Non-planar dynamic rupture modelling across diffuse, deforming fault zones using a spectral finite element method with a non-mesh aligned embedded diffuse discontinuity

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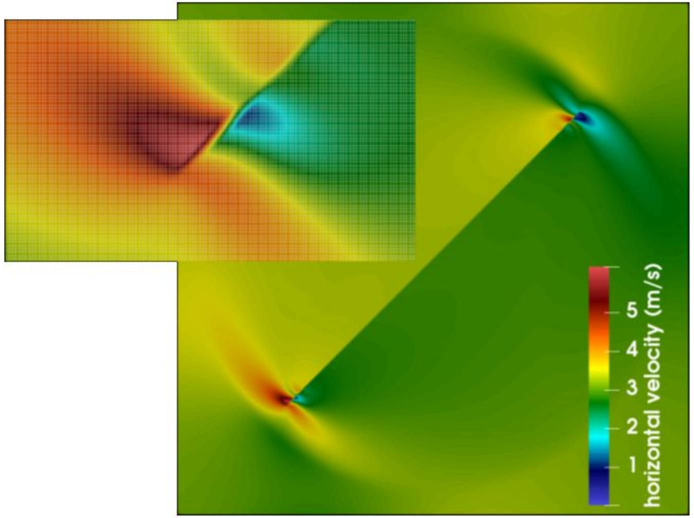

Tilted, kinematically driven Kostrov self-similar crack rupturing not aligned to the mesh. The cell size used is 50 m, with polynomial degree of 3 and a diffuse fault width of 200 m. The inset highlights the solution at the rupture tip.

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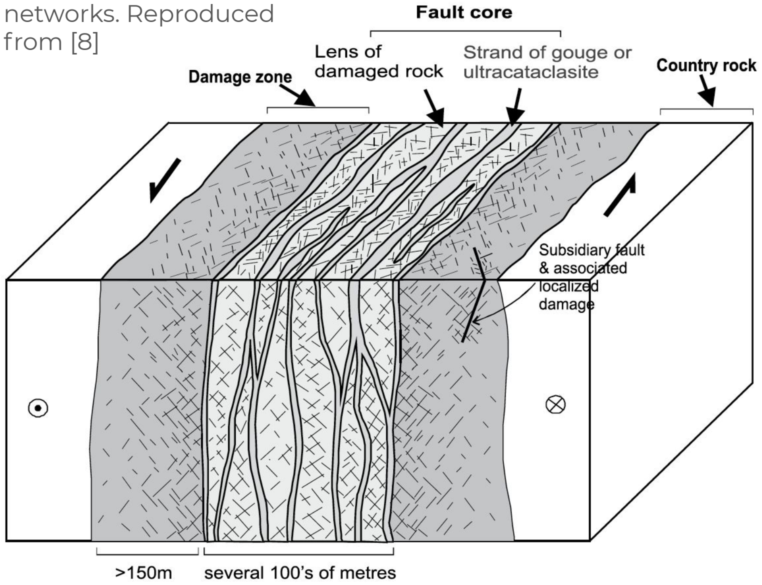
1. Introduction

Complex volumetric failure patterns are observed from well-recorded large and small earthquakes [1,2] as well as in laboratory experiments [3].

To understand the mechanics of slip in extended fault zones, the TEAR project (https://www.tear-erc.eu) aims to model **how faults slip** based on models with increased material and geometrical complexities.

Our method aims to capture arbitrary fault geometries independent of the mesh via a volumetric representation. For this, we use a **PETSc** [4,5,6] **spectral element adaption of the stress-glut method** [7]

Fig 1. Complex volumetric failure patterns of fault networks. Reproduced from [8]



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2. A spectral element stress glut approach (1/4)

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2.1 Method: se2dr (https://bitbucket.org/dmay/se2wave)

Our approach uses se2dr, a rupture dynamics extension inspired by the stress-glut method, originally developed for the finite difference method [9, 10]. The stress glut approximates the fault-jump conditions through inelastic increments to the stress components in a one grid step width inelastic zone. Here we extend the stress glut method to a 2D wave propagation spectral element (continuous Galerkin) method (SEM) built via the high-level library PETSc [4,5,6] as our linear algebra backend.

We use a structured hexahedral mesh as a spatial discretization. The SEM nodal basis is given by a Lagrange polynomial, which in combination with a Gauss-Legendre-Lobatto quadrature rule, the discretization results in a diagonal mass matrix M. The latter translates, by construction, into the flexibility of having locally (element-wise) defined material coefficients (ρ,λ,μ) over the domain, and hence, localized stresses element-wise.

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4. Balay, S., Gropp, W. D., McInnes, L. C., & Smith, B. F., 1997. Efficient management of parallelism in object oriented numerical softwar e libraries, in Modern Software Tools in Scientific Computing, pp. 163–202, Birkh¨auser Press.
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2. A spectral element stress glut approach (2/4)

2.2 Fault representation

The fault is defined via a signed distance function φ(x), a member of the level set family of functions which is, in turn, used to define a fault indicator function with compact support H. The signed distance function represents the fault zone independently of the mesh discretization and provides a straightforward manner to compute a fault local reference frame.

The compact support H is the domain where the tangential shear stress component τ transitions from the zero level set location. At the zero level set, τ is dominated by the frictional sliding law to the outer locations that follow $|\varphi| \geq \delta$, where the material behaves as pure elastic solid. δ is the prescribed minimum transversal distance from the zero level set, which defines the inelastic zone width.

We use the signed distance function to extract the values from the velocity and the displacement field to calculate the slip and slip rate relative to the fault to compare against the reference.

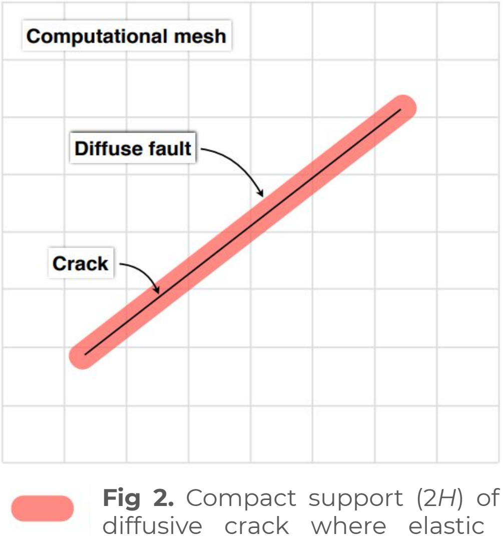

Fig 2. Compact support (2*H*) of the diffusive crack where elastic and tangential stress from the friction law are blended.

2. A spectral element stress glut approach (3/4)

2.3 Shear stress yielding condition

A critical shear stress $\tau_c \geq \tau$ acts as a limiter of the shear traction component of the stress τ within the compact support H interpreted as our plasticity model. This yield criterion uses a time-dependent friction coefficient $\mu(t)$ following the Kostrov self-similar crack problem [11] (Fig. 3 shows a comparison of the Kostrov crack for the reference and our mesh aligned method). When the yielding condition is met, we impose an antiparallel condition between the shear stress and the shear traction, and then we update the shear component of the stress.

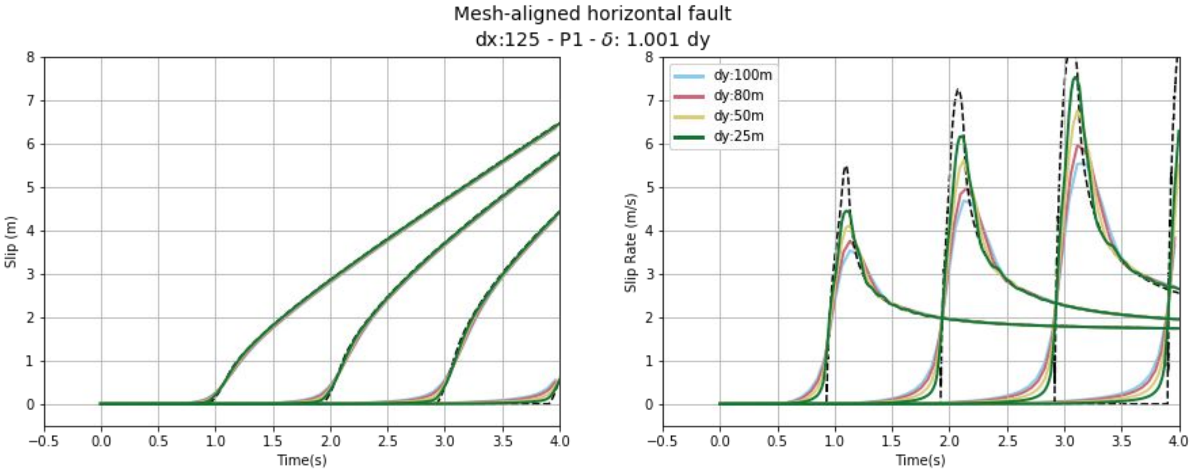

Fig 3. Slip and slip rate profile depicting kinematically driven Kostrov self-similar crack rupturing aligned to the mesh. Continuous lines show uniaxial h-refinement (dy) with polynomial order 1, and a varying inelastic thick zone with constant ratio of 1.001dy. The dashed line shows the reference from SEM2DPACK (https://github.com/jpampuero/sem2dpack).

11. Kostrov, B. V. Self similar problems of propagation of shear cracks. *J. Appl. Math. & Mech.*, 28(5), 1077-1087, 1964.

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2. A spectral element stress glut approach (4/4)

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se2dr allows us to change the friction law of the yielding function to resemble other benchmarks. One example is the SCEC benchmark TPV3 [12], which includes a linear slip weakening friction law.

se2dr also allows for alternative non-local plasticity model functionals. We can consider a functional that transitions the elastic domain into the plastic domain within the compact support H for higher order polynomials. We define such functional as

$$w(\tau, \tau_c, \varphi) = (1-\omega(\varphi, \varphi_o, A))\|\tau\|+\omega(\varphi, \varphi_o, A)\tau_c$$

which uses the weighting function

$$\omega(\varphi, \varphi_o, A) = (\tanh(A(\|\varphi\|-\varphi_o)) + 1)/2,$$

with A, φ_o as the blending parameters.

This blending applied to the TPV3 benchmark results in Fig. 4

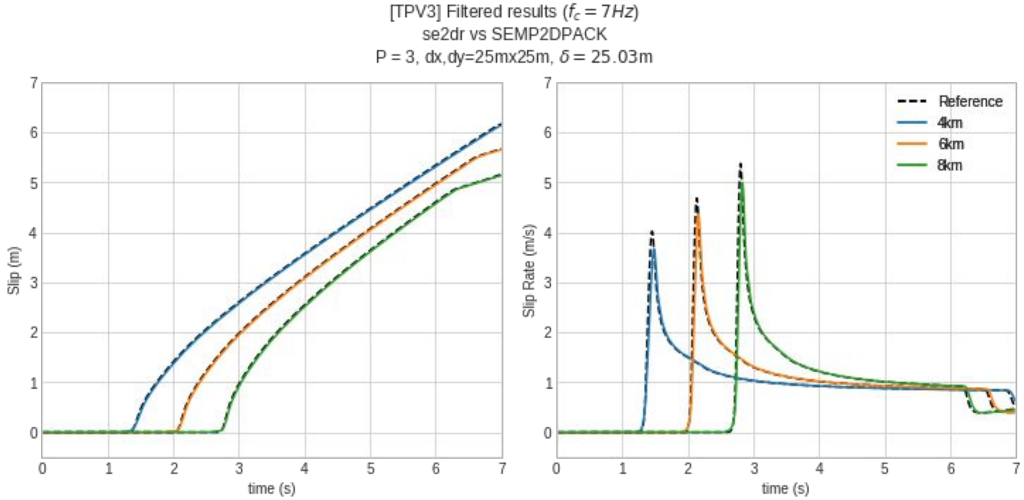

Fig 4. Comparison of the SEM diffuse fault se2dr vs. the SEM split-node reference solution of SEM2DPACK for slip (left) and slip rate (right) at three along-fault stations for the SCEC TPV3 community benchmark [15]. se2dr here uses a cell size of 25 m, with p = 3, a diffuse thickness of 50 m, and a blending function with parameters $A = 18/\delta$, $\varphi_o = 0.65\ \delta$. All results shown were banded pass filtered.

12. Harris, R. A., Barall, M., Aagaard, B., Ma, S., Roten, D., Olsen, K., Duan,B., Liu, D., Luo, B., Bai, K., Ampuero, J., Kaneko, Y., Gabriel, A., Duru,K., Ulrich, T., Wollherr, S., Shi, Z., Dunham, E., Bydlon, S., Zhang, Z.,Chen, X., Somala, S. N., Pelties, C., Tago, J., Cruz-Atienza, V. M., Koz-don, J., Daub, E., Aslam, K., Kase, Y., Withers, K., & Dalguer, L., 2018.A Suite of Exercises for Verifying Dynamic Earthquake Rupture Codes,Seismological Research Letters,89(3), 1146–1162

3. Reference problem

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We solve the reference problem of a Kostrov-like kinematic self-similar shear crack [13] in different geometrical setups. The 2D problem [11] consists of a homogeneous and isotropic elastic medium, and the crack propagating along the pre-defined fault. The initial conditions are [13]:

- Density (ρ) = 2500 kg/m^3
- P-wave velocity (V_p) = 4000 m/s
- Normal stress (S_{yy}) = -40 MPa
- Characteristic distance (L) = 250 m
- μ_s= 0.5, μ_d= 0.25
- S-wave velocity (V_s) = 2309 m/s
- Shear stress (S_{xy}) = 20 MPa
- Sliding speed (V) = 2000 m/s

This reference uses an externally imposed traction, and while it does not include the fully dynamic behaviour, it allows to verify the relation between slip, slip rate and traction [13]. We compare our results at on-fault receivers located at 2, 4, 6 km along-strike.

11. Kostrov, B. V. Self similar problems of propagation of shear cracks. *J. Appl. Math. & Mech.*, 28(5), 1077-1087, 1964.
13. de la Puente, J., J.-P. Ampuero and M. Käser. Dynamic rupture modeling on unstructured meshes using a discontinuous Galerkin method. JGR: Solid Earth, 114(B10), 2009

4. Planar self-similar crack (1/3)

Here we apply our Kostrov-like kinematic model to non-mesh aligned geometrical setups.

As illustrated in Fig. 5, we first show the results of the kinematic crack under different tilting from the mesh aligned case.

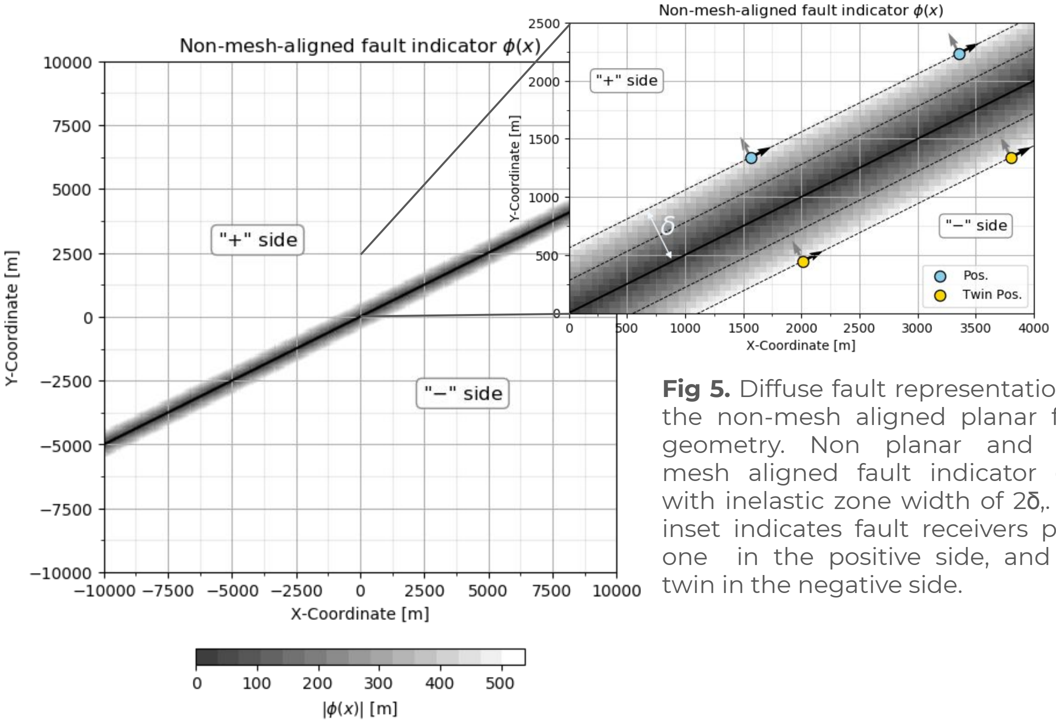

Fig 5. Diffuse fault representation of the non-mesh aligned planar fault geometry. Non planar and non mesh aligned fault indicator φ(x) with inelastic zone width of 2δ,. The inset indicates fault receivers pairs: one in the positive side, and the twin in the negative side.

4. Planar self-similar crack (2/3)

Using our method on tilted cases, we obtain the domain fields of displacement and velocity as depicted in Fig. 6 for polynomial order three and Fig. 7 for low order polynomial one.

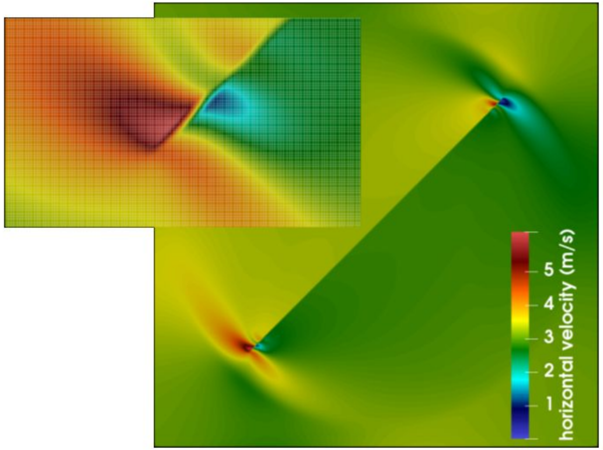

Fig 6. Tilted, kinematically driven Kostrov self-similar crack rupturing not aligned to the mesh. The cell size used is 50 m, with polynomial degree of 3 and a diffuse fault width 2δ=200 m. The inset highlights the solution at the rupture tip.

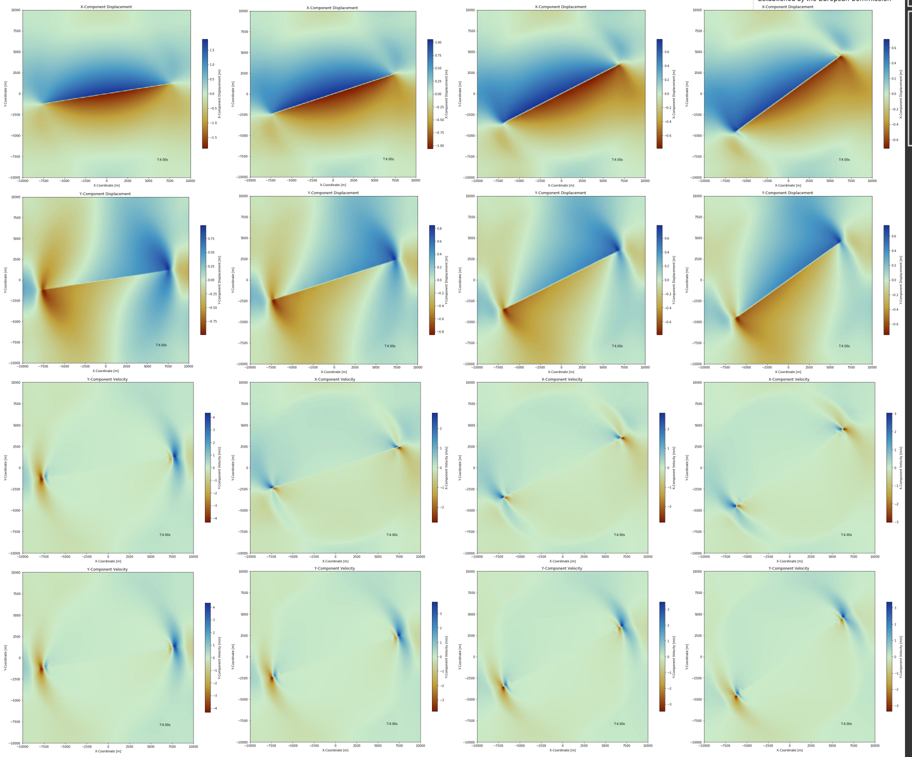

Fig 7. Self-similar crack simulations for the tilted crack with 9, 18, 27, 36 degree tilt. X,Y components of the displacement, and velocity fields are depicted on each row respectively. All simulations were ran under a polynomial order of 1, cell dimensions 50mx50m, and δ=100m

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4. Planar self-similar crack (3/3)

We extract the slip and slip rate profiles from each simulation. These profiles can be seen in Fig. 8; they resemble the case of the mesh aligned linear fault, however, with reduced amplitude.

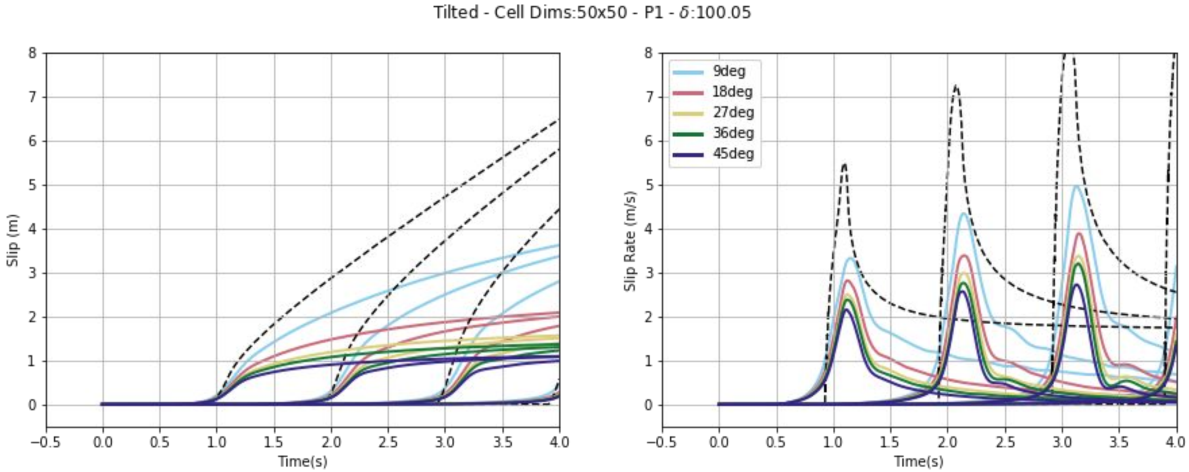

Fig 8. Slip and slip rate profiles for the planar non-mesh aligned geometries. The profile plot includes, as continuous lines, the planar simulations with a tilting of 9, 18, 27, 36, 45 degrees tilt from the mesh aligned case. Similar to figure 3, the dashed lines reflect the SEM2DPACK reference. The simulations here use cell dimensions of 50m x 50m, and a polynomial order of 1. Here, δ = 100.05m, or 2.001 dy.

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5. Curved self-similar crack (1/3)

For the sigmoid case, the respective domain and fault geometry schematic is shown in Fig. 9.

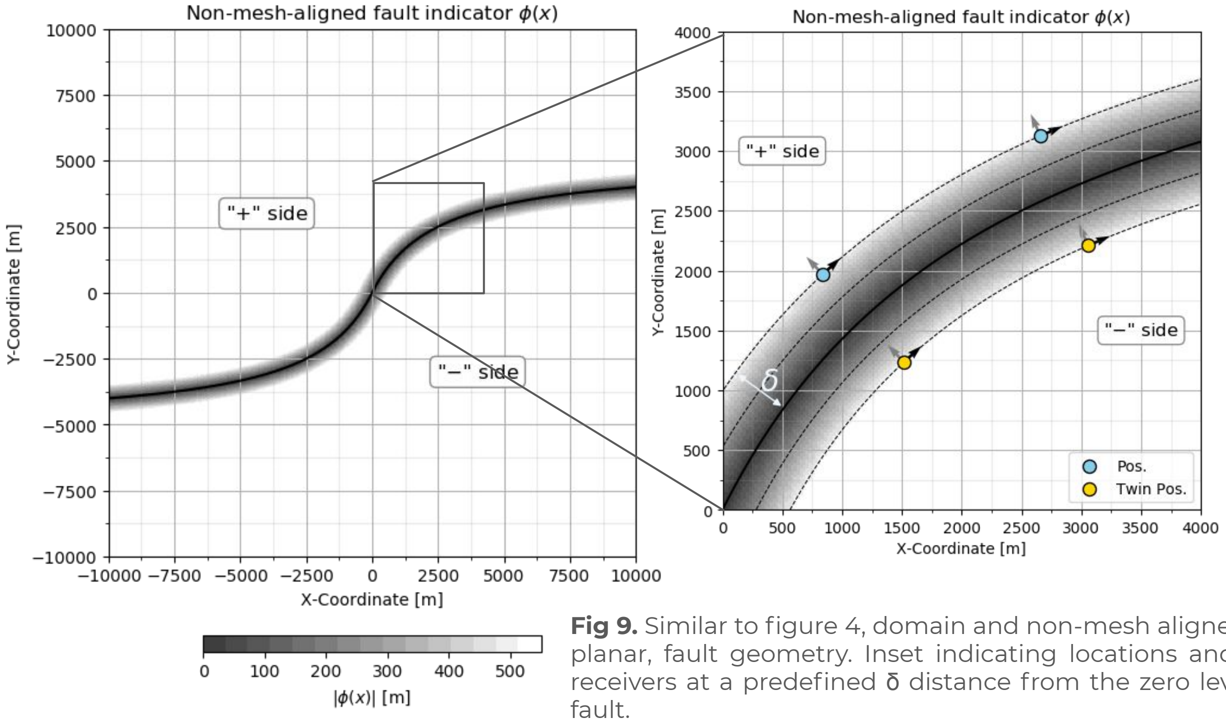

Fig 9. Similar to figure 4, domain and non-mesh aligned, nor planar, fault geometry. Inset indicating locations and twin receivers at a predefined δ distance from the zero level set fault.

5. Curved self-similar crack (2/3)

The displacement and velocity field components of the low polynomial order simulation are shown in Fig. 10.

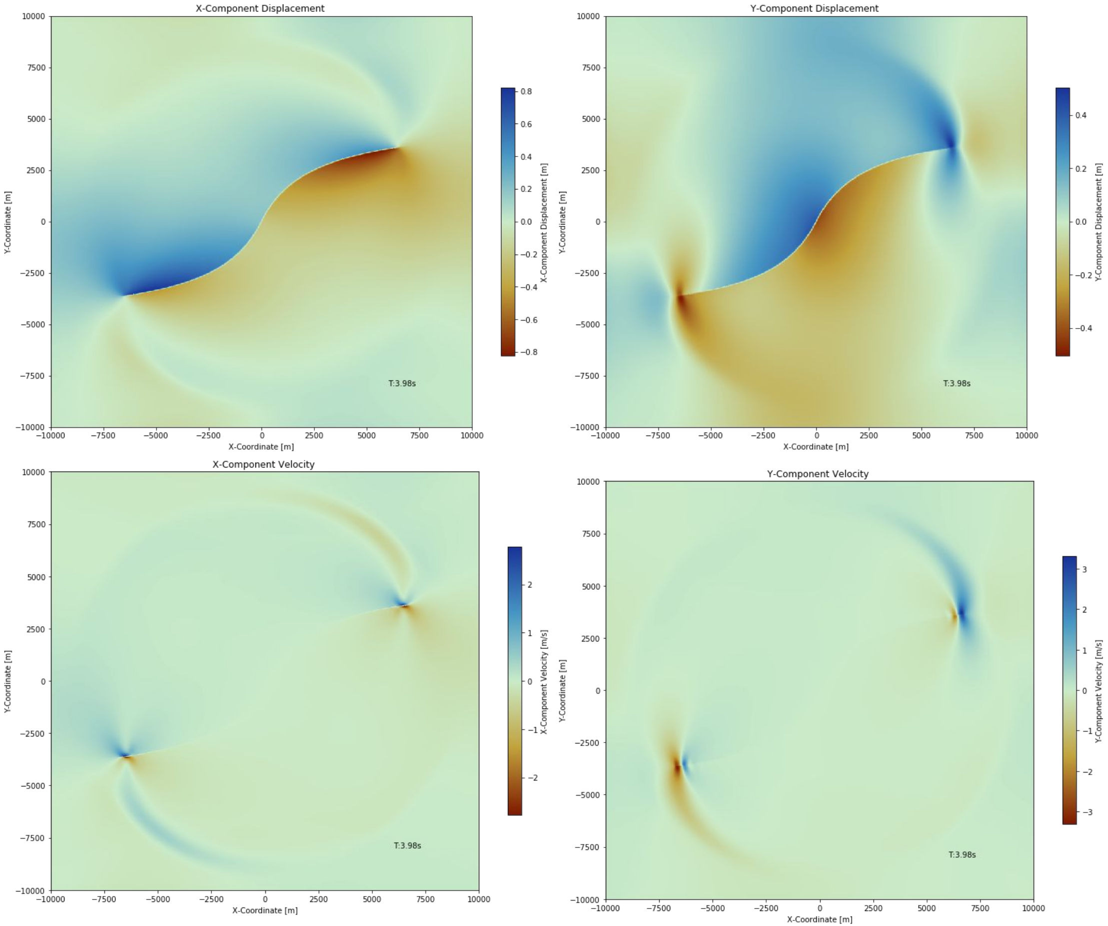

Fig 10. Self-similar crack simulation in a sigmoid geometry. (Top row) Displacement and (Bottom row) velocity field components X and Y in the domain reference frame. Polynomial order of 1 and cell dims of (25mx25m).

5. Curved self-similar crack (3/3)

The extracted slip and slip rate profiles for the low polynomial order simulation of the sigmoid curved kinematic crack also resembles the mesh aligned case with reduced amplitude.

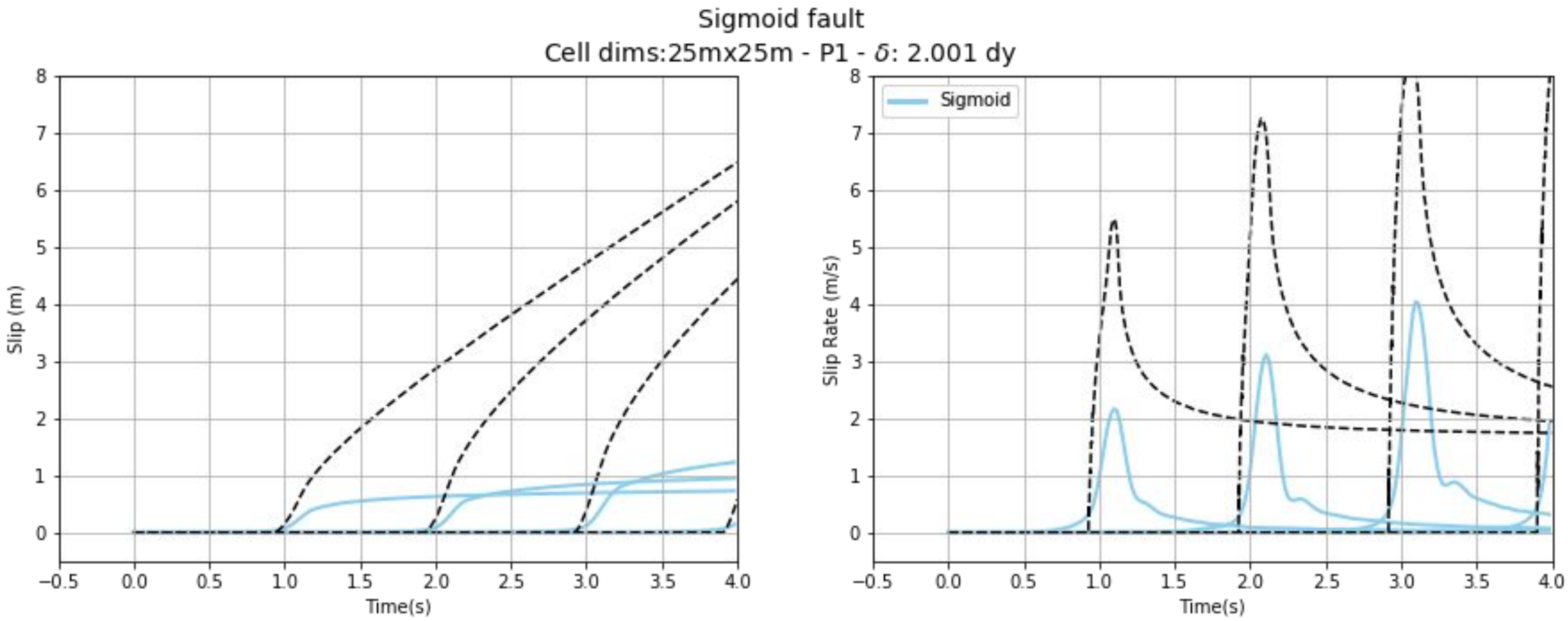

Fig 11. Slip and slip rate profile of kinematic Kostrov self similar crack problem following a sigmoid geometry. The cell dimensions here are 25m x 25m, polynomial order 1, and δ here is 2,001 dy.

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6. Discussion

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Sharp discontinuities manifest spurious oscillations in the proximity, which reduces spectral convergence to low order accuracy. In dynamic rupture problems, the representation of the friction law from a plasticity model can originate such discontinuities, e.g., such as in the case for the linear slip-weakening function as friction law. Additional non-smoothness may be introduced from enforcing conditions on the stresses.

We can choose a functional that defines the non-local plasticity model and transitions into the pure elastic domain within our fault-local compact support. This flexibility can allow us to choose, among other plasticity models, a blending that resembles the solution to the diffusion equation used in diffuse interface methods [14]. The parameters in this function require to be further constrained by a physical quantity or behaviour, as these parameters affect the magnitude of the slip and slip rate and help overcome the sharp transitions from a dynamic rupture within SEM.

14. Sun, Y. & Beckermann, C., 2007. Sharp interface tracking using the phase-field equation, Journal of Computational Physics, 220 (2), 626–653

7. Summary and outlook

A diffuse fault zone description collapses fault volumetric complexities onto a distribution within a compact support.

The mesh aligned case of our stress glut SEM extension matches the split-node spectral element dynamic reference.

Our diffuse interface alternative to dynamic rupture is also tested against non-mesh aligned fault geometries. The results resemble the timing of the reference peak slip rates with reduced amplitudes for low polynomial orders.

We consider exploring alternative non-local plasticity model functionals that can extend our method to high order cases.

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