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Determining gravity potential difference via VLBI time comparisons

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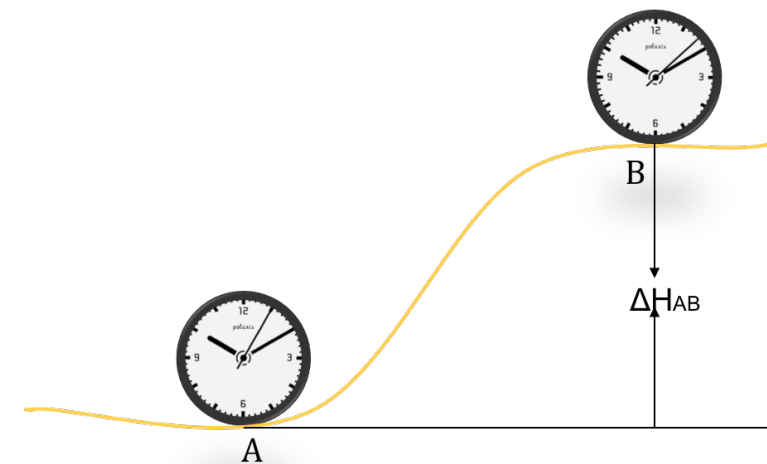


Abstract

VLBI technique plays important role in both astronomy and geodesy due its fantastic ability to determine the position of celestial bodies and the length of baseline on Earth. Moreover it also presents excellent work on time comparisons between atomic clocks located in remote positions where optical fiber links are not accessible. Due to its high reliability and stability, the information of Earth's gravity field can be extracted from VLBI time comparisons in the framework of general relativity. In this study, we provide a formulation to determine the gravity potential difference by VLBI time comparisons. In fact the precision of the estimated gravity potential depends on the performance of participated clocks and the accuracy of time comparison technique. Thus we present simulation experiments using clocks with 10^{-16} @1d stability and broadband VLBI observation and determine gravity potential difference within a VLBI network around world with $10 \text{ m}^2/\text{s}^2$ precision which is equivalent to 1 m in height. The results could be greatly improved using optical atomic clocks with much higher stabilities. Furthermore it can be applied to height transfer across oceans and unifying the height system.

Methods

➤ **Basic principle:** A clock runs quicker at higher geopotential position than at lower position.



After a standard time period, the time intervals recorded by two clocks at stations A and B are

$$\Delta T_A = \Delta T_0 \left(1 + \frac{W_A}{c^2} \right) \quad \Delta T_B = \Delta T_0 \left(1 + \frac{W_B}{c^2} \right) \quad (1)$$

The geopotential and orthometric height differences between A and B are expressed as

$$\Delta W_{AB} = \frac{\Delta T_B - \Delta T_A}{\Delta T_A} c^2 \quad (2)$$

$$H_{AB} = -\frac{1}{g} \frac{\Delta T_B - \Delta T_A}{\Delta T_A} c^2 \quad (3)$$

➤ **Time elapse determination from VLBI observations**

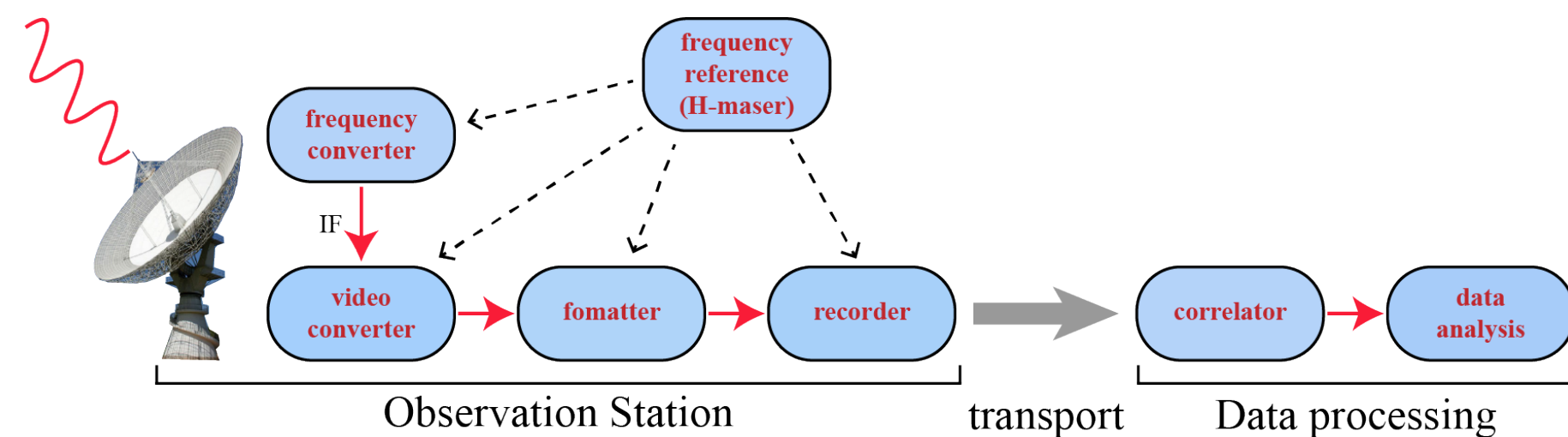


Fig.1 Very Long Baseline Interferometry (VLBI) signal flow. Modified from Hobiger et al (2007).

- ◆ Determine the group delay τ and delay rate $\dot{\tau}$ precisely, then obtain two time series;
- ◆ After eliminating all kinds of delays, take the sum of the adjacent time intervals as time elapse.

Simulation Experiment

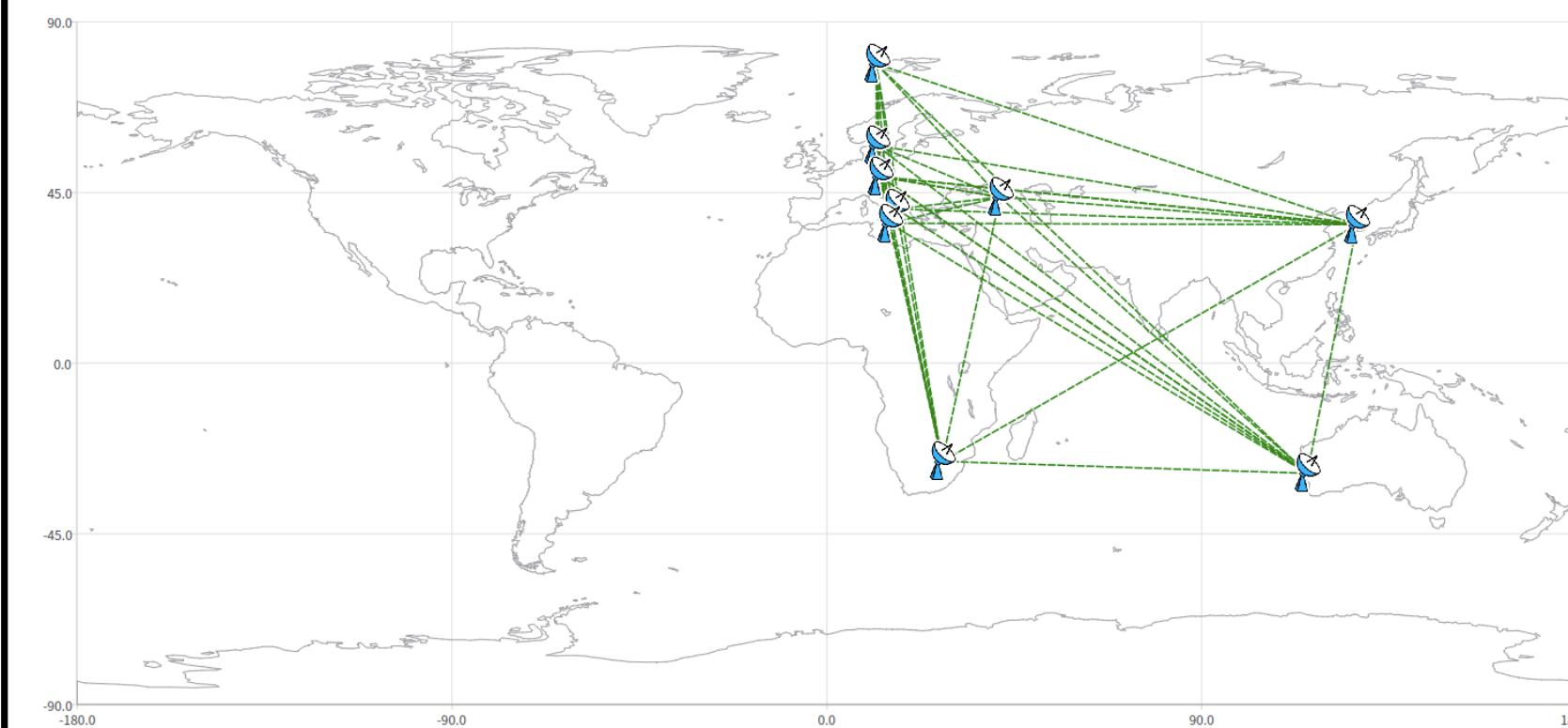


Fig.2 Simulated VLBI observing network which consists of 10 stations. Here we follow the official parameters of IVS-R1997 session which lasts for 24 hours (2021.05.03 17:00-05.04 17:00). The generated VLBI schedule includes 768 scans and 11283 delay observations.

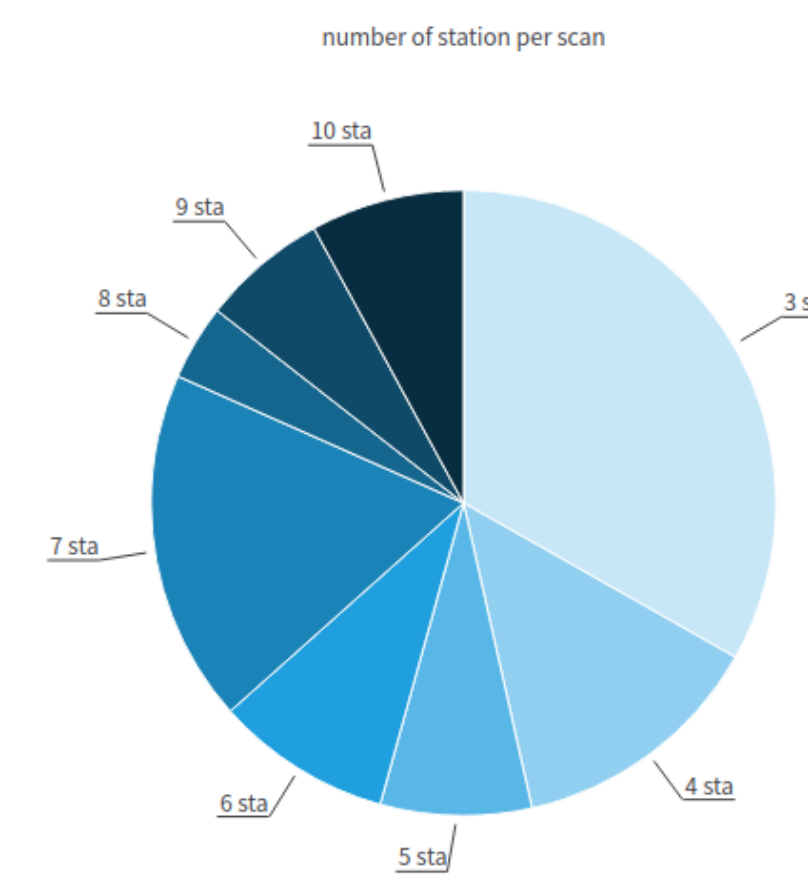


Fig.3 Statistics of the scans after scheduling.

Table 1. Relevant parameters used in simulation experiments in this study.

Parameters	Values
Tropospheric refractive index structure constant C_n	$1.8 \times 10^{-7} \text{ m}^{-1/3}$
Effective wet height of troposphere H	2 km
Wind speed	8 m/s (towards east)
Measurement white noise level	20 ps
Modified Allan deviation (MDEV)	$1.1 \times 10^{-16} @ 1 \text{ d}$
Systematic clock drift	6.0×10^{-15}

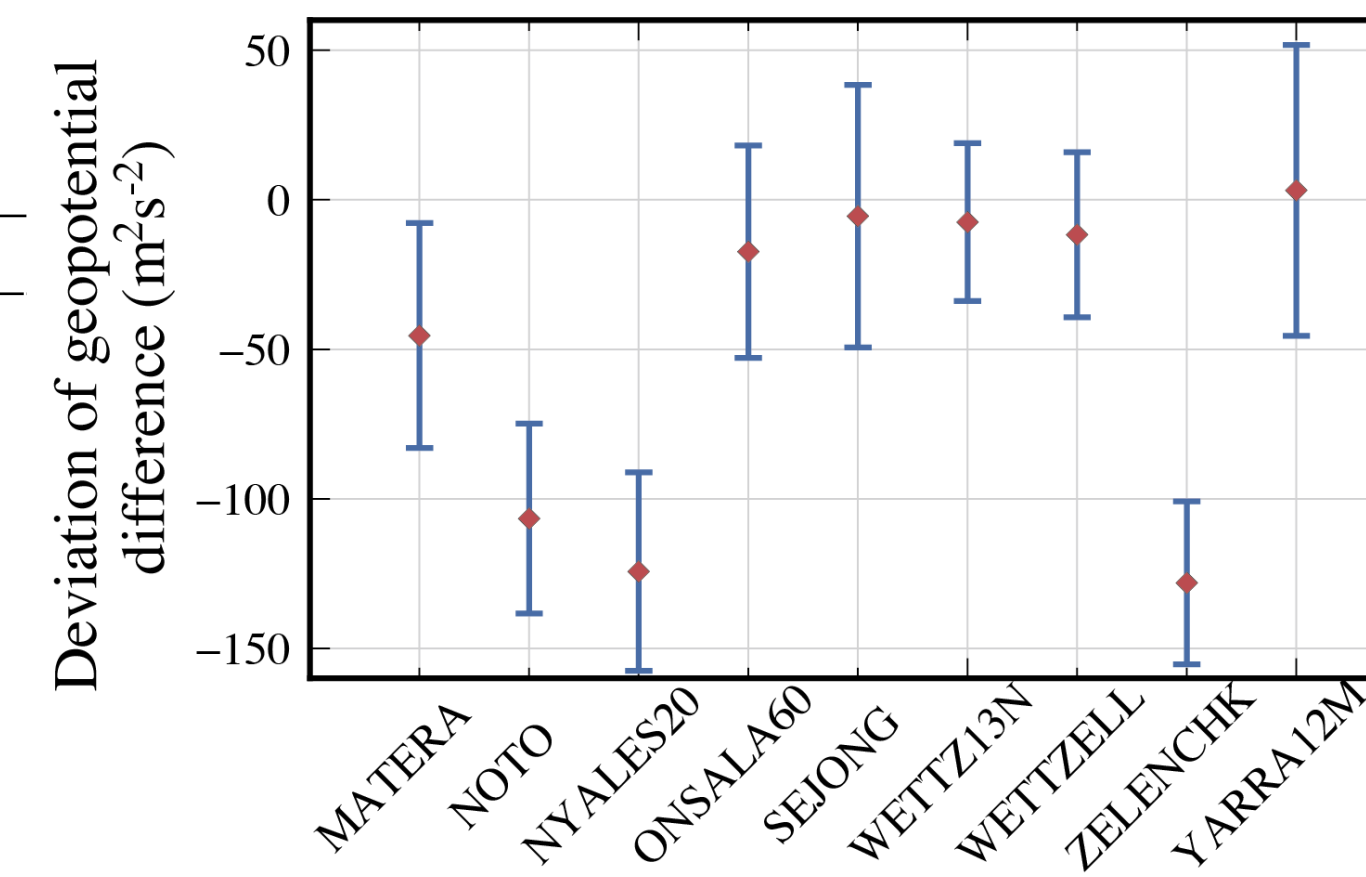


Fig. 4 Deviation of the estimated geopotential differences of the 9 baselines from that of the EGM2008 model.

Conclusions

- We provided a formulation of the VLBI geopotential determination approach (VGDA).
- The simulation experiments show that VGDA is reliable, prospective and has its potential applications in the future.
- One potential application lies in that this new approach could be used to unify the world height datum system (Shen et al, 2018) based on the worldwide distributed VLBI stations.

Results

- The experimental results are displayed in Fig.4 and Table 2.
- Our experiments show that, the orthometric height differences with precision of 1 m level could be obtained via analysis of 24-hour VLBI time comparison.

Table 2. Deviation of the estimated orthometric height differences of 9 baselines from those of the EGM2008 model.

Baseline (refer to HART15M)	Deviation of orthometric height difference (m)
MATERA	4.65 ± 3.83
NOTO	10.88 ± 3.24
NYALES20	12.74 ± 3.38
ONSALA60	1.70 ± 3.62
SEJONG	0.56 ± 4.48
WETTZ13N	0.78 ± 2.69
WETTZELL	1.20 ± 2.81
ZELENCHK	13.07 ± 2.78
YARRA12M	-0.32 ± 4.97

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