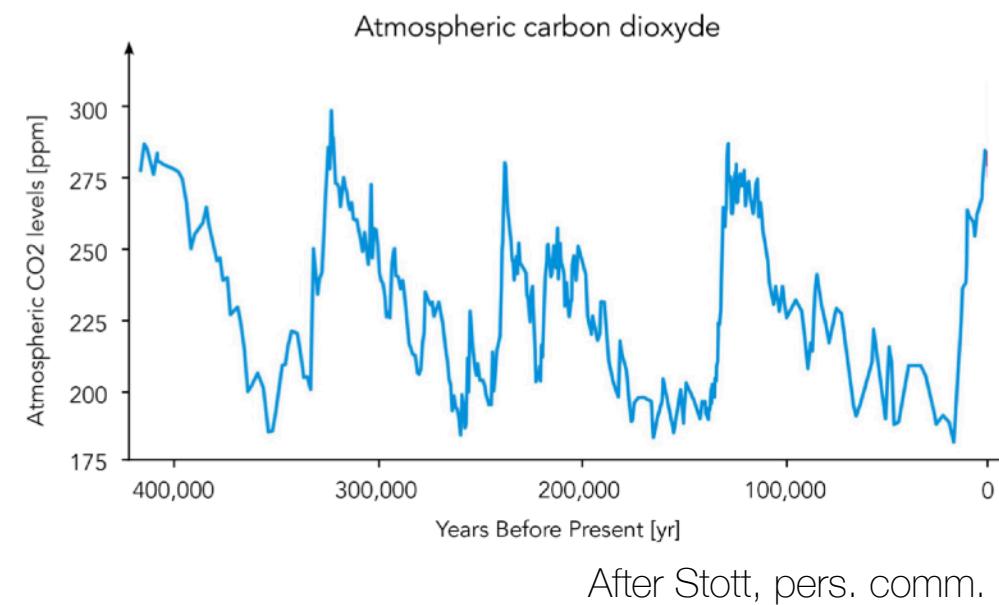


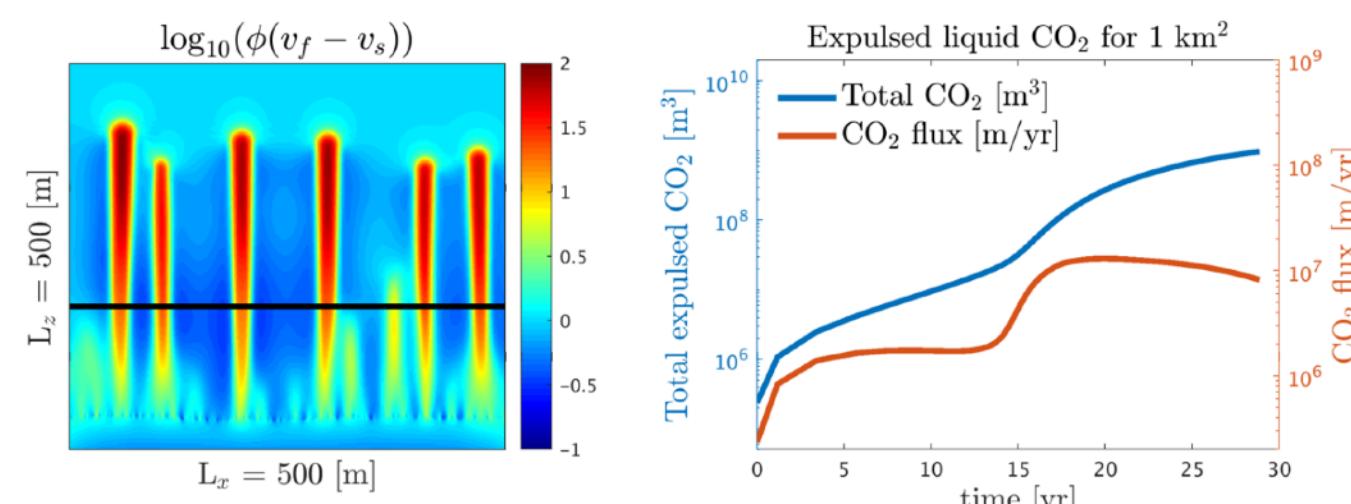
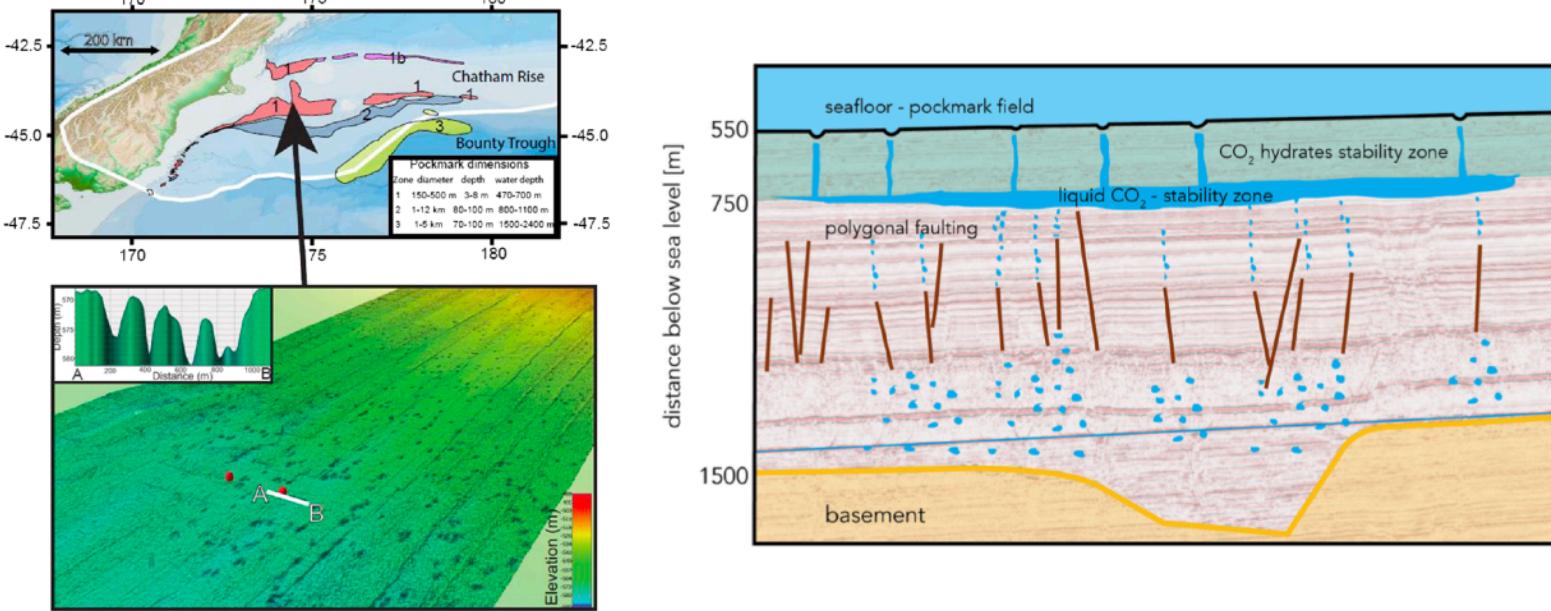
# Modelling reactive two-phase flow: challenges and implications

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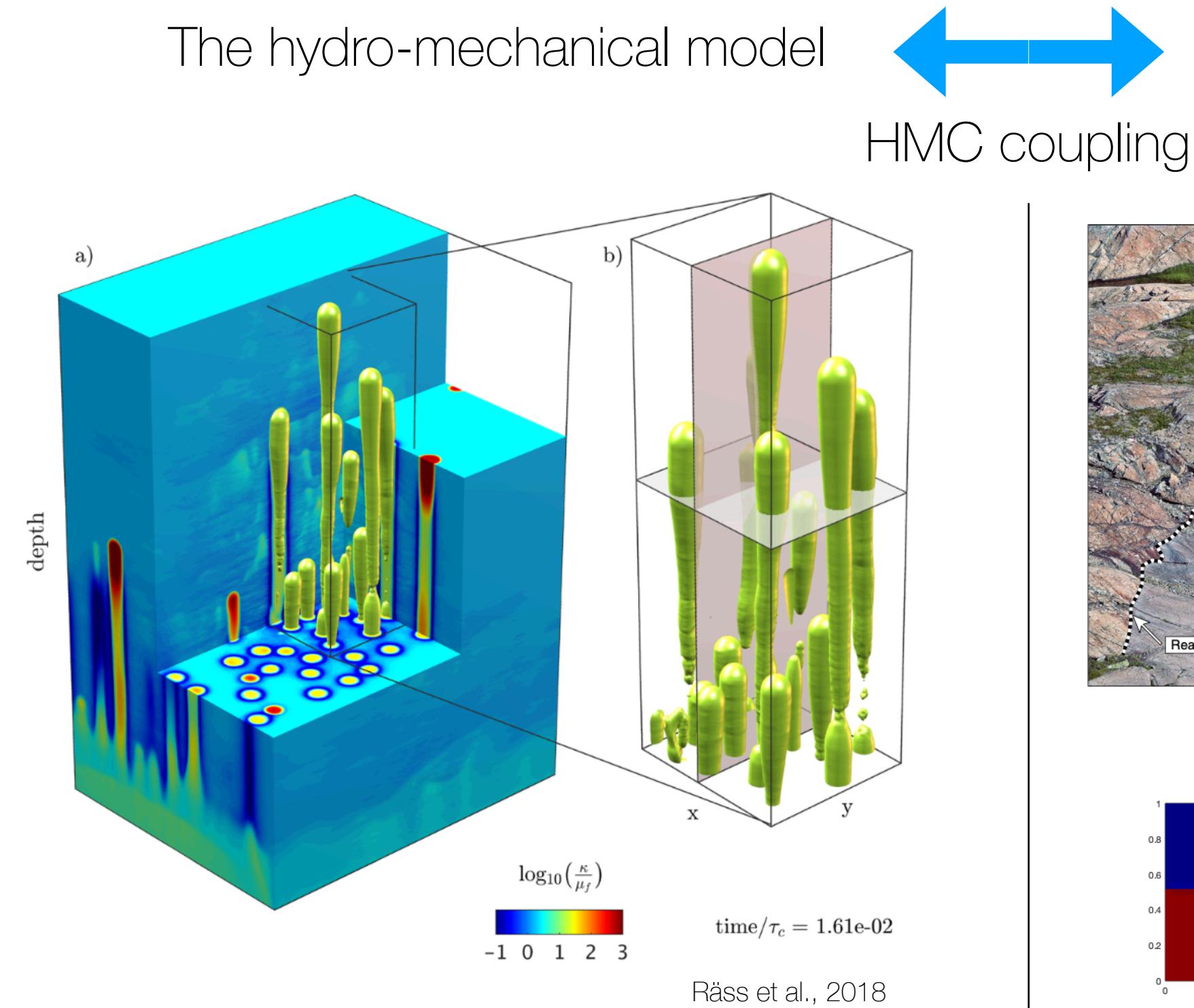
Atmospheric CO<sub>2</sub> variations from ice core records



Understand the mechanisms responsible for CO<sub>2</sub> variations in “recent” geological times.



The hydro-mechanical model



$$0 = \nabla_j (\bar{\tau}_{ij} - \bar{p}\delta_{ij}) - \bar{\rho}g_i \quad \frac{\partial \phi}{\partial t} = (1-\phi)\nabla_k v_k$$

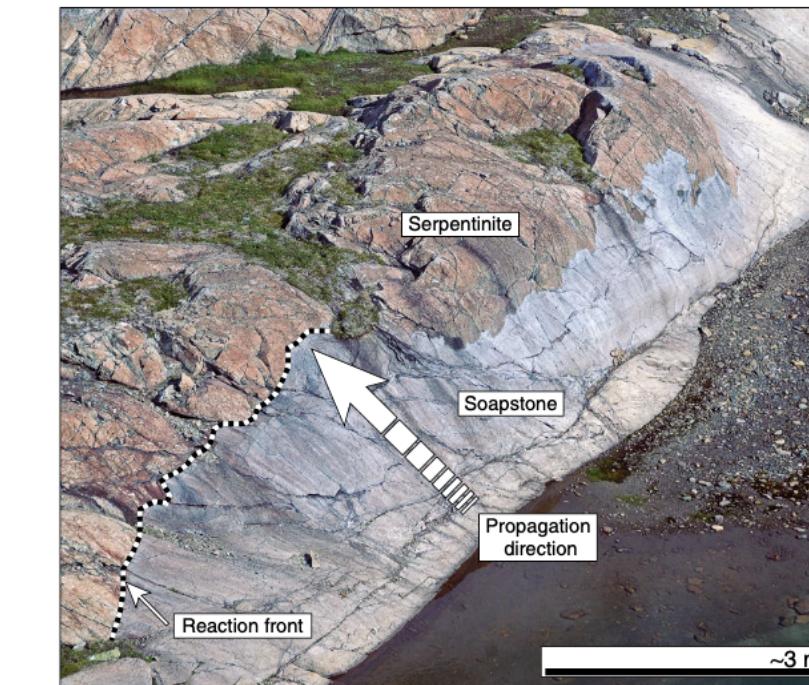
$$0 = \nabla_k v_k + \frac{\bar{p} - p^f}{\eta_\phi(1-\phi)} \quad \eta_\phi = f(\phi, \bar{p}, p^f, \dot{\epsilon}_{II})$$

$$0 = \nabla_k q_k^D - \frac{\bar{p} - p^f}{\eta_\phi(1-\phi)} \quad \mu = f(\phi, \dot{\epsilon}_{II})$$

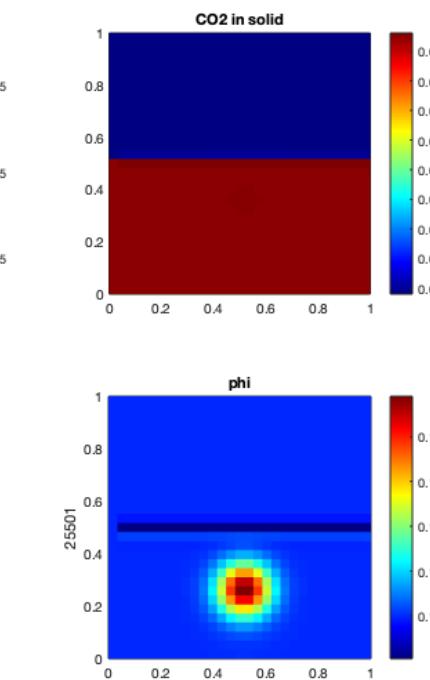
$$q_i^D = -k_\phi(\nabla_i p_i^f + \rho^f g_i) \quad k_\phi = k_0(\phi/\phi_0)^3$$

$$\frac{1}{G} \frac{\partial \bar{\tau}_{ij}}{\partial t} + \frac{\bar{\tau}_{ij}}{\mu_s} = \nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla_k v_k$$

The hydro-chemical model



Beinlich et al., 2021 Nature Geo



$$\rho_f = \rho_f(T, P_f, C_f^{CO_2})$$

$$\rho_s = \rho_s(T, P_f, C_f^{CO_2})$$

$$C_s^m = C_s^m(T, P_f, C_f^{CO_2})$$

$$C_s^{CO_2} = C_s^{CO_2}(T, P_f, C_f^{CO_2})$$

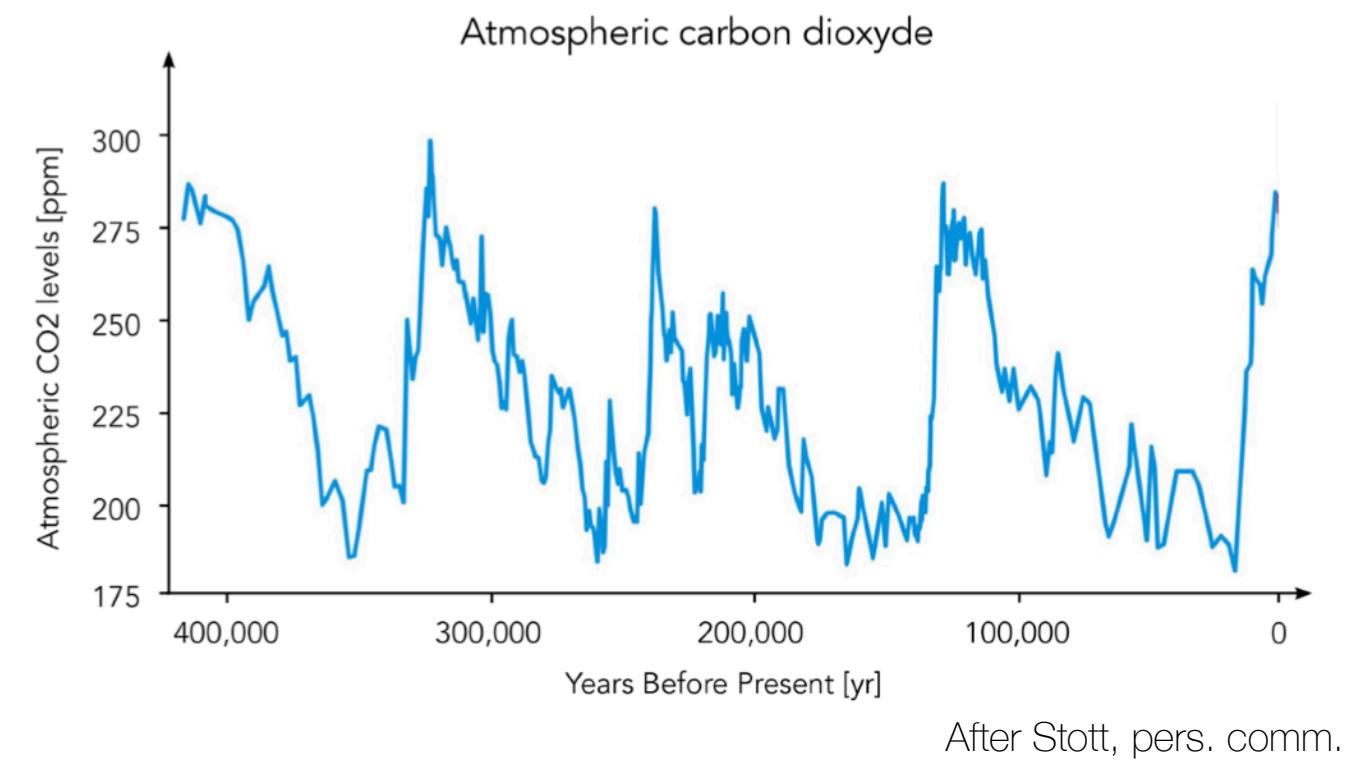
$$\frac{\partial}{\partial t} (\rho_f \phi C_f^{CO_2} + \rho_s (1-\phi) C_s^{CO_2}) - \nabla \left( \rho_f \phi C_f^{CO_2} \frac{k \phi^3}{\mu_f} (\nabla P_f + \rho_f \mathbf{g}) \right) \\ = \nabla (\rho_f \phi D_f^{CO_2} \nabla C_f^{CO_2}) \\ \frac{\partial}{\partial t} (\rho_s (1 - C_s^m) (1 - \phi)) = 0 \\ \frac{\partial}{\partial t} (\rho_f \phi + \rho_s (1 - \phi)) = \nabla \left( \rho_f \frac{k \phi^3}{\mu_f} (\nabla P_f + \rho_f \mathbf{g}) \right)$$

Some details

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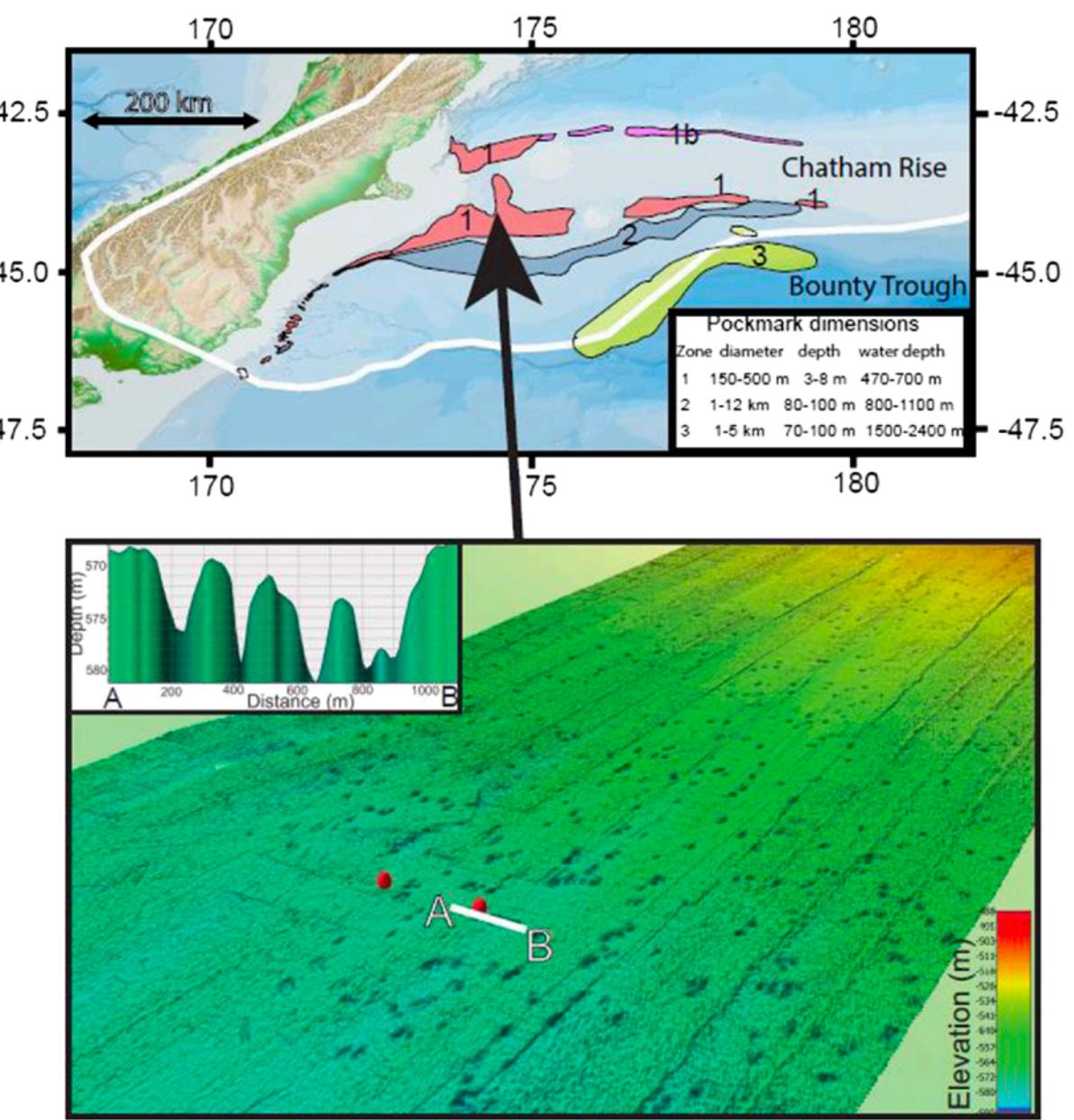
How geologic processes affect the carbon flux from/to the oceans on glacial timescales ?

Atmospheric CO<sub>2</sub> variations from ice core records

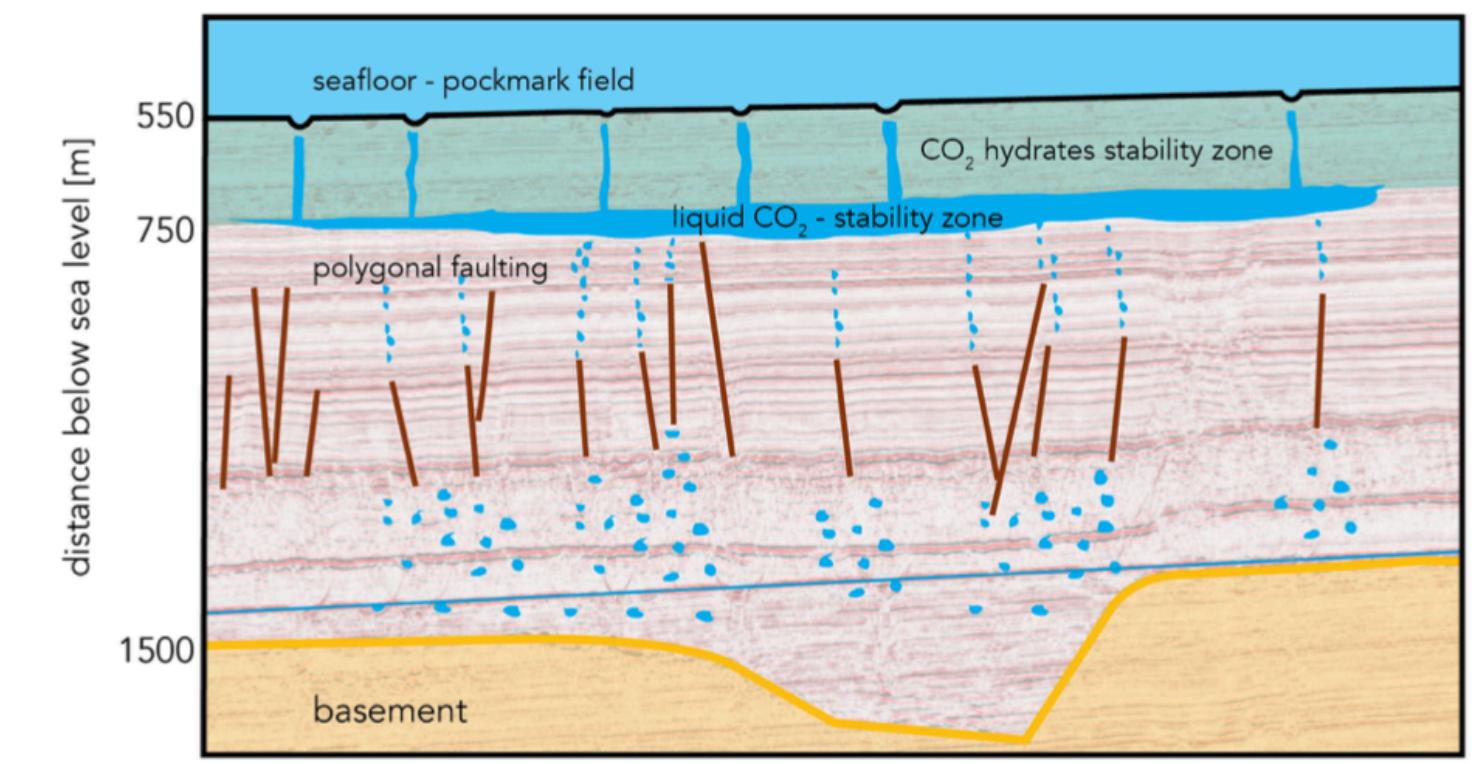


Ice core records show a significant response of Earth's climate to variations in atmospheric CO<sub>2</sub> concentrations.

Pockmark field offshore New Zealand

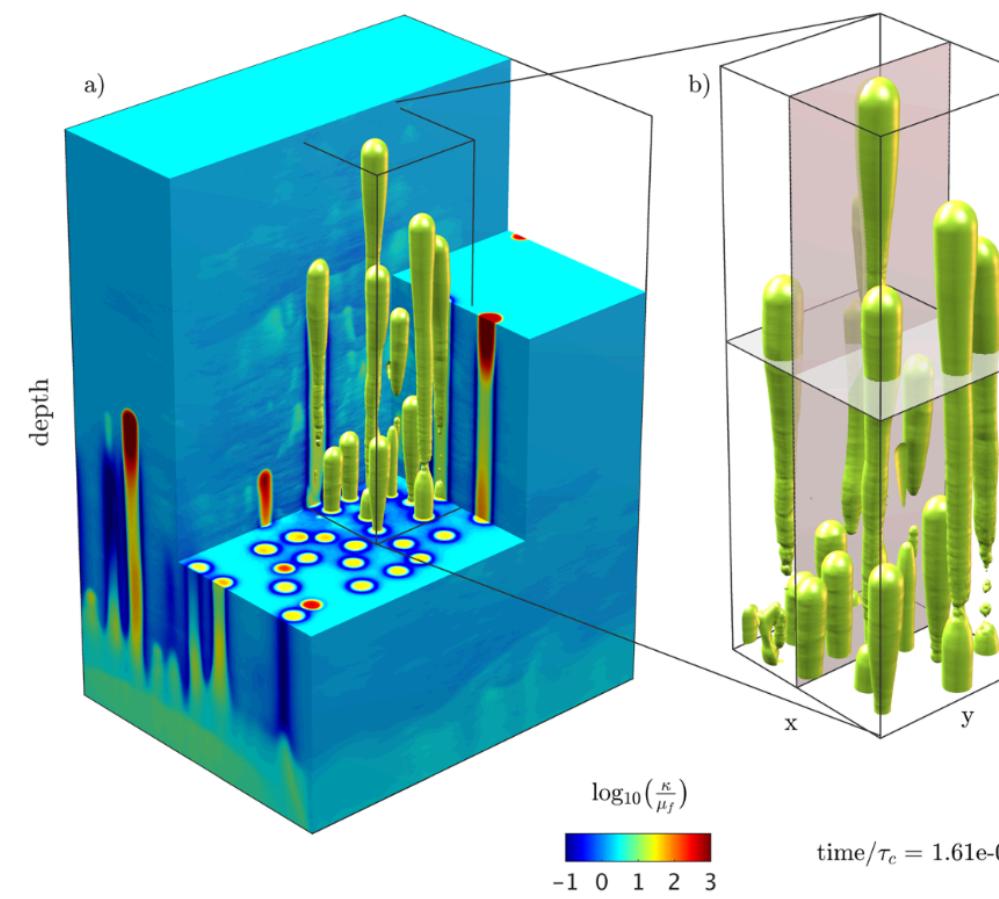


Potential flow pathways



Deep CO<sub>2</sub> leaking through polygonal fault system and accumulating beneath the hydrate «seal».

## The hydro-mechanical model

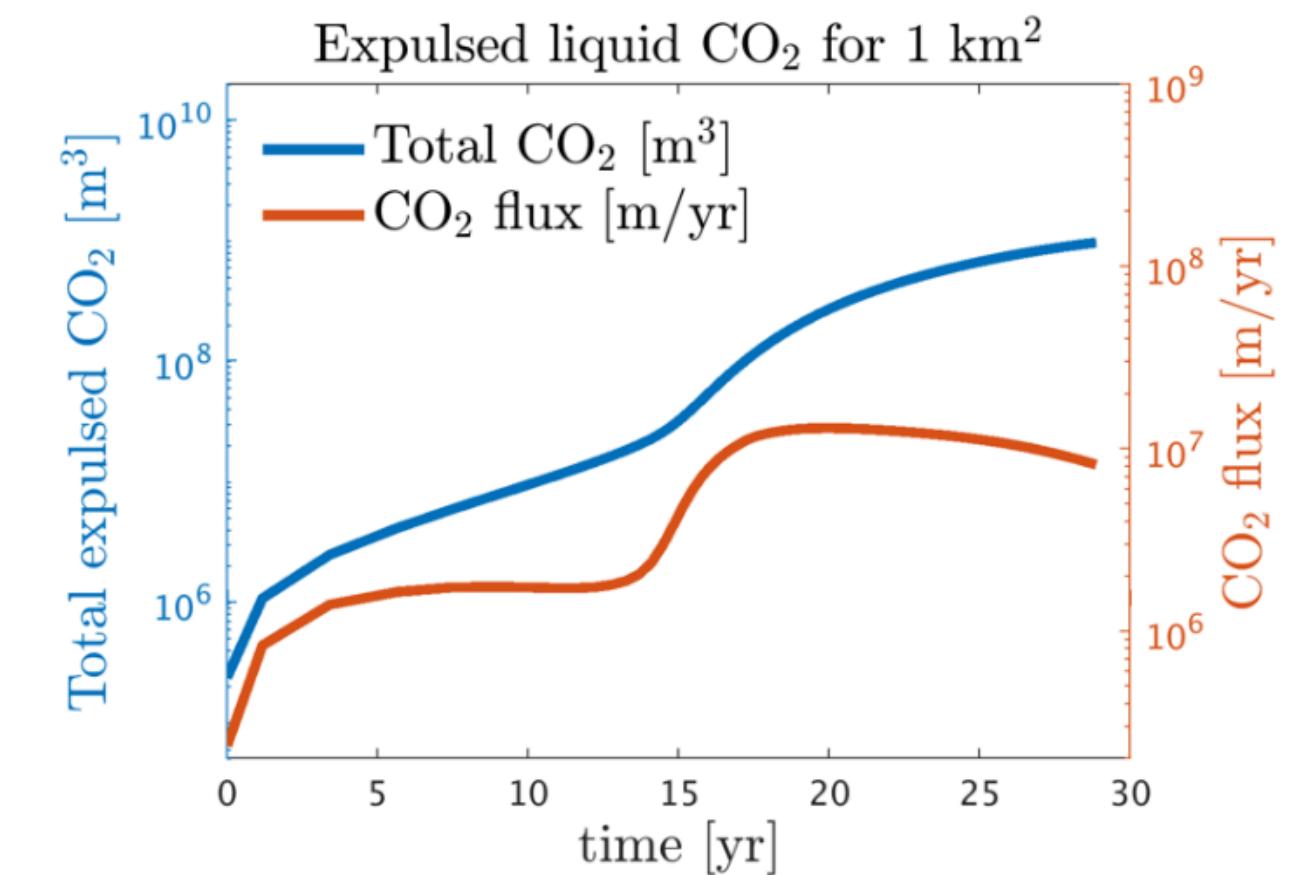
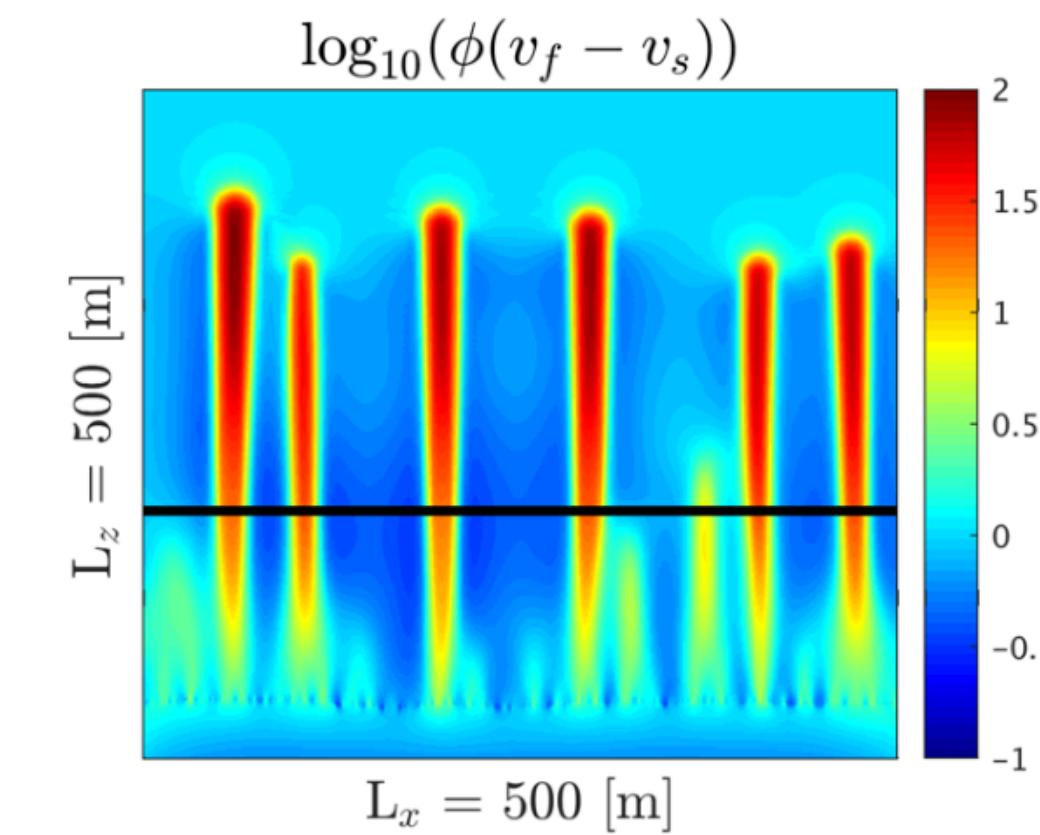
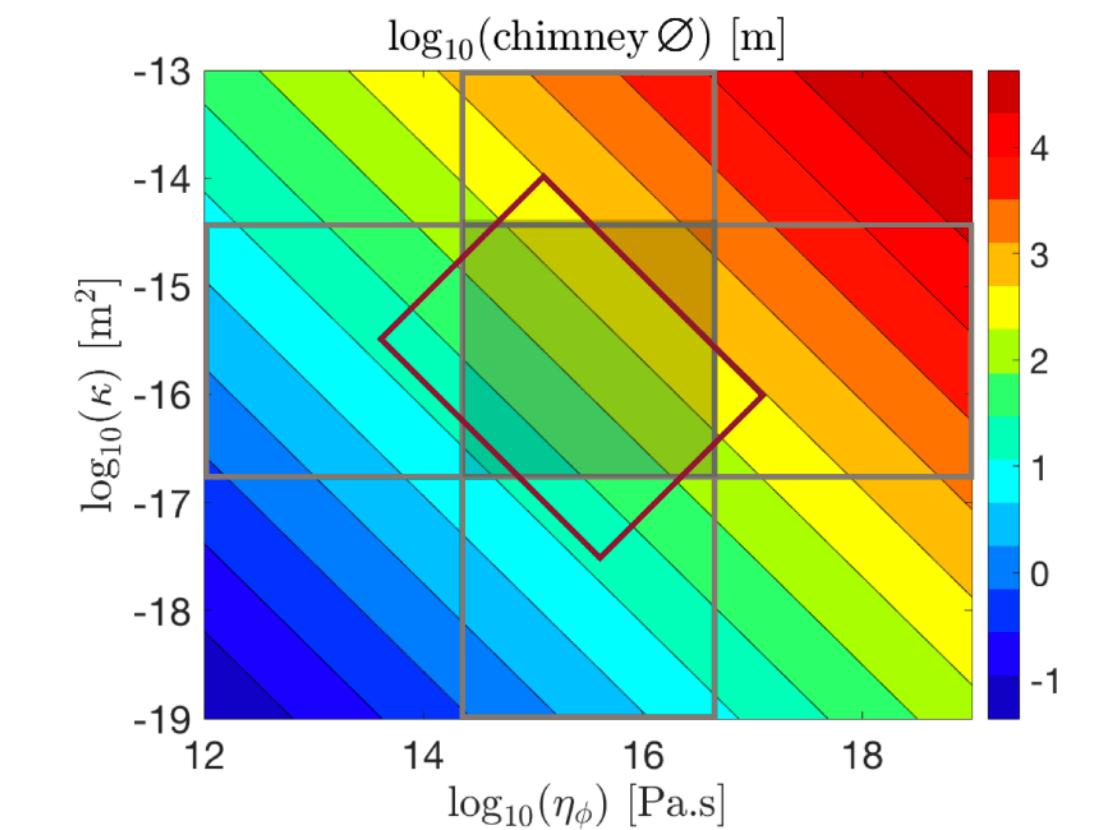
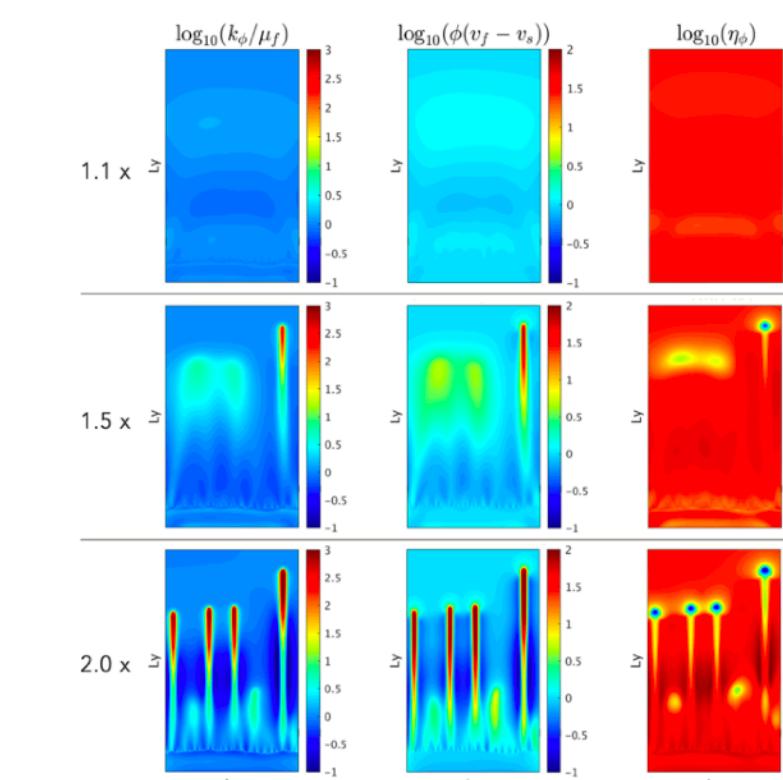


Räss et al., 2018

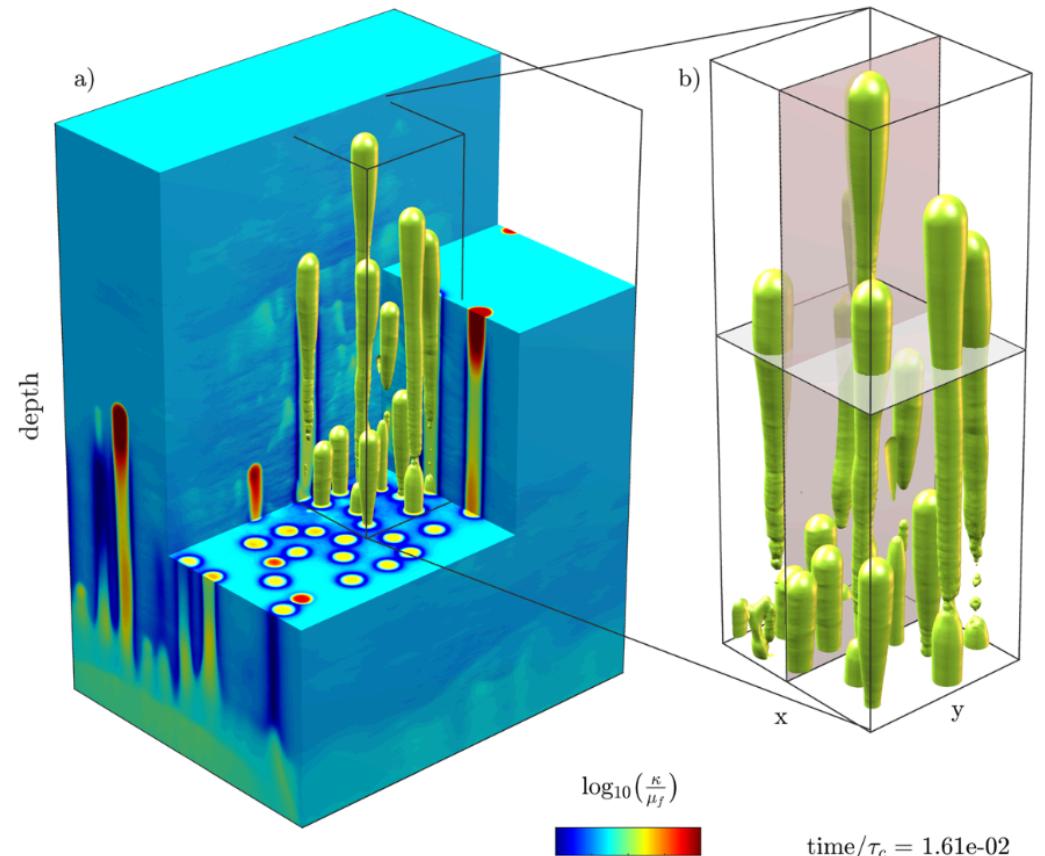
$$\begin{aligned} 0 &= \nabla_j (\bar{\tau}_{ij} - \bar{p}\delta_{ij}) - \bar{\rho}g_i \\ 0 &= \nabla_k v_k + \frac{\bar{p} - p^f}{\eta_\phi(1 - \phi)} \\ 0 &= \nabla_k q_k^D - \frac{\bar{p} - p^f}{\eta_\phi(1 - \phi)} \\ q_i^D &= -k_\phi(\nabla_i p_i^f + \rho^f g_i) \\ \frac{1}{G} \frac{\partial \bar{\tau}_{ij}}{\partial t} + \frac{\bar{\tau}_{ij}}{\mu_s} &= \nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla_k v_k \end{aligned}$$

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= (1 - \phi) \nabla_k v_k \\ \eta_\phi &= f(\phi, \bar{p}, p^f, \dot{\epsilon}_{II}) \\ \mu &= f(\phi, \dot{\epsilon}_{II}) \\ k_\phi &= k_0 (\phi/\phi_0)^3 \end{aligned}$$

## Chimney size and CO<sub>2</sub> flux predictions



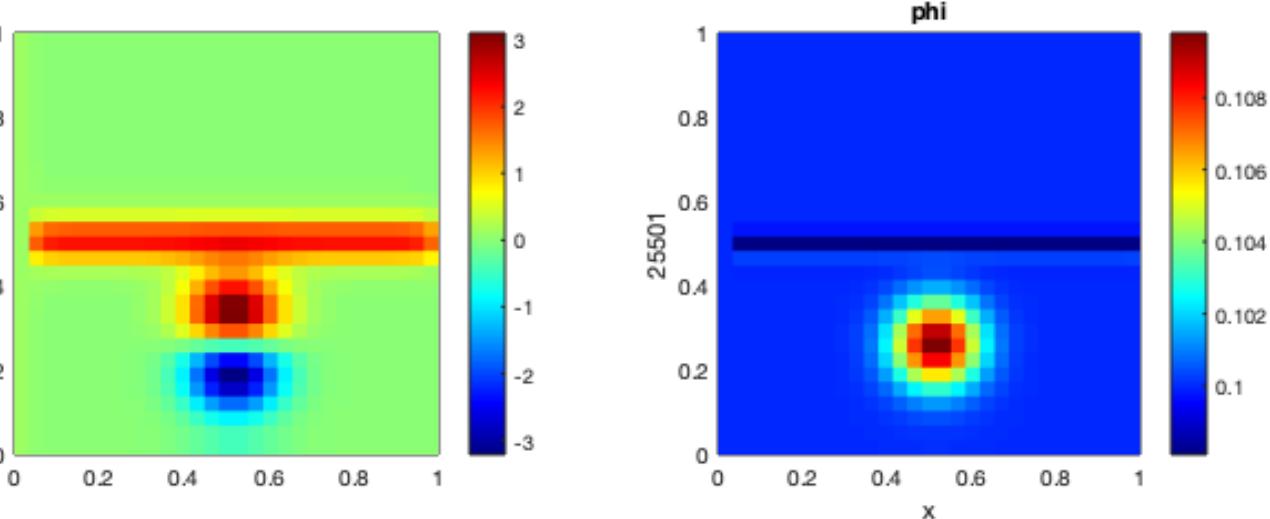
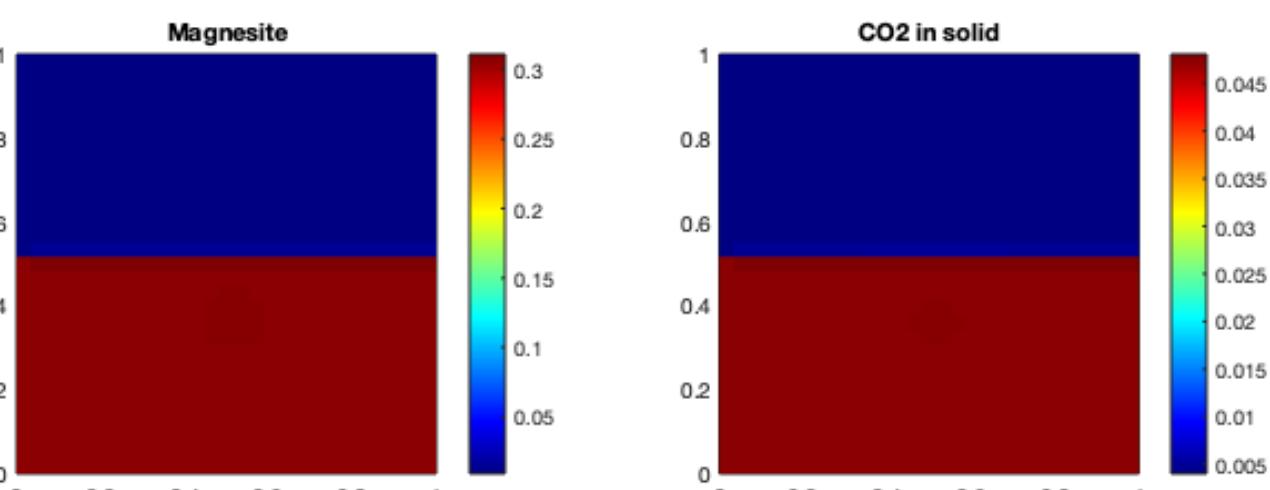
## The hydro-mechanical model



Räss et al., 2018

$$\begin{aligned}
 0 &= \nabla_j(\bar{\tau}_{ij} - \bar{p}\delta_{ij}) - \bar{\rho}g_i \\
 0 &= \nabla_k v_k + \frac{\bar{p} - p^f}{\eta_\phi(1 - \phi)} \\
 0 &= \nabla_k q_k^D - \frac{\bar{p} - p^f}{\eta_\phi(1 - \phi)} \\
 q_i^D &= -k_\phi(\nabla_i p_i^f + \rho^f g_i) \\
 \frac{1}{G} \frac{\partial \bar{\tau}_{ij}}{\partial t} + \frac{\bar{\tau}_{ij}}{\mu_s} &= \nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla_k v_k
 \end{aligned}$$

HMC coupling



Beinlich et al., 2021 Nature Geo

$$\begin{aligned}
 \rho_f &= \rho_f(T, P_f, C_f^{CO_2}) \\
 \rho_s &= \rho_s(T, P_f, C_f^{CO_2}) \\
 C_s^m &= C_s^m(T, P_f, C_f^{CO_2}) \\
 C_s^{CO_2} &= C_s^{CO_2}(T, P_f, C_f^{CO_2}) \\
 \frac{\partial}{\partial t} (\rho_f \phi C_f^{CO_2} + \rho_s (1 - \phi) C_s^{CO_2}) - \nabla \left( \rho_f \phi C_f^{CO_2} \frac{k \phi^3}{\mu_f} (\nabla P_f + \rho_f \mathbf{g}) \right) \\
 &= \nabla \left( \rho_f \phi D_f^{CO_2} \nabla C_f^{CO_2} \right) \\
 \frac{\partial}{\partial t} (\rho_s (1 - C_s^m) (1 - \phi)) &= 0 \\
 \frac{\partial}{\partial t} (\rho_f \phi + \rho_s (1 - \phi)) &= \nabla \left( \rho_f \frac{k \phi^3}{\mu_f} (\nabla P_f + \rho_f \mathbf{g}) \right)
 \end{aligned}$$

