

Towards an Improved Parameter-Free Entrainment Model for Snow Avalanches

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1. INTRODUCTION

- Why should we model entrainment?
- Requirements on consistent entrainment models
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Why Should We Model Entrainment?

- Accumulated entrainment may exceed initial avalanche mass by a large factor ([Sovilla et al., 2006](#)).
- Acceleration of eroded snow slows the avalanche down, but entrainment may prevent starvation and increase flow depth (☞ higher velocity, potentially longer run-out).
- Volume in run-out zone is important for mitigation measures!
- Accounting for entrainment ([Sovilla & Bartelt, 2002](#)) and deposition ([Rauter & Köhler, 2020](#)) may reduce variation of friction parameters significantly.



Requirements on Consistent Entrainment Models (1)

Typical entrainment models in the literature:

$$q_e = \textcolor{yellow}{c} \rho_{sc} |\textcolor{blue}{u}|$$

with q_e entrainment rate ($\text{kg m}^{-2} \text{s}^{-1}$), ρ_{sc} density of snow cover, $\textcolor{blue}{u}$ velocity, $\textcolor{yellow}{c}$ a coefficient to be fitted.

BUT physics tells us that the physical properties of the snow cover (shear strength) and the dynamical parameters of the avalanche (shear stress) determine $q_e \Rightarrow$ NO FREE PARAMETERS...



Requirements on Consistent Entrainment Models (2)

$$\text{Entrainment} = \text{Erosion} + \text{Acceleration}$$

Depends on

- snow-cover strength
- avalanche shear stress



Depends on

- avalanche shear stress

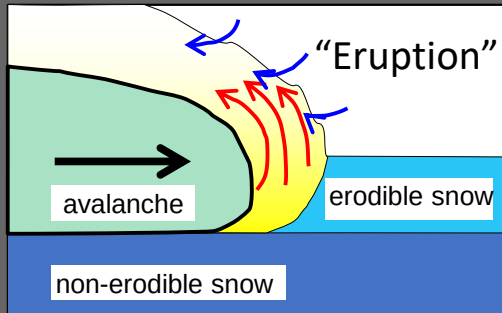
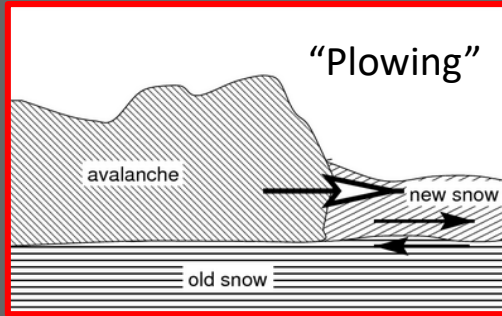


- Entrainment model must respond to avalanche rheology for correct balance between flow and entrainment.
- In simple cases, entrainment rate can be computed ([Issler, 2014](#)).

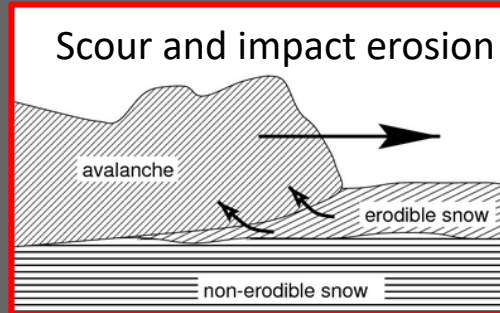
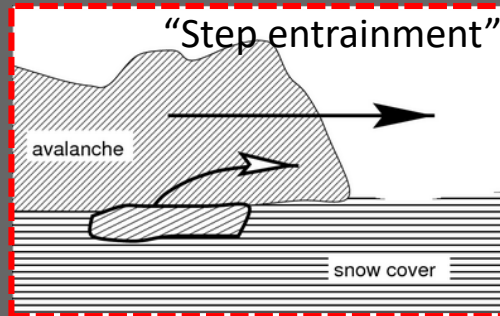


Entrainment Mechanisms (1)

Frontal entrainment



Basal entrainment



- Several mechanisms can occur simultaneously.
- Depending on snow-cover and avalanche properties

Adapted from ([Gauer and Issler, 2004](#))

Entrainment Mechanisms (2)

Experimental hints or evidence exist for all four mechanisms:

- Plowing in slow wet-snow avalanches
- Eruption likely at the front of powder-snow avalanches
- Continuous scour in dry-snow avalanches without pronounced weak layer in snow cover
- Step erosion if weak layer is buried in snow cover

N.B. Need for improved interpretation techniques to decide whether profiling radar shows entrainment or only erosion.

Entrainment Terms in the Balance Equations

Typical form of depth-averaged quasi-3D balance equations:

$$\rho_0 \partial_t \delta = -q_e + q_d \quad \text{Mass bal. snow cover}$$

$$\rho_1 \partial_t h + \rho_1 \nabla \cdot (h \mathbf{u}) = q_e - q_d \quad \text{Mass bal. avalanche}$$

$$\begin{aligned} \rho_1 \partial_t (h \mathbf{u}) + \rho_1 \nabla \cdot (h \overline{\mathbf{u} \mathbf{u}} + h \overline{\mathbf{p}}) \\ = \rho_1 [\mathbf{g} - (\mathbf{g} \cdot \hat{\mathbf{n}}) \hat{\mathbf{n}}] - \frac{\bar{u}}{\|\bar{\mathbf{u}}\|} \tau_b + q_e \mathbf{u}_e - q_d \mathbf{u}_d \end{aligned} \quad \text{Momentum bal. aval.}$$

$= 0$ in this case!

q_e, q_d : Entrainment / deposition rate

$\mathbf{u}_e, \mathbf{u}_d$: Speed of mass that enters/leaves the avalanche (in general $\neq \bar{\mathbf{u}}$)



Basic Assumptions (1)

Fundamental difference in failure criteria:

- Non-cohesive granular beds: $\tau_c \propto \sigma_n$ (normal stress)
 - Solids (snow cover, clay, ...): τ_c approx. independent of σ_n
- ⇒ Entrainment formulas used in purely granular models are not directly applicable to snow avalanches!

Key property of snow: Brittle failure at shear rates above $\sim 10^{-3} \text{ s}^{-1}$

- ⇒ Low specific erosion energy $O(0.1\text{--}1 \text{ kJ m}^{-3})$
- ⇒ Shear stress at erosion interface locked to τ_c (basal entrainment).



Basic Assumptions (2)

Example energetics of different subprocesses in an avalanche:

- Flow depth $h \sim 1.5$ m, density $\rho_1 \sim 200$ kg/m³, velocity $u \sim 30$ m/s
- Eroded snow $\delta \sim 1.0$ m in 10 s, $\rho_0 \sim 150$ kg/m³, particles ~ 1 mm, erosion rate $q_e \sim 15$ kg/(m² s), erosion speed $\omega \sim 0.1$ m/s

⇒ Gravitational power: $P_g = \rho_1 h u g \sin \theta \quad \sim 45 \frac{\text{kW}}{\text{m}^2}$

⇒ Erosion power: $P_c \sim \omega \times 6000 \frac{\text{m}^2}{\text{m}^3} \times 1 \frac{\text{J}}{\text{m}^2} \sim 1 \frac{\text{kW}}{\text{m}^2}$

⇒ Entrainment power: $P_e = \frac{1}{2} q_e u^2 \quad \sim 7 \frac{\text{kW}}{\text{m}^2}$



Basic Assumptions (3)

Consider

- Frontal entrainment: only plowing, no eruption
- Basal entrainment: mainly scour/impact erosion

Step entrainment may be included if averaged over sufficiently large space and time intervals.

Assume a [depth-averaged avalanche flow model](#).



Entrainment Described as a Shock (1)

Brittle failure:

⇒ Instantaneous transition from bonded snow to free particles

⇒ Mesoscopic description with sharp interface snow cover–avalanche

↗ Density and velocity jump across interface

↗ In frontal entrainment, also rapid change of depth across front similar to hydraulic jump

⇒ Idealize transition zone as infinitesimally short.



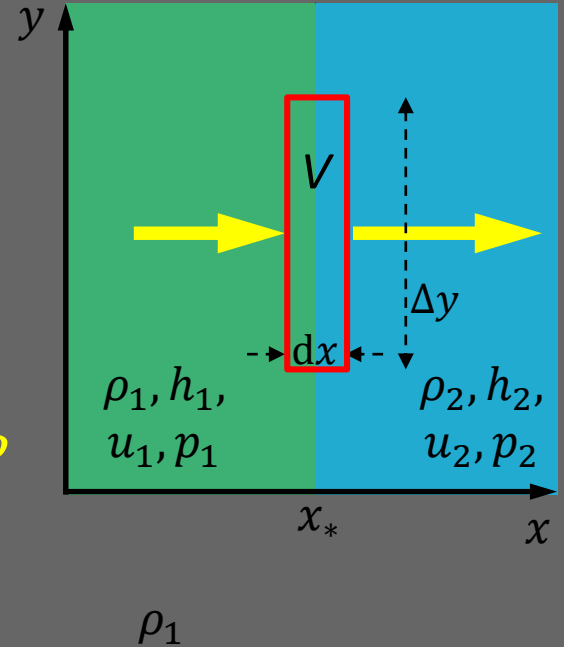
Entrainment Described as a Shock (2)

Conserved quantities $\Phi = \text{mass } \rho h$, momentum $\rho h u$ (and energy) must be conserved across shock!

Consider infinitesimally thin control volume V enclosing the discontinuity:

Rate of change of $\Phi = \text{Influx of } \Phi - \text{Outflux of } \Phi$

$$\partial_t \bar{\Phi}(x_*) \Delta y \underbrace{dx}_{\rightarrow 0} = [J_{\Phi,x}(x_-) - J_{\Phi,x}(x_+)] \Delta y$$



2. FRONTAL ENTRAINMENT

Observable in wet-snow avalanches.

Flow density must be high.

Avalanche pushes (plows) the snow cover into motion.

Freshly plowed snow

Snow entrained earlier

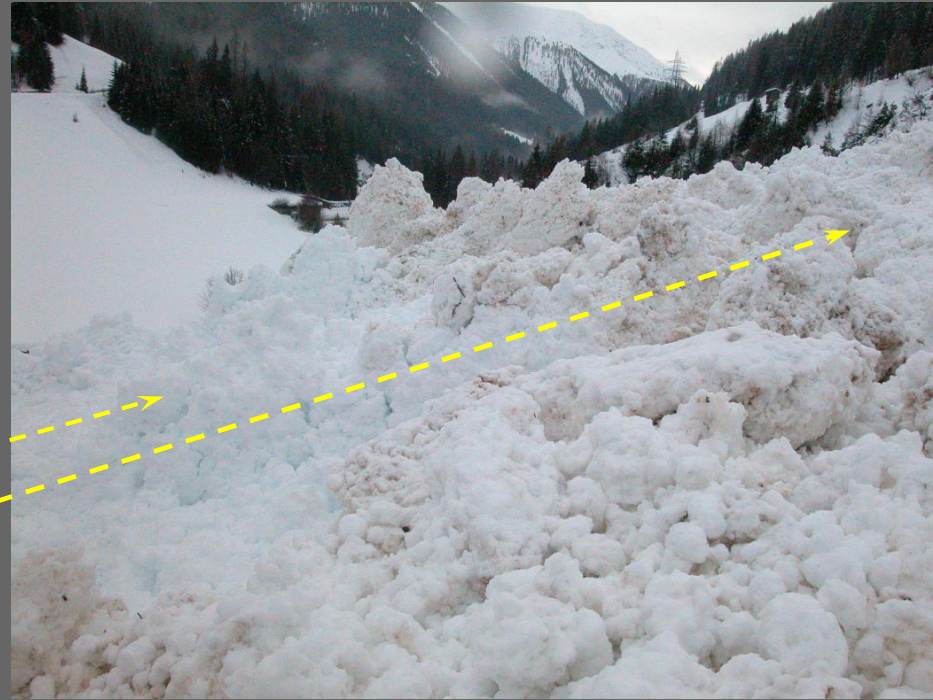
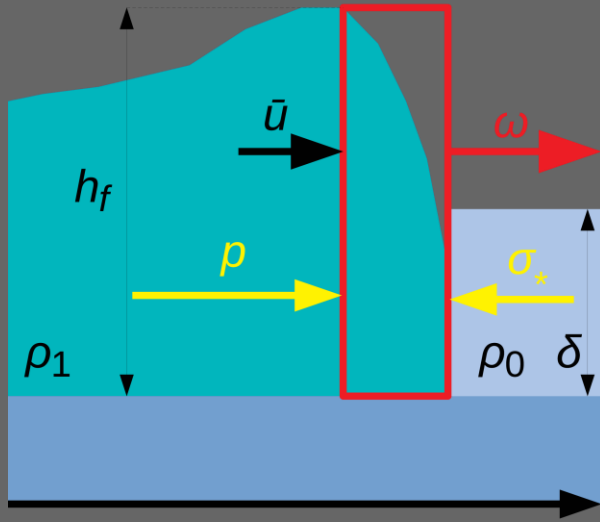


Photo B. Teufen, 2004

The Eglit–Grigorian–Yakimov Model (1)



From ([Issler, 2020](#)).

Avalanche front is idealized as an (infinitesimally thin) discontinuity of density, flow depth and velocity

- Entrainable snow cover has depth δ , density ρ_0 and strength σ_* .
- Avalanche with density ρ_1 and velocity $\bar{u}$ exerts hydrostatic pressure p .
- Front velocity $\omega > \bar{u}$.

The Eglit–Grigorian–Yakimov Model (2)

The EGYEM is a *boundary* condition, there is no entrainment rate q_e !
Jump conditions for mass and longitudinal momentum across control volume:

$$\rho_0 \omega \delta = \rho_1 h_f (\omega - \bar{u})$$

$$(\sigma_* + \frac{1}{2} \rho_0 g_z \delta) \delta = \rho_1 h_f \left[-\bar{u} (\omega - \bar{u}) + \frac{1}{2} g_z h_f \right]$$

Obtain propagation velocity of the front (=boundary):

$$\omega = \frac{1}{\bar{u}} \left[\frac{1}{2} g_z \left(\frac{\rho_1}{\rho_0} \frac{h_f^2}{\delta} - \delta \right) - \frac{\sigma_*}{\rho_0} \right]$$



The Eglit–Grigorian–Yakimov Model (3)

What happens if the avalanche pressure is too weak to entrain the entire erodible snow layer? 3 options proposed by [Eglit \(1982\)](#):

- a) Avalanche slides on snow cover ($\delta_{\text{eff}} = 0$).
- b) Avalanche front stops ($\omega = 0$), snow piles up, front may move again when hydrostatic pressure overcomes snow-cover strength.
- c) Partial entrainment (assuming $\rho_0 = \rho_1$ for simplicity here),

$$\delta_{\text{eff}} = \frac{\rho_1 h_f^2 g_z}{\sigma_*} < \delta .$$



Possible Extensions of the EGYEM (1)

Take closer look at options a)–c) if avalanche fails to entrain entire erodible layer:

- Option a) applies if snow-cover strength exceeds maximum avalanche stress (shear and normal stress combined).
- Option b) may apply after initial «spill-over» due to finite shear strength of avalanching snow. Effectively, $\omega < 0$ (shock propagates upstream).
- What about the shock propagation velocity ω with option c)?



Possible Extensions of the EGYEM (2)

Assume $\rho_0 = \rho_1$ in jump conditions and use formula for δ_{eff}

$$\Rightarrow \omega = 0 \quad !?!$$

Q: *What is missing?*

A: Proposed formula for δ_{eff} balances avalanche pressure entirely against snow-cover resistance. Nothing left for accelerating the eroded snow!

Q: *How to solve this?*

A: Solve jump conditions with ω and δ_{eff} as unknowns (homework 😊)!



Possible Extensions of the EGYEM (3)

Further refinements to consider ([Issler, 2020](#)):

1. How does the “compressive strength” σ_* relate to measurable snow properties?
2. Can one *calculate* rather than assume the flow density ρ_1 ?
3. Is there enough energy dissipation across the shock to account for snow compression (if $\rho_1 > \rho_0$) and lifting ($h_f \gg \delta$)?
4. Can one replace the infinitesimally short shock front by a finite-length transition zone and account for internal friction in the flow?

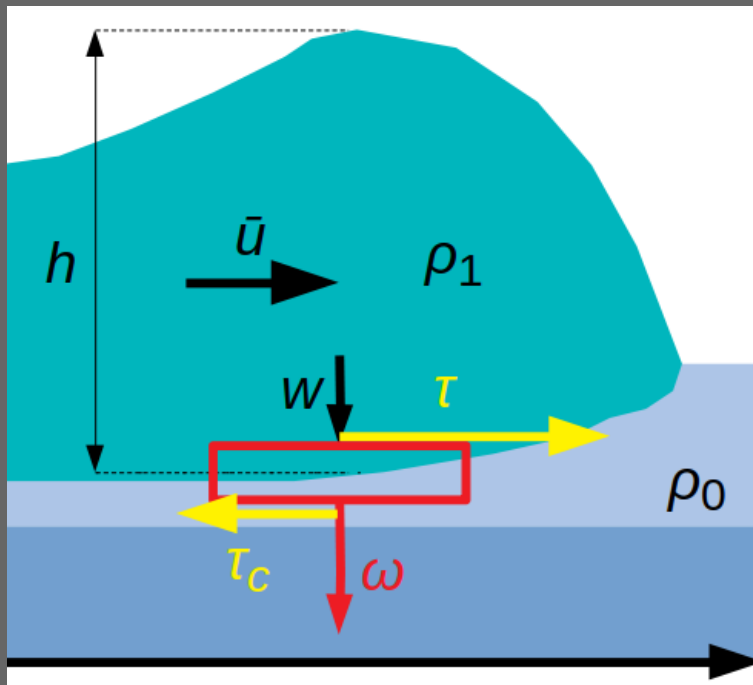


3. BASAL ENTRAINMENT

- Consider only *entrainment* from the snow cover into an avalanche. Deposition requires separate discussion ([Rauter and Köhler, 2020](#)).
- Entrainment from the dense/fluidized layer into the powder-snow cloud can be treated similarly but is more complicated.
- For simplicity, assume the bed strength to increase with depth as $\tau_c(\delta - z) = \tau_c(\delta) + (\rho_0 + \epsilon) \cdot (\delta - z)g \sin \theta$ to prevent it from complete failure when the avalanche arrives.
- Assume a [typical depth-averaged model](#).



The Tangential-Jump Model (Issler, [2014](#), [2020](#))



Shock surface assumed parallel to ground.

Consider jump in shear stress across shock surface:

- $\tau_- = \tau_c$ in snow cover
- $\tau_+ = \tau(\rho_1, h, \bar{u})$ in avalanche

Difference $\tau_+ - \tau_-$ available to accelerate the eroded snow to speed $\bar{u}$:

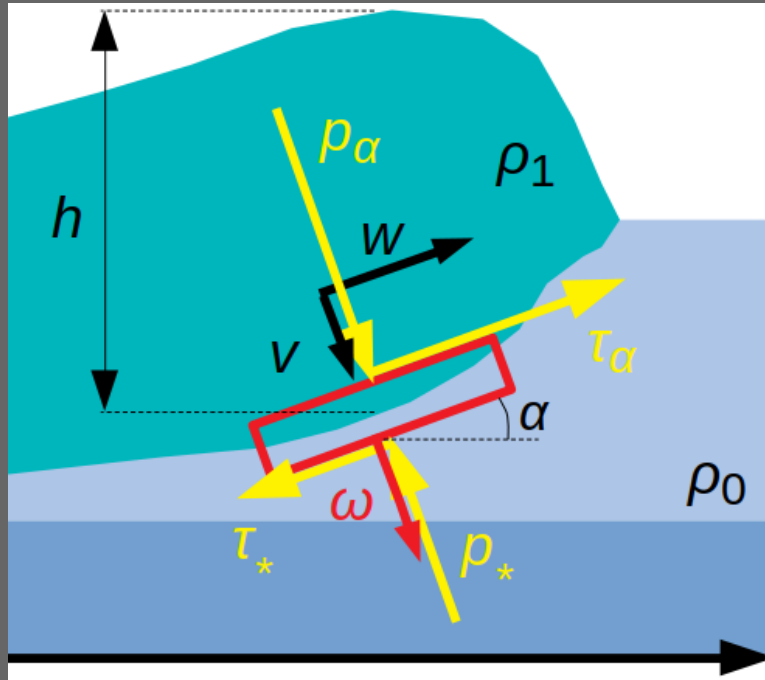
$$q_e = \Theta(|\tau| - \tau_c) \frac{\tau(\rho_1, h, \bar{u}) - \tau_c}{\bar{u}}$$

Hidden Assumptions in the TJEM

1. Erosion depth increases from avalanche head to tail, thus erosion surface *cannot* be parallel to the ground. But typically, the inclination is small: $\alpha \approx \omega/\bar{u} \lesssim 1^\circ \Rightarrow$ excellent approximation.
2. Entrainment may modify the friction law to some degree (modified velocity profile). Expect that q_e is over-estimated.
3. Jump condition for z-momentum is not used.
4. Strength τ_c is not exactly *shear* strength of the snow cover, the normal stress σ_n should also be taken into account.

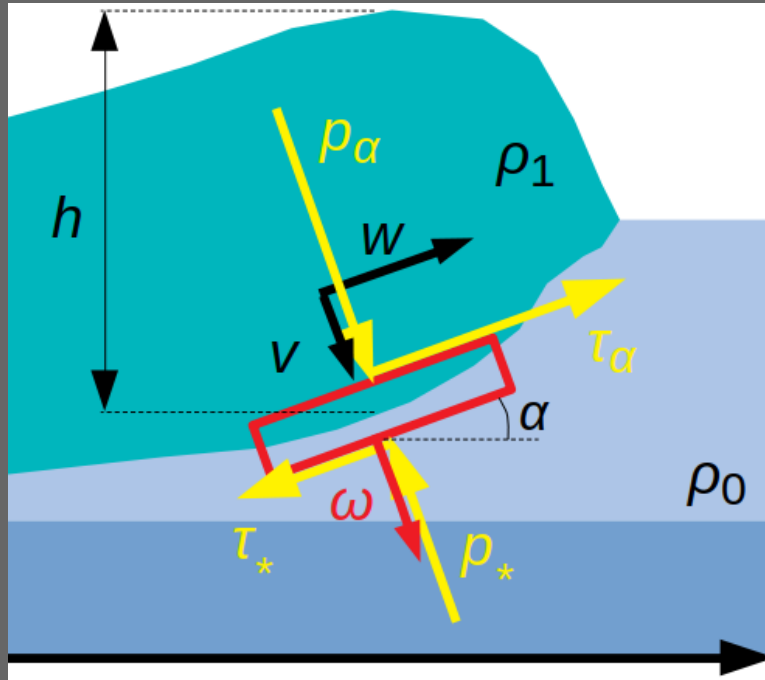


The Grigorian–Ostroumov Model—Original Formulation



- Shock surface inclined at angle $\alpha = \arctan(\partial_s \delta(s, t))$
- Use jump condition for momentum normal to shock surface (similar to EGYEM).
- Compressive snow-cover strength p_*
- Densities ρ_0, ρ_1 assumed given

The Grigorian–Ostroumov Model—Original Formulation (2)



- Original derivation amounts to ([Grigorian and Ostroumov, 2020](#)):

$$\omega = \Theta(p_\alpha - p_*) \sqrt{\frac{p_\alpha - p_*}{\rho_0(1 - \rho_0/\rho_1)}}$$

- Can also be expressed as

$$\omega = \Theta(p_\alpha - p_*) \frac{p_\alpha - p_*}{\rho_0 v}$$

to show similarity with EGYEM.

The Grigorian–Ostroumov Model—Pressure

- [Grigorian and Ostroumov \(2020\)](#) use expression for pressure with hydrostatic *and dynamic* components:

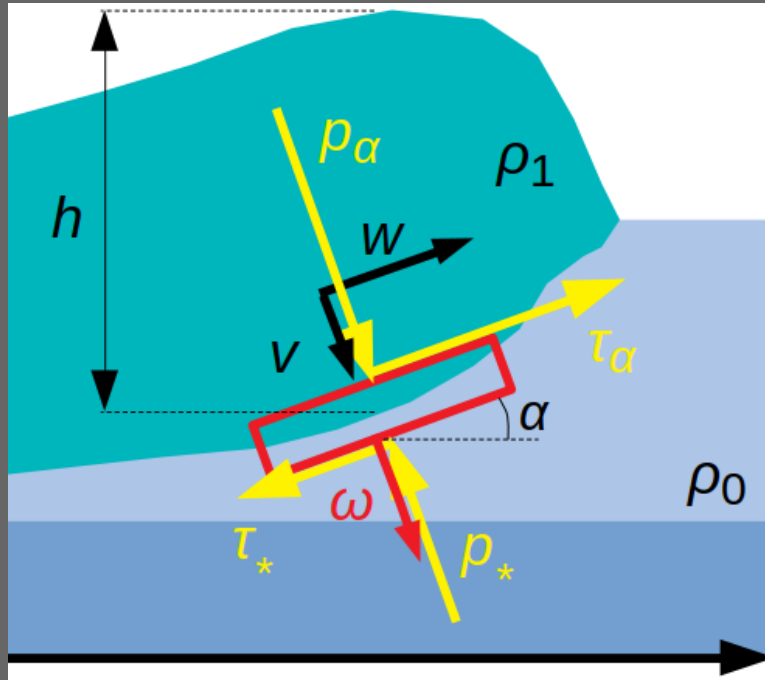
$$p_{\alpha} = \rho_1 g_z h \cos \alpha + C \rho_1 \bar{u}^2$$

C : undetermined parameter (often set to 1 in simulations).

- [Eglit and Demidov \(2005\)](#) point out that this is incorrect from a fluid-dynamical point-of-view.
- With $C = O(1) \Rightarrow p_{\alpha} = O(100 \text{ kPa}) \gg p_*, \rho_1 g_z h$
 $\Rightarrow$ over-estimated entrainment rates, quasi-frontal entrainment.

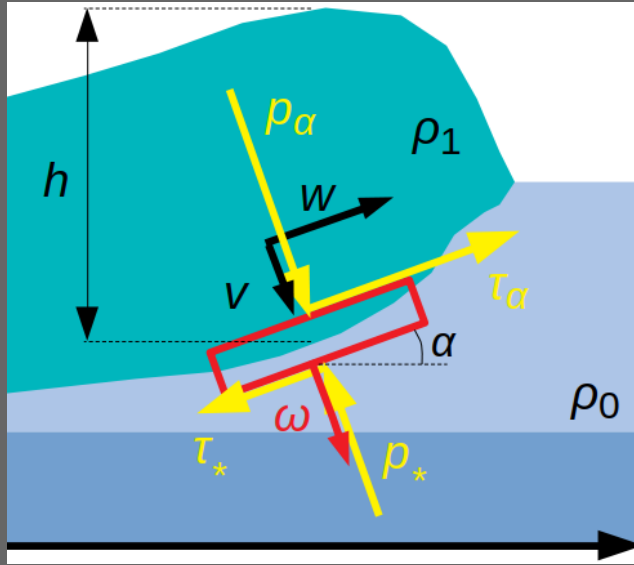


The Grigorian–Ostroumov Model—Is It Consistent?



- Consistency requires *erosion* speed ω to be adapted to *entrainment* capacity of flow.
- Acceleration due to horizontal components of p , p_* , τ and τ_* :

The Grigorian–Ostroumov Model (GOEM)—Is It Consistent? (2)



- Consistency requires *erosion* rate q_e to be adapted to *entrainment* capacity of flow.
- GOEM considers momentum balance only in one direction (mostly bed-normal).
- Shear stresses are decisive for accelerating eroded snow in forward direction.
- Erosion threshold depends both on shear stress and normal stress.

⇒ GOEM is not fully self-consistent without tangential momentum component!

Can One Unify the TJEM and the GOEM?

- The GOEM emphasizes pressure-driven erosion, the TJEM shear-driven entrainment.
- For a more complete model, use both momentum jump conditions (and only hydrostatic contribution to pressure).
- To be investigated whether the combined model will give substantially different entrainment rates from the TJEM.

Conceptual and practical challenge:

Entrainment models cannot be validated independently of the rheological assumptions!



4. CONCLUDING REMARKS

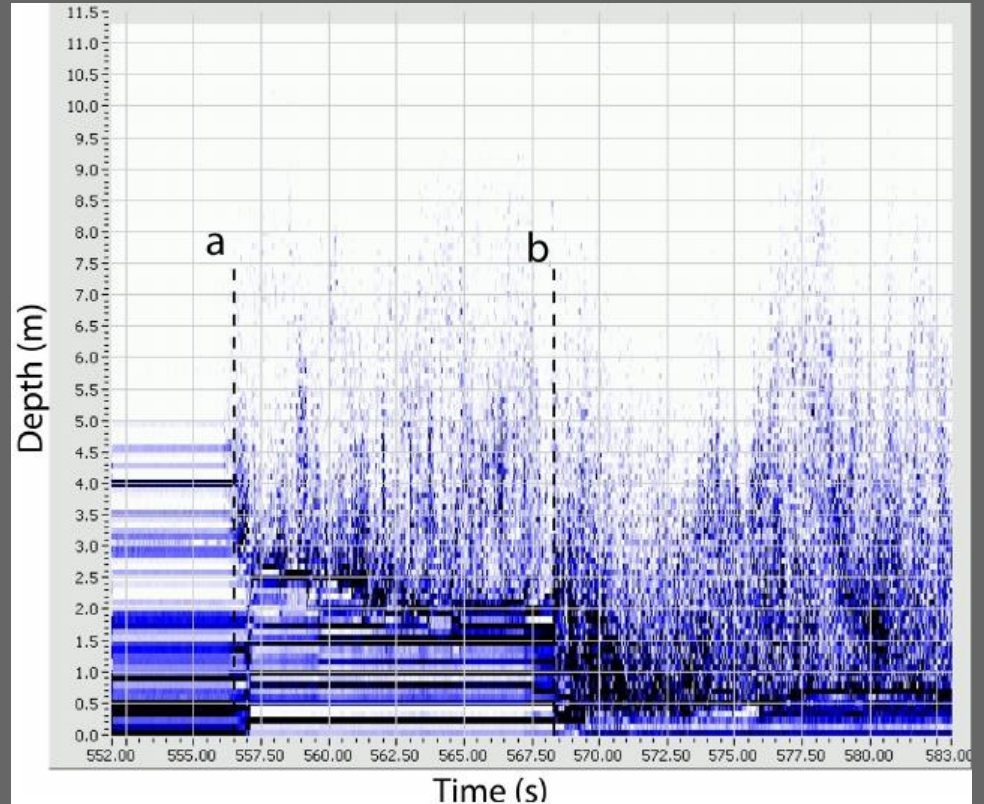
Effect of snow-cover layering?

- Failure surface may immediately jump to a buried weak layer ([step-entrainment](#) mechanism).
- Likely explanation for “step-wise” erosion seen in measurements with profiling radar ([Sovilla, 2004](#)).
- **Avalanche is not able to accelerate the eroded mass to flow speed instantaneously but drags it along; may erode it from the top.**
- Do we need to model this explicitly? If so, how?
(Option: Add momentum balance for snow cover?)



Radar reflectivity profile, avalanche of 1999-12-27 at Vallée de la Sionne, showing quasi-instantaneous erosion of approx. 1 m and 1.5 m of snow at times *a* and *b*, with gradual entrainment (~ 0.1 m/s) in-between.

Figure from ([Sovilla et al., 2006](#)).



Recalibration of Run-Out Models

Entrainment term $-q_e \bar{u}$ acts like an extra (velocity-dependent) resistance term in the equation of motion.

⇒ Friction parameters must be recalibrated!

Preliminary tests with Voellmy-type block model with entrainment:

$$\mu \approx 0.25-0.3, \quad k = g / \xi \approx 10^{-3}$$

for avalanches with return period $T \geq 100$ y (P. Gauer, priv. comm.).

Range of friction parameters may be much smaller than without entrainment ([Sovilla & Bartelt, 2002](#)).



Input Data for Prognostic Calculations

In hazard-mapping applications, *predictions* must be made. Such calculations require additional input data:

➤ *Erodible snow depth $b(x,y)$:*

Depends on climate and altitude. Near the release area related to fracture depth. A critical input only if avalanche can erode it all.

➤ *Shear strength $\tau_c(x,y)$ of erodible snow layer:*

Also depends on climate and altitude. Typical values 0.5–1.5 kPa. May need dedicated measurement campaigns to make reliable recommendations.



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