Ensemble Kalman Filter for non-conservative moving mesh solvers with a joint physics and mesh location update

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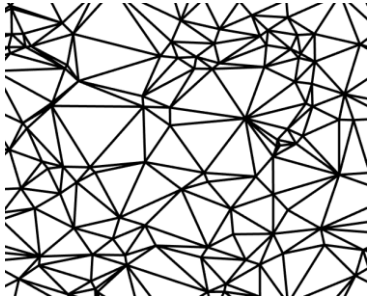
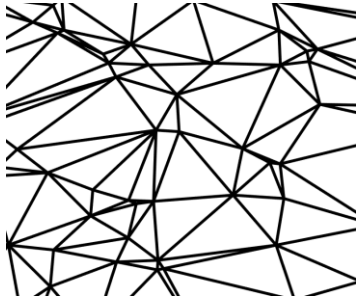
Adaptive moving mesh (AMM) solvers can leverage computational power by focusing resolution on areas with important dynamics.

However, this can present difficulties for popular ensemble data assimilation methods -such as the EnKF- since ensemble members may have meshes with different numbers of nodes in different locations.

This can make the calculation of needed error statistics difficult.



Member 1



Member 2

Adaptive moving meshes can be driven by physics….

- In many AMM models mesh structure is driven directly by the physics or to satisfy a physical principle.

- In sea ice, for example, ice velocity is directly tied to ice thickness, drag, stress, concentration, etc..

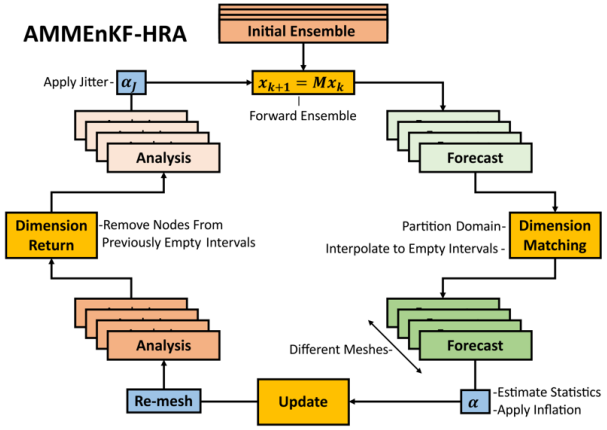
- This leads one to ask: if the mesh is dynamic and dependent on the physics, should we update the mesh as well? Can we do it from just the physical observations?

Three Main Challenges

Dimension Matching: each ensemble member must have the same number of state variables.

Component Paring: since nodes can be in different locations decisions must be made on which nodes to compare to which.

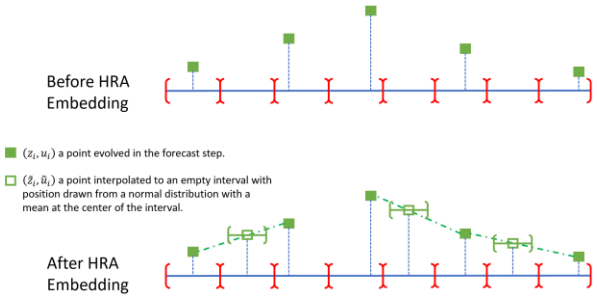
Dimension Return: ensemble meshes may need be returned to a mesh format suitable for integration with the AMM.



In the case where node locations are driven by a physical flow, we may augment the state vector with the node locations.

$$\vec{x} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ \tilde{u}_i \\ \vdots \\ u_N \\ z_1 \\ z_2 \\ \vdots \\ \tilde{z}_i \\ \vdots \\ z_N \end{bmatrix}$$

u: *physical value*
z: *node location*



Through the inclusion of the node locations information about the physical mesh drivers is brought into the analysis update.

PDE Setup:

$u_t = f(u, u_z, \dots) \quad z \epsilon [0, L]$

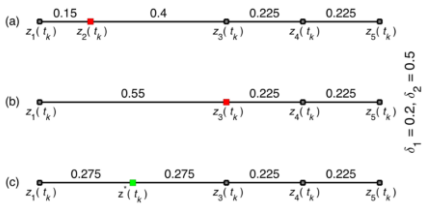
BC $\quad u(t,0) = u(t,L)$ perioidic

IC $\quad u(0,z) = u_0(z)$

mesh: $\quad 0 = z_0 < \cdots < z_{N_0} = L$

mesh dynamics: $\quad z = z_i(t)$

Remeshing



Important Estimators

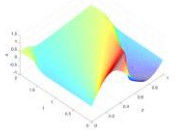
Error Anomaly Matrix

$$E_k^f = \left[\vec{x}_k^{f_1} - \overline{\vec{x}_k}^f, \cdots, \vec{x}_k^{f_N} - \overline{\vec{x}_k}^f \right]$$

Background Error Covariance $\mathrm{B}_k^f = \frac{1}{N-1} E_k^f \left(E_k^f\right)^T$

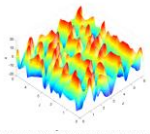
$$\begin{bmatrix} \sigma_{u_i u_j} & \sigma_{u_i z_j} \\ \sigma_{u_i z_j} & \sigma_{z z_j} \end{bmatrix}$$

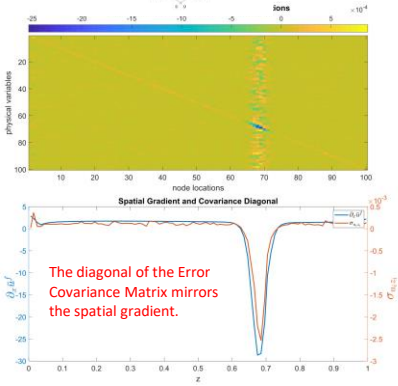
Burgers

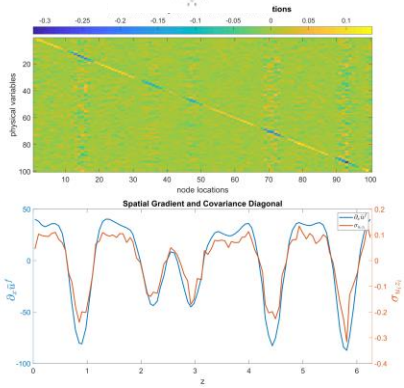


Test Bed Problems

Kuramoto-Sivashinsky







- The EnKF can function successfully for non-conservative adaptive moving meshes.
- Augmenting the state vector with node location brings in the underlying physical drivers into the analysis update and does improve assimilations in our test bed cases.
- The full study may be found here: https://doi.org/10.1002/qj.3980