GOCE SGG filtering with FIR, IIR and wavelet MRA

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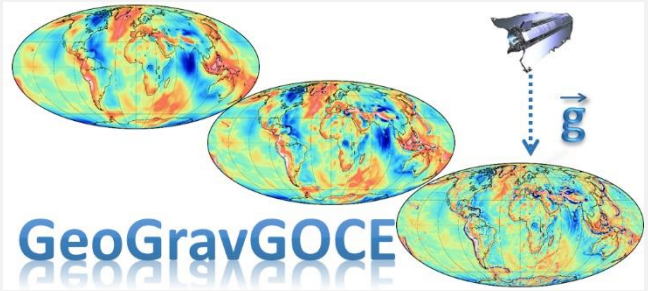

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Filtering of GOCE gradients

The aim of this study is to filter GOCE gravity gradients in order to reduce the high-frequency noise and long-wavelength correlated errors, while preserving the signals in the GOCE measurement bandwidth (MBW) (5 - 100 mHz).

Three methods were applied for this purpose:

- Finite Impulse Response filters (FIR)
- Infinite Impulse Response filters (IIR)
- Wavelet Multi-Resolution Analysis (WL-MRA)

FIR (Finite Impulse Response filter)

An FIR filter is non-recursive

The output after an FIR filter is computed using the current and previous inputs.

FIR is a bandpass filter that attenuates the signal outside of GOCE MBW, while a Hamming window is used to minimize the spectral leakage error in the boundaries of the determined MBW.

Several filtering options and tests were performed in order to choose the proper filter order which will be able to preserve the pass-band signals.

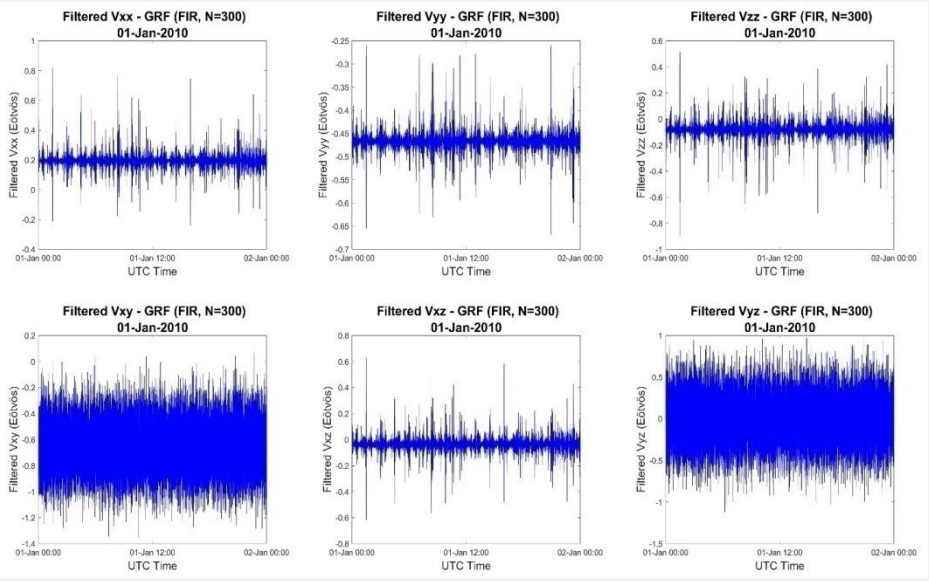
FIR filter evaluation

- FIR band-pass filtering was carried out for reduced (to a spectrally enhanced GGM) GOCE SGG data, for filter orders 300, 500, 700, 1000, 1500, 1700, and 2500.

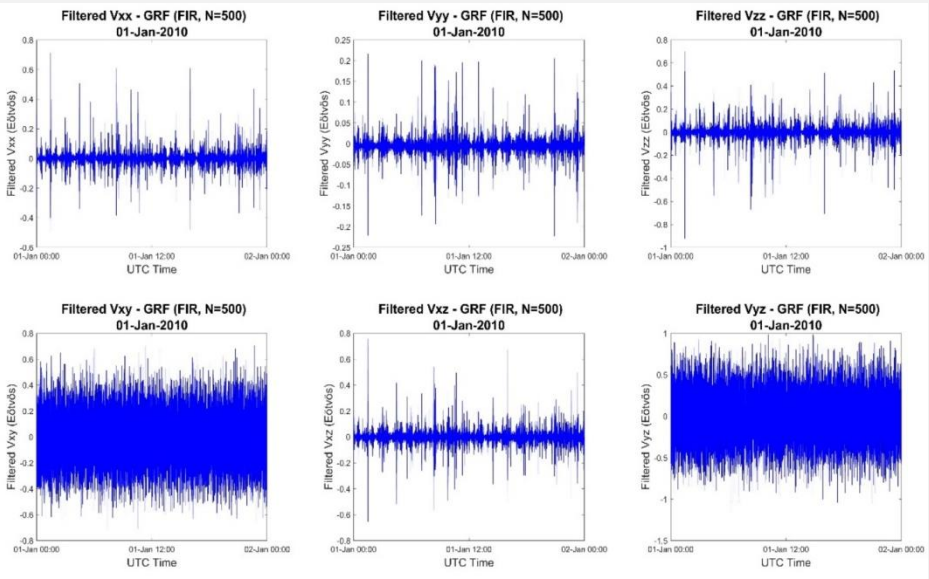
- GO_CONS_GCF_2_TIM_R6 complete to degree and order (d/o=300) was used for the data reduction.

- The N^{th} filter performance evaluation is done in terms of the Power Spectral Densities (PSDs) of the original unfiltered and filtered signal.

Comparison of various FIR filters (reduced data)

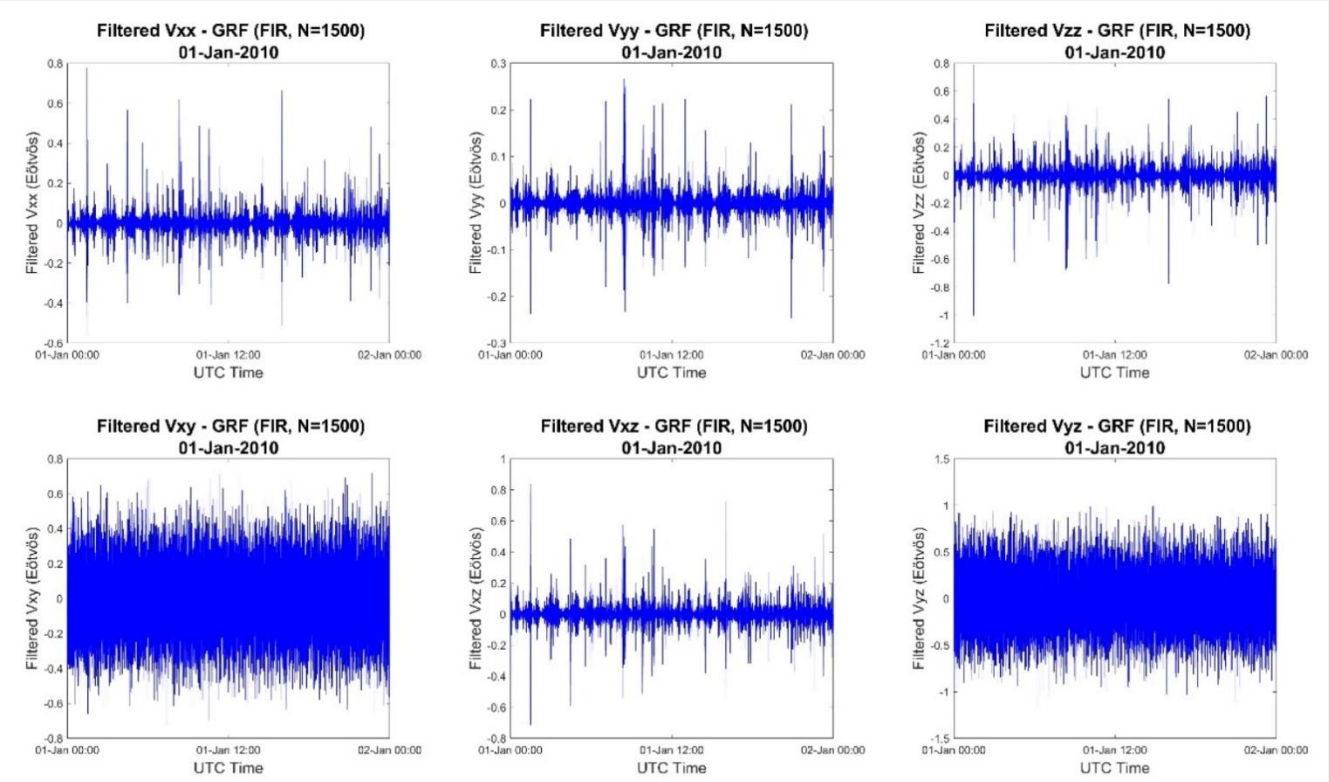


N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
300	min	-0.241	-0.669	-0.897	-1.353	-0.618	-1.118
	max	0.822	-0.260	0.517	0.065	0.629	0.978
	mean	0.195	-0.467	-0.080	-0.650	-0.035	0.000
	std	0.058	0.024	0.074	0.176	0.064	0.262
	rms	0.204	0.467	0.109	0.674	0.073	0.262



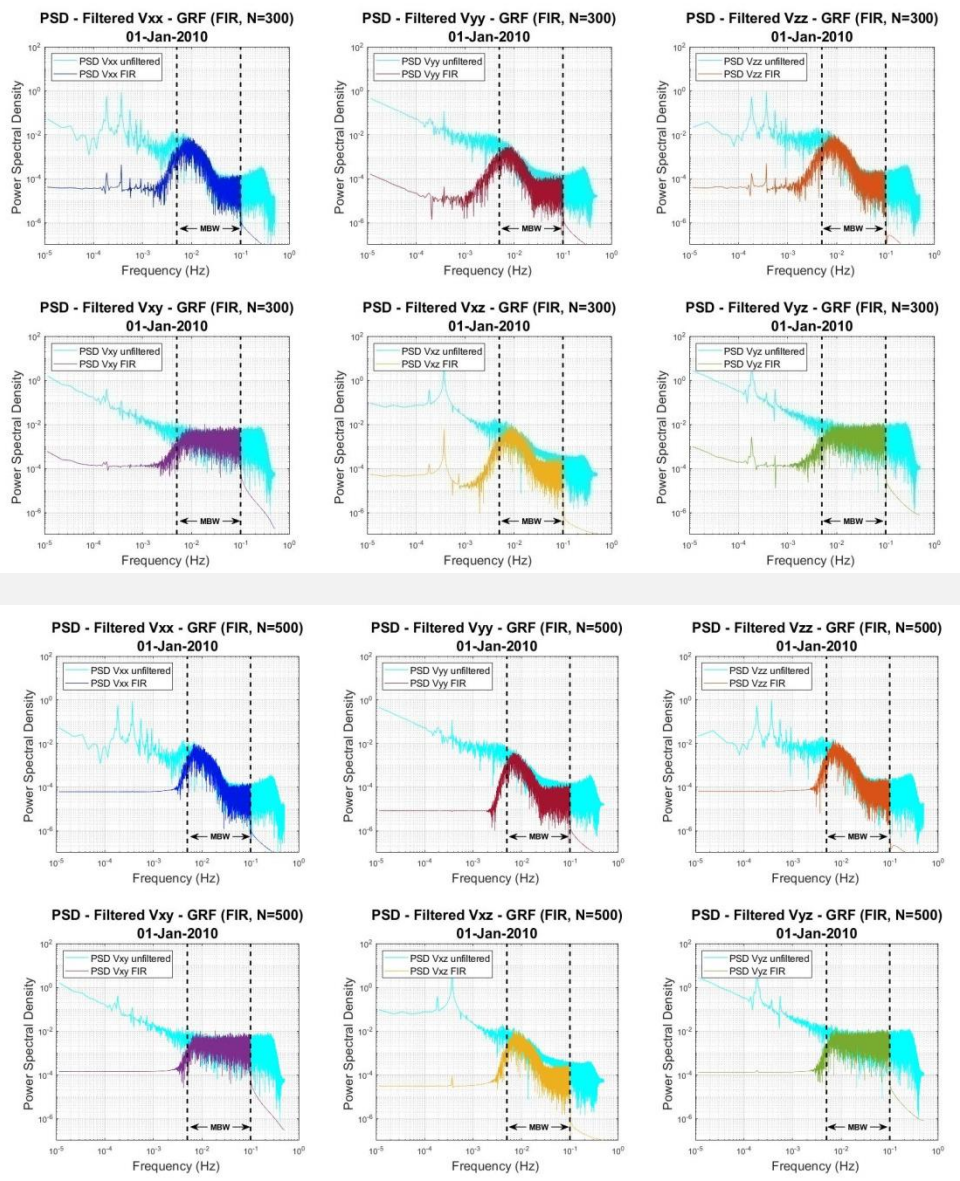
N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
500	min	-0.482	-0.223	-0.924	-0.707	-0.652	-1.138
	max	0.711	0.217	0.700	0.705	0.757	0.987
	mean	0.002	-0.005	-0.001	-0.007	0.000	0.000
	std	0.065	0.027	0.083	0.178	0.072	0.265
	rms	0.065	0.028	0.083	0.178	0.072	0.265

Comparison of various FIR filters (reduced data)

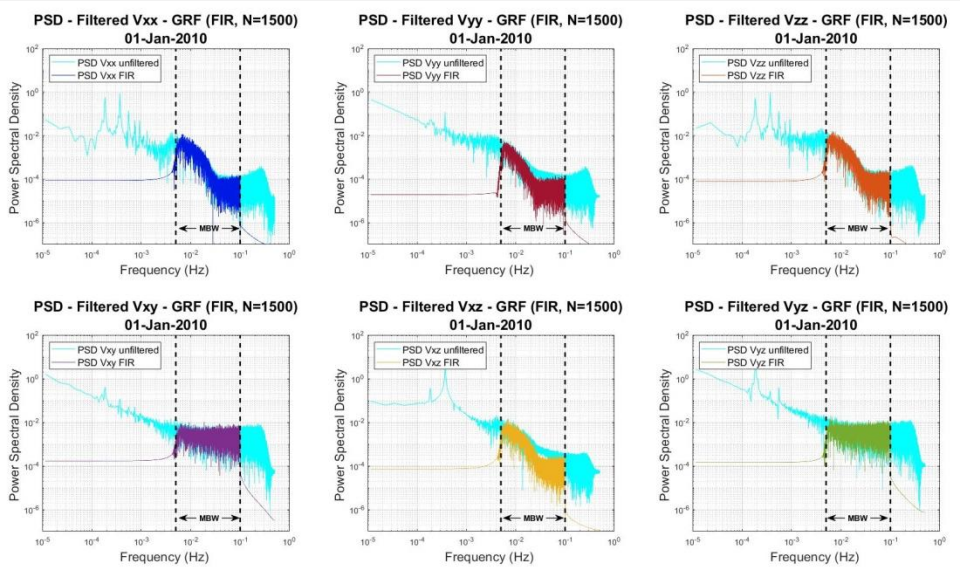


N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
1500	min	-0.554	-0.246	-1.004	-0.724	-0.714	-1.165
	max	0.777	0.266	0.791	0.717	0.837	1.001
	mean	0.000	0.000	0.000	0.000	0.000	0.000
	std	0.072	0.033	0.094	0.180	0.080	0.268
	rms	0.072	0.033	0.094	0.180	0.080	0.268

Comparison of PSDs of various FIR filters (reduced data)

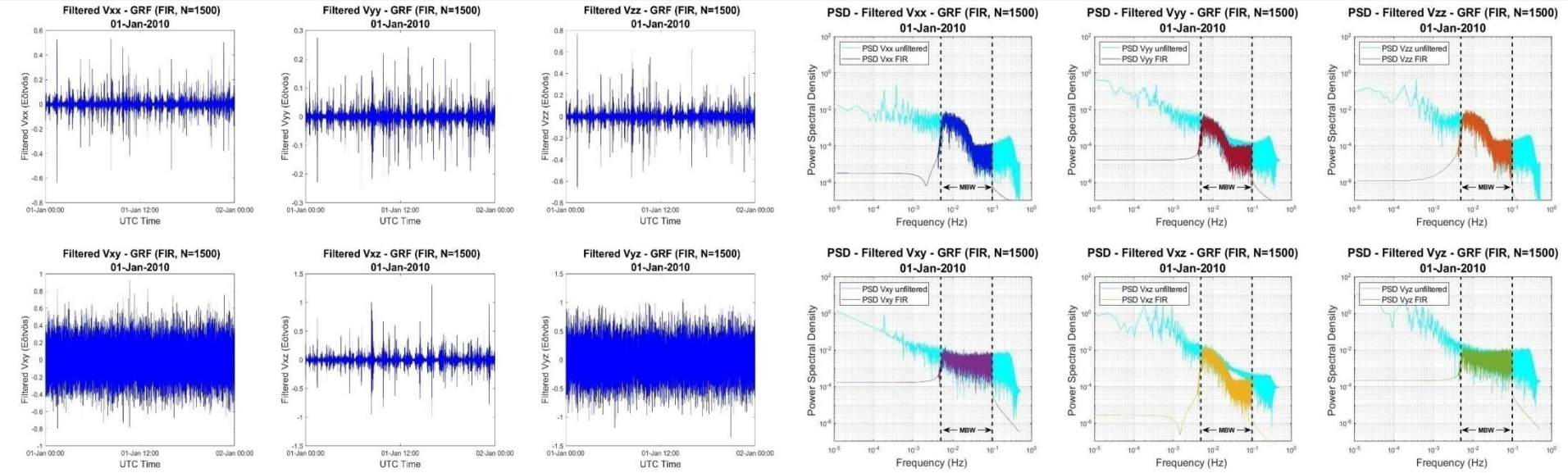


For the 1500th order the filter is stable in the GOCE MBW, while the signals attenuate well outside the MBW.



FIR Filtering of the enhanced* GOCE gravity gradients

The filter with order N=1500 responds optimally to the filtering requirements, with the best statistical and spectral performance.



	Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
min	-0.636	-0.250	-0.677	-0.801	-1.015	-1.352
max	0.534	0.274	0.765	0.925	1.308	1.064
mean	0.000	0.000	0.000	0.000	0.000	-0.001
std	0.060	0.033	0.072	0.187	0.113	0.273
rms	0.060	0.033	0.072	0.187	0.113	0.273

* The final reduced data used in the GeoGravGOCE project as a result of a GGM validation process: $V_{ij}^{enhanced} = \mathrm{V}_{\mathrm{ij}}^{GOCE} - \mathrm{V}_{\mathrm{ij}}^{TIM\ R6_{n=2}^{195}} - \mathrm{V}_{\mathrm{ij}}^{EGM08_{\mathrm{n=166}}^{2190}}$

IIR (Infinite Impulse Response filter)

IIR is a recursive filter

The filter output is computed using the current and former inputs and outputs

The IIR filter is based on the equivalent low pass Butterworth filter, providing a flat response in the pass-band

Several filtering tests have been performed to choose the proper filter order for successful filtering of GOCE SGG data, which will be able to preserve the pass-band signals, based on the IIR filters.

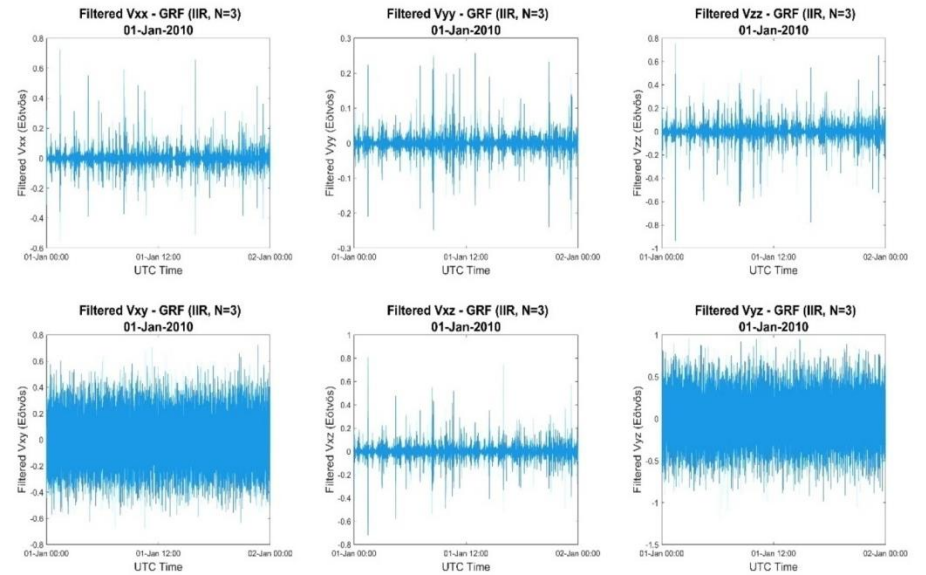
IIR filter evaluation

- The IIR Butterworth filter was used to process the reduced GOCE SGG data for filter orders 3, 4, 5, 6, & 7.

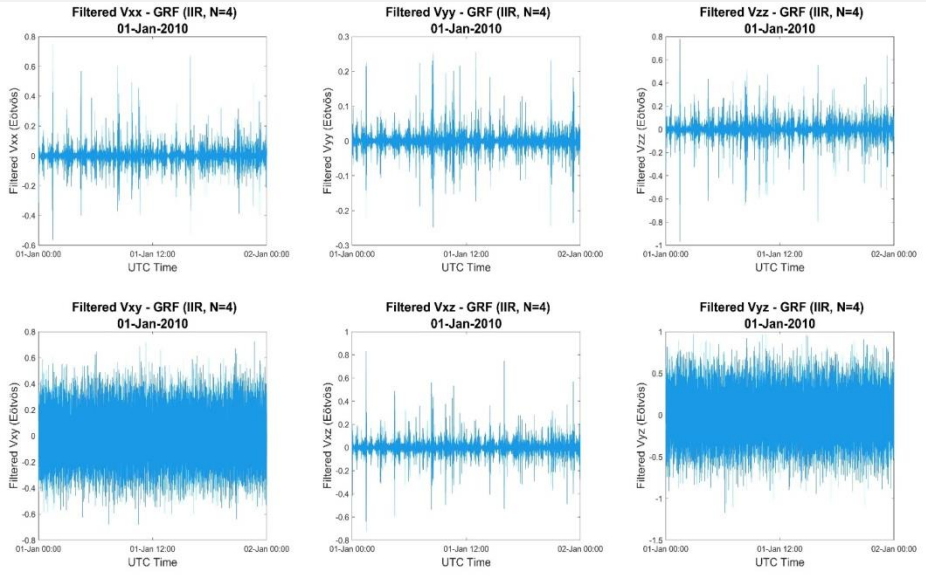
- The reduced data were based on the GO_CONS_GCF_2_TIM_R6 GGM complete to degree and order (d/o=300).

- The N^{th} filter performance evaluation is done in terms of the PSDs of the original unfiltered and filtered signal

Comparison of various IIR filters (reduced data)

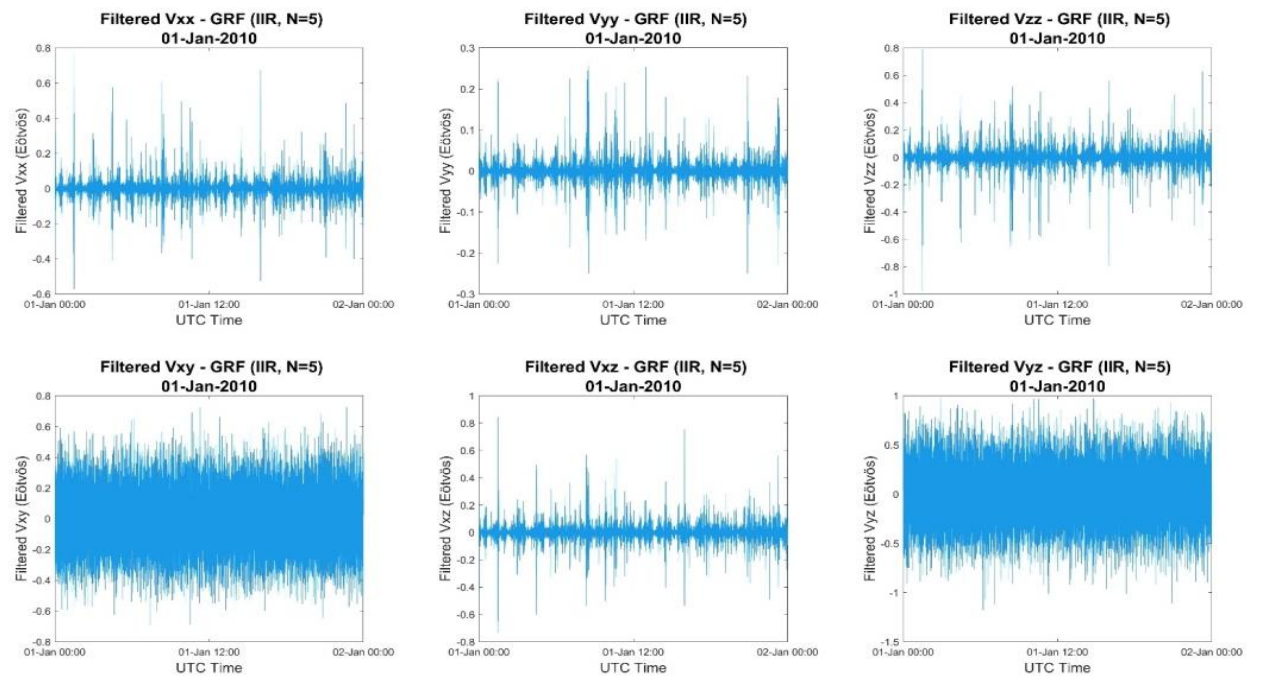


N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
3	min	-0.549	-0.248	-0.939	-0.671	-0.717	-1.164
	max	0.723	0.258	0.756	0.722	0.809	0.979
	mean	0.000	0.000	0.000	0.000	0.000	0.000
	std	0.069	0.032	0.091	0.170	0.077	0.253
	rms	0.069	0.032	0.091	0.170	0.077	0.253



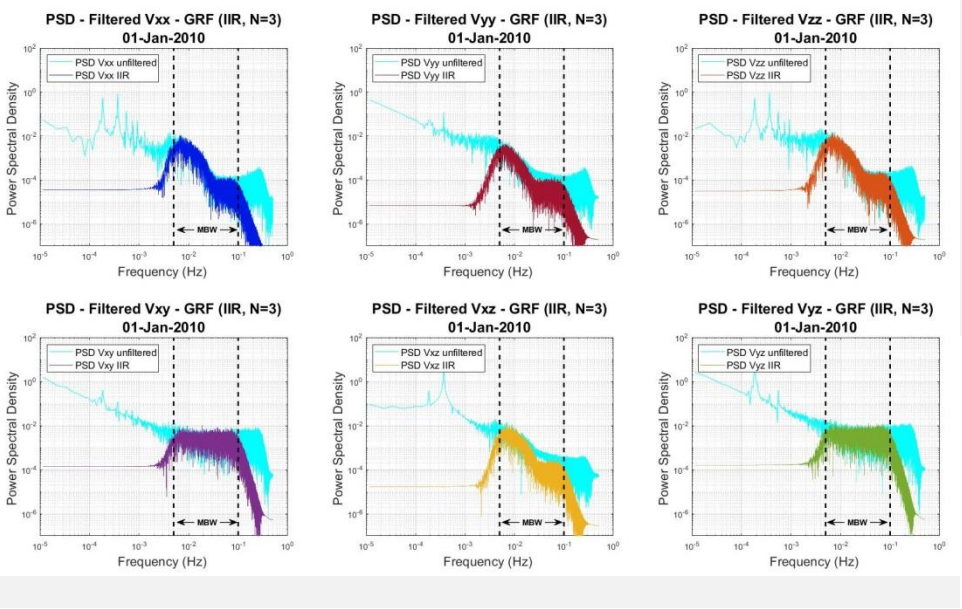
N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
4	min	-0.564	-0.249	-0.967	-0.681	-0.729	-1.173
	max	0.746	0.257	0.780	0.724	0.830	0.972
	mean	0.000	0.000	0.000	0.000	0.000	0.000
	std	0.071	0.033	0.093	0.172	0.079	0.257
	rms	0.071	0.033	0.093	0.172	0.079	0.257

Comparison of various IIR filters (reduced data)

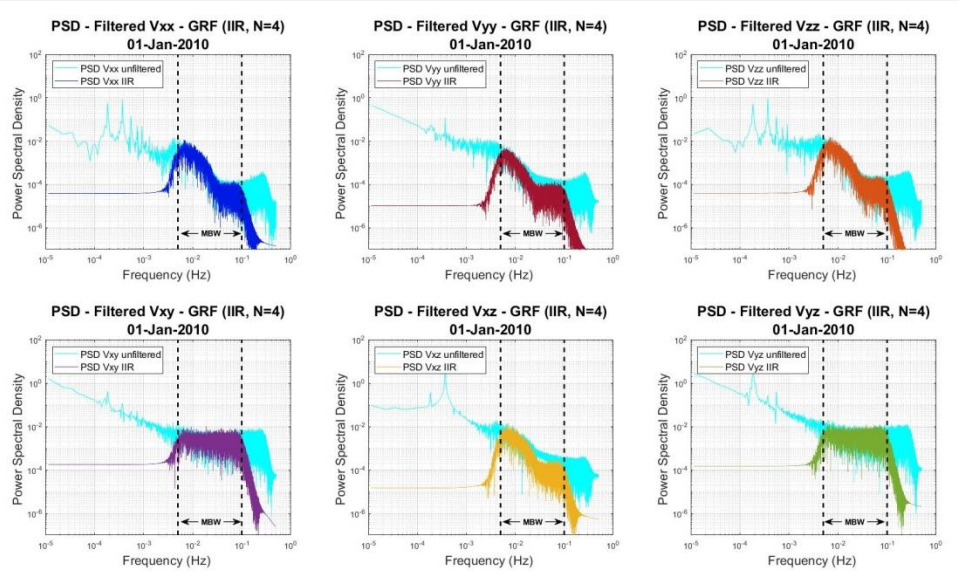


N		Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
5	min	-0.571	-0.250	-0.983	-0.692	-0.734	-1.176
	max	0.759	0.261	0.793	0.726	0.841	0.983
	mean	0.000	0.000	0.000	0.000	0.000	0.000
	std	0.072	0.033	0.094	0.174	0.080	0.259
	rms	0.072	0.033	0.094	0.174	0.080	0.259

Comparison of PSDs of various IIR filters (reduced data)

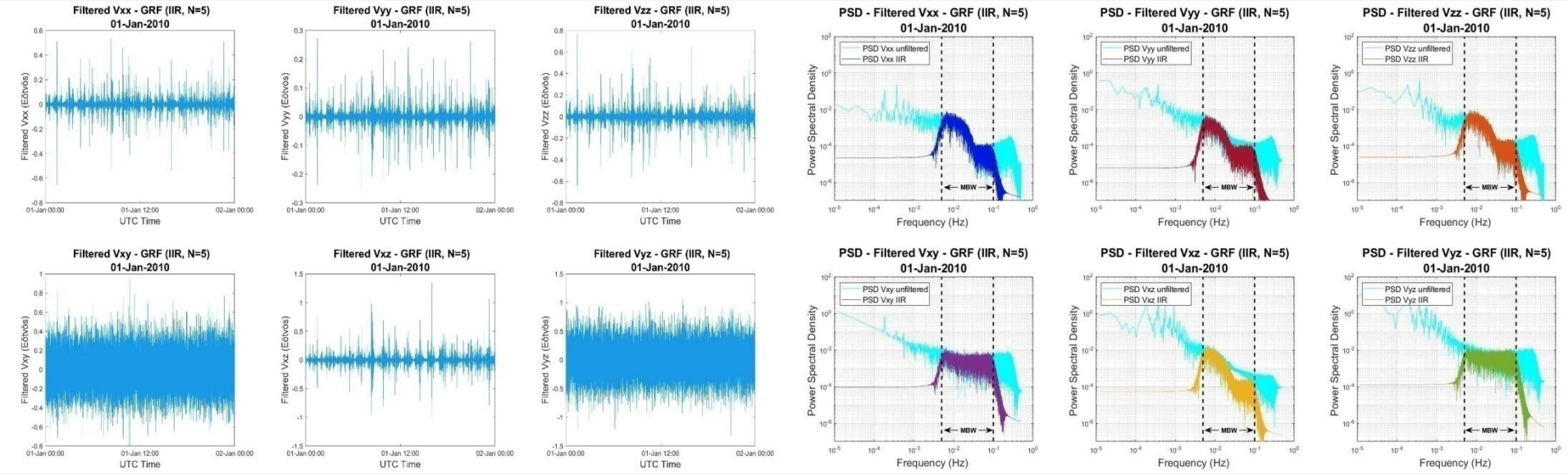


The 5th order IIR filter performs optimal outside the MBW boundaries



IIR Filtering of the enhanced* GOCE gravity gradients

The 5th order IIR filter has been adopted to remove errors beyond the MBW as it presents the best spectral performance



	Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
min	-0.649	-0.257	-0.683	-0.800	-1.006	-1.326
max	0.533	0.273	0.763	0.946	1.358	1.061
mean	0.000	0.000	0.000	0.000	0.000	0.000
std	0.060	0.033	0.072	0.181	0.113	0.264
rms	0.060	0.033	0.072	0.181	0.113	0.264

* The final reduced data used in the GeoGravGOCE project as a result of a GGM validation process: $V_{ij}^{enhanced} = \mathrm{V}_{\mathrm{ij}}^{GOCE} - \mathrm{V}_{\mathrm{ij}}^{TIM\ R6_{n=2}^{195}} - \mathrm{V}_{\mathrm{ij}}^{EGM08_{\mathrm{n=166}}^{2190}}$

Wavelet MRA

- A Daubechies family wavelet (1D wavelet) with 10 vanishing moments (db10) at twelve (12) levels of decomposition was applied per GOCE track for along-track analysis.

- Wavelet MRA was applied both for the original unfiltered SGGs and for the reduced ones in the GRF (original gravity gradients referred to a GGM).

- The levels of decomposition were chosen to be 12 in accordance to the prerequisite that the longest wavelengths should correspond to the Earth's perimeter (in spherical approximation).

- Each level of decomposition consists of three detail (H, V, & D) and an approximation coefficient.

Wavelet MRA coefficients

detail coefficients

correspond to higher frequencies of the signal

approximation coefficients

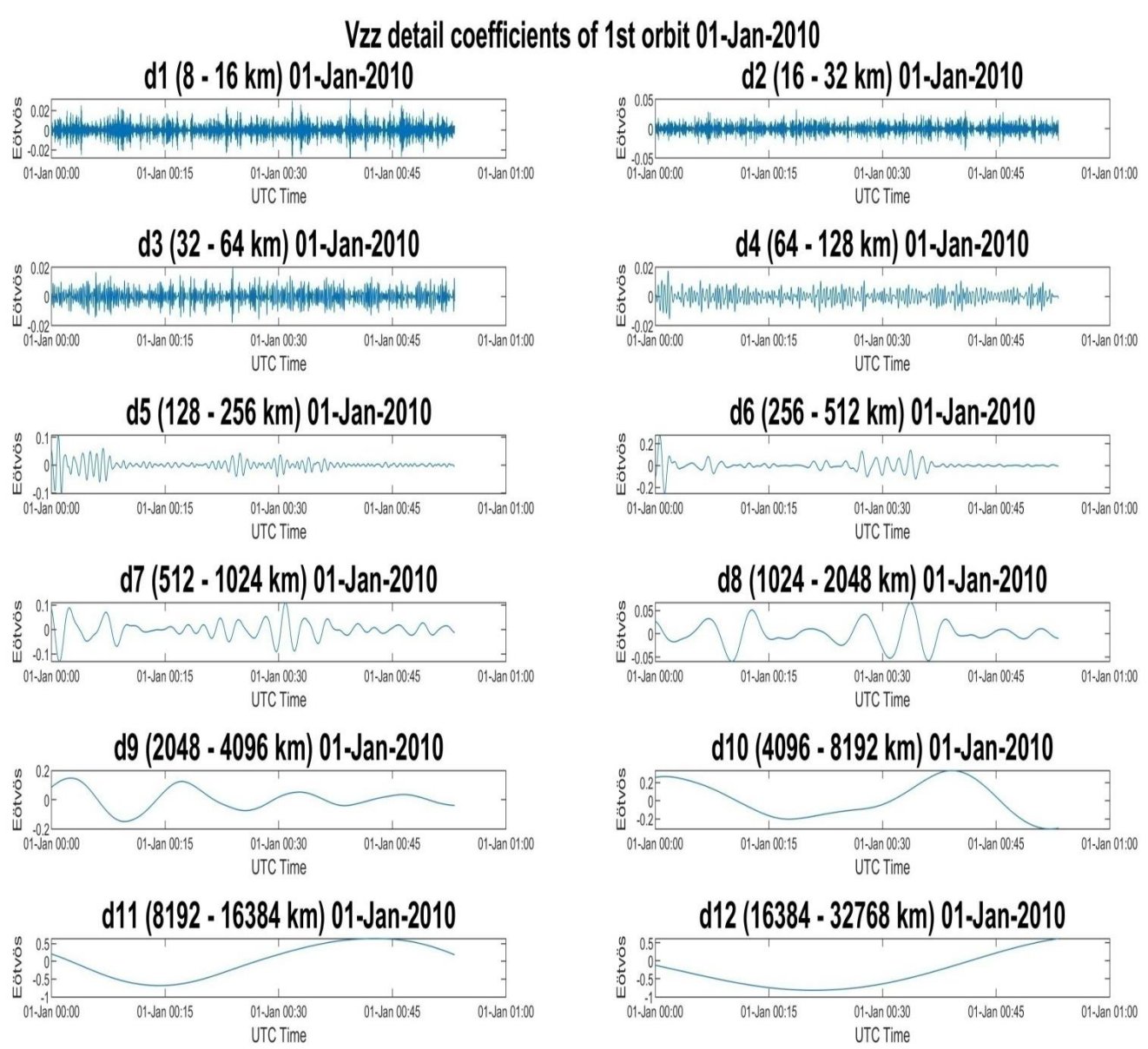
indicative of the lower frequencies
- showing the main trend of the signal

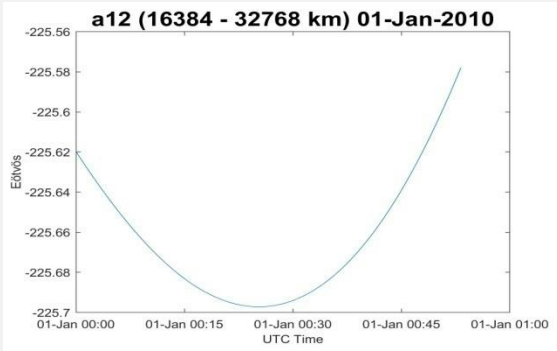
Levels of decomposition

- Each level of decomposition corresponds to different spatial resolutions
- The nominal boundaries of the GOCE MBW are mainly found in the detail coefficients of levels 4 and 7, with level 3 being of (probably) interest

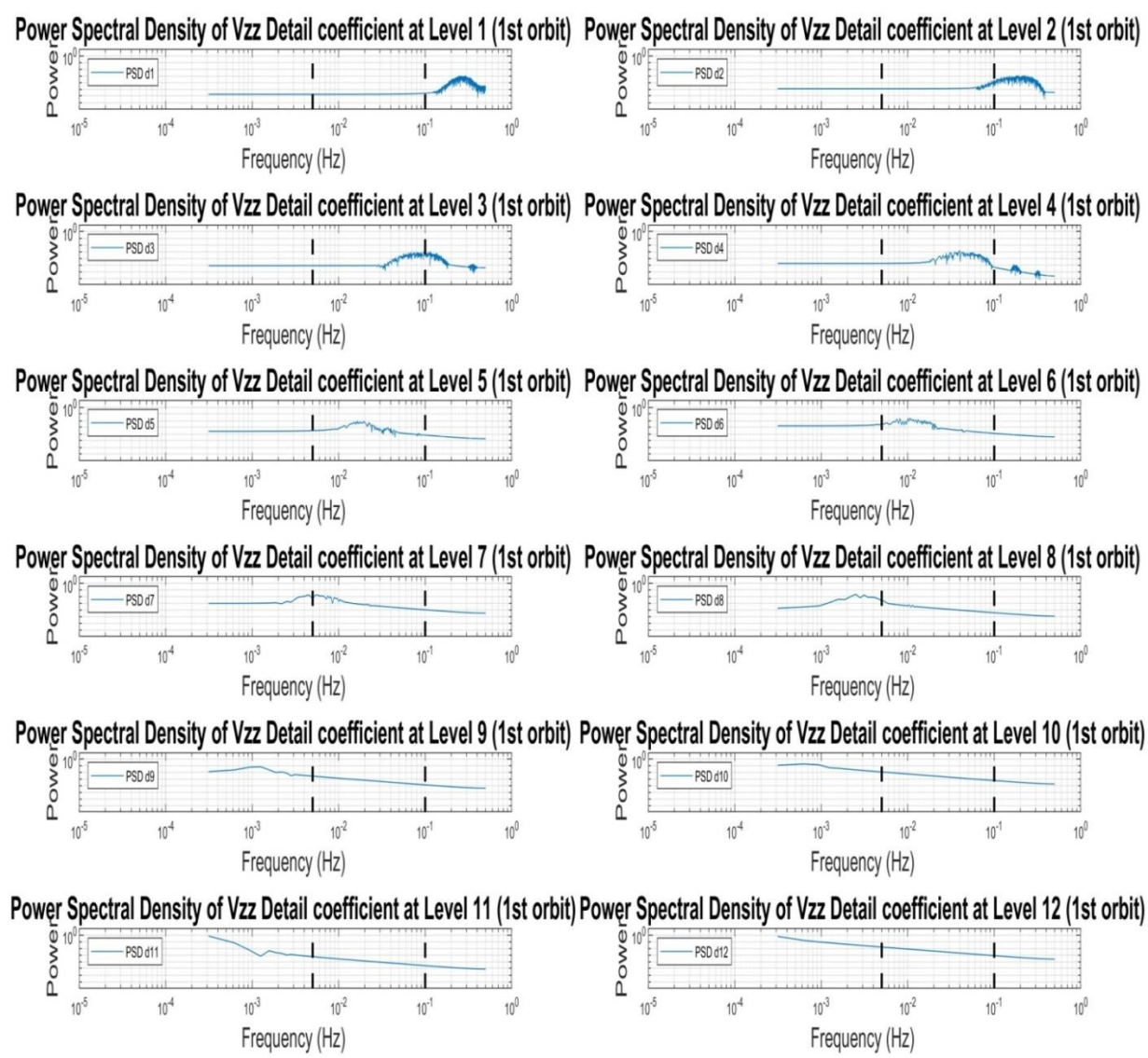
Levels of Decomposition	Resolution (km)	
	Lower Bound	Upper Bound
Level 1	8	16
Level 2	16	32
Level 3	32	64
Level 4	64	128
Level 5	128	256
Level 6	256	512
Level 7	512	1024
Level 8	1024	2048
Level 9	2048	4096
Level 10	4096	8192
Level 11	8192	16384
Level 12	16384	32768

Vzz detail and approximation coefficients (reduced data)





PSDs of the Vzz detail coefficients (reduced data)



WL selective reconstruction of GOCE SGG data

- Spectral information outside the MBW is mainly noise, with error increase below and above the MBW

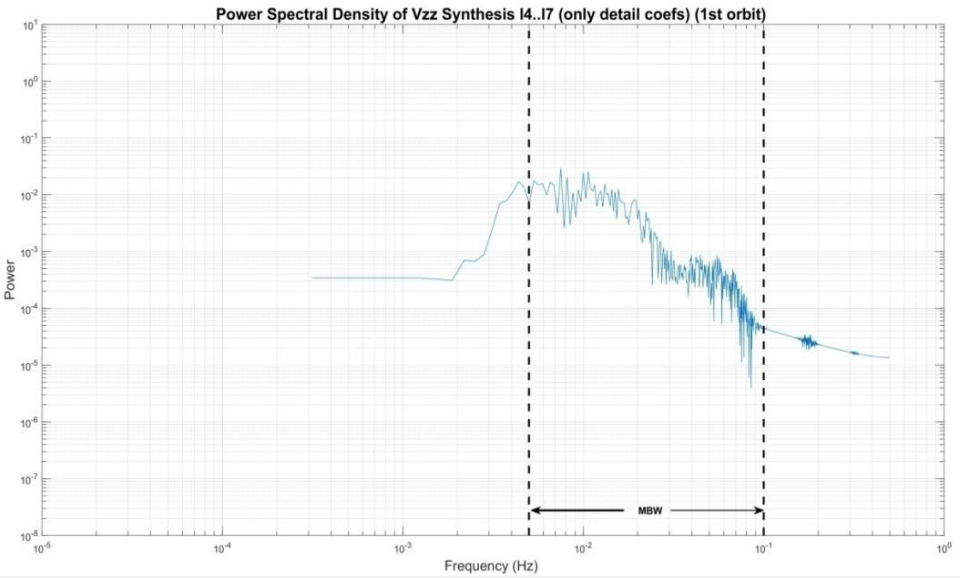
- If information outside the MBW exists in our data in the GRF, errors will be propagated through their transformation to the LNOF

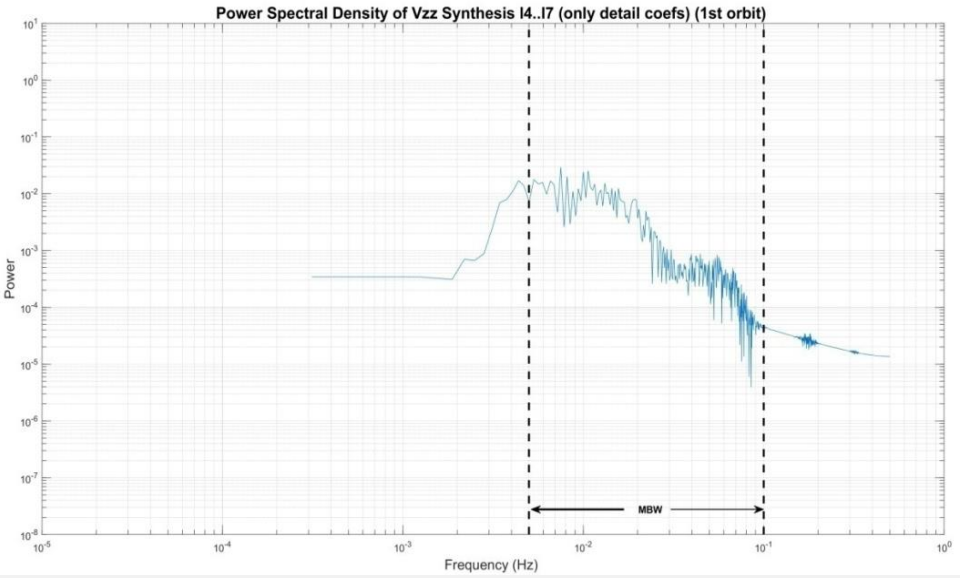
- The ideal reconstruction is the one removing noise and retaining signal carrying useful information

- The reconstruction of the signal is the summation of the detail coefficients of each level with the approximation coefficient of the last level of decomposition

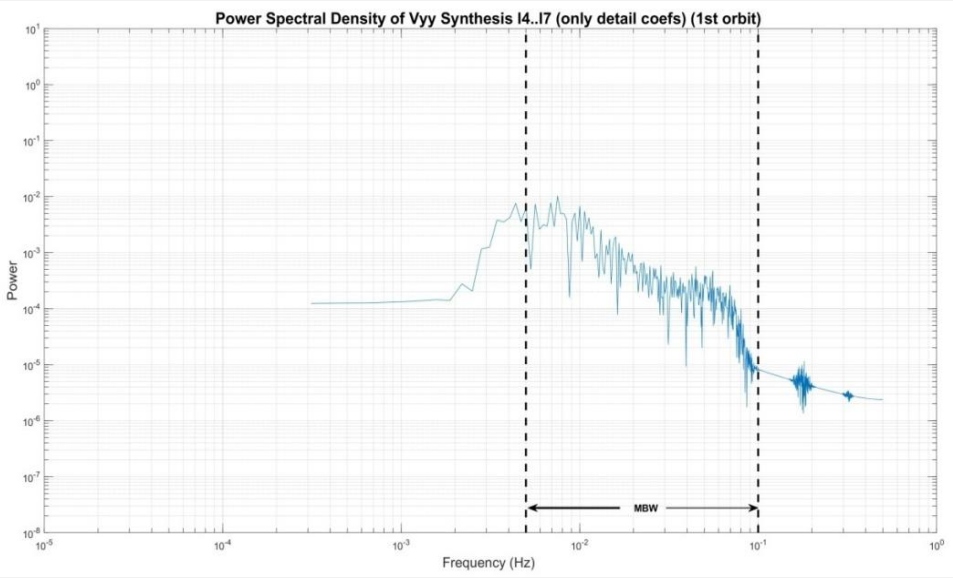
$$d_i = H_i + V_i + D_i \qquad s = (d_1) + (\mathrm{d}_2) + \ldots + (a_\mathrm{n} + \mathrm{d}_n)$$

PSDs selective reconstruction GOCE reduced data (one orbit)

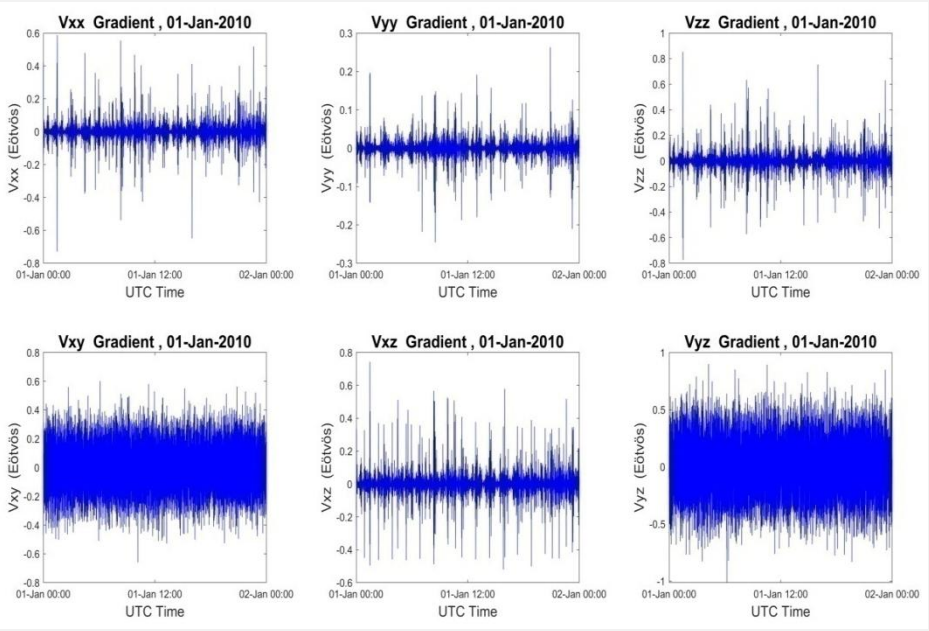




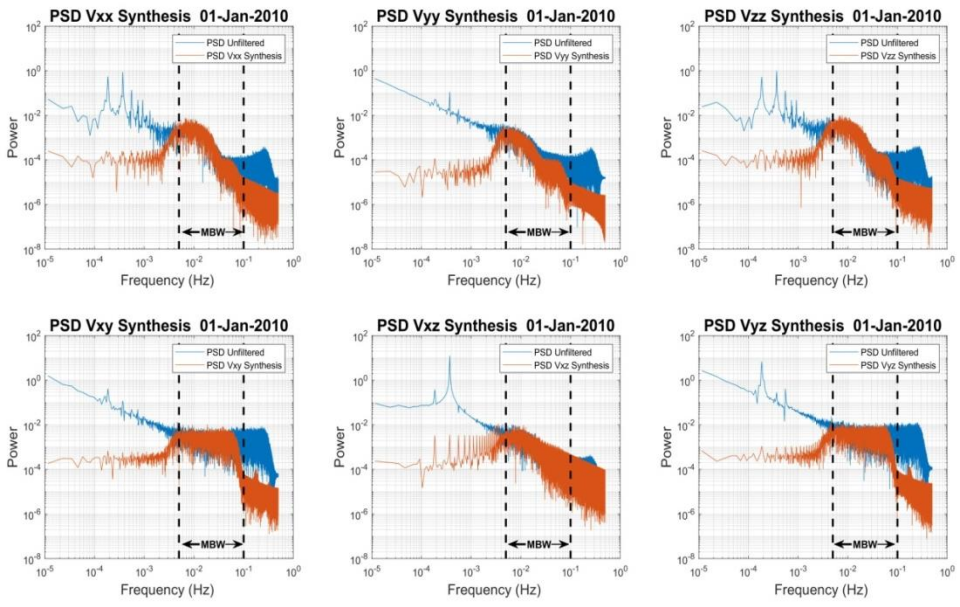
PSDs of reconstructions using the detail coefficients of levels 4-7



WL selective reconstruction of GOCE reduced data (one day)

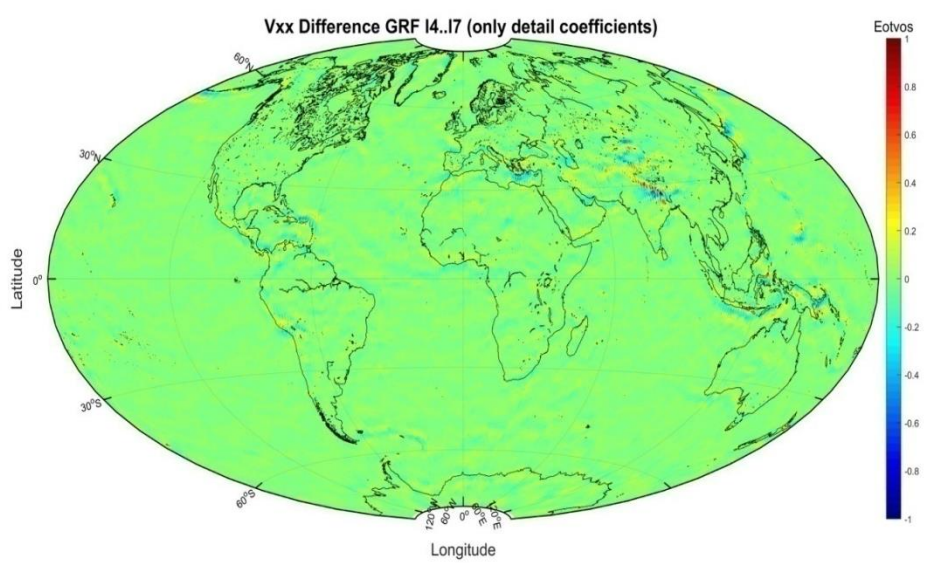


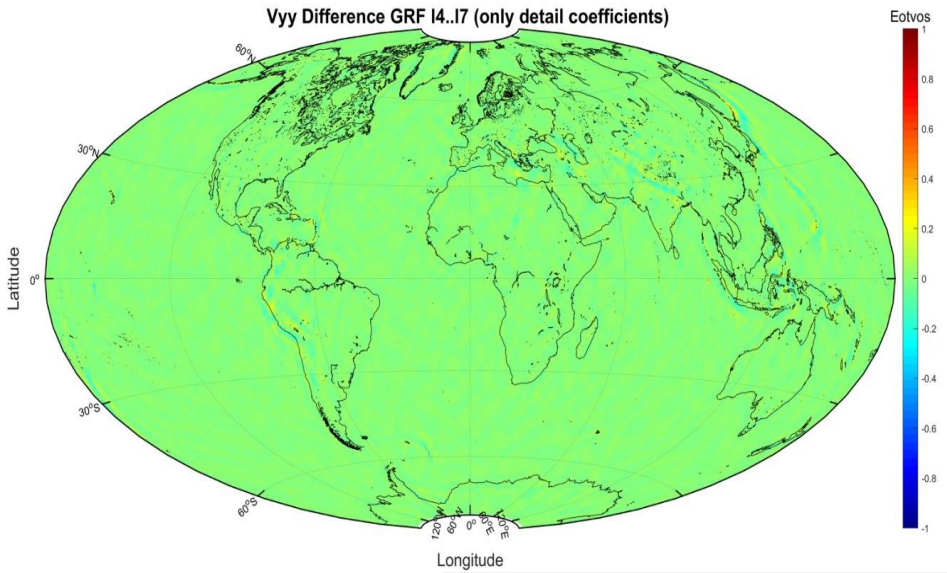
Reconstruction using the detail coefficients (levels 4-7) for one day data

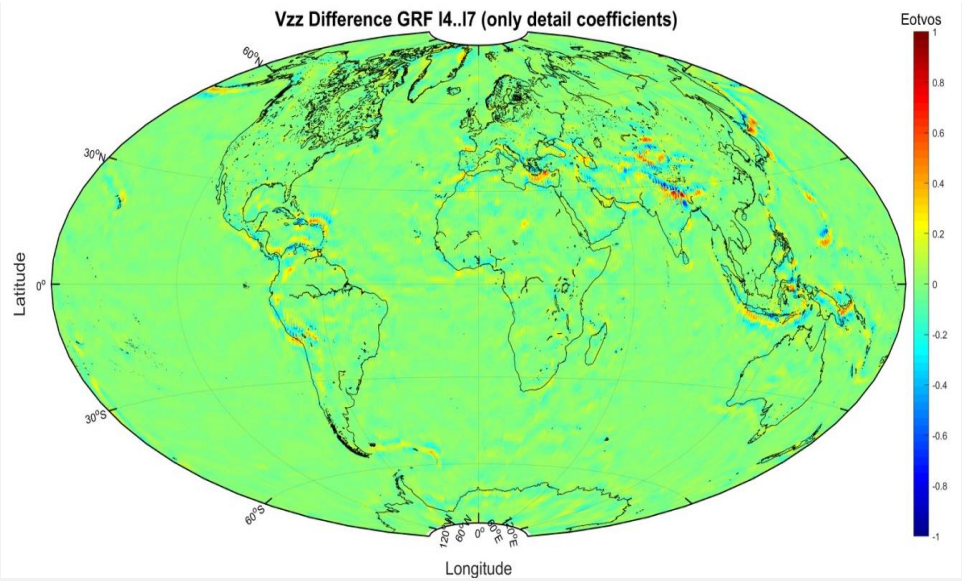


l4..l7	Vxx	Vyy	Vzz	Vxy	Vxz	Vyz
Min	-0.730	-0.246	-0.774	-0.659	-0.520	-1.013
Max	0.587	0.263	0.854	0.602	0.744	0.899
Mean	0.000	0.000	0.000	0.000	0.000	0.000
Std	0.063	0.026	0.080	0.143	0.080	0.215
Rms	0.063	0.026	0.080	0.143	0.080	0.215

WL selective reconstruction GOCE reduced data (one month)

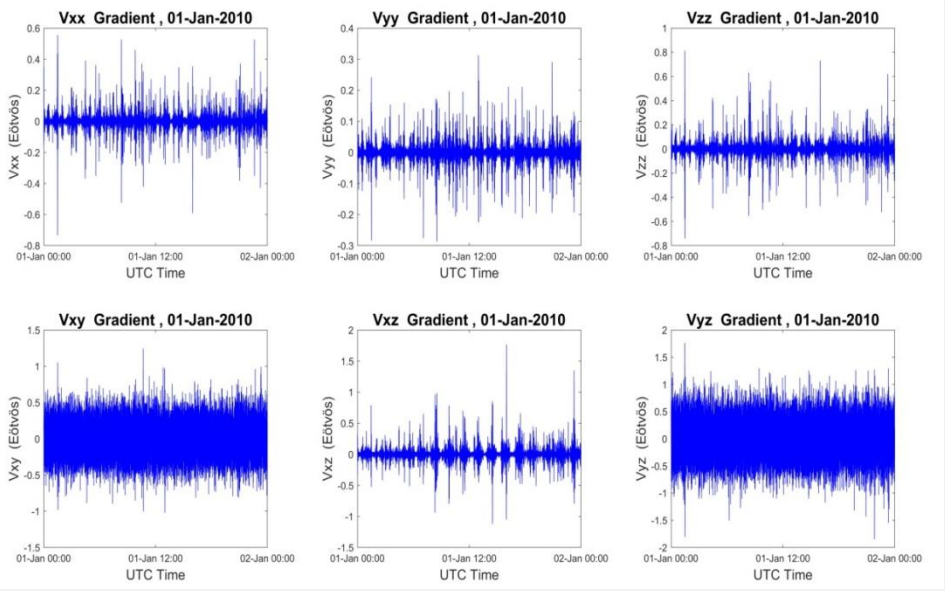




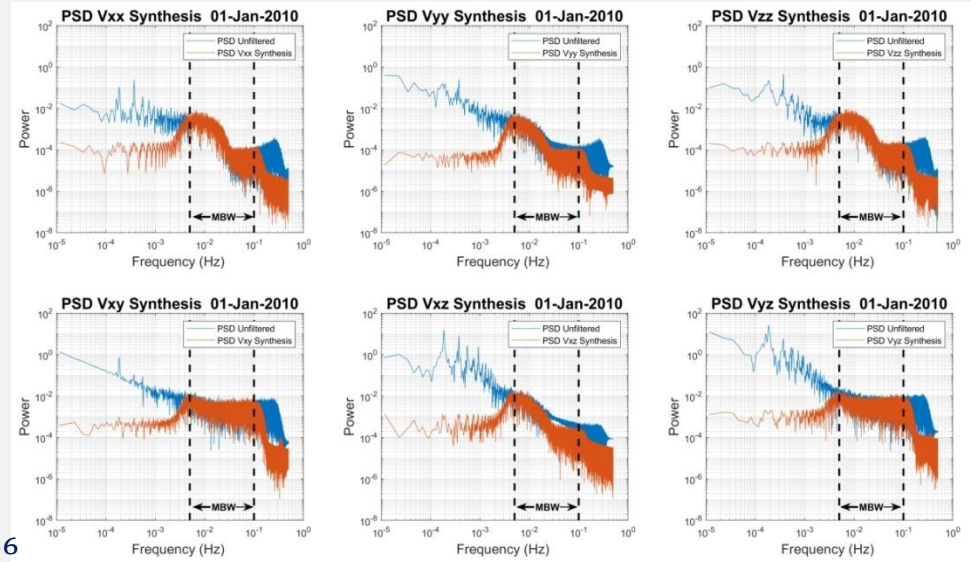


Reconstructed monthly gravity gradients (levels 4 to 7)

WL selective reconstruction GOCE enhanced* data (one day)



Reconstruction using the detail coefficients (levels 4-7) for one day data

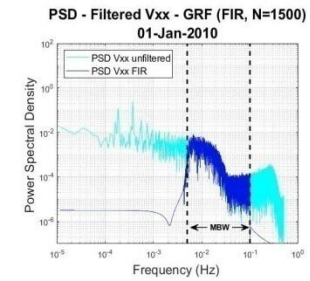


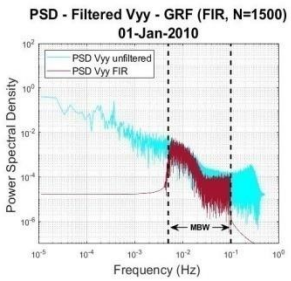
* The final reduced data used in the GeoGravGOCE project as a result of a GGM validation process:

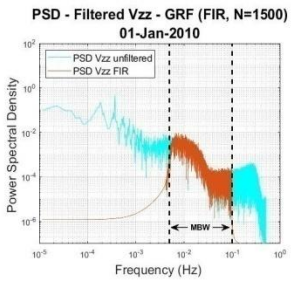
$$V_{ij}^{enhanced} = \mathrm{V}_{\mathrm{ij}}^{GOCE} - \mathrm{V}_{\mathrm{ij}}^{TIM\ R6_{n=2}^{195}} - \mathrm{V}_{\mathrm{ij}}^{EGM08_{\mathrm{n=166}}^{2190}}$$

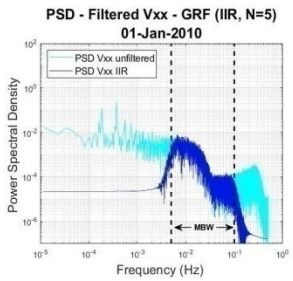
l4..l7 (Eötvös) - 01 January 2010						
	Vxx	**Vyy**	**Vzz**	**Vxy**	**Vxz**	**Vyz**
Min	-0.734	-0.287	-0.742	-1.020	-1.121	-1.847
Max	0.555	0.312	0.815	1.242	1.766	1.760
Mean	0.000	0.000	0.000	0.000	0.000	0.000
Std	0.065	0.040	0.078	0.215	0.141	0.318
Rms	0.065	0.040	0.078	0.215	0.141	0.318

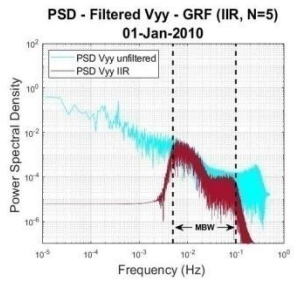
Comparison of the PSDs of the three methods

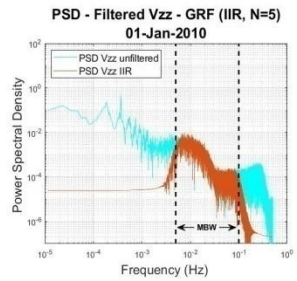


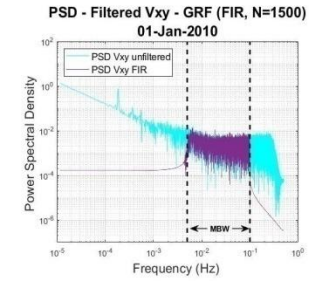


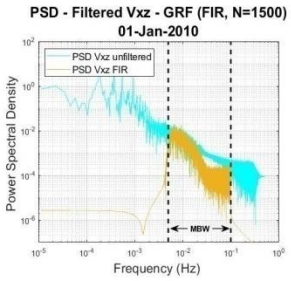


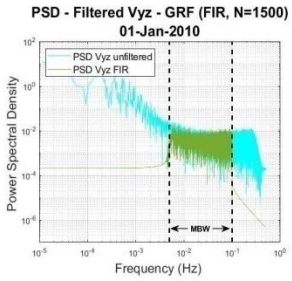


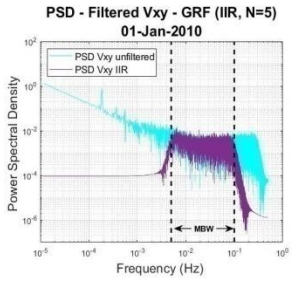


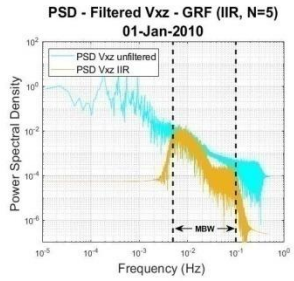


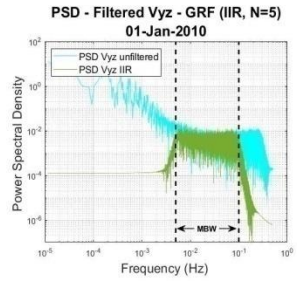








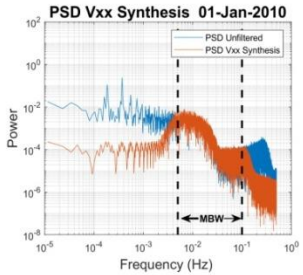


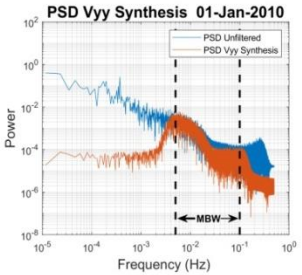


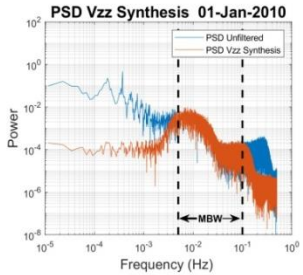
FIR(N=1500)

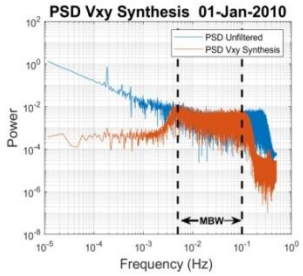
IIR (N=5)

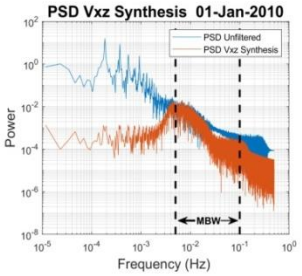
Wavelet MRA
(l4..l7)

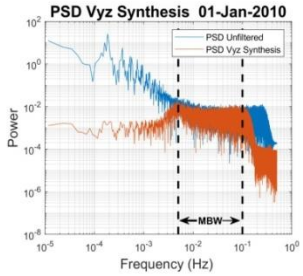












Conclusions

- FIR filter with N=1500 is selected to be the optimal one among all presented methods. It is thus applied for GOCE SGG data for the entire mission duration. It shows ideal filtering results in the enhanced gravity gradients both in terms of the statistics and spectrally.

- The filtering experiments of the reduced gravity gradients with the FIR and IIR filters, showed that the FIR filter with order N=1500 and the IIR filter with order N=5, respond optimally to the filtering requirements.

- In the Wavelet MRA, the determination of the bandwidths of each level of decomposition gives a significant advantage in the filtering procedure. A selective reconstruction using only the detail coefficients of levels 4-7, represented mainly signal inside the MBW.

Acknowledgements

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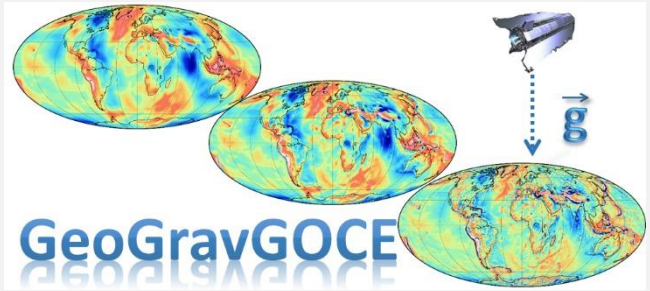

gRAVLAB