

Frequency Dependent Mantle Viscoelasticity via the Complex Viscosity: cases from Antarctica and Western North America

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Rick O'Connell
(1941-2015)

MEASURES OF DISSIPATION IN VISCOELASTIC MEDIA

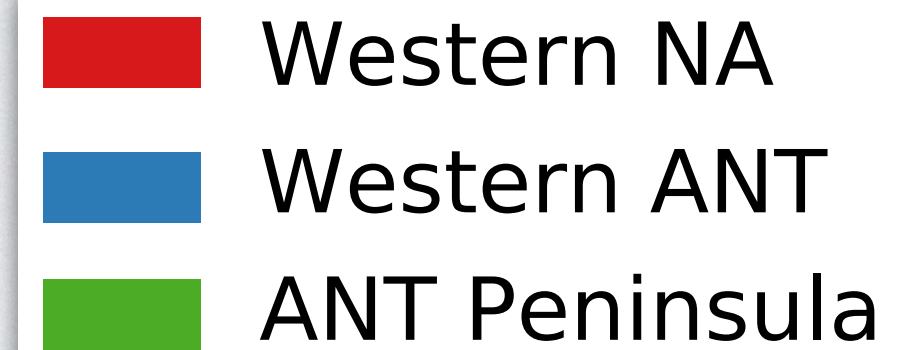
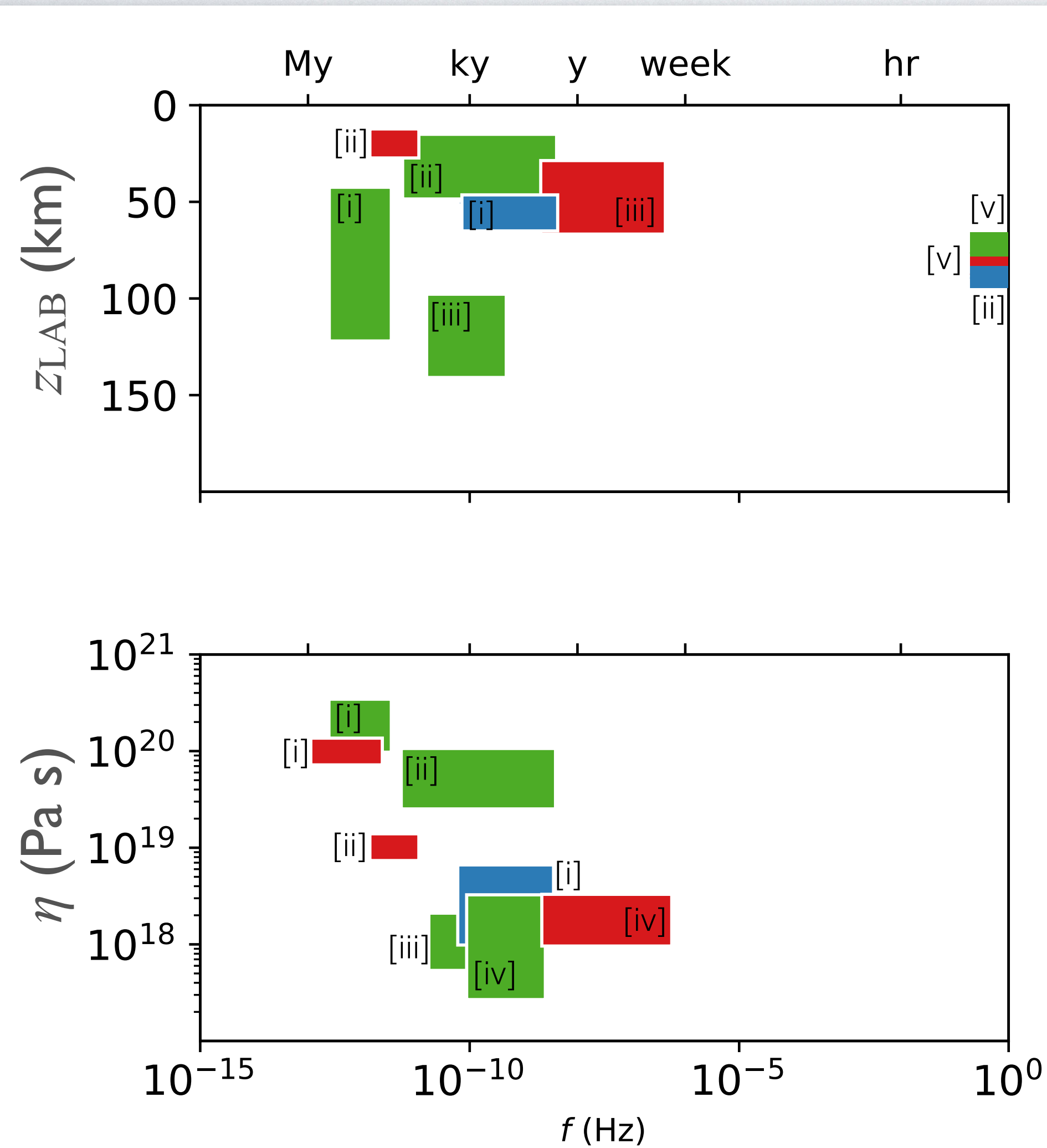
R. J. O'Connell and B. Budiansky

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Introduction

The intrinsic dissipative behavior of viscoelastic materials can have various phenomenological consequences, such as the attenuation in time of free vibrations or the spatial attenuation of driven plane waves. It has become commonplace to use the "quality" parameter Q , borrowed from electric circuit theory, to denote a non-dimensional measure of dissipation in terms of some ratio of stored to dissipated energy. Unfortunately, the literature suffers from a lack of uniformity concerning the precise definition of Q , and there is occasional confusion between some ill-defined Q of a process (such as modal decay) and an intrinsic Q of the material. In this note we propose as a standard definition for the Q of a viscoelastic material one that is simple and unambiguous, and indicate its connection with the phenomenological features of several dissipative processes.

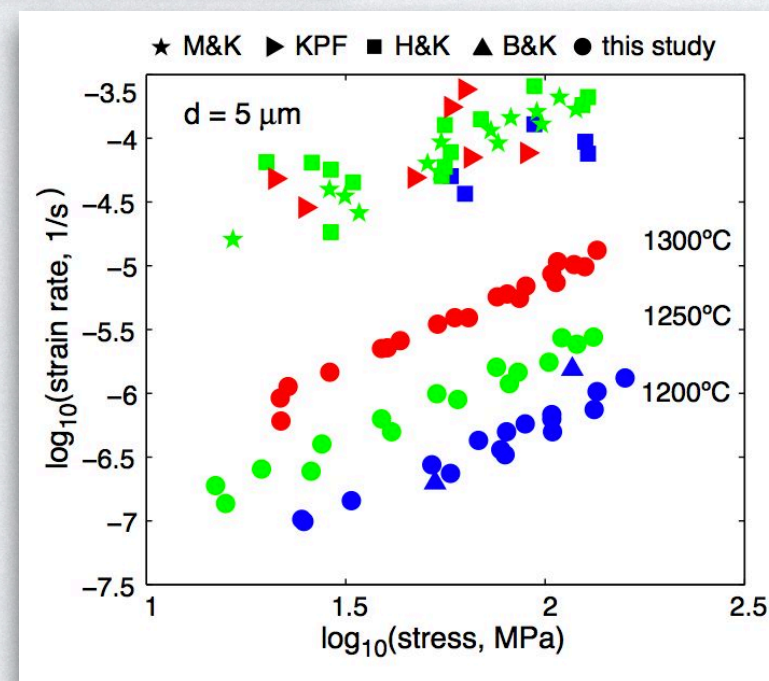
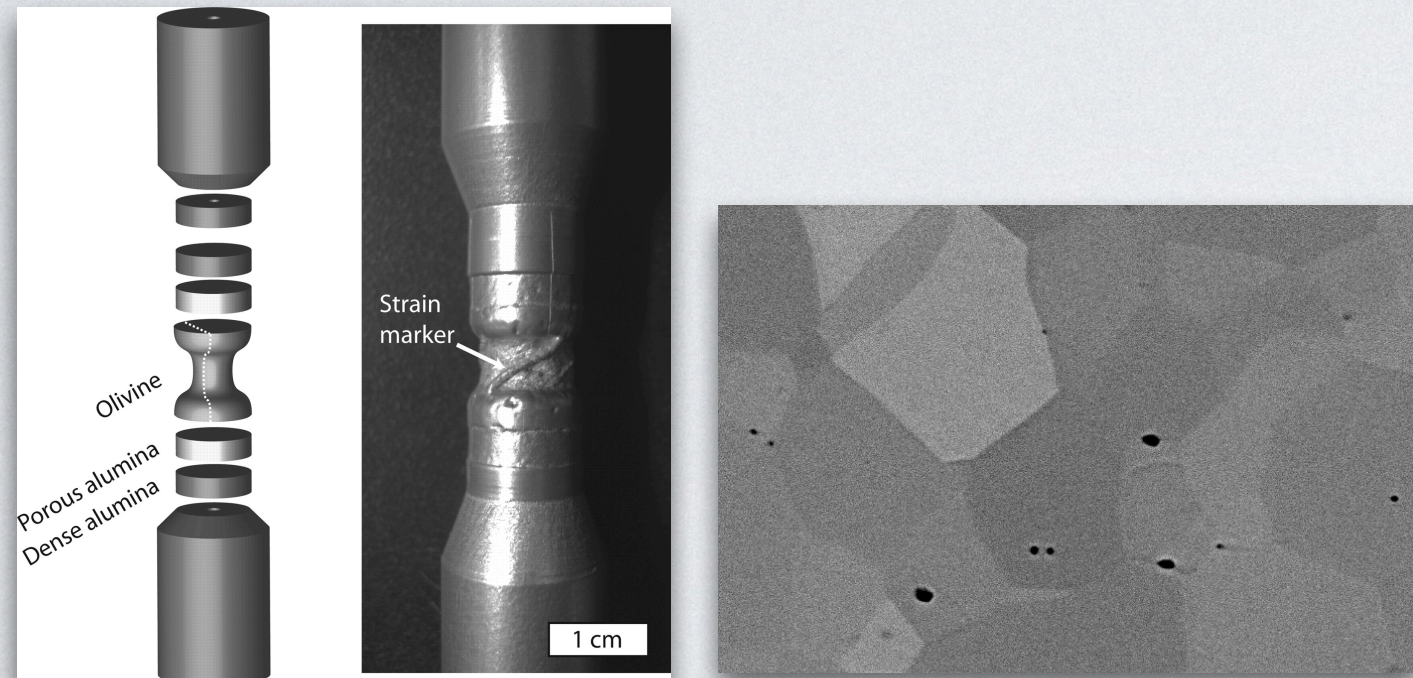
Estimates of Viscosity



GIA-based estimates
grouped into geographic area
all estimates of asthenospheric viscosity

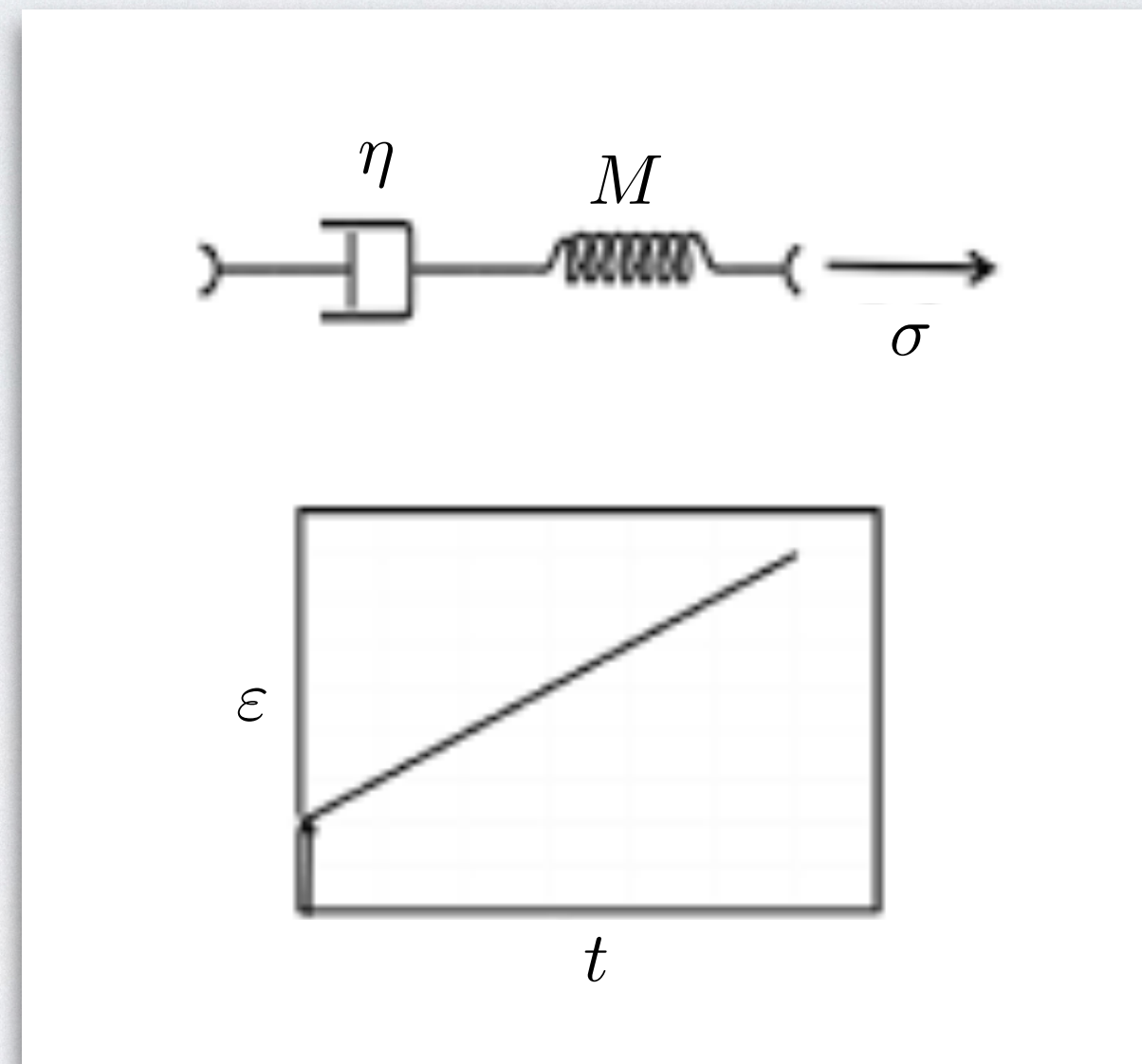
discrepancy in estimates remain!

Parameterizing Dissipation



Faul & Jackson (2007)

Grain-scale



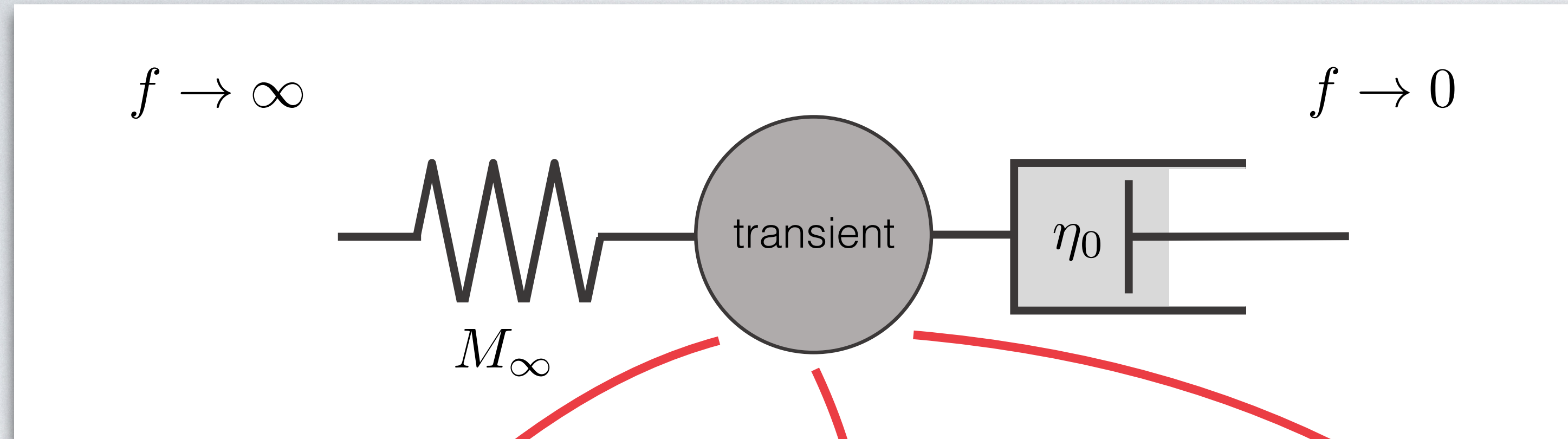
*Phenomenological
Model*

$$\begin{matrix} M^*(\omega) \\ Q^{-1}(\omega) \end{matrix}$$

results frequency
dependent wave-
speeds
 $v(\omega)$

*Frequency dependent
moduli and
attenuation*

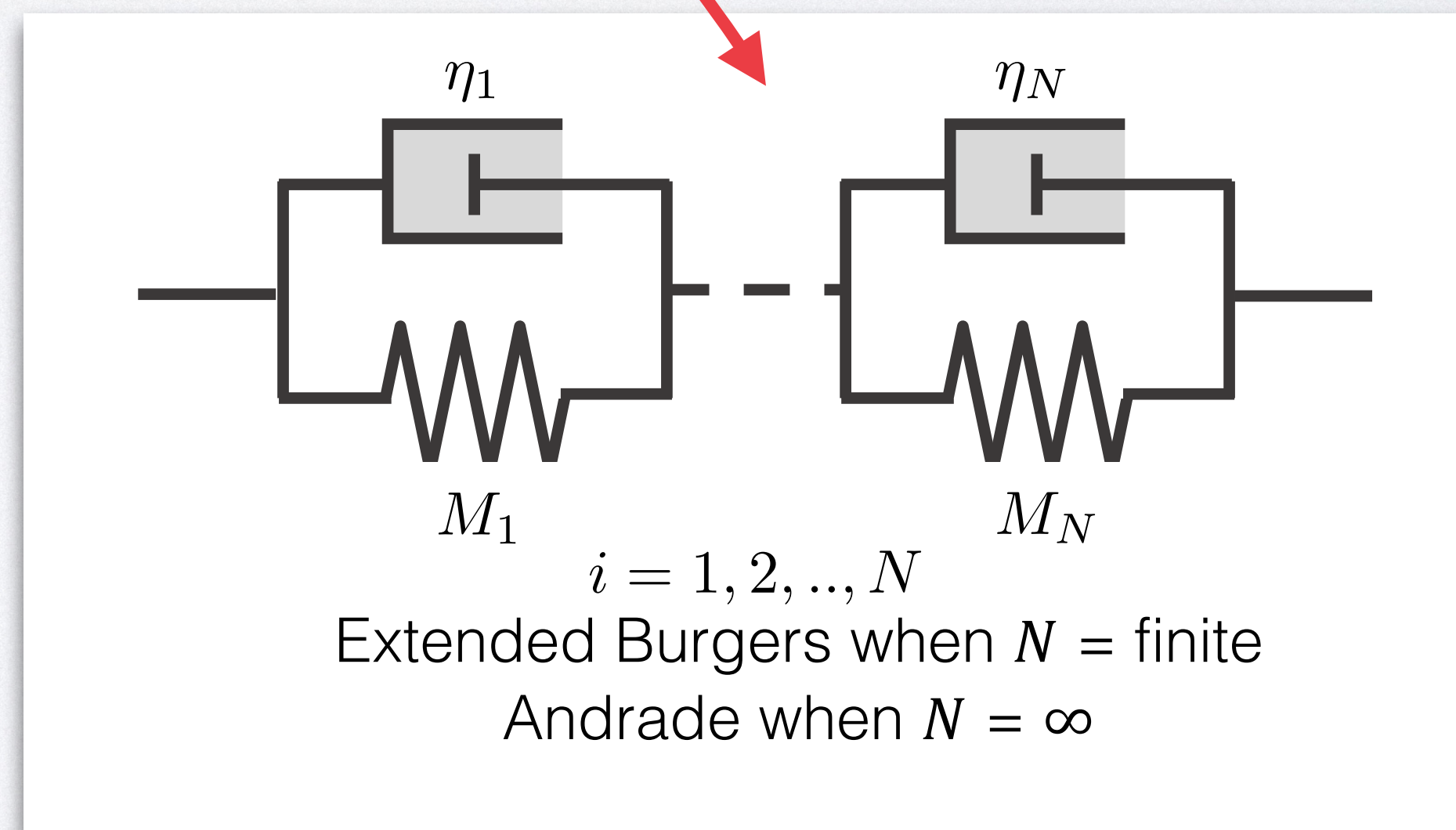
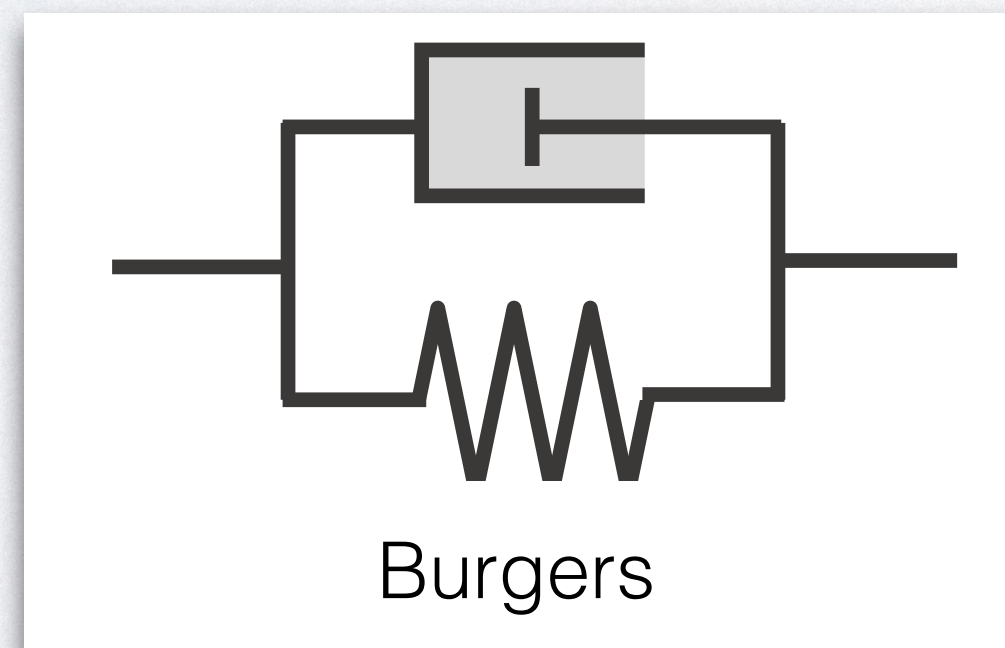
Grain-scale to Phenomenological Models



With **any** phenomenological model we can derive the modulus, $M^*(\omega)$, a continuous complex function in frequency.

Other frequency dependent parameters are determined from this: $Q(\omega)$, $\nu_s(\omega)$

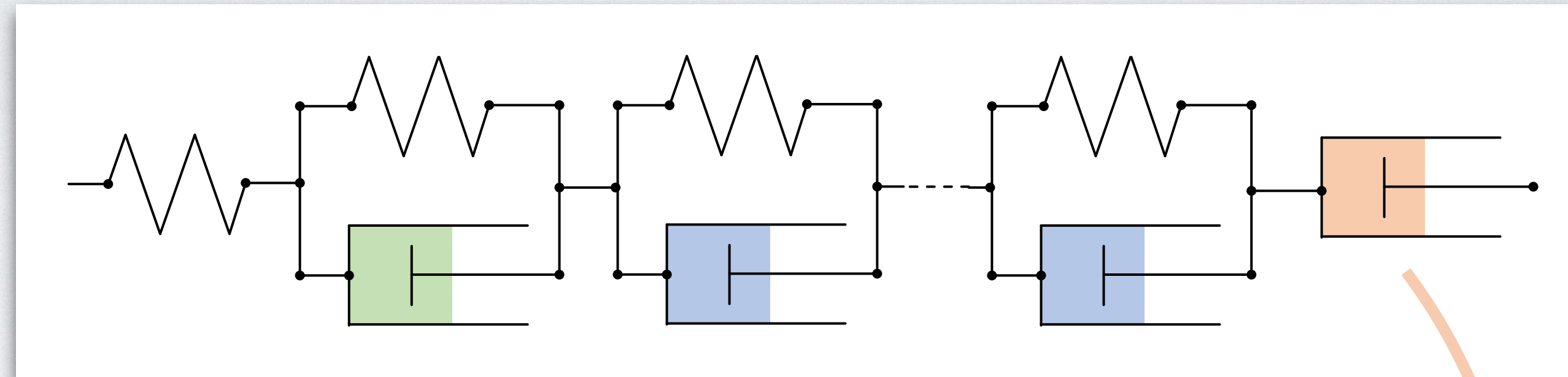
Maxwell
(no transient)



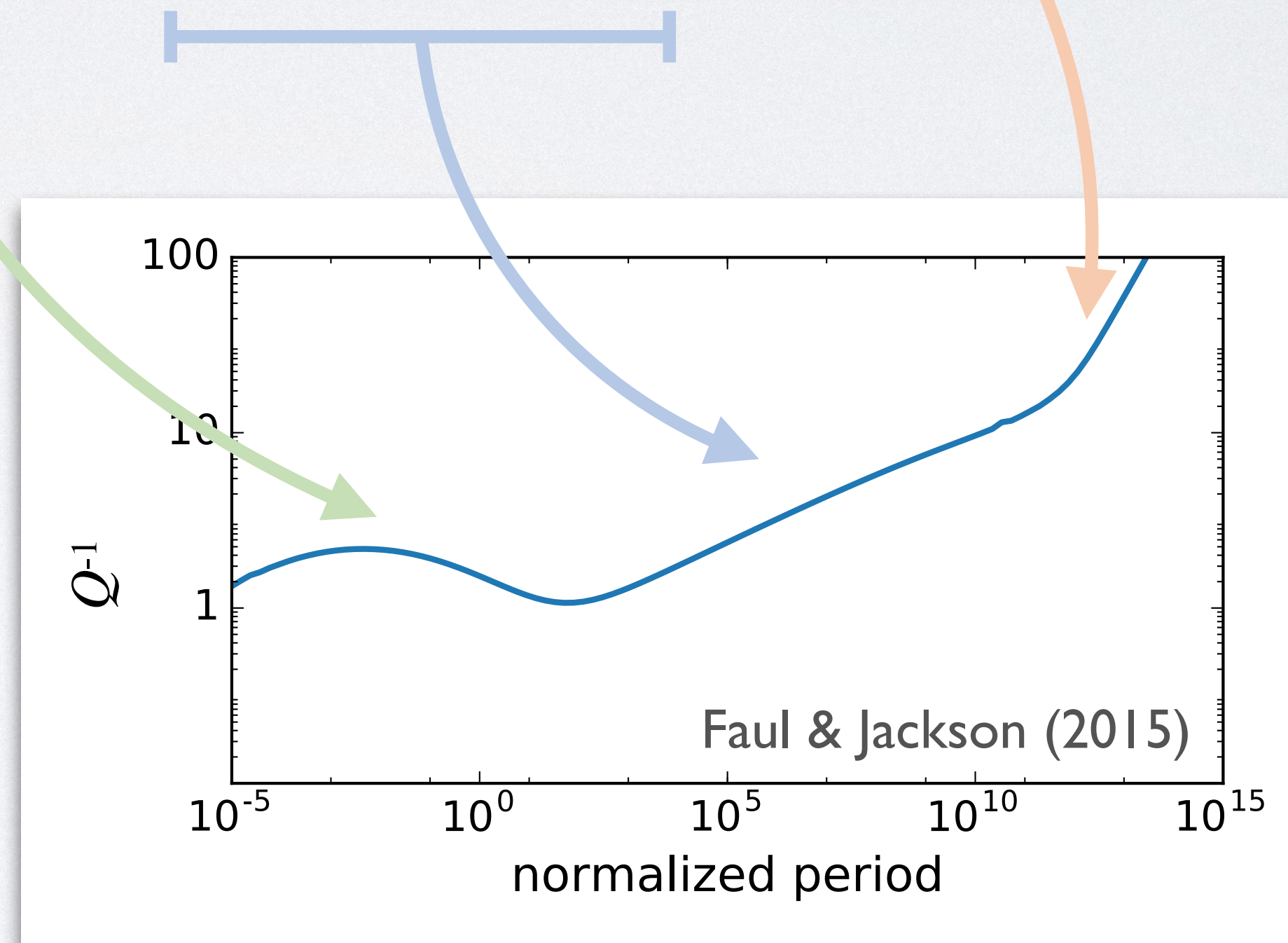
Phenomenological Models to Measures of Dissipation

“Extended Burgers” Viscoelastic Model

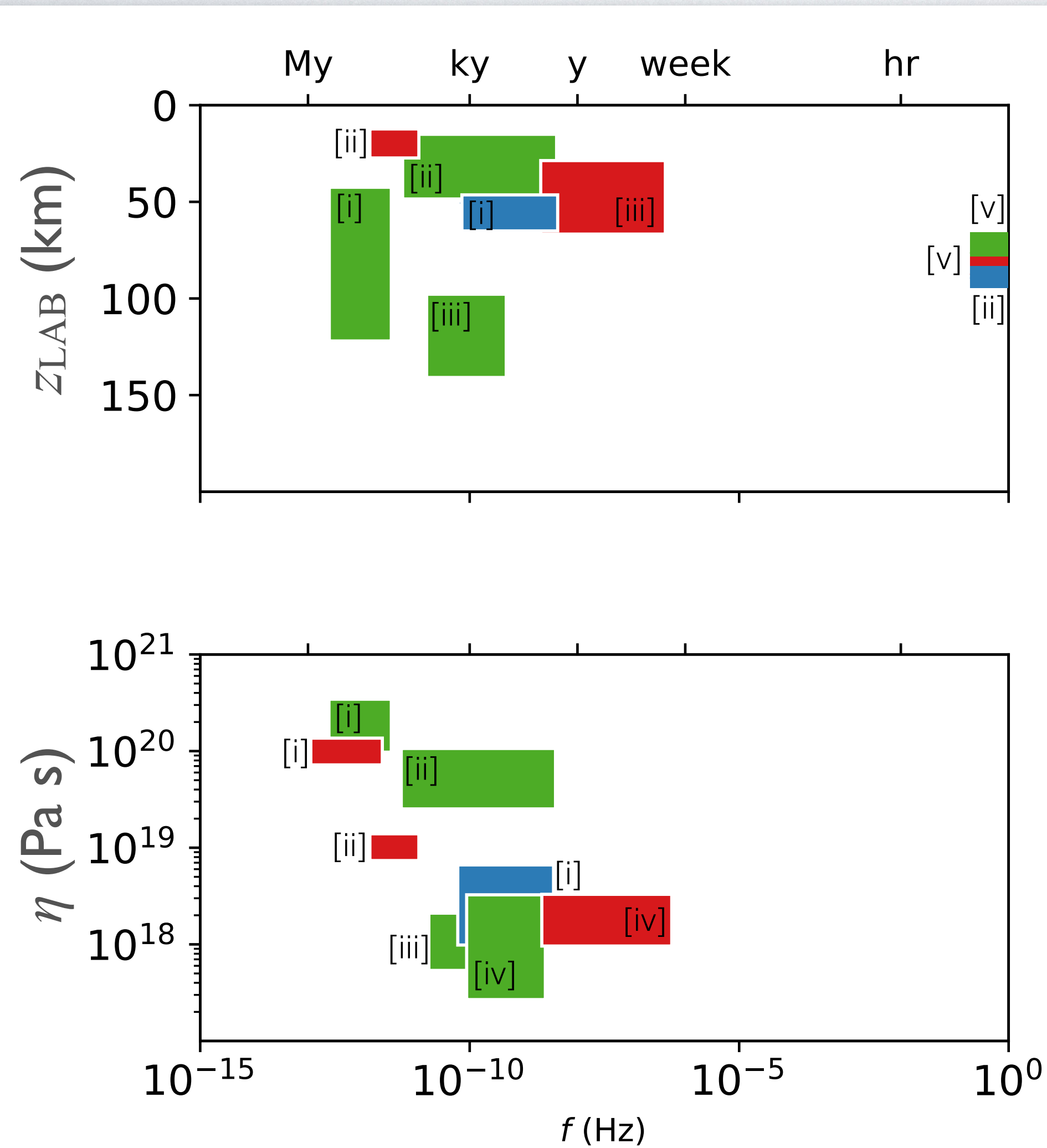
$\omega \rightarrow \infty$



steady state
viscous
behavior



Estimates of “Viscosity”

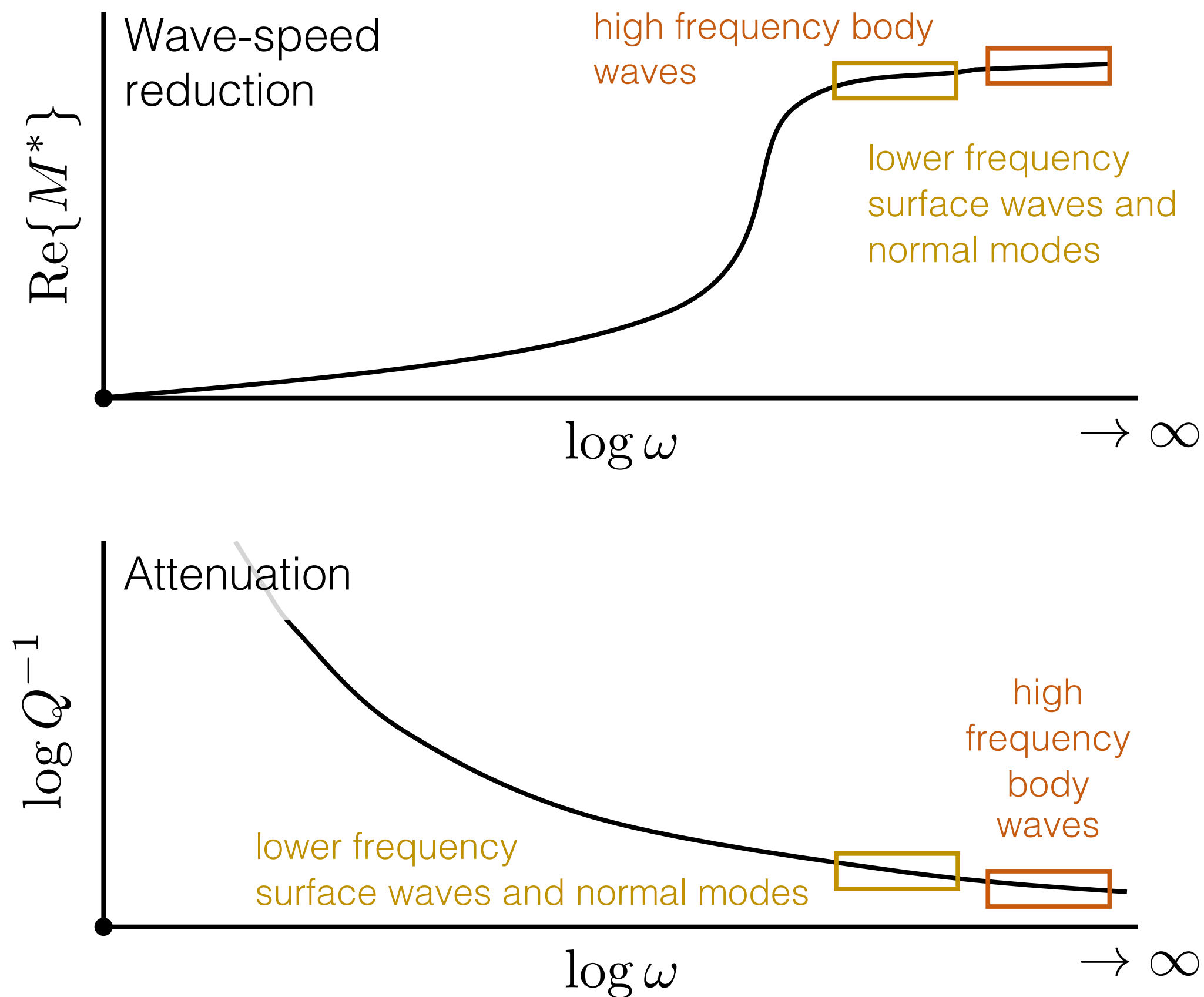


■ Western NA
■ Western ANT
■ ANT Peninsula

all assumed a Maxwell rheology
so, η estimates here are really η_0

*Does it make sense to have all these values
assigned to a single dashpot, η_0 ?
What does this mean when we relate micro- to
macro-scale deformation?*

Sampling frequency dependent parameters



analogue in seismology

wave-speeds reduce at low frequency

(Kanamori & Anderson, 1977)

attenuation increases at low frequency

(Lekic et al., 2009; Lau & Faul, 2018)

data sample the underlying frequency dependent parameters

Apparent Viscosity

Complex viscosity, $\eta^*(\omega)$

$$\eta^*(\omega) \equiv -i \frac{M^*(\omega)}{\omega}$$

Just as $M^*(\omega)$
(a complex function that
smoothly varies in frequency)
may be derived for **any**
viscoelastic model
(Maxwell, Andrade, etc.),
 $\eta^*(\omega)$ can be.

Apparent Viscosity

Complex viscosity, $\eta^*(\omega)$

$$\eta^*(\omega) \equiv -i \frac{M^*(\omega)}{\omega}$$

We propose that
estimates we see
probe the
apparent viscosity

$$\tilde{\eta}(\omega) = \|\eta^*(\omega)\|$$

a measure of the
degree of viscous
dissipation

Apparent Plate Thickness

Maxwell Timescale, τ_M

$$\tau_M \equiv \frac{\eta_0}{M_\infty}$$

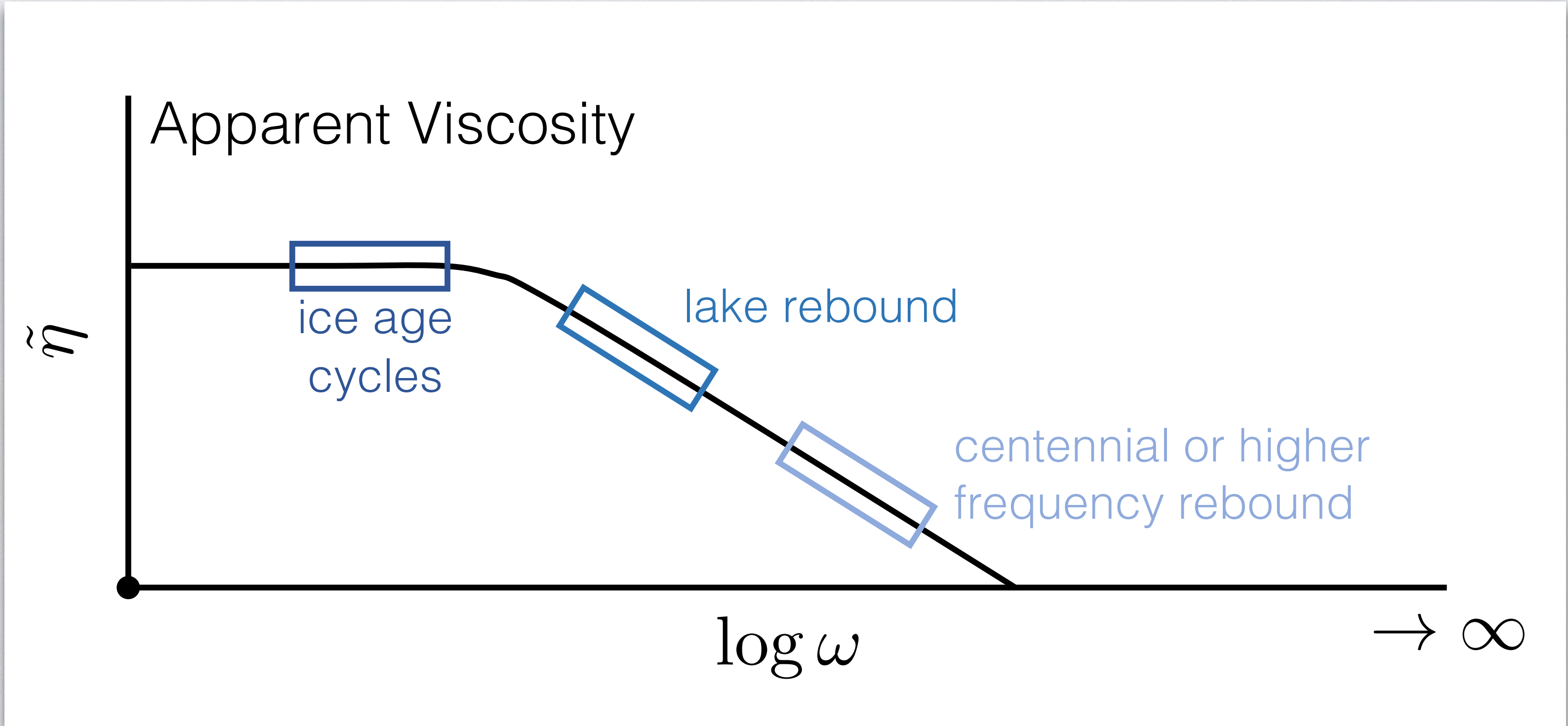
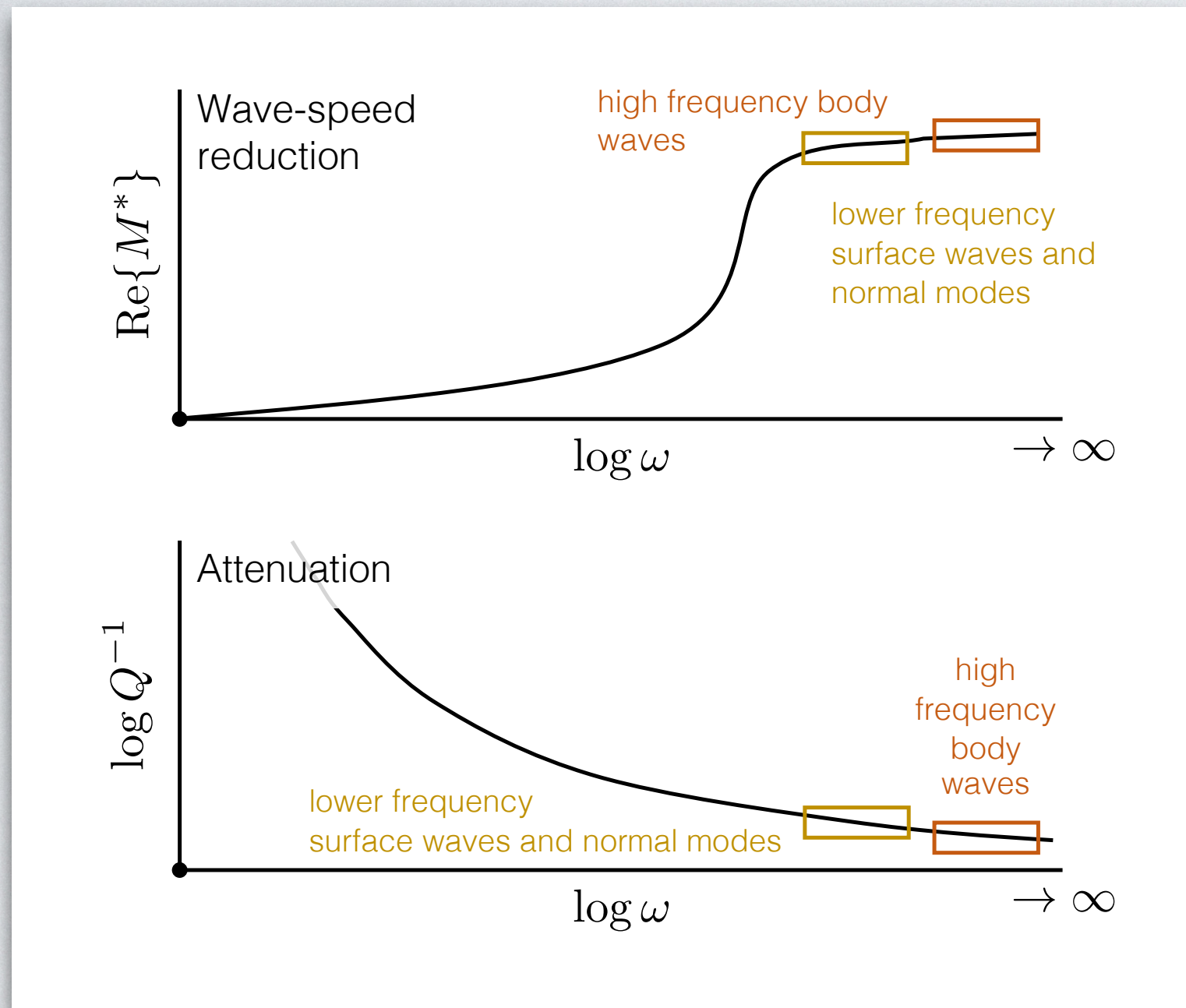
$$\tau_M(z) \equiv \frac{\eta_0(z)}{M_\infty(z)}$$

We propose a
frequency dependent
measure
apparent LAB depth

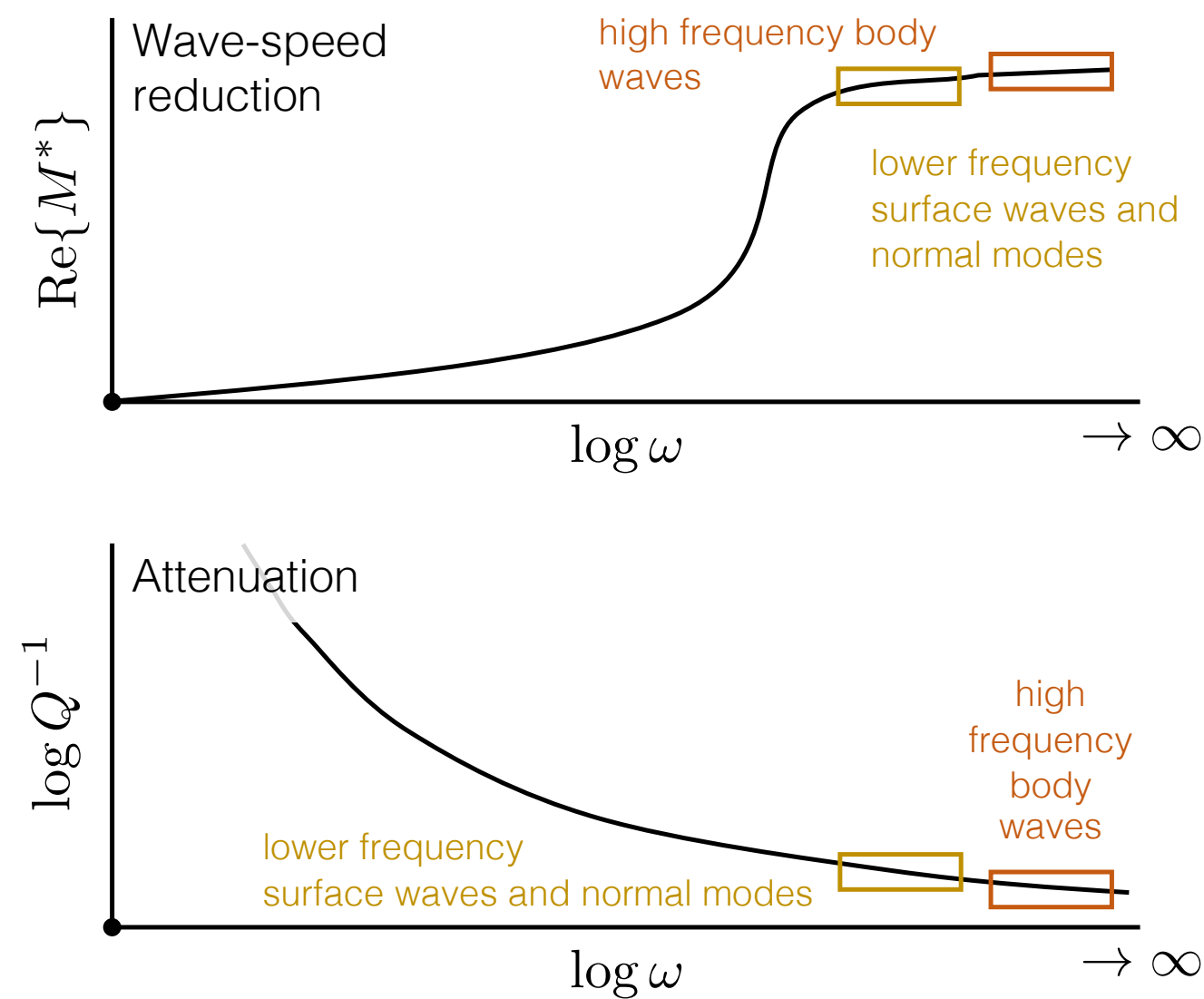
$$\tilde{z}_{\text{LAB}}(\omega) \equiv z|_{\tau_M=\omega^{-1}}$$

at any given timescale, the
depth at which the elastic
to viscous transition
occurs

Sample *Apparent Viscosity* and *Apparent Lithospheric Thickness*



Sample *Apparent Viscosity* and *Apparent Lithospheric Thickness*



Question

If we account for the frequency dependence of viscous dissipation, can we map this frequency dependence?

Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

Determining
thermodynamic
conditions of subsurface
at a given locations

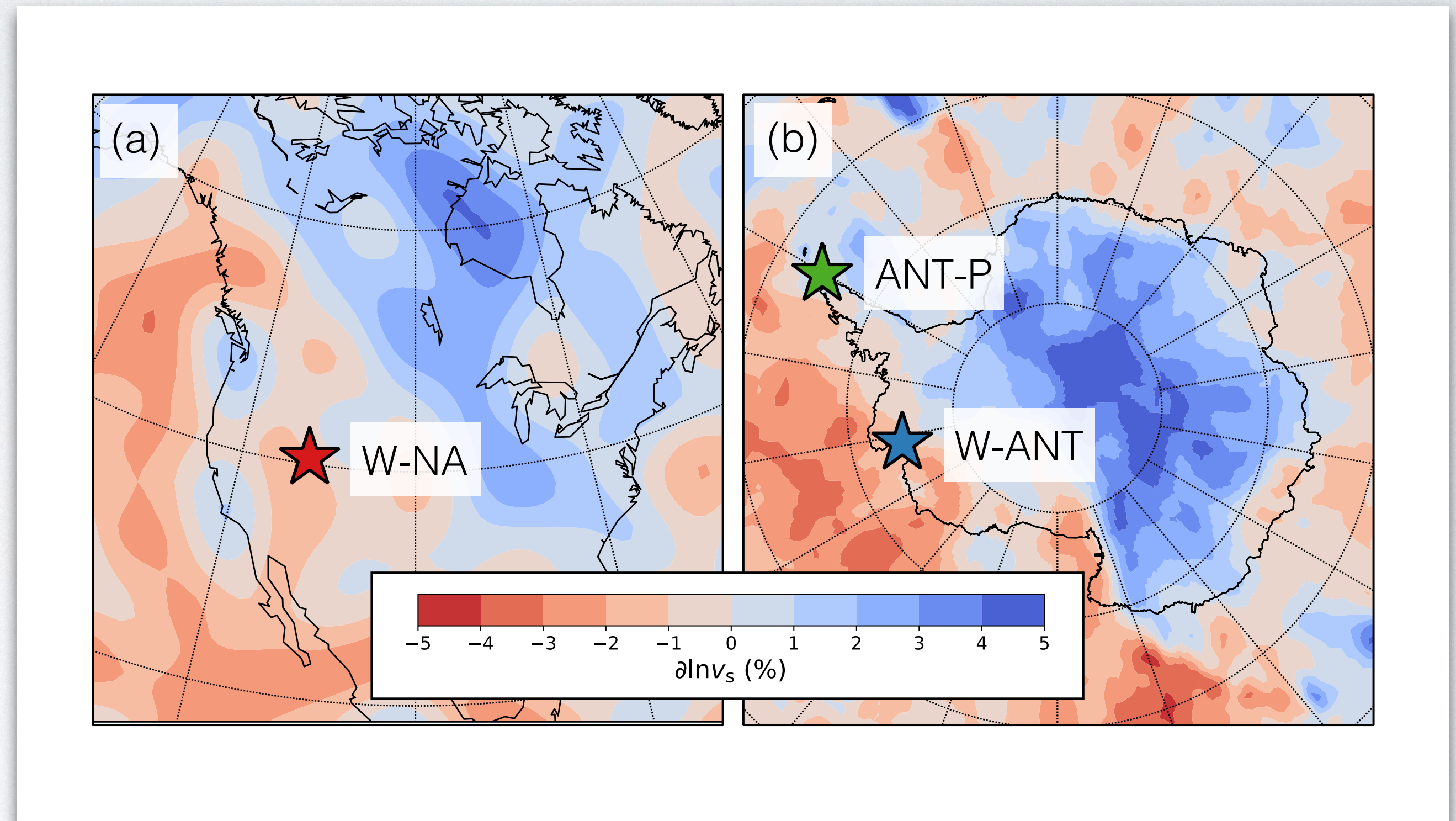
Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

Determining
thermodynamic
conditions of subsurface
at a given locations

Use tomography models from
each location

W. USA: Shen & Ritzwoller (2016)
Antarctica: Lloyd et al. (2019)

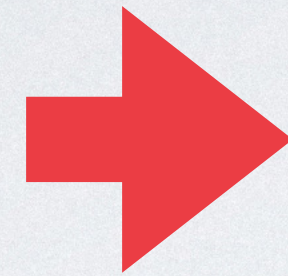
Use constitutive laws to map
mechanical (ν_s) observations
(at seismic timescale) to
temperature, pressure



W-NA: Western North America
W-ANT: Western Antarctica
ANT-P: Antarctic Peninsula

Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

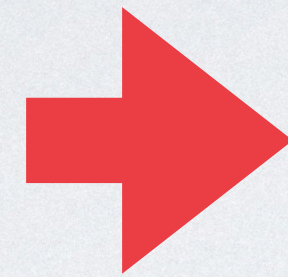
Determining
thermodynamic
conditions of subsurface
at a given locations



Using experimentally
determined constitutive
laws, predict $M^*(\omega)$ as a
function of depth

Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

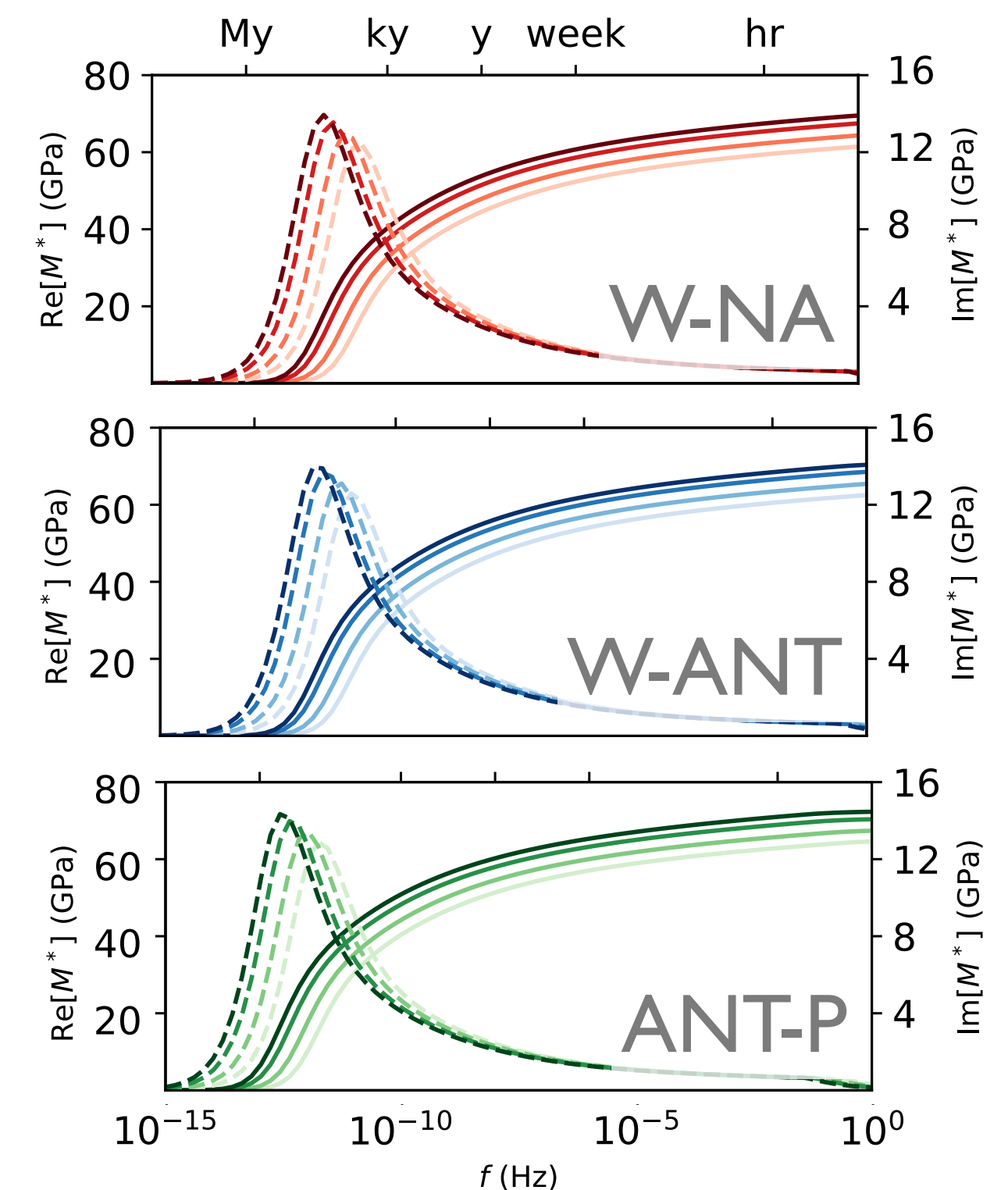
Determining
thermodynamic
conditions of subsurface
at a given locations



Using experimentally
determined constitutive
laws, predict $M^*(\omega)$ as a
function of depth

Self-consistently tie elastic,
anelastic, and viscous constitutive
laws to predict $M^*(\omega)$ across the
entire spectrum,

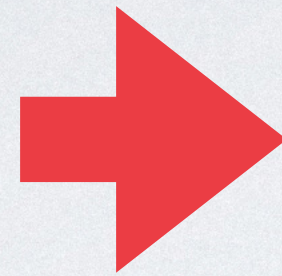
Elastic: Stixtude & Lithgow-Bertelloni (2005)
Anelastic: MacCarthy et al. (2011)
Viscous: Hirth & Kohlstedt (2004)



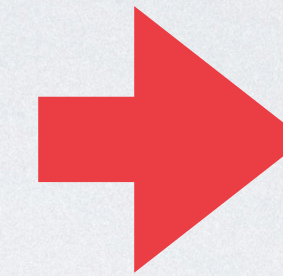
darker colors deeper
 $z = (50-350)$ km

Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

Determining
thermodynamic
conditions of subsurface
at a given locations



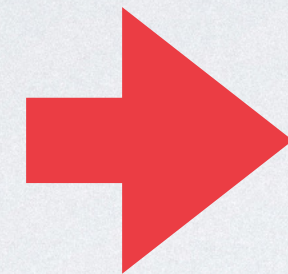
Using experimentally
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function of depth



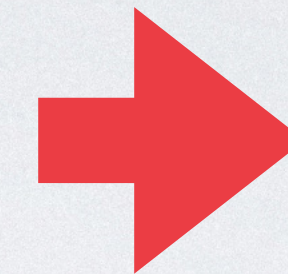
*Extract apparent
asthenospheric viscosity
and plate thickness for
each*

Predicting *Apparent Viscosity* and *Apparent Lithospheric Thickness*

Determining
thermodynamic
conditions of subsurface
at a given locations



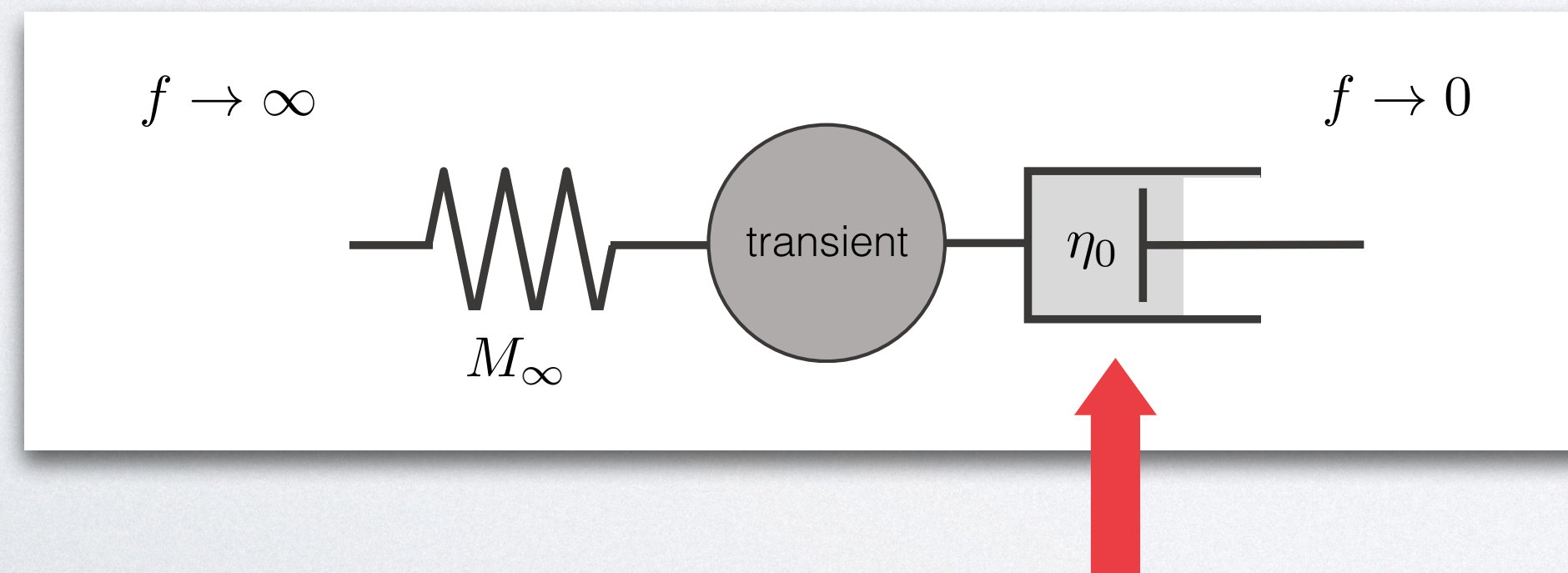
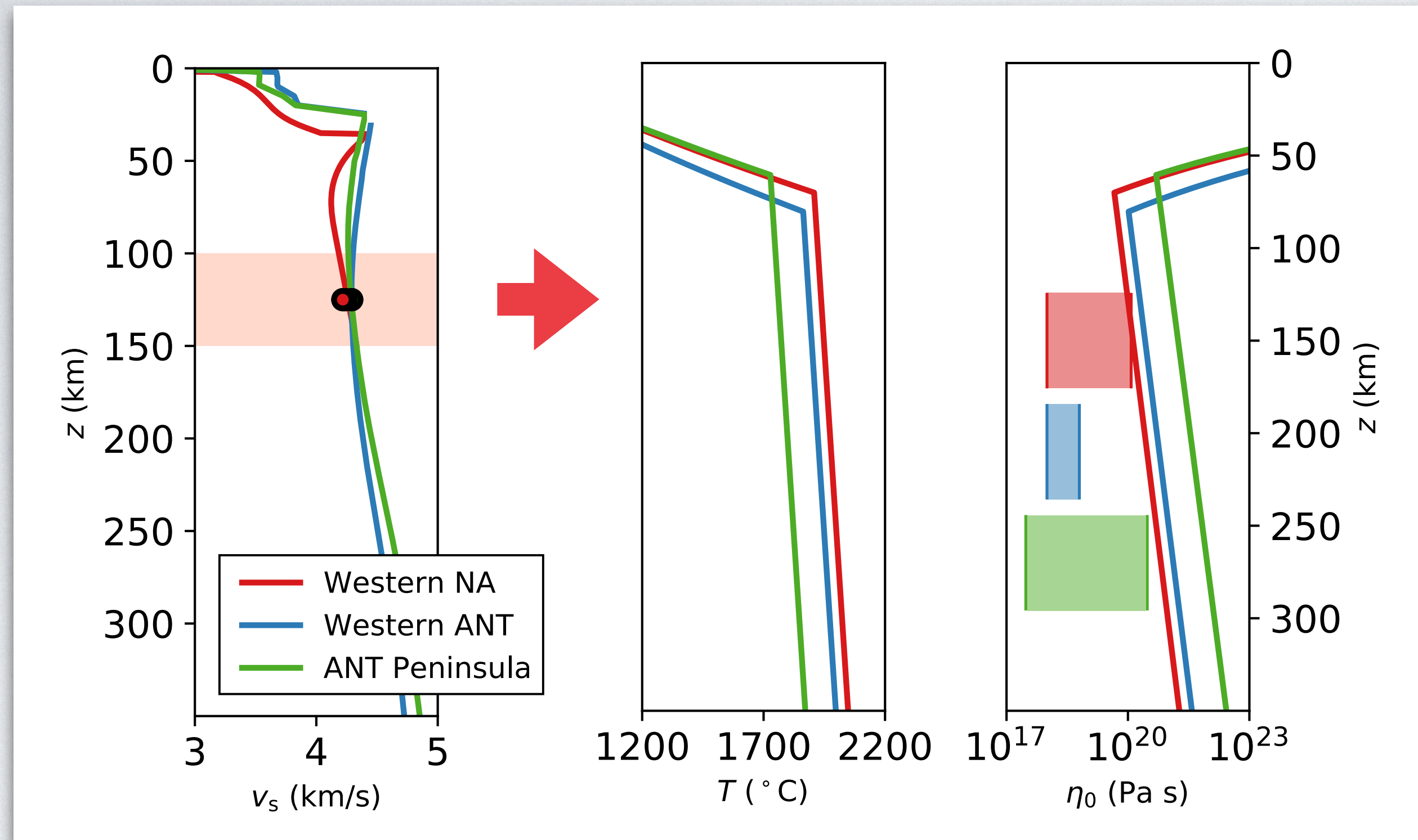
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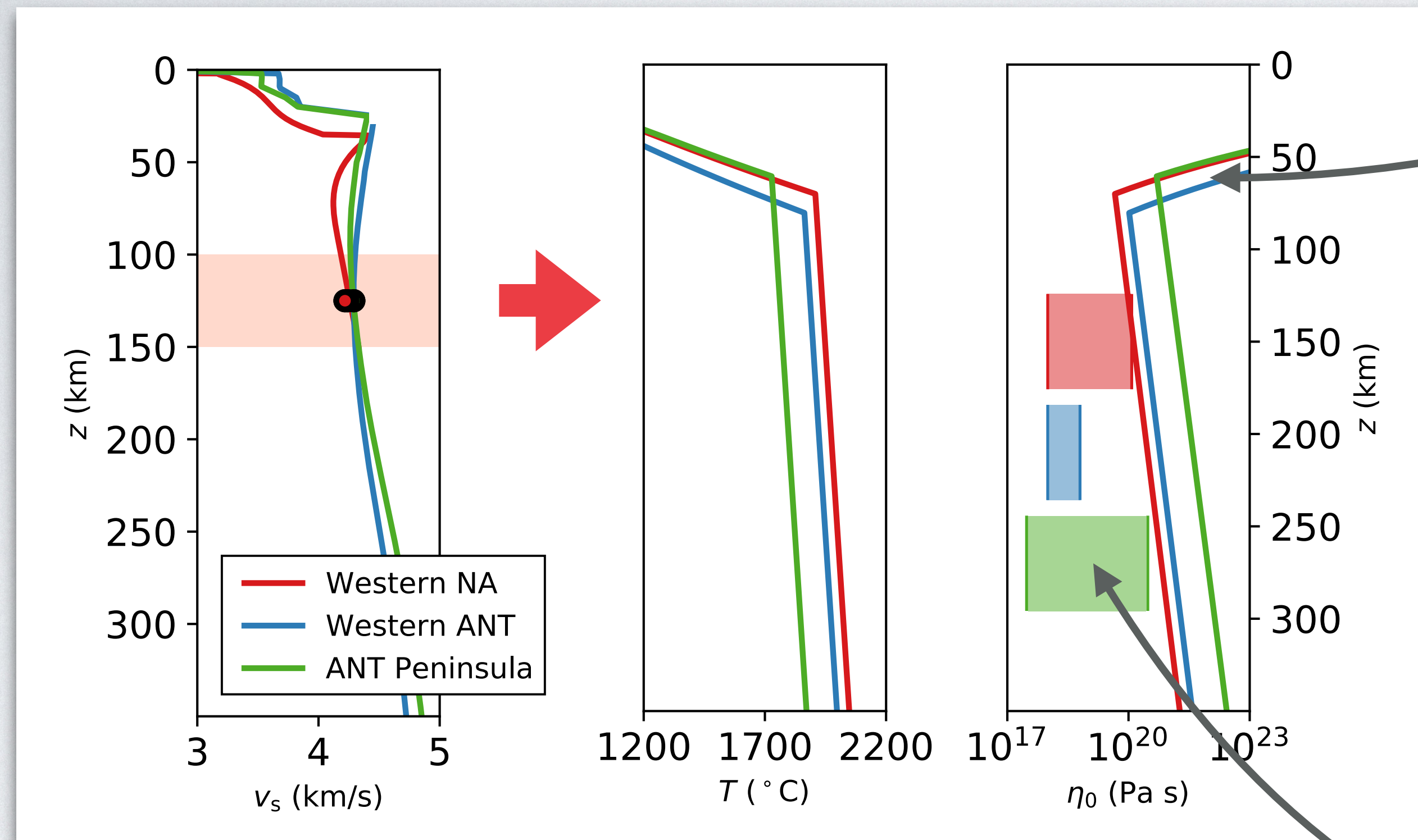
*Extract apparent
asthenospheric viscosity
and plate thickness for
each*

*For all conversions between
thermodynamic state to
mechanical parameters, we used
the open source software
package: the “Very Broadband
Rheology Calculator”
(Havlin et al., 2020)*

Results: *Steady State Viscosity*



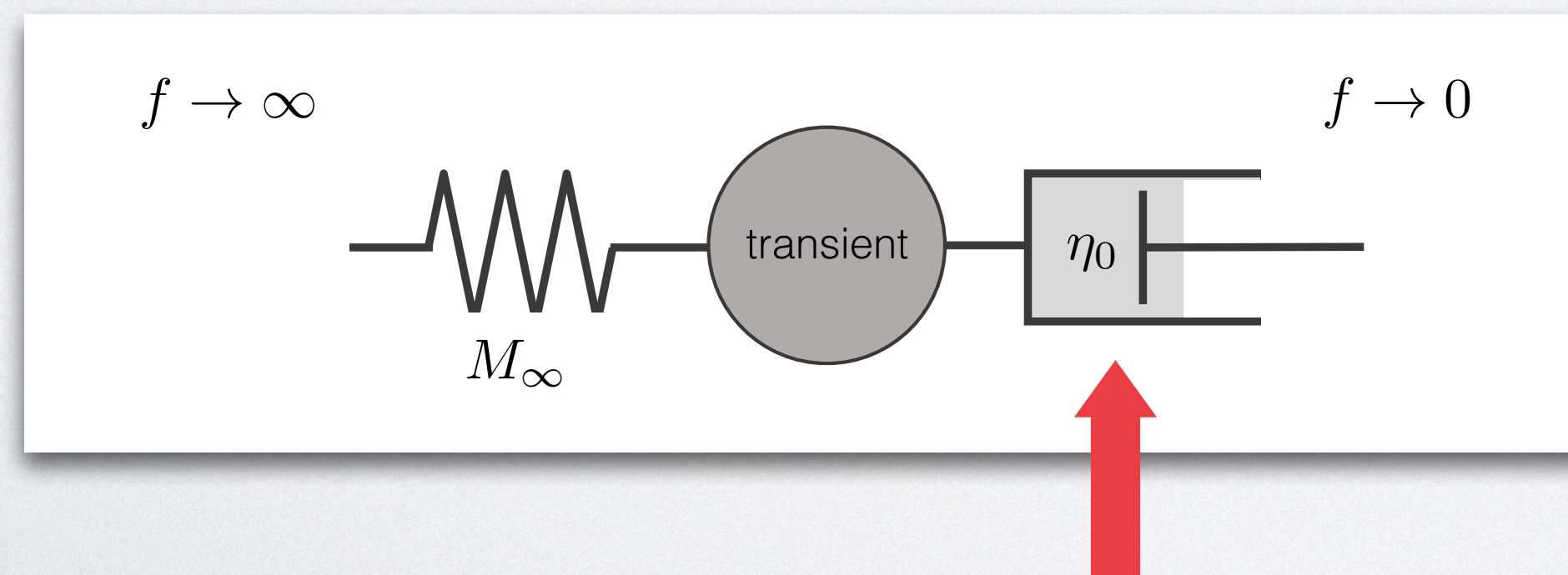
Results: *Steady State Viscosity*



synthetic
predictions

predicted η_0 values are too
large and their range over
depth does not cover the
range of observations.

observations

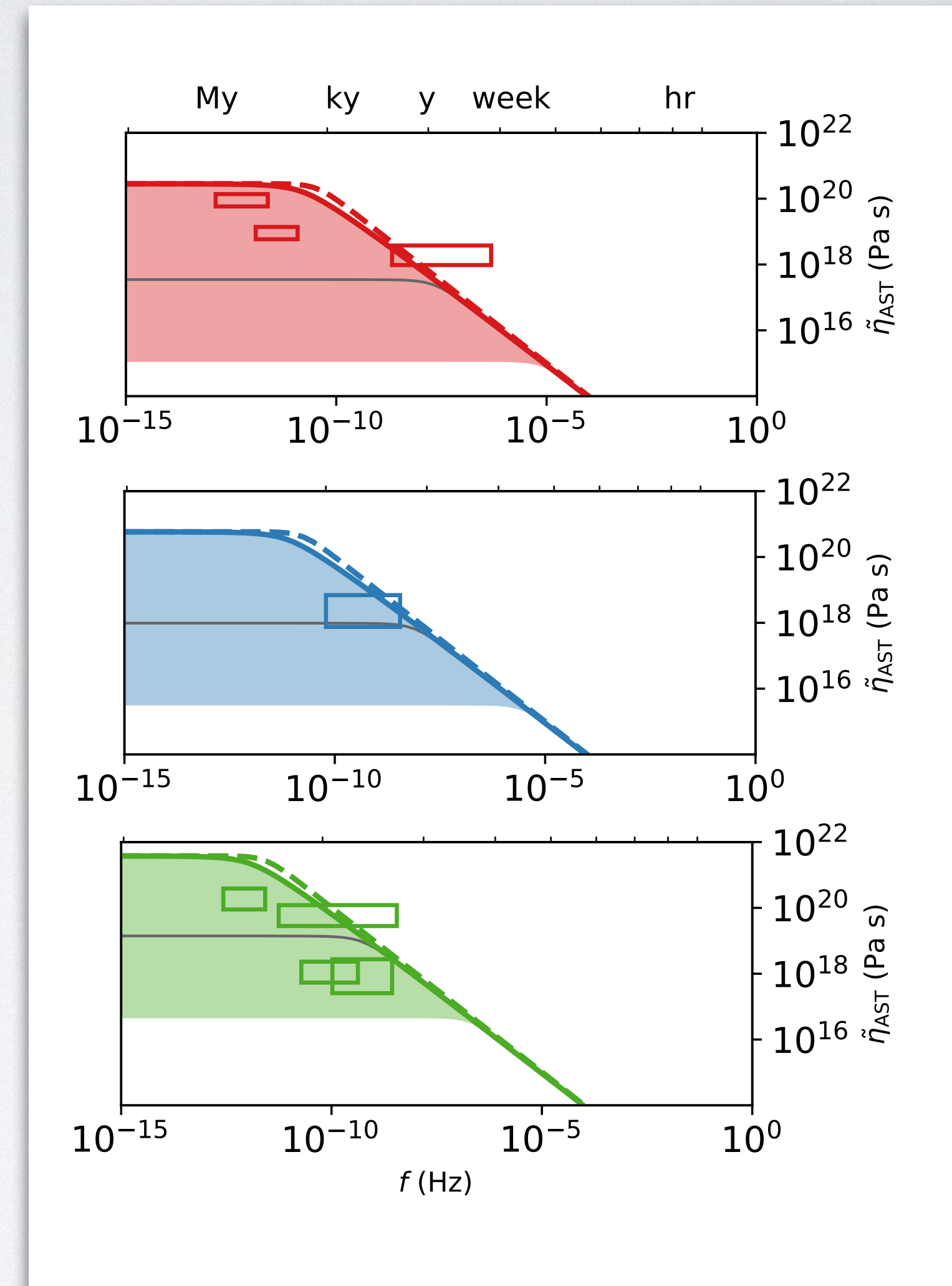


Results: *Apparent Viscosity*

Solid Colored line:
Apparent Asthenospheric Viscosity

Increases towards longer timescales
and plateaus to η_0 as $\omega \rightarrow 0$

At high frequency, meaningless as
elasticity takes over



Western
North
America

Western
Antarctica

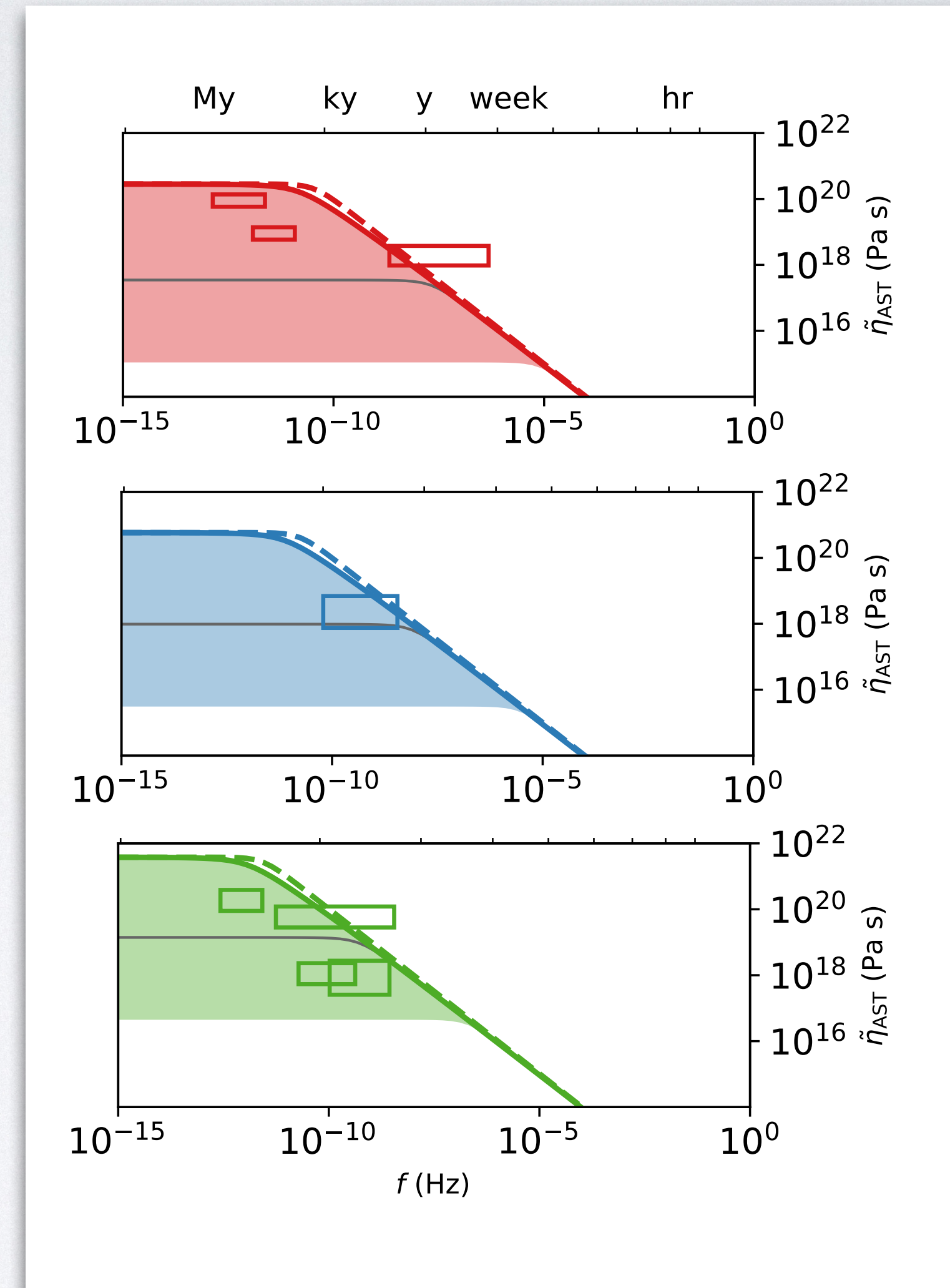
Antarctic
Peninsula

Results: *Apparent Viscosity*

Boxes:

Observations *placed at their frequency band*

Explains observations well when frequency band is well



Western
North
America

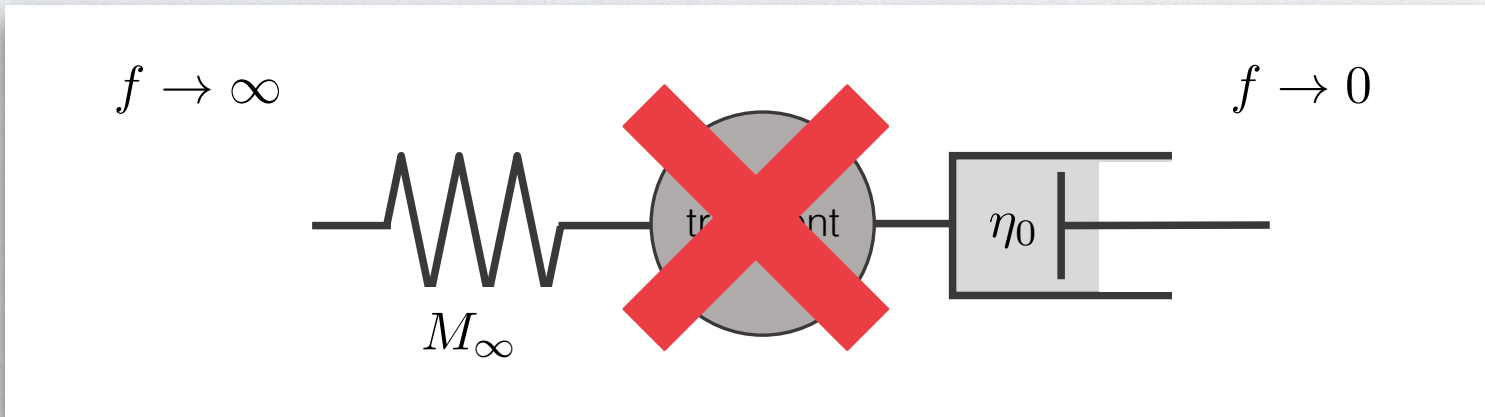
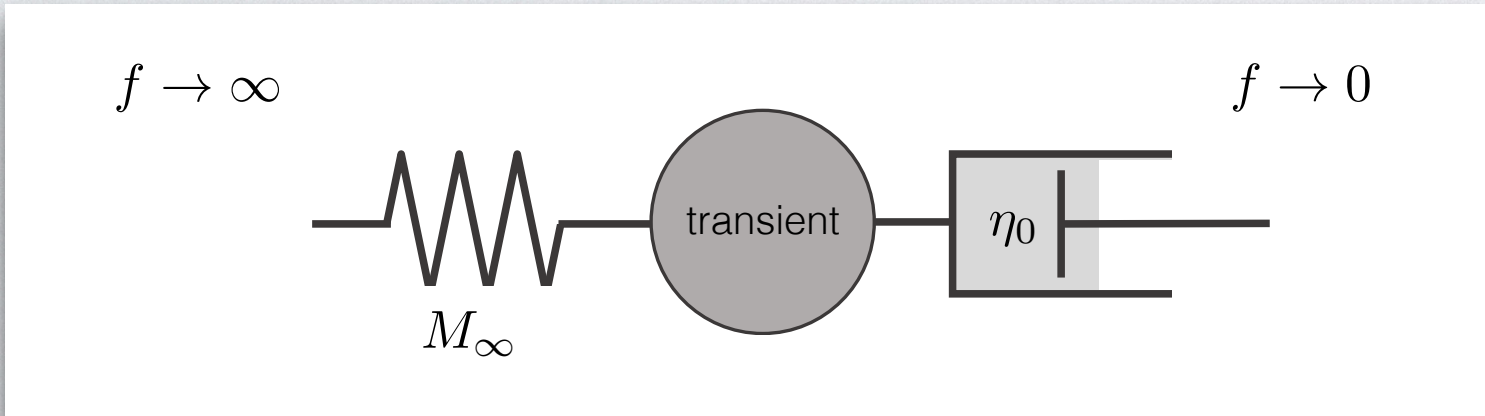
Western
Antarctica

Antarctic
Peninsula

Results: *Apparent Viscosity*

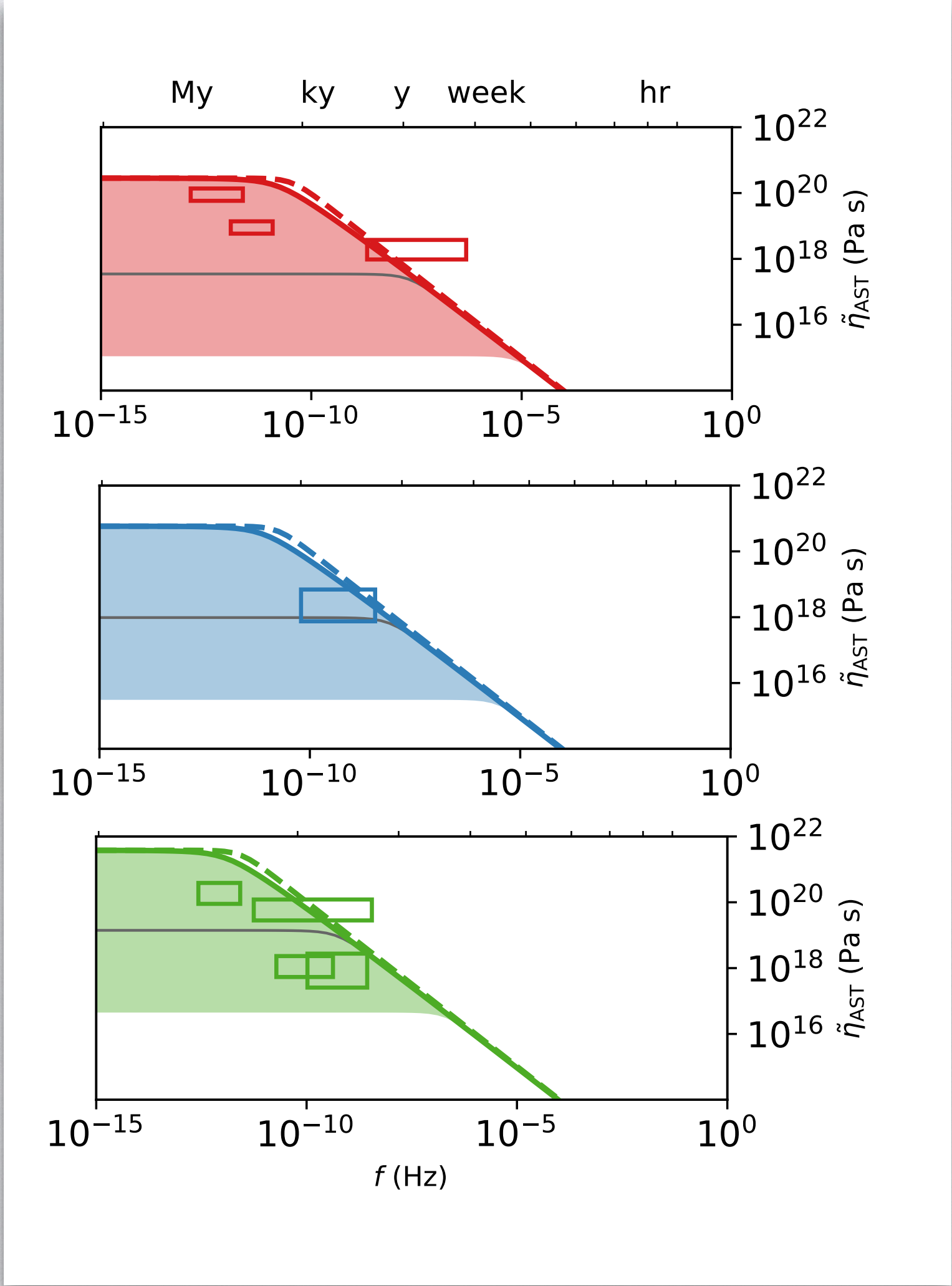
Dashed colored line:
Equivalent Maxwell Viscoelastic Model
Changes ~ 50% of Apparent Viscosity

Data not fine enough to distinguish



Solid colored lines

Dashed colored lines



Western
North
America

Western
Antarctica

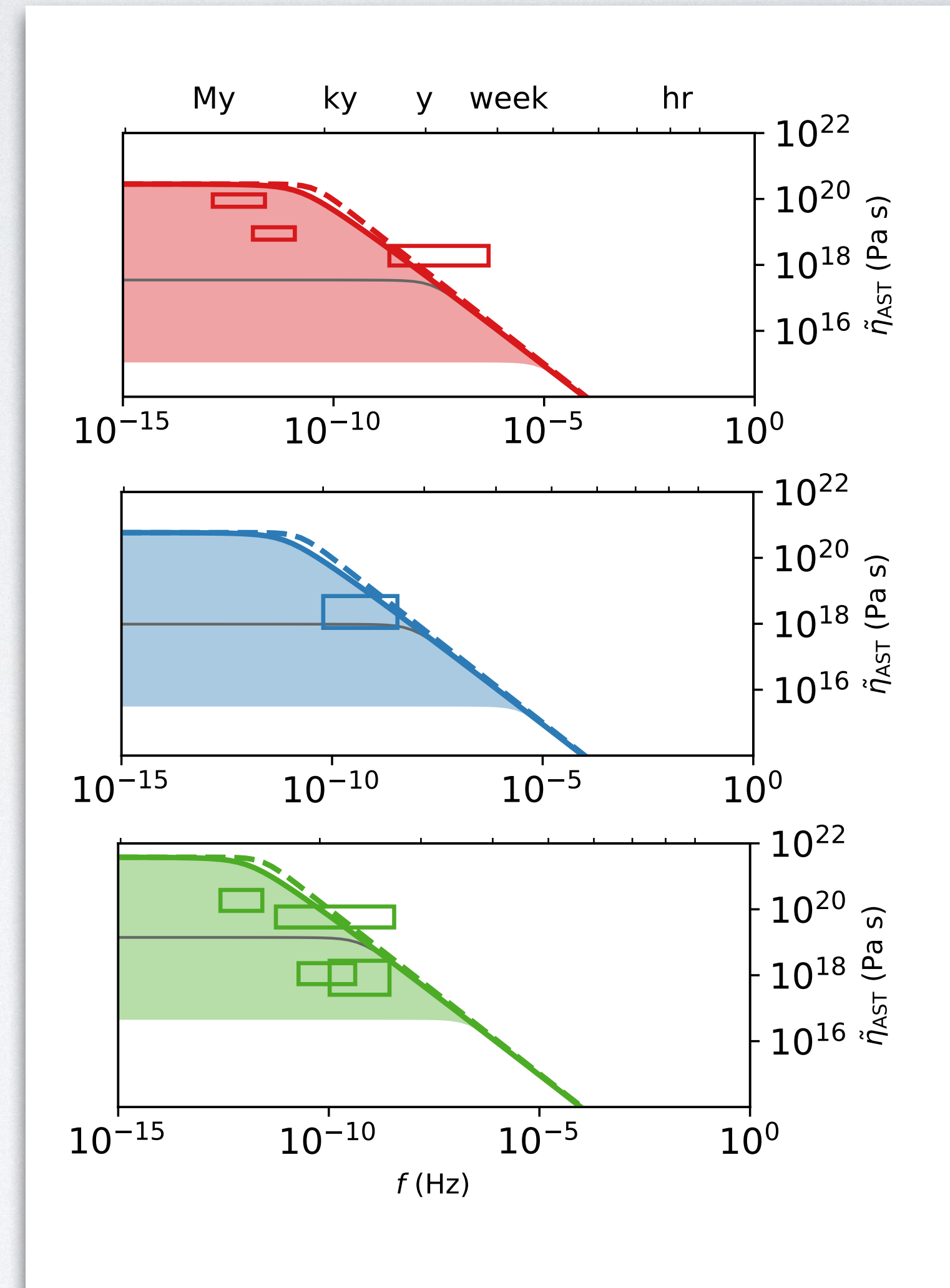
Antarctic
Peninsula

Results: *Apparent Viscosity*

Shaded Area:

Effect of steady state dislocation creep affects
apparent viscosity

Suggests some observations may require
dislocation-related deformation

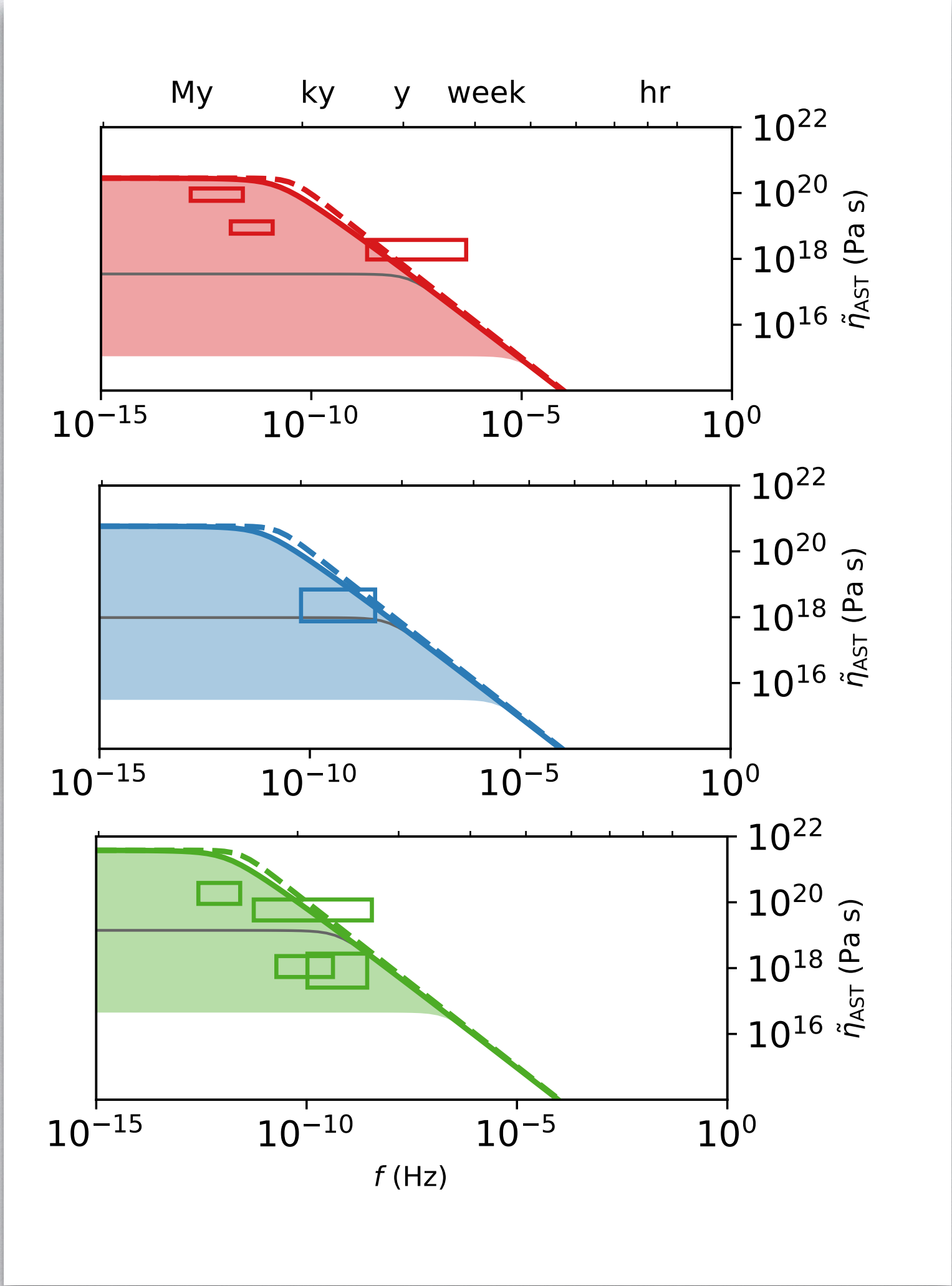
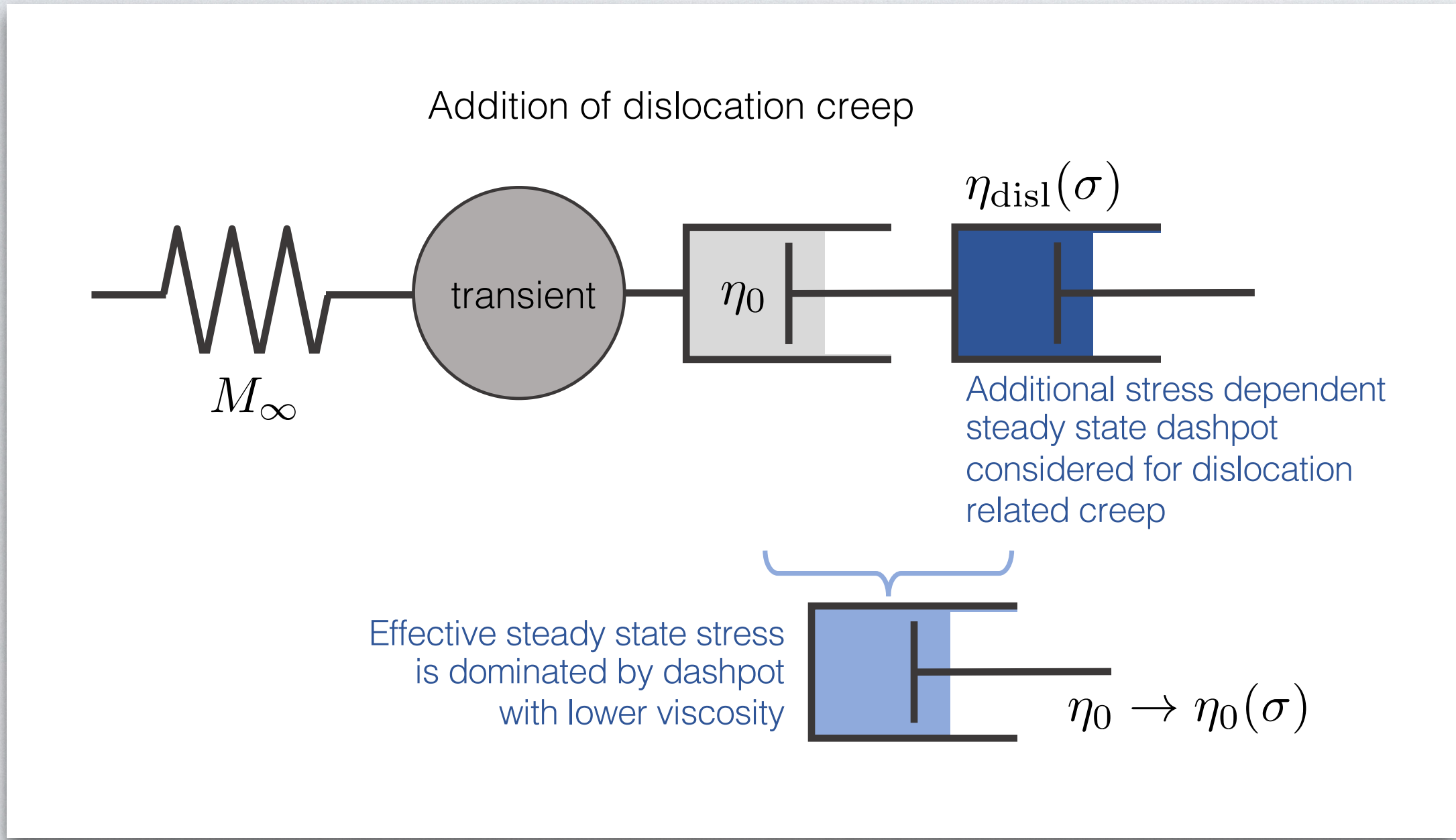


Western
North
America

Western
Antarctica

Antarctic
Peninsula

Results: *Apparent Viscosity*



Western North America

Western Antarctica

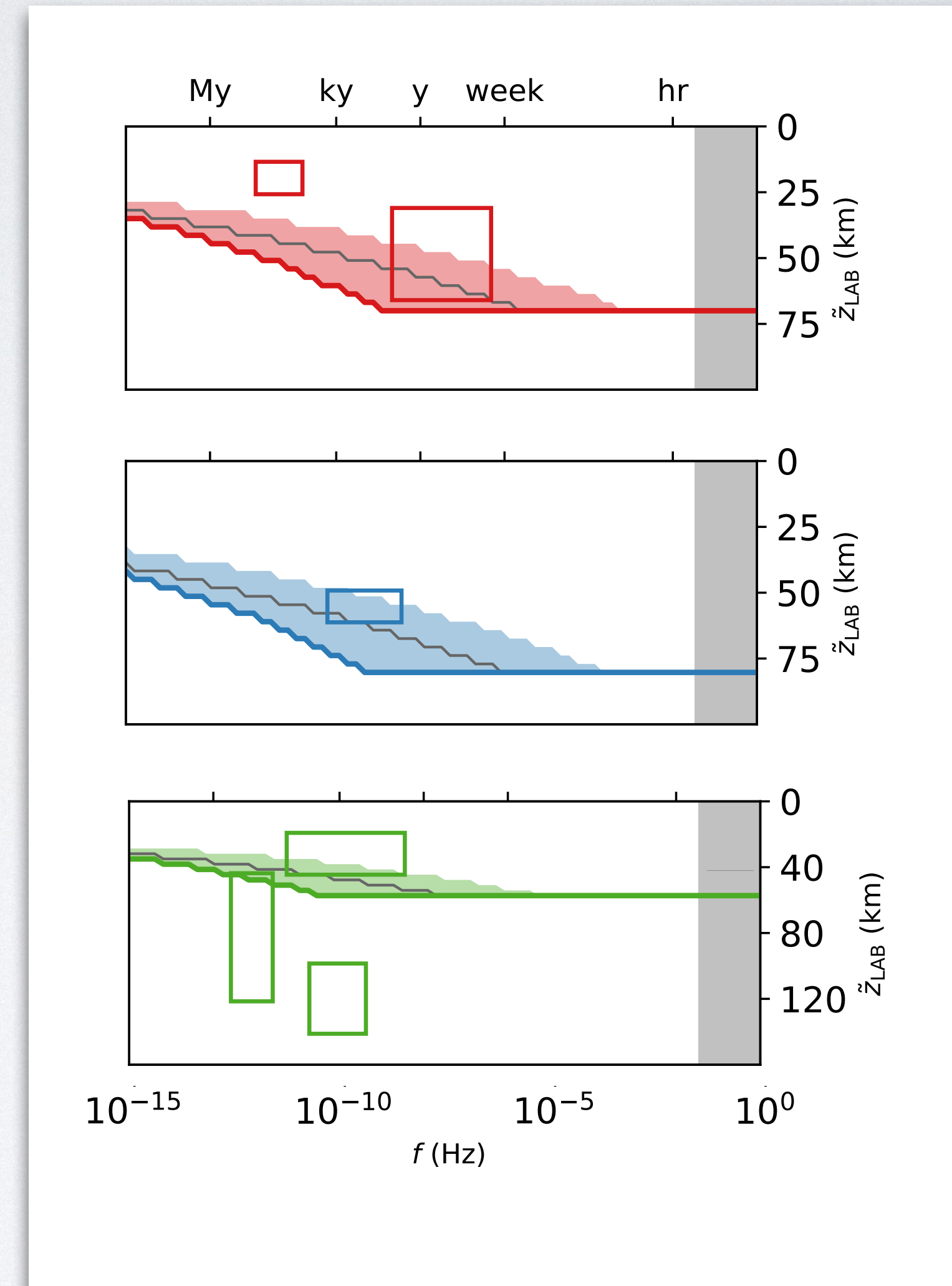
Antarctic Peninsula

Results: *Apparent Plate Thickness*

Solid Colored line:
Apparent Plate Thickness

Relaxes to shallower depths towards
longer timescales

At high frequency, hotter (deeper)
conditions are required to reach the
elastic-viscous transition



Western
North
America

Western
Antarctica

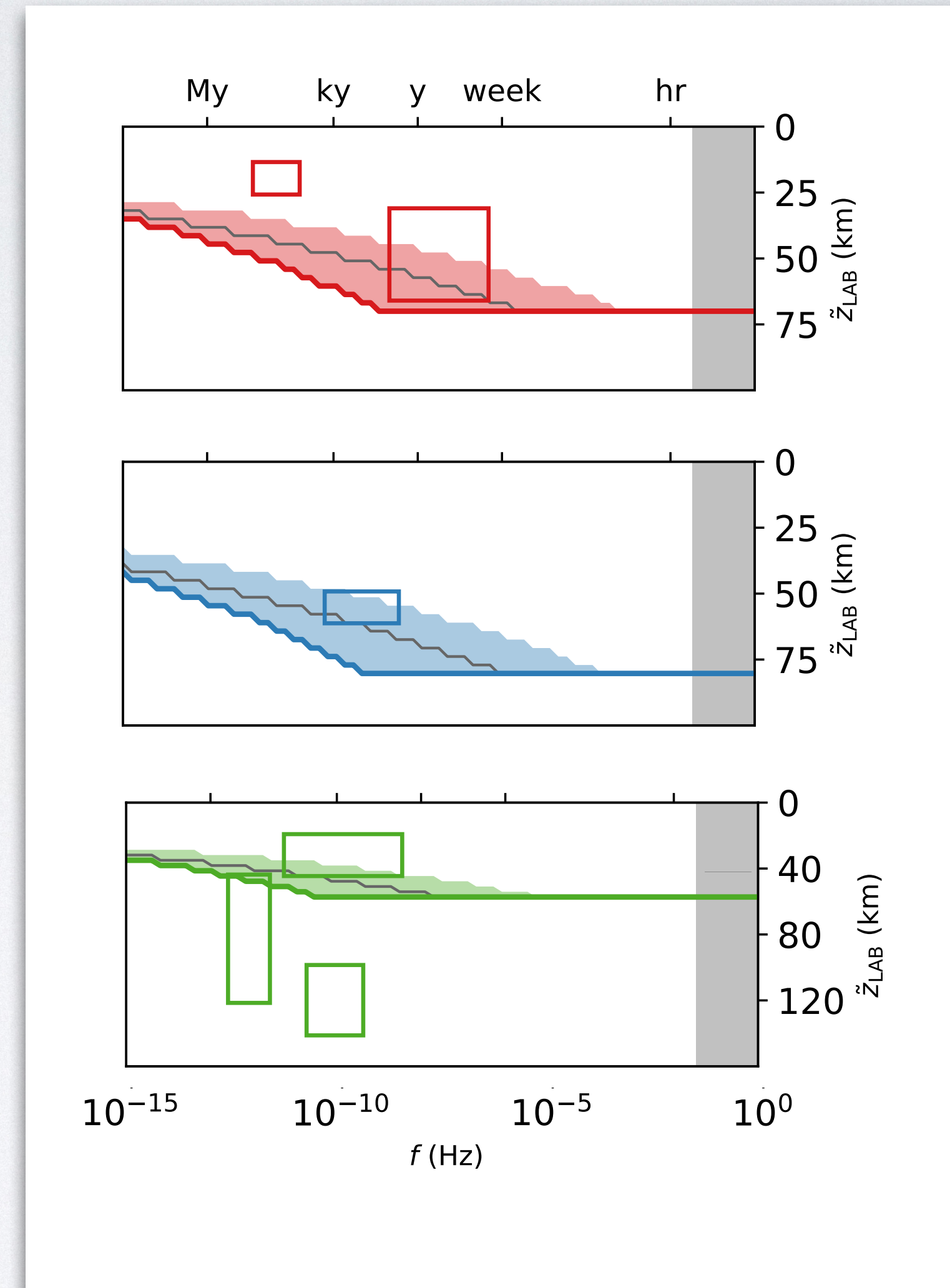
Antarctic
Peninsula

Results: *Apparent Plate Thickness*

Boxes:

Observations *placed at their frequency band*

Observations somewhat mixed.



Western
North
America

Western
Antarctica

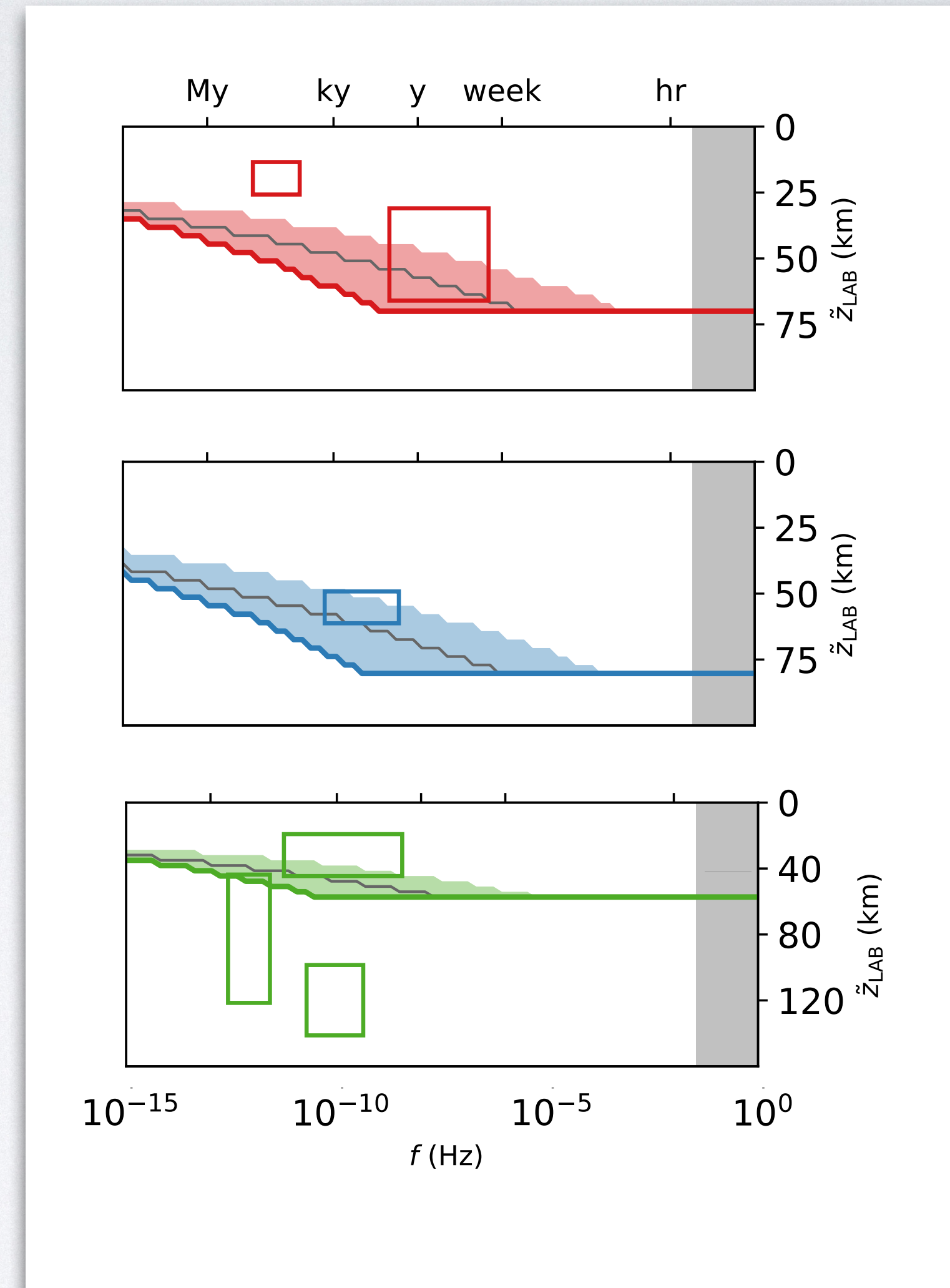
Antarctic
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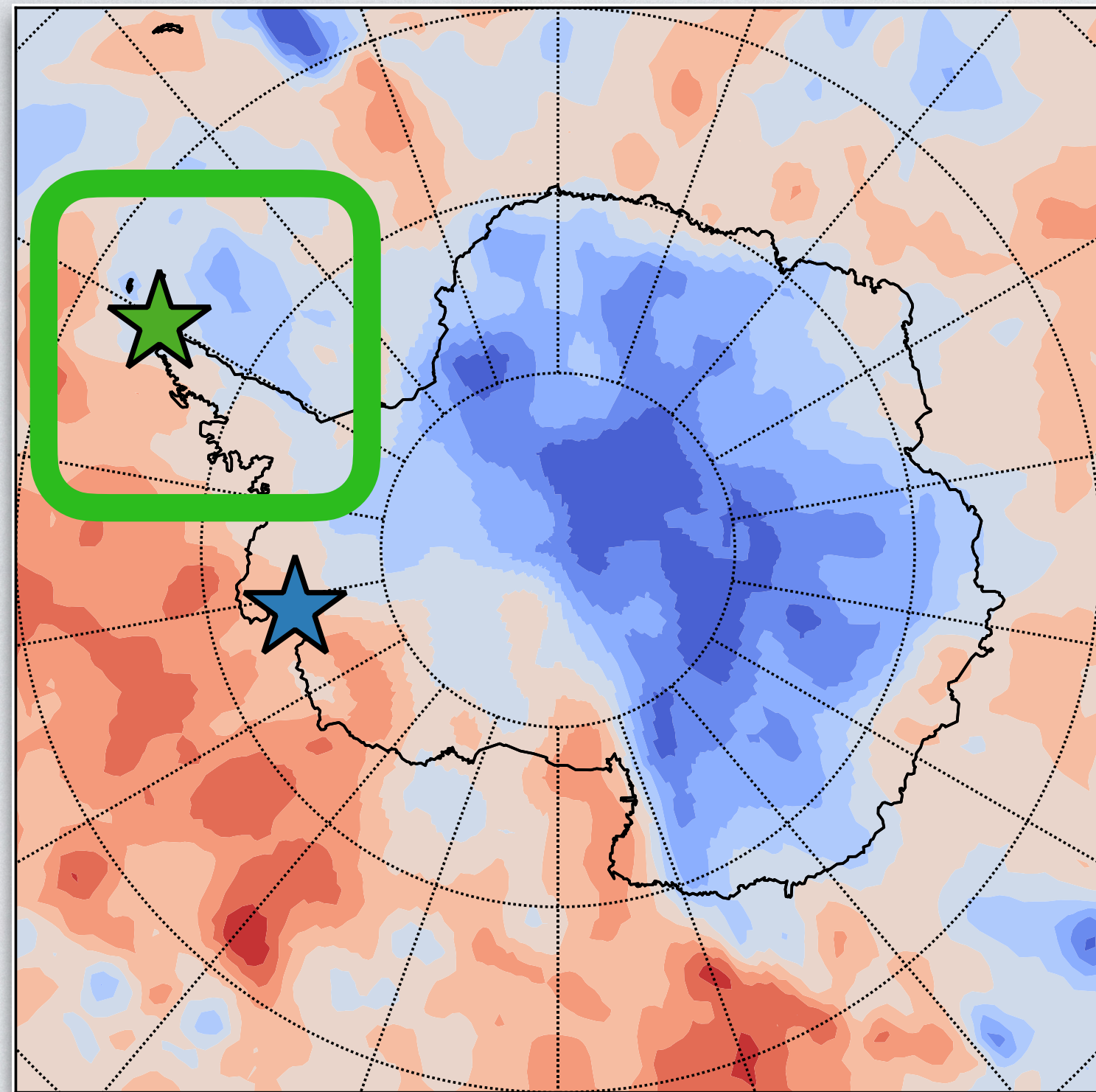


Western
North
America

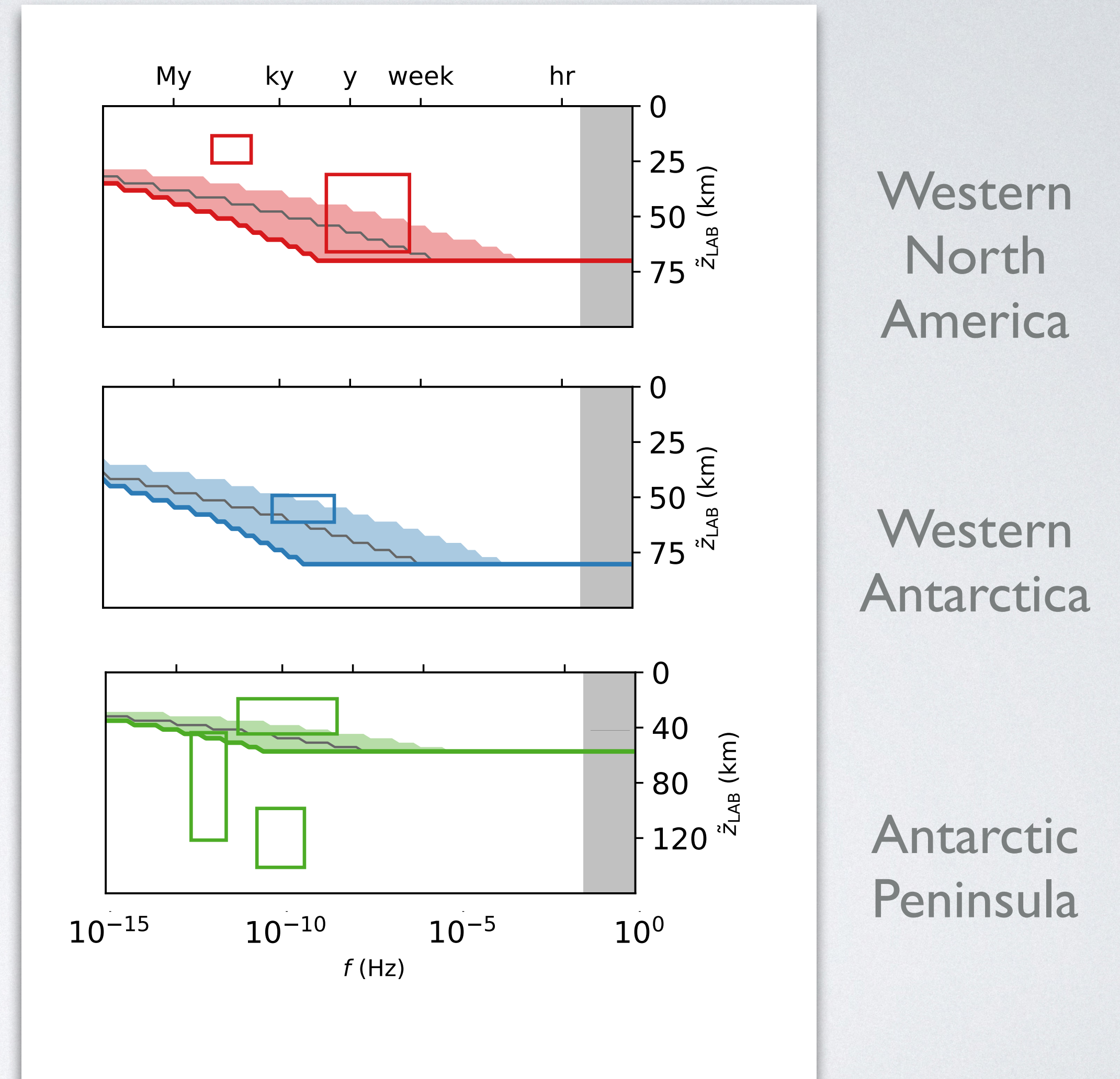
Western
Antarctica

Antarctic
Peninsula

Results: *Apparent Plate Thickness*



Complicated structure by peninsular
Slab beneath
Both continental and oceanic
environments



Western
North
America

Western
Antarctica

Antarctic
Peninsula

Summary

Each of the observations used were from independent studies and each come with their own assumptions but we took their asthenospheric viscosity estimates at face value.

A more thorough exploration would be to repeat each of these studies under a consistent framework.

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Laboratory derived (involving significant extrapolation) constitutive laws do remarkably well fitting observations when interpreted as apparent quantities.

Are we at a place to go between the two to understand rheology?

Summary

Each of the observations used were from independent studies and each come with their own assumptions but we took their asthenospheric viscosity estimates at face value.

A more thorough exploration would be to repeat each of these studies under a consistent framework.

With the wealth of observations we have now, is it possible to map out how dissipation as a function of frequency if we mindfully attempt to fit apparent quantities than discrete dashpots?

Post seismic rebound, ice sheet collapse, centennial melt, ice age melt, lake rebound

Laboratory derived (involving significant extrapolation) constitutive laws do remarkably well fitting observations when interpreted as apparent quantities.

Are we at a place to go between the two to understand rheology?

Submitted work by
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Benjamin Holtzman
Jacqueline Austermann
Chris Havlin
Cameron Book
Emily Hopper
Andrew Lloyd