \\ \qquad + \left(\frac{\rho_s(1-\phi)}{\rho_w} + \phi\,(1+\beta_{sr})\right)\frac{1}{1-X_s}\frac{dX_s}{dt} \\ \frac{1}{(1-\phi)}\frac{d\phi}{dt} = \left(\beta_{e_t} - \frac{\beta_s'\phi}{1-\phi}\right)\left(-\frac{d\overline{P}}{dt} + \frac{dP_w}{dt}\right) - \frac{\overline{P}-P_w}{(1-\phi)\eta_\phi} \end{cases}$$

Complete reaction



Force balance $\quad \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0$

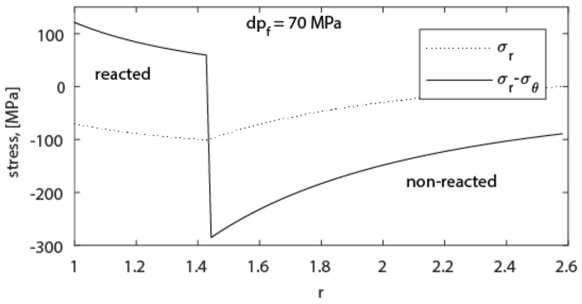
Incompressibility $\quad \frac{\partial v}{\partial r} + \frac{v}{r} = 0$

Elastic/ viscous rheology $\quad \frac{v}{r} = \frac{\dot{\sigma}_\theta - \dot{\sigma}_r}{4G}$ (elastic), $\quad \frac{v}{r} = \frac{\sigma_\theta - \sigma_r}{4\mu}$ (viscous)

Jump condition at the reaction front $\quad \rho_1 v_1 - \rho_2 v_2\big|_{r=c} = \frac{dc}{dt}(\rho_1 - \rho_2)$

Stresses around the pore at different levels of pore pressure increase

a)



b)