

A three-dimensional version of the free surface capturing discretization for staggered grid finite difference schemes: Implementation into StagYY

Paul J. Tackley
ETH Zurich

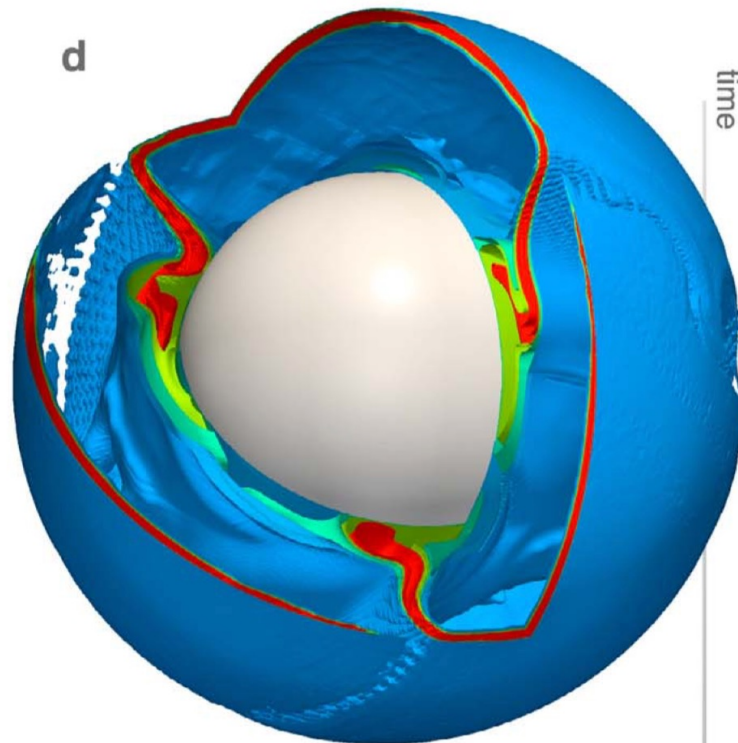
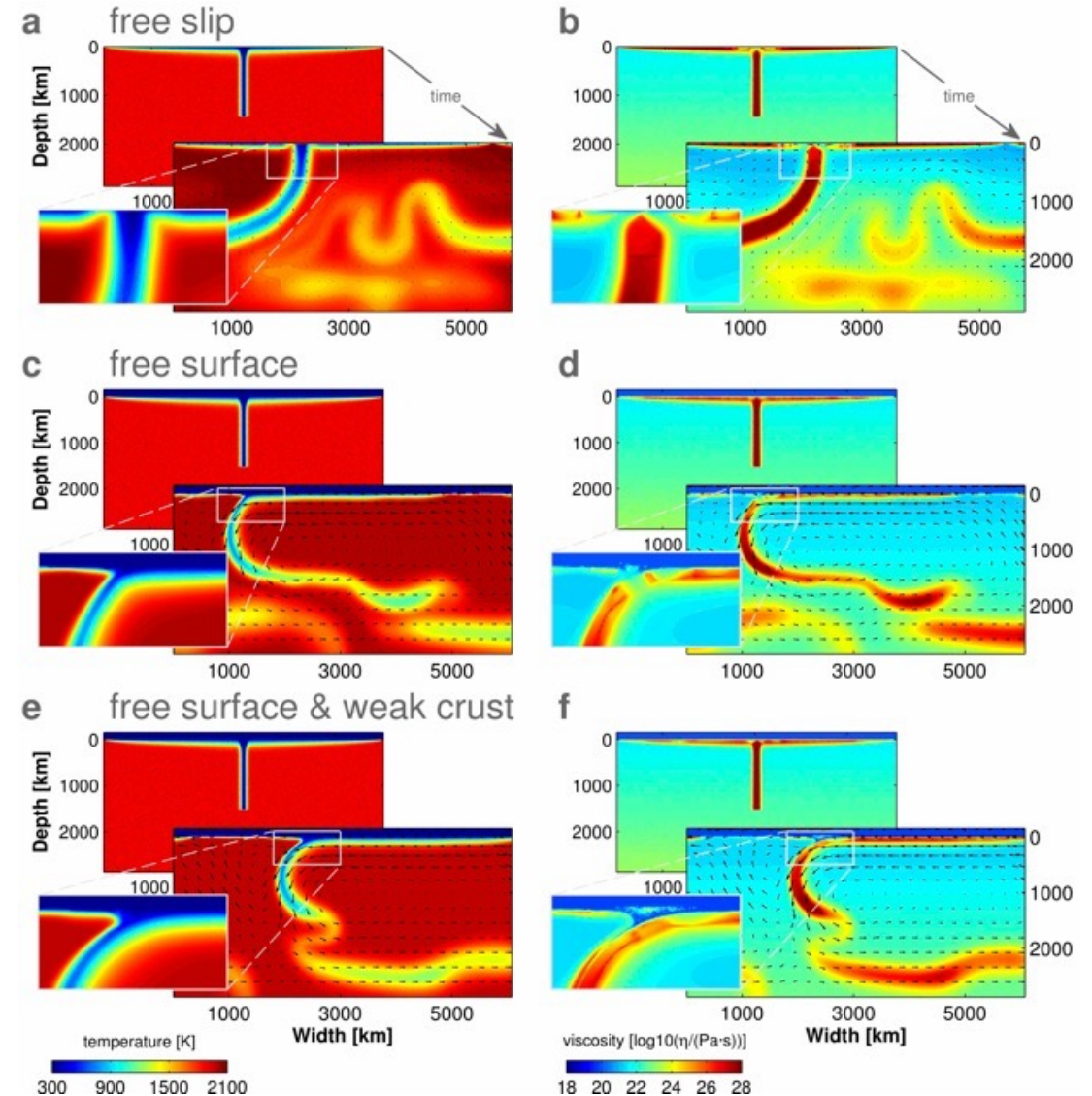


A **free surface** instead of **free-slip (flat)** surface can make an important difference even in global models!

Helps **single-sided subduction** (Cramer, F., P. J. Tackley, I. Meilick, T. V. Gerya, and B. J. P. Kaus (2012), *Geophys. Res. Lett.*, doi:10.1029/2011GL050046)



Fabio Cramer



“Sticky air” is a widely-used technique to approximate a free surface

- The numerical grid does not conform to the free surface, instead it extends into the air, which has low density and viscosity.
- It can give similar results to using a true free surface, as in the comparison on the right and in *Cramer et al. (GJI 2012, doi 10.1111/j.1365-246X.2012.05388.x)*

Physics of the Earth and Planetary Interiors 171 (2008) 198–223

A benchmark comparison of spontaneous subduction models—Towards a free surface

H. Schmeling^{a,*}, A.Y. Babeyko^{a,b}, A. Enns^a, C. Faccenna^c, F. Funiciello^c, T. Gerya^d, G.J. Golabek^{a,d}, S. Grigull^{a,e}, B.J.P. Kaus^{d,g}, G. Morra^{c,d}, S.M. Schmalholz^f, J. van Hunen^h

211

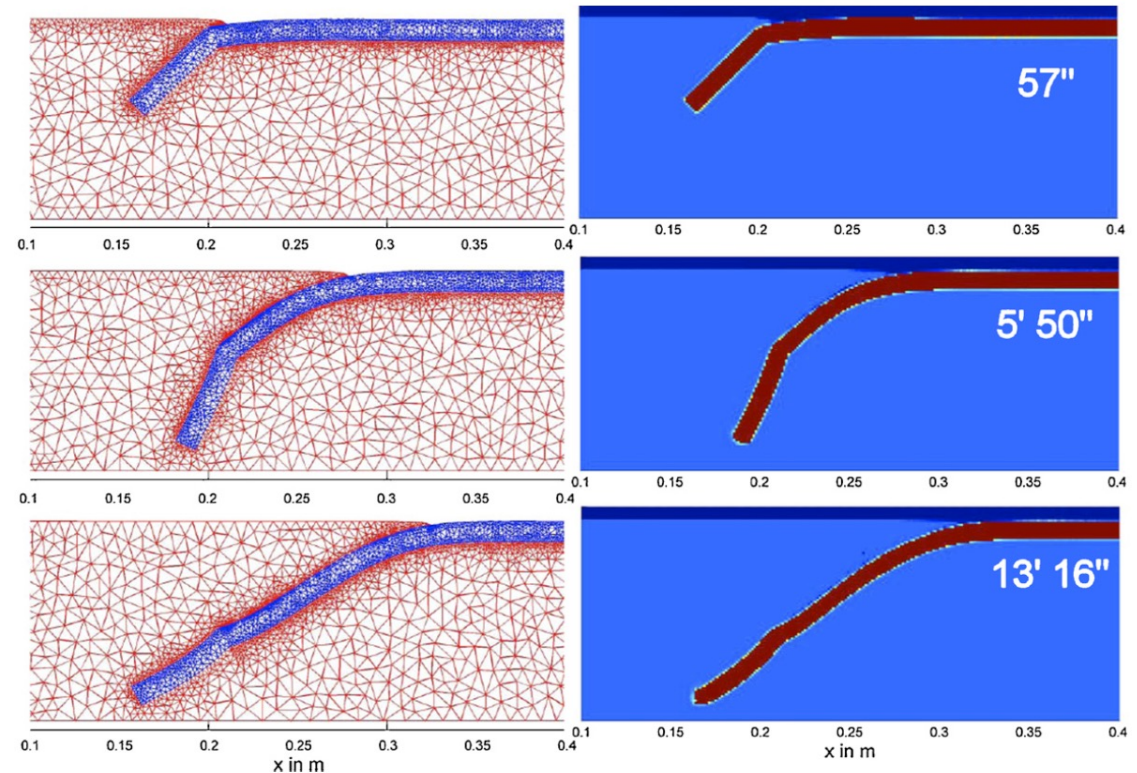


Fig. 16. Zoom in for viscosity snapshots of the FEMS-2D (left), FDCON (right) numerical models for times 57s, 5' 50'', and 13' 16'' which are comparable to the time steps presented for the laboratory experiment. For FDCON the harmonic mean for viscosity is used.

Sticky air has some problems

- Huge viscosity jump @ free surface => poor iterative solver convergence. To alleviate this, a not-very-low air viscosity is typically chosen – usually similar to the viscosity in the asthenosphere.
- High velocities in low-viscosity “air” => small time-step (Courant)
- “Air” still has finite viscosity => not a true stress-free boundary

Why not solve the equations only in the rock and use appropriate boundary conditions at the free surface?

Like *Harlow & Welch (1965)* did for water / Navier-Stokes equations:

Staggered grid, like we use today:

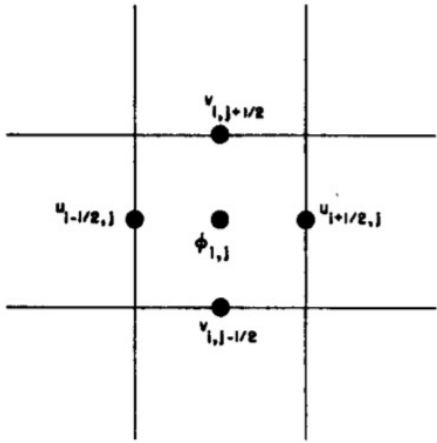


FIG. 2. Field variable value placement about a computational cell. Velocities are defined at cell boundaries while pressures are defined at cell centers.

2188

F. H. HARLOW AND J. E. WELCH

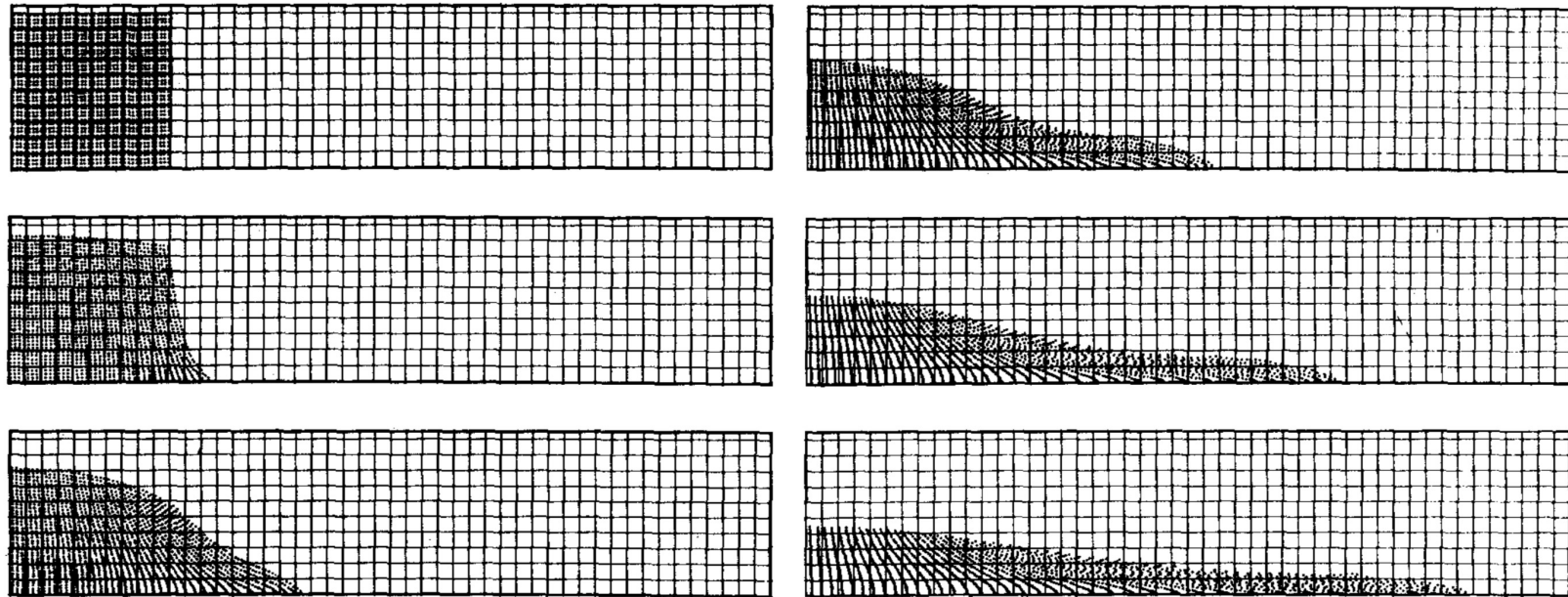


Fig. 5. Configuration of marker particles for the broken dam at times $t = 0, 0.5, 1.0, 1.5, 2.0, 2.5$. There were four computational cells for each square of the grid shown here.

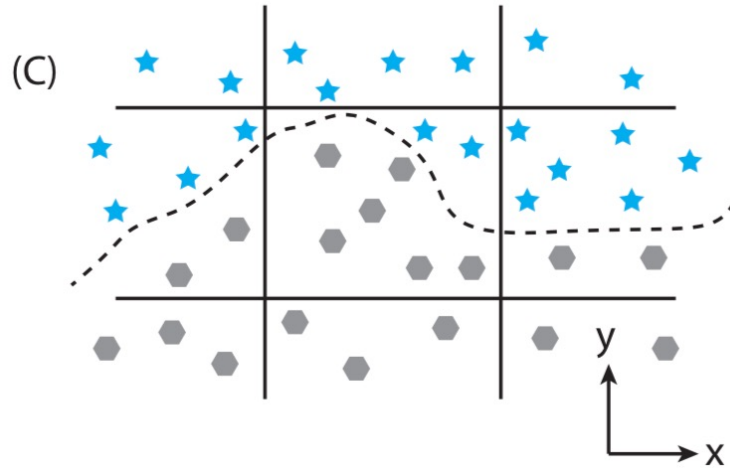
Harlow & Welch (Phys. Fluids 8, 2182-2189, 1965)



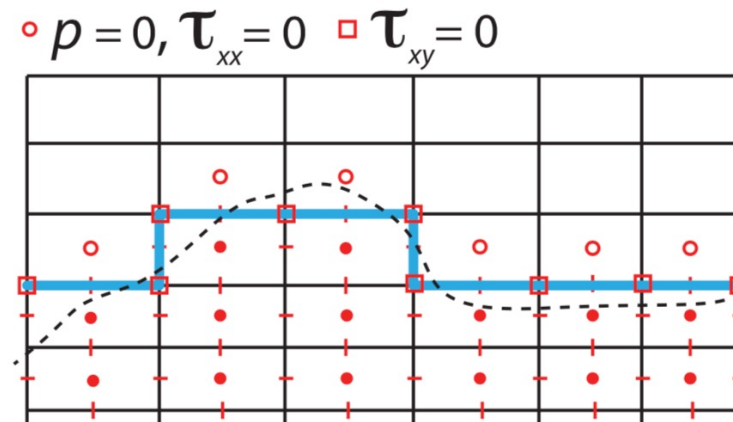


Duretz et al. (2016) introduced such a treatment for the Stokes equations in 2-D

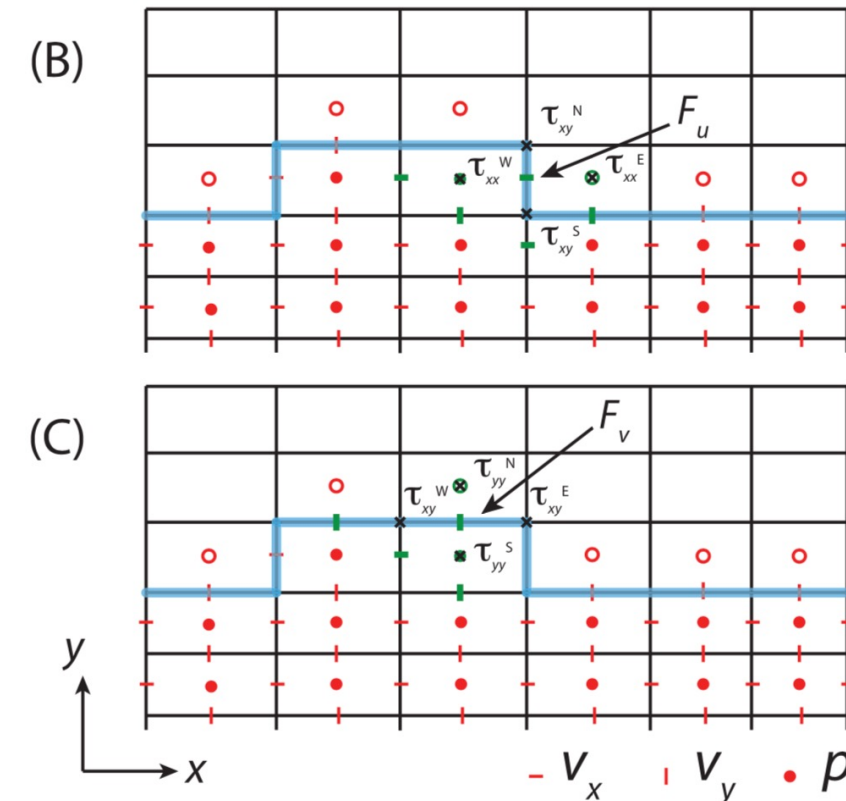
Smooth free surface



Discretised free surface



Details of modified x- and y-momentum stencils



Figures from: Duretz, T., D. A. May, and P. Yamato (2016), A free surface capturing discretization for the staggered grid finite difference scheme, *Geophysical Journal International*, 204(3), 1518-1530, doi:10.1093/gji/ggv526.

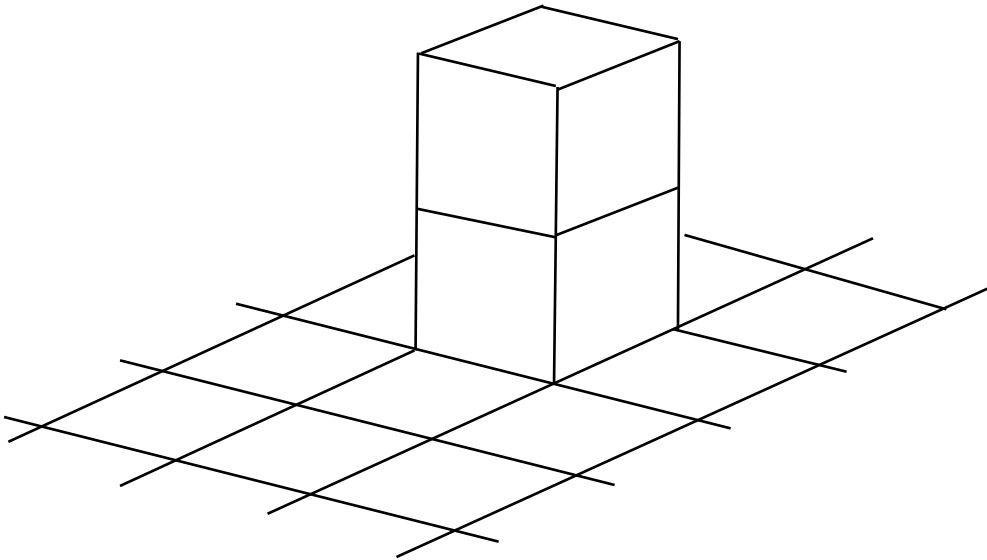
Implementation in 3-D cartesian or spherical

- In principle, very similar.
- For (v-p) solution the surface follows cell boundaries, i.e. coincident with velocity and shear-stress points.
- v- or p- values that are in the air are flagged using 0 viscosity, and are not solved for.
- The momentum stencils at boundary points are modified as in Duretz et al. (2016).
- The free-surface stabilization method (Duretz et al. 2011) is used to prevent “drunken sailor” oscillations (by modifying the stencil).

Duretz, T., D. A. May, T. V. Gerya, and P. J. Tackley (2011), Discretisation errors and free surface stabilization in the finite difference and marker-in-cell method in geodynamic applications: A numerical study, *Geochem. Geophys. Geosyst.*, 12(Q07004), doi:10.1029/2011GC003567.

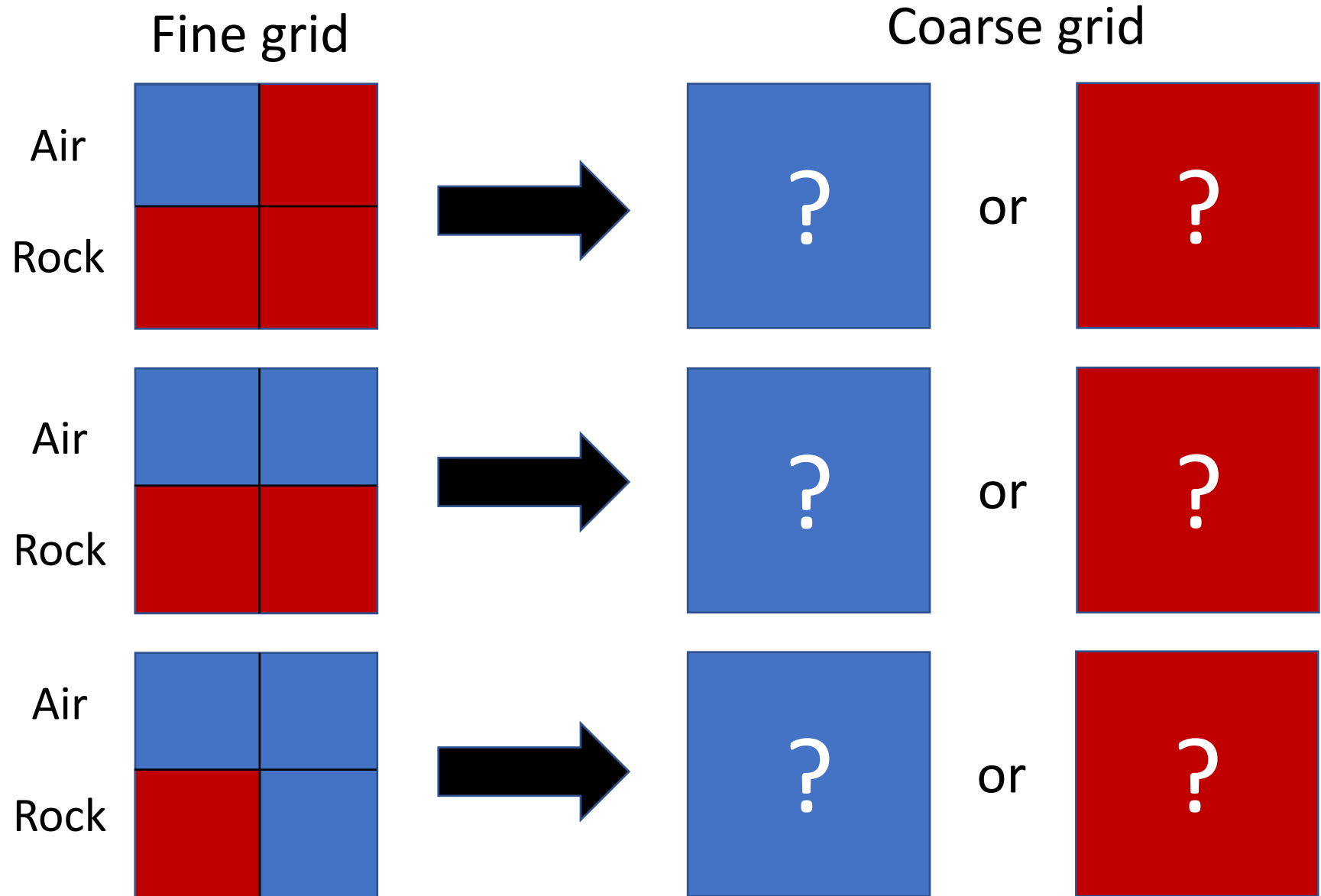
Issues: (i) Pathological topography

Ideally, the surface should be ~smooth with only single steps but what happens if, locally, the surface looks like this? (or the inverse – a hole?) Is the solution unique, not ill-conditioned?



The rules/stencils described in the text of Duretz et al. (2016) seem to ensure unique solutions in such cases.

(ii) Multigrid solver: How to coarsen mixed cells?

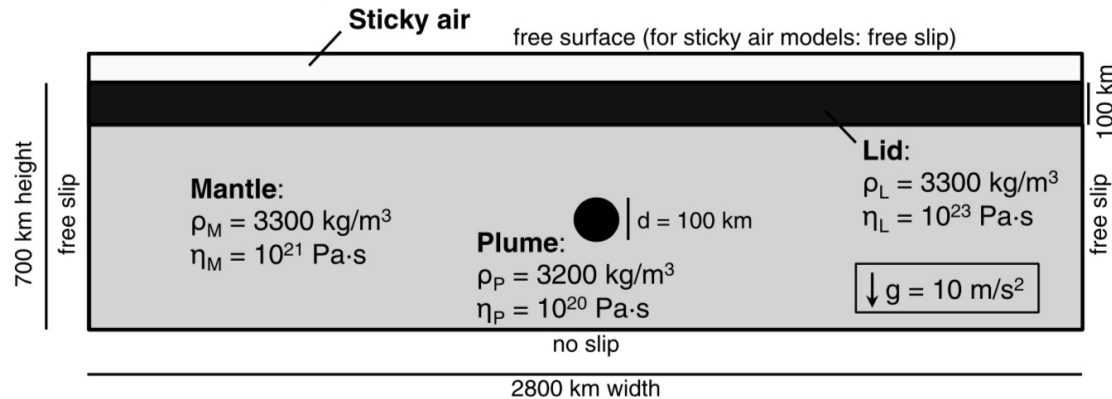
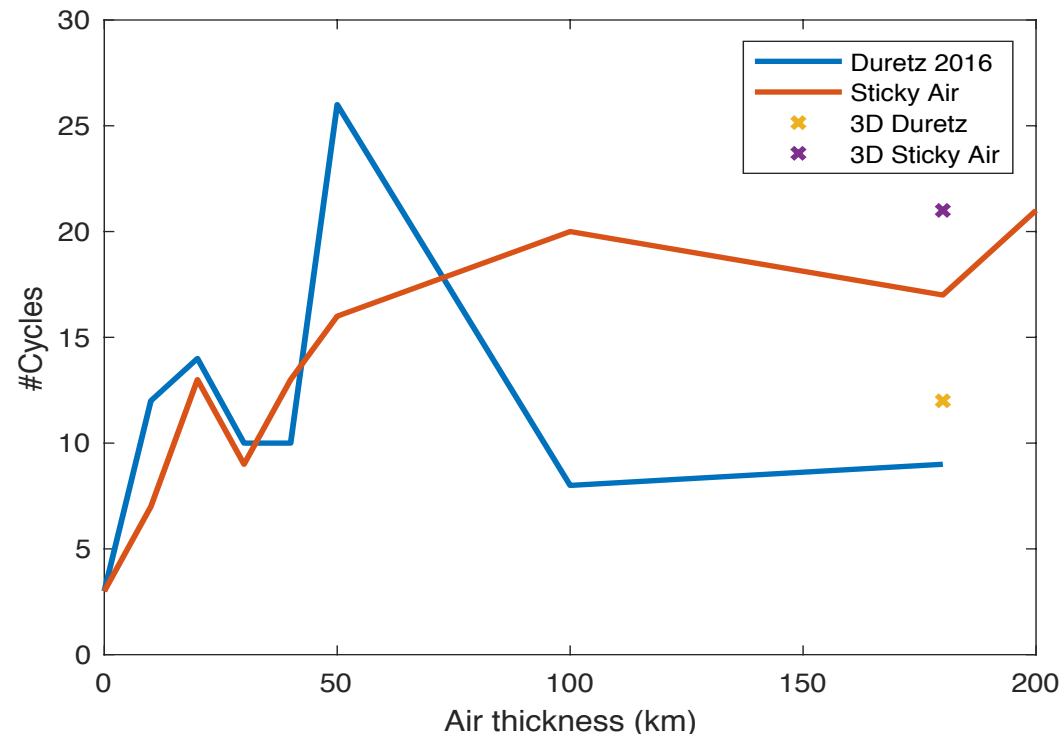


Reduced convergence is expected because the surface of the coarse grid does not (in general) match the surface of the fine grid. How to minimise this issue?

Finding: Usually converges best when “air wins”:
If ≥ 1 fine cell is air, make the coarse cell air.
However, sometimes it is better to use a “half” criterion: when $> \text{half}$ the fine cells are air, the coarse cell is air.

Multigrid solver convergence

(b) 2-D Stokes cylinder:



Crameri et al. (2012 GJI) case 2

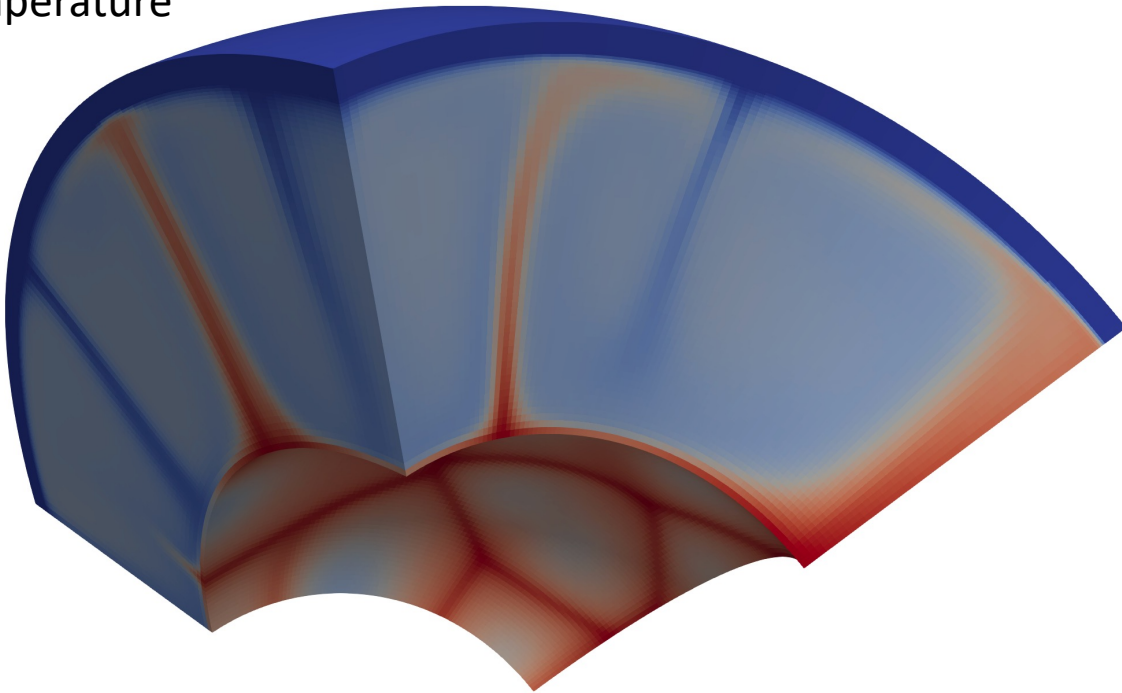
- 256x64 cells with vertical refinement.
- Multigrid f-cycles down to 32x8 cells.
- 25 relaxations at each level.
- #Cycles for factor 10^4 convergence.

- #Cycles depends strongly on air thickness for the Duretz (2016) approach, less so for sticky air.
- This is related to the alignment (or lack of) of the free surface with the coarse grid cell boundaries: 8, 10, 16 or 20 is a good number of cells to have in the air. E.g. going from 180 km (20 cells) to 200 km (21 cells) results in going from 9 cycles to infinity (non-convergence).
- 3D is similar; slightly more cycles required.

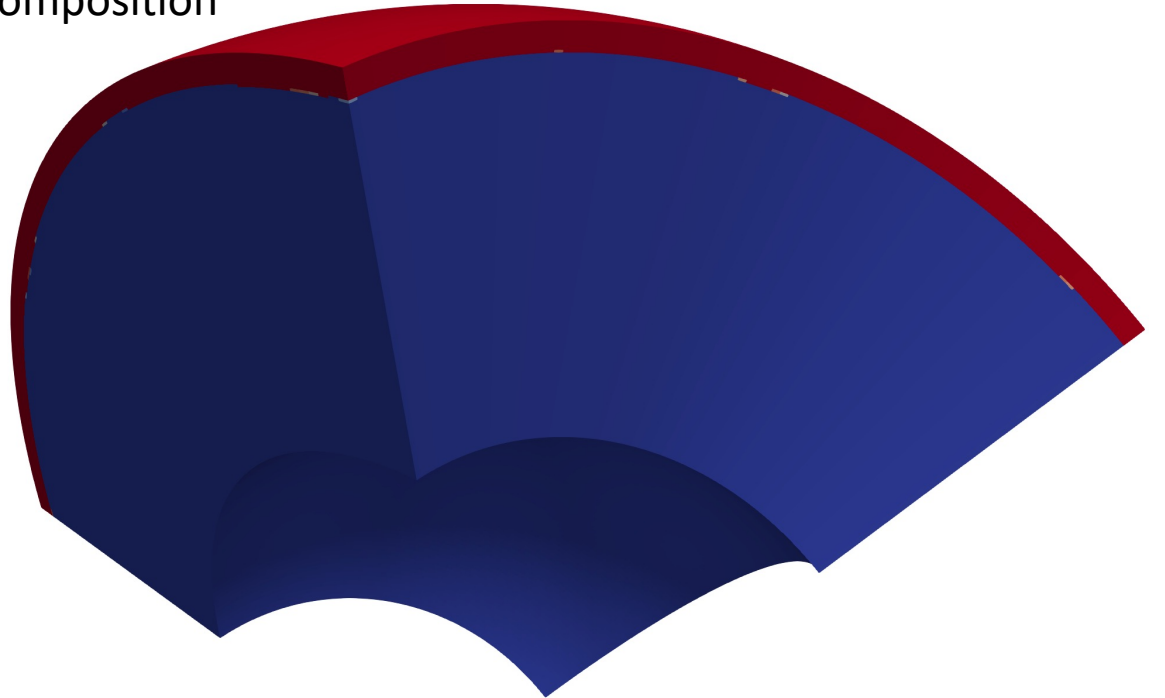
Example 3D spherical result

- 3D spherical patch, constant viscosity convection with $Ra=10^6$.
- Made 'easier' by the surface deflecting by < 1 grid cell

Temperature



Composition



Remaining issues

- Marker-in-cell => noisy free-surface position. Plan to implement marker chain instead for tracking the free surface.
- Multigrid convergence with a 3-D spherical yin-yang grid does not match that of a single 3-D spherical (or cartesian) patch => more testing and development needed.