

An improved approach for testing gravitational redshift via spacecraft base three frequency links combination

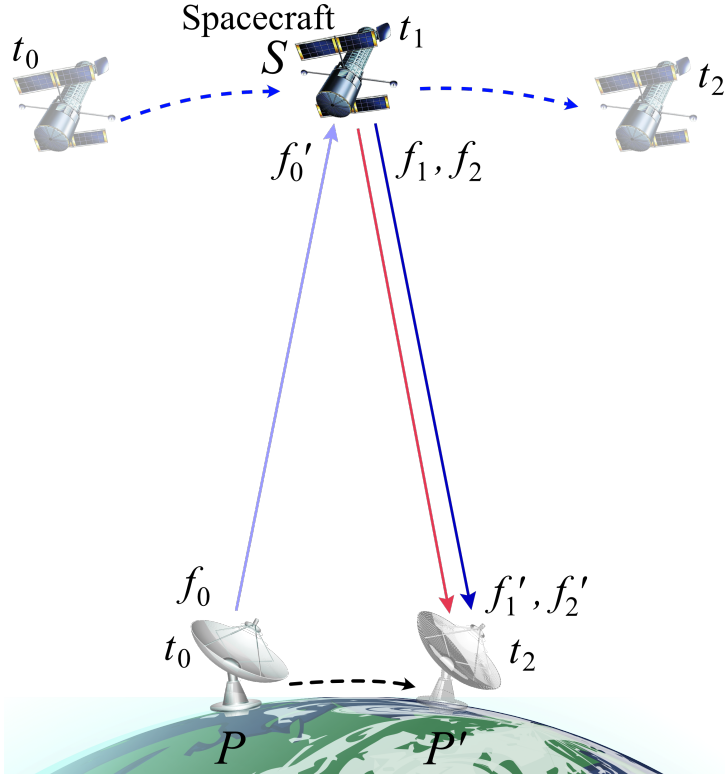
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Abstract: We propose a novel approach for testing the gravitational redshift based on frequency signals transmitted between a spacecraft and a ground station. The main idea is integrating one uplink signal from ground to spacecraft and two downlink signals from spacecraft to ground. Based on the integration and certain correction models, the gravitational shift of the signals between spacecraft and ground station can be detected at high precision level. The gravitational redshift effect can be used to validate the Einstein equivalence principle (EEP), which can be tested at about $10^{-6} \sim 10^{-8}$ levels in different cases for a short period (less than a month) if the stability of on-board atomic clock reach $10^{-17}/\text{day}$.



We represent the frequency shifts of the three links as $\Delta f_0 = f_0' - f_0$, $\Delta f_1 = f_1' - f_1$, $\Delta f_2 = f_2' - f_2$, and set f_0 as reference frequency presenting $f_1 = af_0$, $f_2 = bf_0$ (a and b are known scaling factors). Thus, we take the following combination of frequency shifts as an output value f_{out} :

$$f_{out} = b^{-1} \cdot \Delta f_2 - \frac{\Delta f_0 + a^{-1} \cdot \Delta f_1}{2} = b^{-1} \cdot f_2' - 0.5a^{-1} \cdot f_1' - 0.5f_0'$$

The combination is useful for errors cancellation. After some error corrections models are adopted, the EEP can be tested at a high precision level.

Result: The theoretical uncertainties of various error sources and the ultimate precision of EEP test experiment, based on three-link combination. The experiment period is from several days to a month.

We estimated three different cases:

- (1) A GPS satellite using 1.227 GHz and 1.575 GHz as downlink frequencies, where the uplink frequency is assumed as 1.4 GHz;
- (2) The ACES project on ISS employing 2.248 GHz and 14.7 GHz as downlink frequencies, and 13.475 GHz as the uplink frequency;
- (3) An assumed satellite using 8 GHz and 12 GHz as downlink frequencies, and 10 GHz as the uplink frequency, for which the orbit height are similar to a GPS satellite (about 20 000 km).

Uncertainty of frequency shift components	Symbol	Residual magnitudes for different cases		
		GPS	ACES	Assumed
Classical Doppler shift	$\delta f_{dop}/f_0$	Neglectable	Neglectable	Neglectable
Relativistic shift	$\delta f_{rel}/f_0$	$\sim 2.9 \times 10^{-18}$	$\sim 5.0 \times 10^{-17}$	$\sim 2.9 \times 10^{-18}$
Ionospheric shift	$\delta f_{ion}/f_0$	$\sim 7.9 \times 10^{-15}$	$\sim 8.8 \times 10^{-18}$	$\sim 2.2 \times 10^{-17}$
Tropospheric shift	$\delta f_{trop}/f_0$	Neglectable	Neglectable	Neglectable
Path discrepancy	Doppler $\delta f'_{dop}/f_0$	$< 4.4 \times 10^{-19}$	$< 6.2 \times 10^{-19}$	$< 4.4 \times 10^{-19}$
	Ionosphere $\delta f'_{ion}/f_0$	$< 1.4 \times 10^{-18}$	$< 1.8 \times 10^{-21}$	$< 2.7 \times 10^{-20}$
	Troposphere $\delta f'_{trop}/f_0$	$< 1.1 \times 10^{-17}$	$< 5.3 \times 10^{-18}$	$< 1.1 \times 10^{-17}$
Iono. shift uncertainty caused by δt	$\delta f''_{ion}/f_0$	$\sim 4.7 \times 10^{-17}$	$\sim 6.9 \times 10^{-19}$	$\sim 2.7 \times 10^{-17}$
Clock errors and other minor effects	$\delta f_{oth}/f_0$	$\sim 1.2 \times 10^{-17}$	$\sim 1.2 \times 10^{-17}$	$\sim 1.2 \times 10^{-17}$
Total uncertainty of frequency shift	$\delta f_{all}/f_0$	$\sim 3.5 \times 10^{-15}$	$\sim 1.1 \times 10^{-16}$	$\sim 3.7 \times 10^{-17}$
Uncertainty of EEP test	$\delta \alpha$	$\sim 6.5 \times 10^{-6}$	$\sim 2.2 \times 10^{-6}$	$\sim 7.3 \times 10^{-8}$

Method: The combination of three frequency links. Ground station P emits a frequency signal f_0 at time t_0 . The signal is received by spacecraft S as f_0' at time t_1 . Spacecraft S also emits two frequency signal f_1 and f_2 at time t_1 . Furthermore, the ground station receives the two signals as f_1' and f_2' at time t_2 , when its position is changed to P' due to the Earth's rotation. The gravitational redshift can be detected after the combination of the frequencies