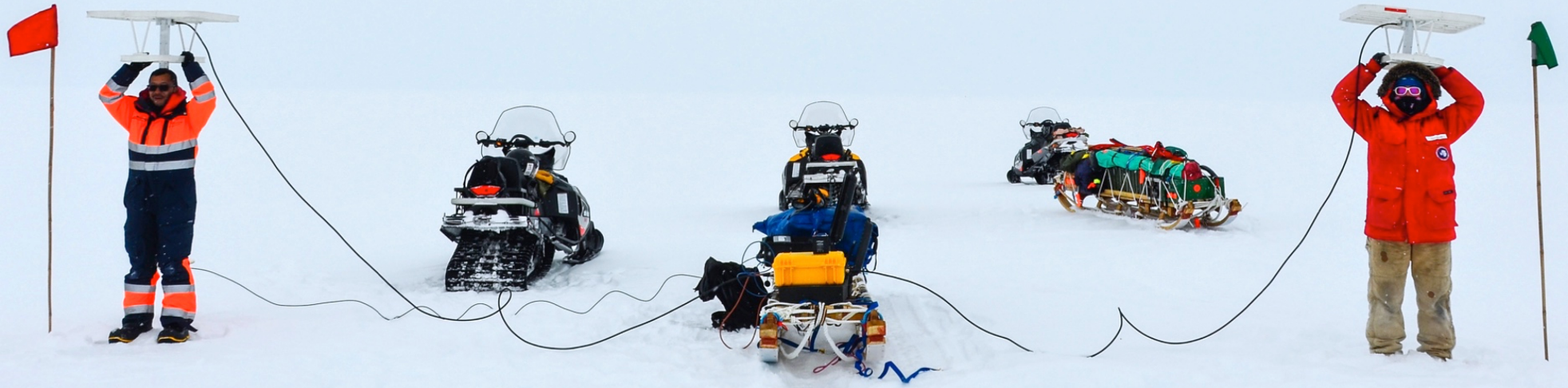
Polarimetric radar sounding to infer and quantify shear margin ice fabric anisotropy

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Scott Polar Research Institute | British Antarctic Survey | Plymouth Marine Laboratory | Stanford University | University of California, Santa Cruz

CR1.2 | **EGU21-2107** | Mon 26 Apr | 13:30-15:00 CEST



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Please contact me if you would like to collaborate, or are just interested in learning more about radar and fabric!

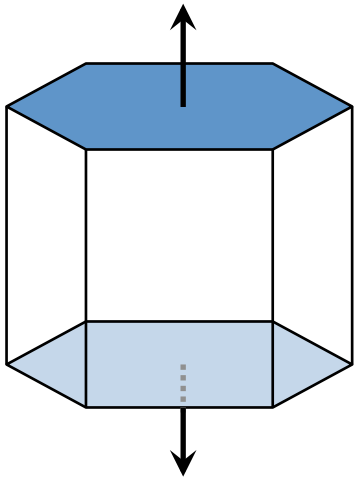
What is ice fabric?

- Ice fabric represents the collective orientations of ice crystals
- COF (radar, ice cores) v. CPO (seismics)
- Represented by a 2nd order orientation tensor with eigenvalues *a* along eigenvectors *x*:

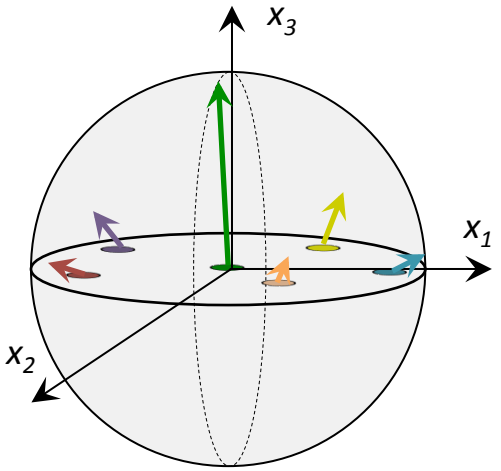
$a_1 + a_2 + a_3 = 1$

$a_1 < a_2 < a_3$ (radar, seismics)

$a_1 > a_2 > a_3$ (ice cores)



The *c*-axis of a single ice crystal



Ice fabric: *c*-axes of *many* ice crystals

Running Commentary

❄ The orientation of a single ice crystal is described by the direction of its c-axis.

❄ The aggregate orientation of the c-axes of many ice crystals represents the overall ice fabric.

❄ Orientation of the fabric c-axes can be represented by a 2nd order orientation tensor.

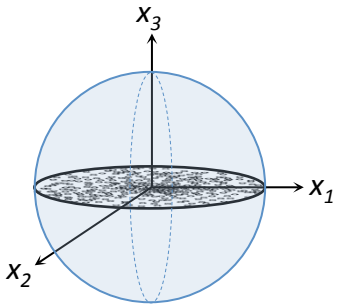
❄ The eigenvalues (*a*) describe the relative concentration of c-axes projected along each of the three eigenvectors (*x*).

❄ Higher eigenvalues indicate greater concentrations than lower eigenvalues.

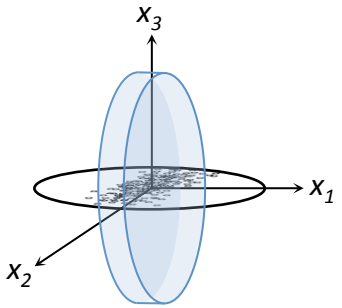
❄ The nomenclature, while similar between radar and seismics, differs from that used in ice core studies.

Fabric anisotropy

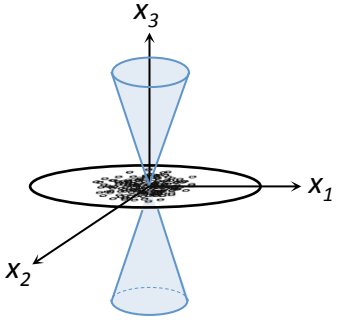
- Fabric represents the deformation history of glacier ice
- "Complex history → complex fabric"



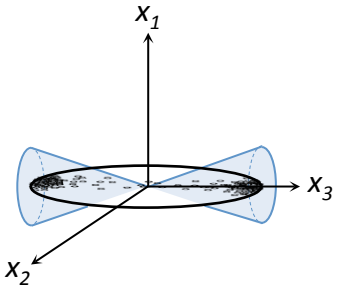
Isotropic (near-random)
$a_1 \approx a_2 \approx a_3 \approx 1/3$
Near-surface firn/ice
Uniform deformation



Vertical girdle
$a_1 << a_2 \approx a_3$
Centre of glacier
Uniaxial compression, longitudinal extension



Vertical cluster
$a_1 \approx a_2 << a_3$
Near-bed ice
Planar simple shear



Horizontal cluster
$a_1 \approx a_2 << a_3$
Glacier shear margin
Lateral simple shear

* Shown are *general* examples (deviations occur!)

Further reading

Azuma & Higashi (1985) AoG
Alley (1988) Nature

Running Commentary

❄ Fabrics evolve as a result of their deformation history because the c-axes tend to rotate towards the direction of max. compression.

❄ A complex flow history leads to a complex fabric.

❄ In general, minimal deformation produces a near-isotropic fabric, such as those observed in young ice near the ice surface.

❄ At ice divides, lateral tension with flow divergence produces vertical girdle fabrics.

❄ Near the bed, resistance from basal friction causes planar shearing that causes vertical clustering.

❄ Similar processes occur at shear margins that separate stagnant from streaming ice, but the cluster is orthogonally oriented.

Birefringence in ice

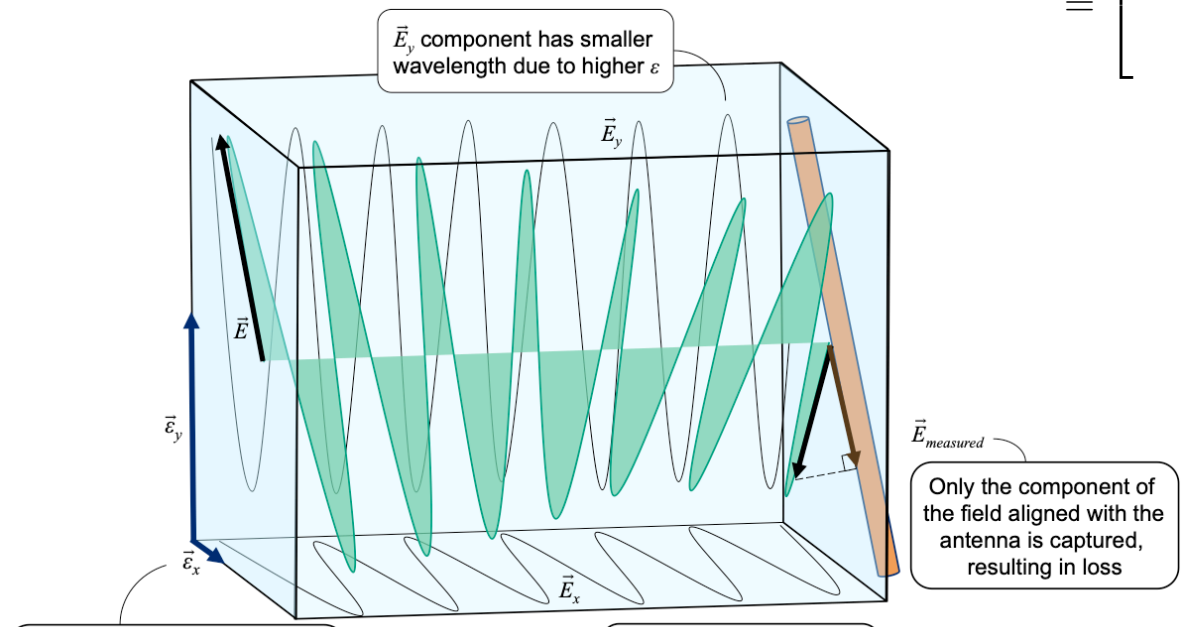
Radar is able to detect the horizontal components of fabric anisotropy due to the birefringence of ice as an effective medium

$$\varepsilon = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$= \begin{bmatrix} \varepsilon_\perp + a_1 \Delta\varepsilon' & 0 & 0 \\ 0 & \varepsilon_\perp + a_2 \Delta\varepsilon' & 0 \\ 0 & 0 & \varepsilon_\perp + a_3 \Delta\varepsilon' \end{bmatrix}$$

$$\Delta\varepsilon = \varepsilon_y - \varepsilon_x = (a_2 - a_1)\,\Delta\varepsilon'$$

This equation relates the bulk (macroscopic) birefringence $\Delta\varepsilon$ to the microscopic (crystal) birefringence $\Delta\varepsilon'$



I can't release this yet, but please contact me if you would like a draft!

Further reading

Hargreaves (1978) JGlac
Doake (1981) GJI
Fujita et al. (2006) JGlac
Young et al. (*accepted*) JGR

❄ Polar ice is known to be birefringent at radar frequencies.

❄ Waves undergoing birefringence are decomposed into two orthogonally-oriented waves that travel at slightly different speeds due to differences in their dielectric permittivities.

❄ Radar antennas are linearly-polarised, which means that it only captures a single polarisation plane.

❄ The observed birefringence in the antenna plane can be expressed in terms of the permittivity difference.

❄ Radar can directly resolve only the horizontal components of fabric asymmetry (a_1, a_2), but not the vertical component (a_3).

Polarimetric backscatter model

$$S(\theta) = \underbrace{\left[\frac{\exp(jk_0 z)}{4\pi z}\right]^2}_{\text{free-space propagation}} \cdot \underbrace{\left[\prod_{i=1}^{N} (RTR')_{\theta,\varepsilon,N+1-i}\right]}_{\text{received (upward) propagation}} \cdot \underbrace{[R\Gamma R']_{\theta,\beta,N}}_{\text{boundary scattering}} \cdot \underbrace{\left[\prod_{i=1}^{N} (RTR')_{\theta,\varepsilon,i}\right]}_{\text{transmitted (downward) propagation}}$$

$$T = \underbrace{\begin{bmatrix} T_x & 0 \\ 0 & T_y \end{bmatrix}}_{\text{Transmission}} \quad R = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}_{\text{Rotation}} \quad \Gamma = \underbrace{\begin{bmatrix} r & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Reflection}}$$

(full equation in Eqs. 5-7 in Fujita et al. 2006 JGlac)

Further reading

Fujita et al. (2006) JGlac
Young et al. (2020) TCD
Ershadi et al. (2021) TCD
Young et al. (*accepted*) JGR

❄ Polarisation of the radar signal can be modelled by accumulating the birefringence, the polarimetric phase shift, and the fabric strength and observing the signal orientation dependence at each depth step.

❄ The backscatter model applies basis transforms to the signal (*R*) as it propagates downwards from the transmitted antenna (*T*), scatters at boundaries (*Γ*), and reflects back towards the received antenna.

❄ Transmission is anisotropic due to differences in x- and y-permittivities and conductivities.

❄ Reflection is also anisotropic, the amount depends on the properties of the scattering interface.

Polarimetric backscatter model

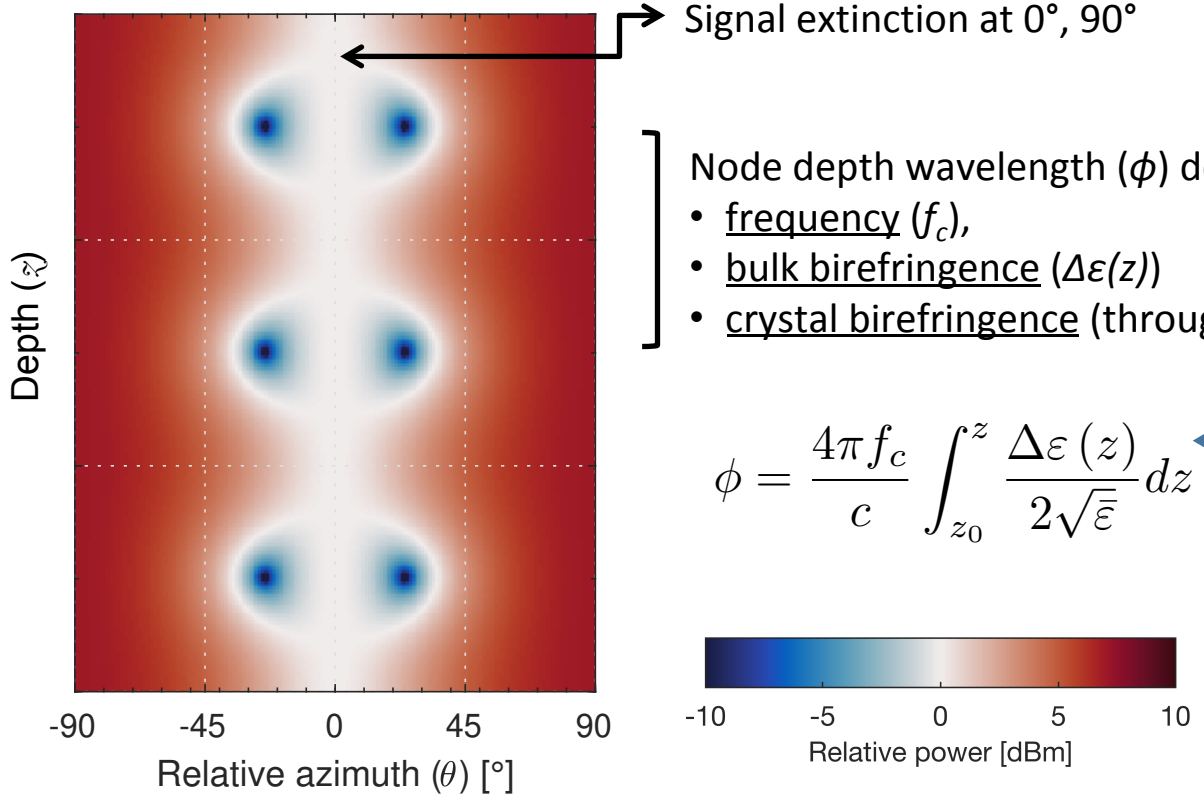
Node azimuth separation dependent on anisotropic scattering (*r*)

Signal extinction at 0°, 90°

Node depth wavelength (ϕ) dependent on:

- frequency (f_c),
- bulk birefringence ($\Delta\varepsilon(z)$)
- crystal birefringence (through $\bar{\varepsilon}$)

$$\phi = \frac{4\pi f_c}{c} \int_{z_0}^{z} \frac{\Delta\varepsilon(z)}{2\sqrt{\bar{\varepsilon}}} dz \quad \leftrightarrow \quad \Delta\varepsilon = (a_2 - a_1)\,\Delta\varepsilon'$$



Further reading

Fujita et al. (2006) JGlac
Young et al. (2020) TCD
Young et al. (*accepted*) JGR

Running Commentary

❄ The resulting depth-azimuth plot shows the location of "co-polarised nodes".

❄ The azimuthal separation of these nodes is dependent on the amount of bulk anisotropic scattering present in the ice.

❄ The depth separation of these nodes is dependent on the radar frequency, crystal birefringence, and fabric asymmetry through bulk birefringence.

❄ Because the frequency is known from the radar system, and the crystal birefringence is largely constrained from previous studies, we can then directly quantify the horizontal difference in fabric asymmetry.

Polarimetric radar sounding

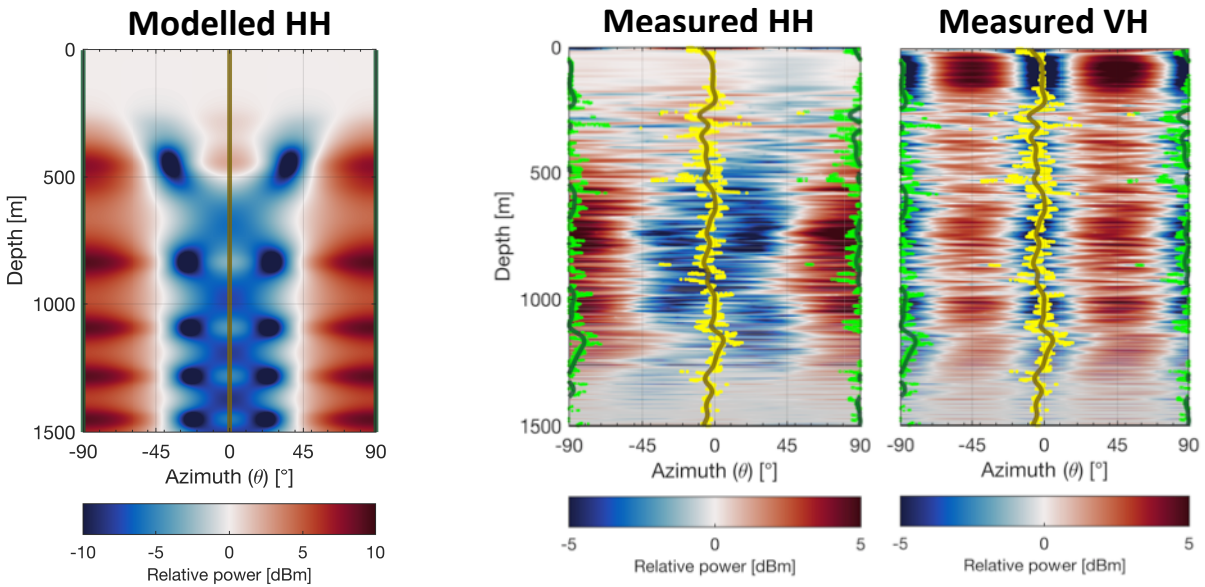
Application to radar sounding using linearly-polarised antennas can detect azimuthal (*x-y*) fabric asymmetry

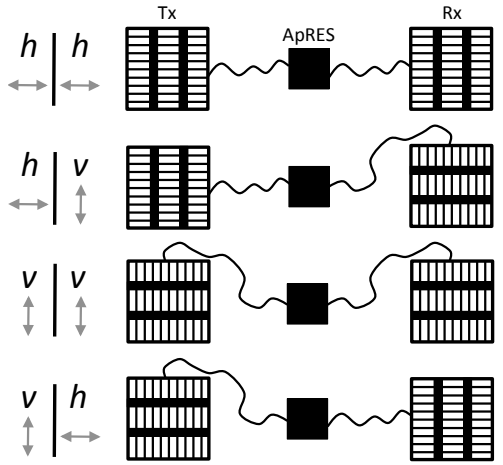
Further reading

Brisbourne et al. (2019) JGR
Young et al. (2020) TCD

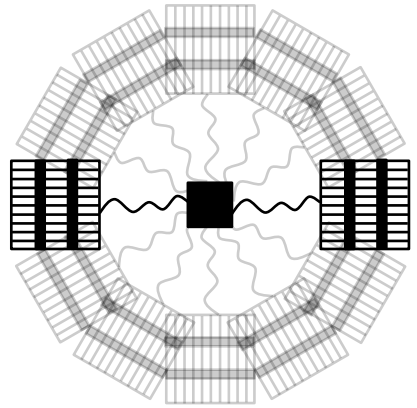
$$S(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} s_{hh}\cos^2\theta - (s_{vh}+s_{hv})\sin\theta\cos\theta + s_{vv}\sin^2\theta & s_{hv}\cos^2\theta + (s_{hh}-s_{vv})\sin\theta\cos\theta - s_{vh}\sin^2\theta \\ s_{vh}\cos^2\theta + (s_{hh}-s_{vv})\sin\theta\cos\theta - s_{hv}\sin^2\theta & s_{vv}\cos^2\theta + (s_{vh}+s_{hv})\sin\theta\cos\theta + s_{hh}\sin^2\theta \end{bmatrix}$$





Quad-pol setup



Azimuthal rotation setup

Running Commentary

❄ In practice, we can reconstruct the polarimetric backscatter using a fully-polarimetric dataset composed of four antenna combinations of orthogonal orientations.

❄ Quad-polarimetry produces essentially the same output as if the antenna were circularly rotated.

❄ The use of the cross-polarised (VH) power anomaly can give a rough indication of the fabric eigenvectors without distinguishing between the two.

❄ Because fabric orients itself in relation to deformation, the orientation of the x_1 and x_2 eigenvectors gives directions of principal strain (if a vertical girdle is modelled). To distinguish between the two, we have to rely on the HHVV signal (next slide).

❄ Measured results with an ApRES system at WAIS Divide compare well to modelled results using measurements from the WAIS Divide ice core.

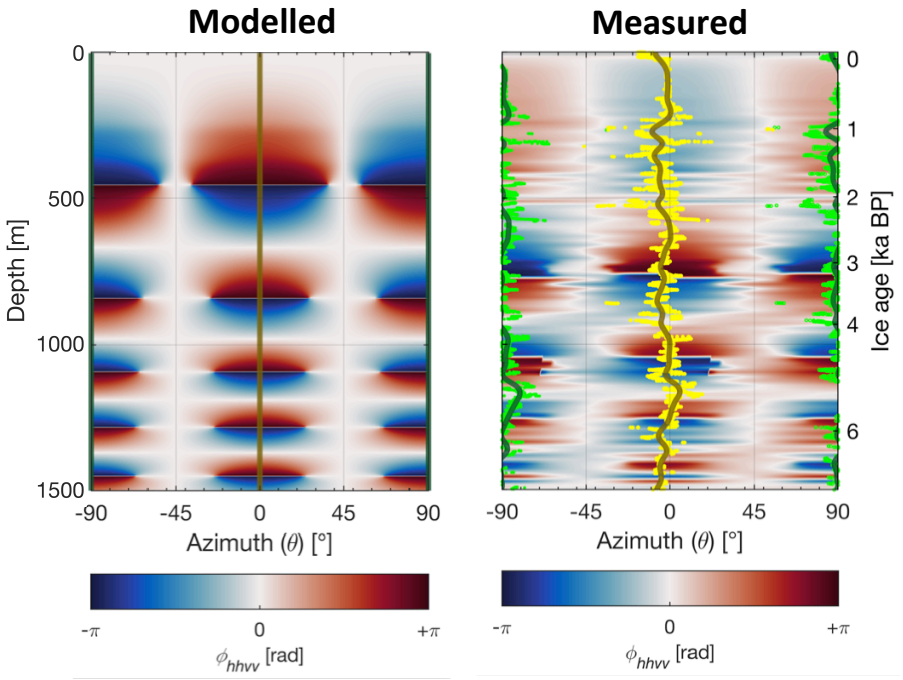
Polarimetric coherence

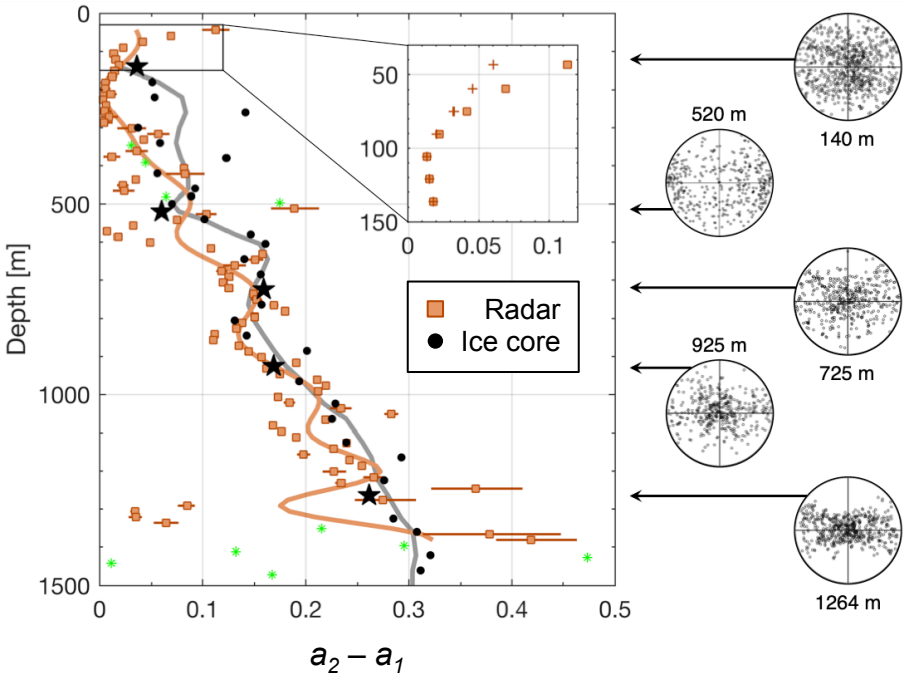
Application of polarimetric coherence to the effective medium model quantifies the azimuthal fabric asymmetry

$$c^{\star}_{hhvv} = \frac{\sum_{i=1}^{N} s_{hh,i} \cdot s^{*}_{vv,i}}{\sqrt{\sum_{i=1}^{N} |s_{hh,i}|^2} \sqrt{\sum_{i=1}^{N} |s_{vv,i}|^2}}$$

$$\phi^{\star}_{hhvv} = \arg\left(c_{hhvv}\right)$$

$$a_2 - a_1 = \frac{c}{4\pi f_c} \frac{2\sqrt{\bar{\varepsilon}}}{f(\nu)\Delta\varepsilon'} \left| \frac{d\phi_{hhvv}}{dz} \right|$$





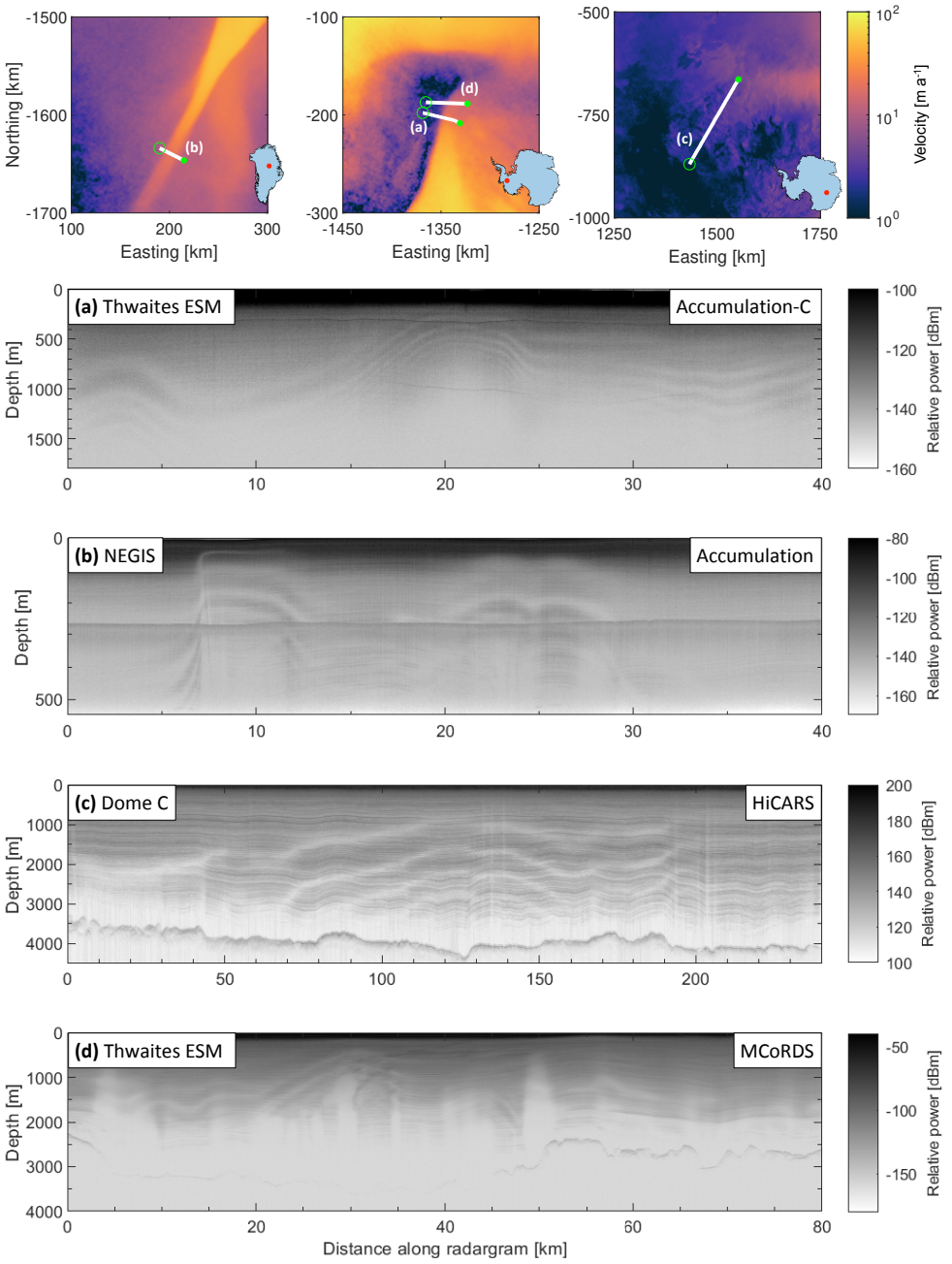
Further reading: Jordan et al. (2019) IEEE; Young et al. (2020) TCD

Running Commentary

- Application of polarimetric coherence to the effective medium model quantifies the amount of azimuthal fabric asymmetry.
- The quad-polarised derivative of the phase angle through depth ($d\phi_{hhvv}/dz$) is proportional to the amount of azimuthal fabric asymmetry.
- The polarity of $d\phi_{hhvv}/dz$ distinguishes between x_1 (negative) and x_2 (positive).
- Comparisons of radar-redived estimates of azimuthal fabric strength with similar measurements from ice cores at WAIS Divide show good agreement.
- Signal strength limits coherence deeper into the ice column.
- These techniques are established in polarimetric remote sensing, but are nascent in radioglaciology.

Birefringence in ice

- In radargrams, periodic patterns appear as a result of birefringence
- Antennas must be angled off-parallel and off-perpendicular to fabric axes!



Further reading:

Young et al. (*accepted*) JGR
Franke et al. (2021) ESSD Discussions

Running Commentary

❄ Birefringence causes periodic power loss that can be visualised if the antenna plane is neither parallel nor perpendicular to the fabric orientation.

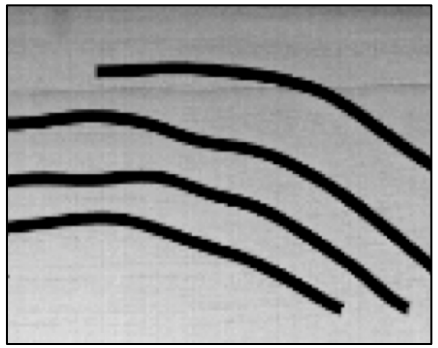
❄ We show example here of what we believe to be birefringence-induced power loss at 2 sites in Antarctica and 1 in Greenland.

❄ These radargrams are produced from co-linear antennas (i.e. a "regular" non-polarimetric setup).

❄ These radargrams were all open-access, and there exists many other radargrams with similar imprinted phenomena.

❄ We argue that these signals at shear margins represent the shear state of its deformational regime.

Application: Thwaites Glacier

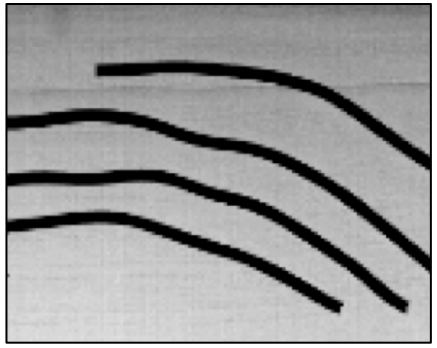


?

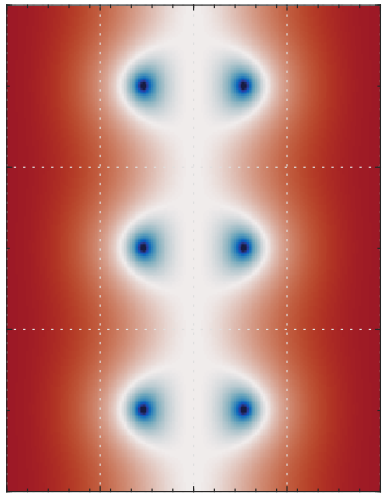
Effective medium model

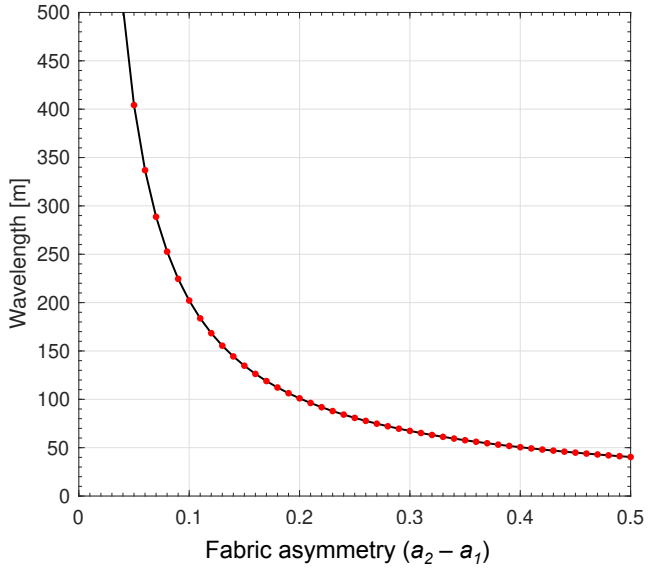
(Fujita et al. 2006)

$$\begin{bmatrix} s_{hh} & s_{vh} \\ s_{hv} & s_{vv} \end{bmatrix} = \left[\frac{\exp(jk_0 z)}{4\pi z}\right]^2 \cdot \left[\prod_{i=1}^{N} (RTR')_{\theta,\varepsilon,N+1-i}\right] \cdot [R\Gamma R']_{\theta,\beta,N} \cdot \left[\prod_{i=1}^{N} (RTR')_{\theta,\varepsilon,i}\right]$$



$a_2 - a_1 = 0.12$





Further reading

Young et al. (*accepted*) JGR

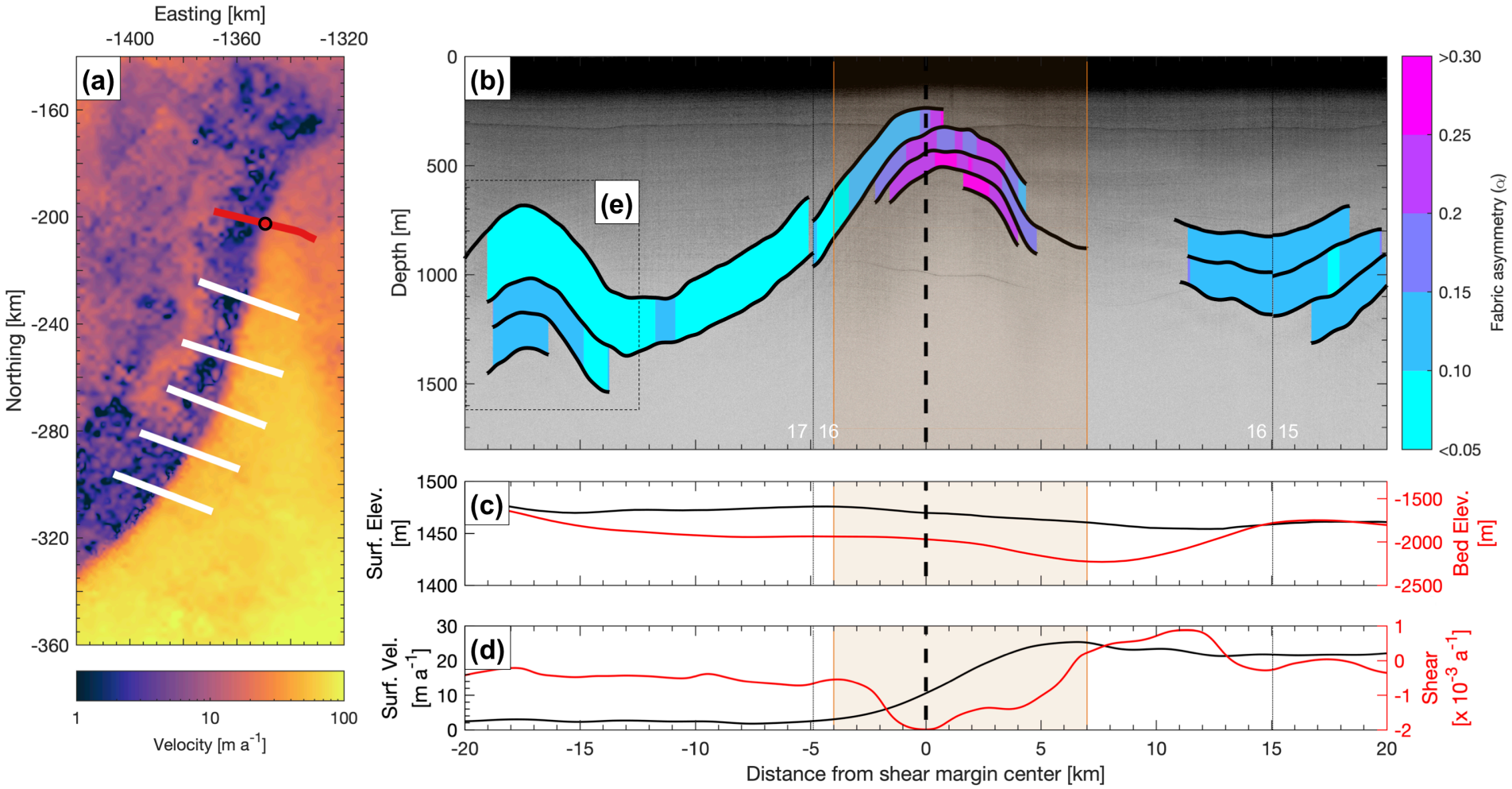
Running Commentary

❄ Although these birefringence traces are often treated as 'artifacts', we show that these patterns can indicate spatial changes in fabric strength.

❄ We relate the wavelength of the observed periodic power loss to fabric strength by inverting the effective medium model.

❄ The presence of birefringence loss also indicates that the fabric axes are oriented neither parallel nor perpendicular to the antenna planes, which gives us some indication as to the fabric orientation.

Application: Thwaites Glacier



Further reading: Young et al. (*accepted*) JGR

Running Commentary

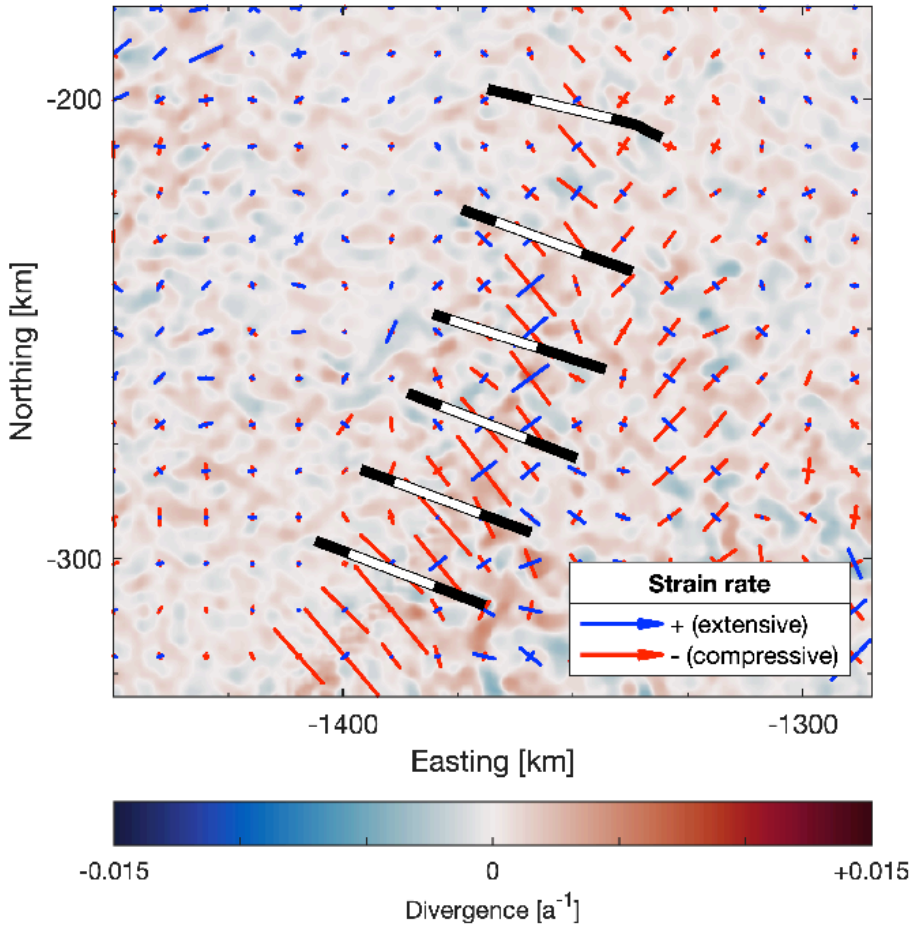
❄ Here, we show the change in fabric strength within radargrams orthogonally crossing the Thwaites Glacier Eastern Shear Margin.

❄ Fabric strength peaks at the location of maximum shear, and relaxes both inwards and outwards of the shear margin centre.

❄ The shape of these patterns is a reflection of the physical state of flow-induced fabric induced through lateral simple shear.

❄ The bump on the outward side (e) may indicate relic shear or historical flow.

Application: Thwaites Glacier

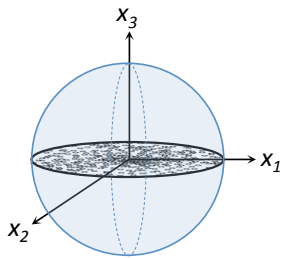
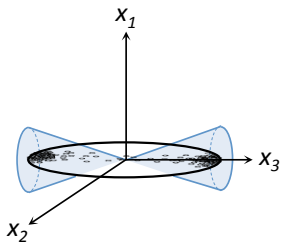
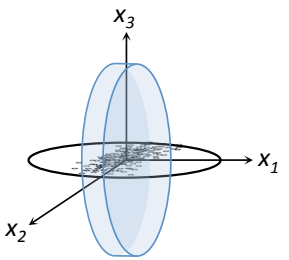
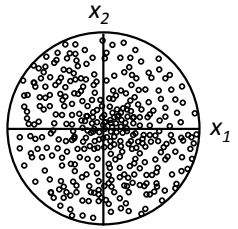
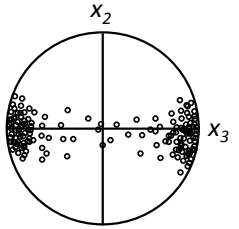
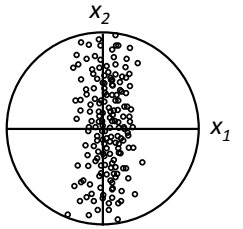
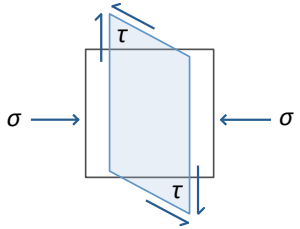
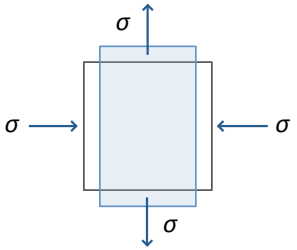


Further reading: Young et al. (*accepted*) JGR

❄ Surface strain axes are all observed to be neither parallel nor perpendicular to transect lines, therefore birefringence loss is expressed.

❄ Note that surface strain axes may vary through depth (to be covered in later slides).

❄ Because the morphology of the observed fabric variations represent its shear state, the detection of birefringent periodicity may help constrain lateral variations in fabric strength, and locate past and present shear margins.

	(a) Outward of ESM (slow/no flow)	(b) Within ESM boundary (high shear strain)	(c) Inward of ESM (fast flow)
Location			
COF type	Near-random	Horizontal pole	Vertical girdle
Fabric Envelope			
Schmidt plot			
Eigenvalues	$a_1 \approx a_2 \approx a_3 \approx 1/3$	$a_1 \approx a_2 << a_3$	$a_1 << a_2 \approx a_3$
Axis alignment	None	45° to shear plane	90° to extensional eigenvector
Deformation Regime	None / uniform 	Lateral simple shear, transverse compression 	Uniaxial compression, longitudinal extension 

Further reading: Young et al. (*accepted*) JGR

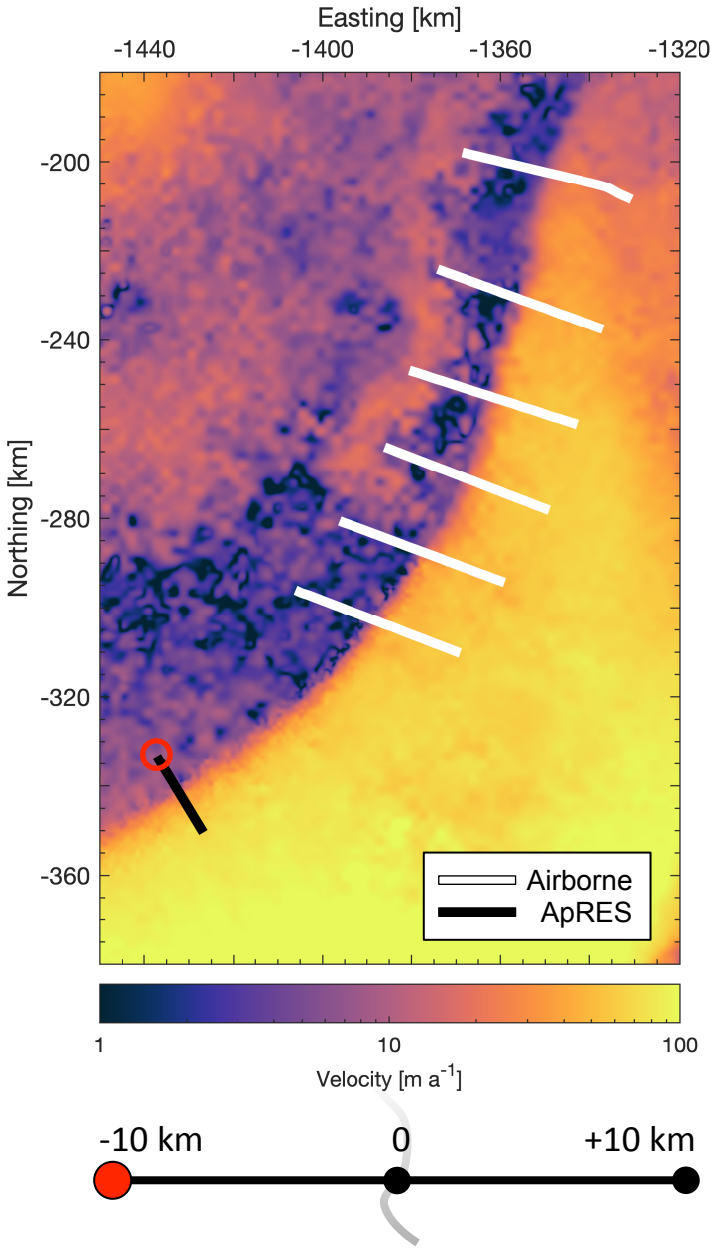
Running Commentary

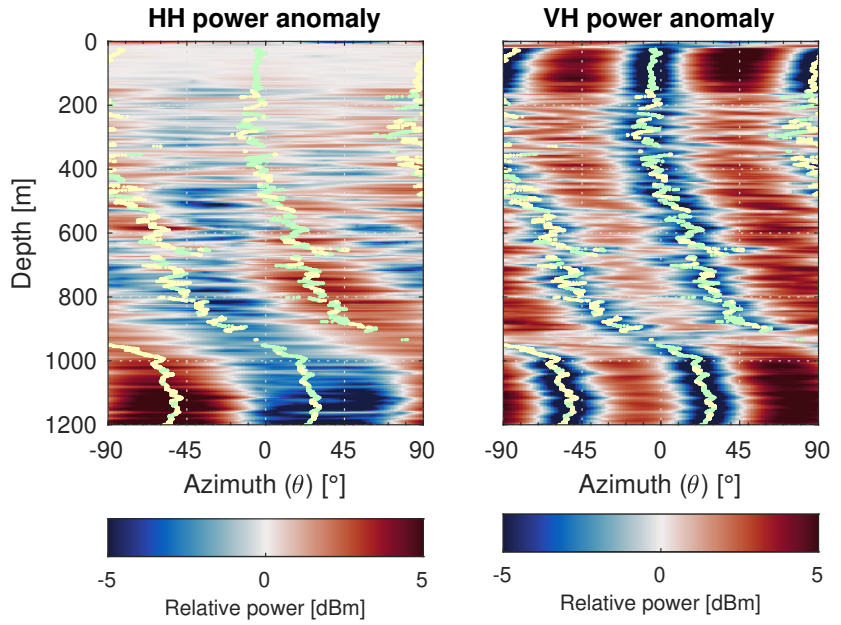
❄ Where ice is stagnant, fabric should be near-isotropic. The fabric strengths should be very weak, with any anisotropy a result of asymmetric strain, no matter how minimal.

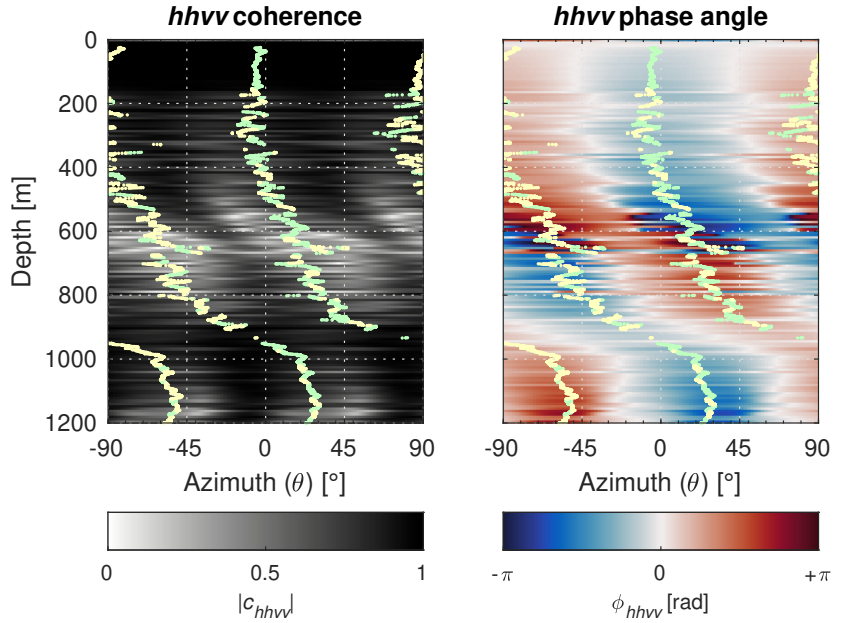
❄ Where ice is streaming, fabric should resemble more of a vertical girdle with moderate fabric strength. Because there may still exist some shearing, some in-column rotation may be evident.

❄ At the shear margin, fabric should tend towards a horizontal pole, aligned oblique to the shear plane. The detected fabric is strong due to high horizontal asymmetry.

❄ Shown here are end members, actual fabric is likely to be intermediate between the three bounds.





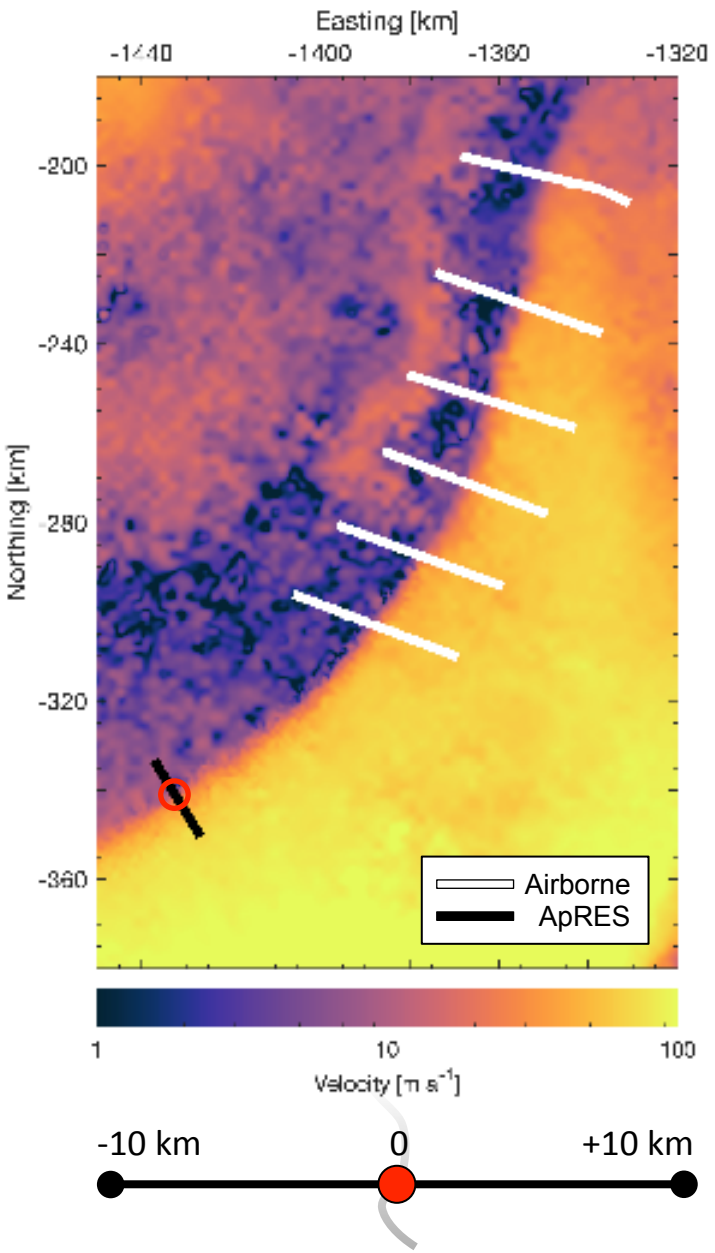


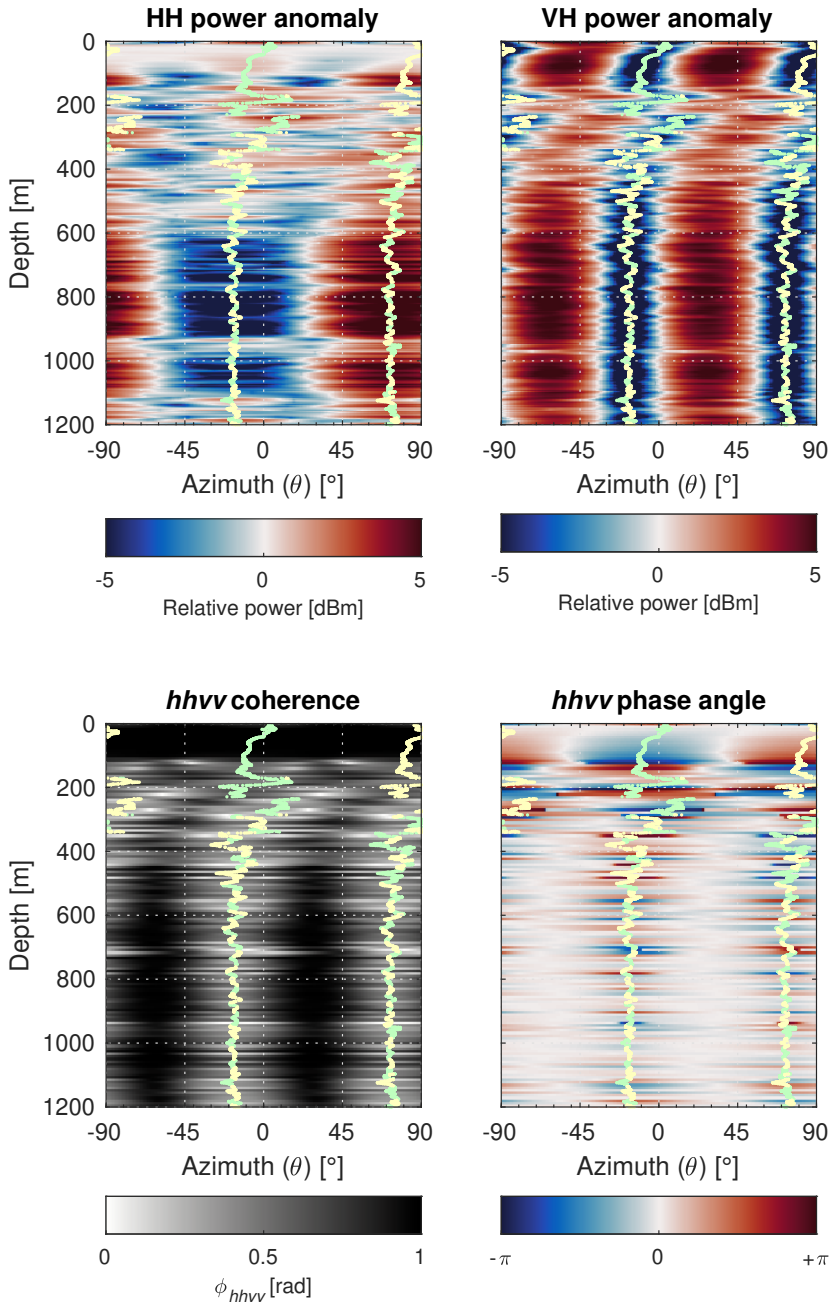
❄ Shown here are PRELIMINARY RESULTS.

❄ Observations (here, made using a quad-pol ApRES) are always more complicated than theory.

❄ Where ice is stagnant, there exists low variation in *hhvv* phase angle. The depth of the shallowest co-polarised node at 650 m indicates low fabric strength.

❄ A gradual rotation of the fabric eigenvectors through depth may indicate depth-dependent shear, or gradual flow reorientation (or other yet-identified mechanism).



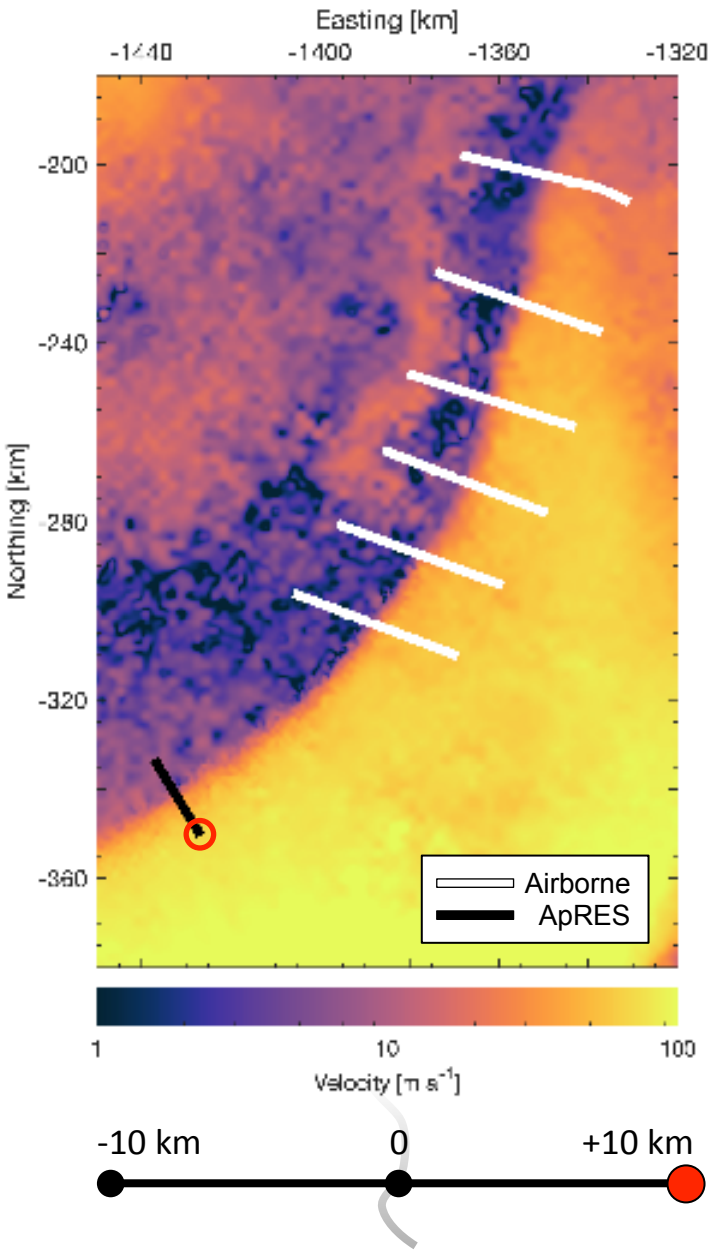


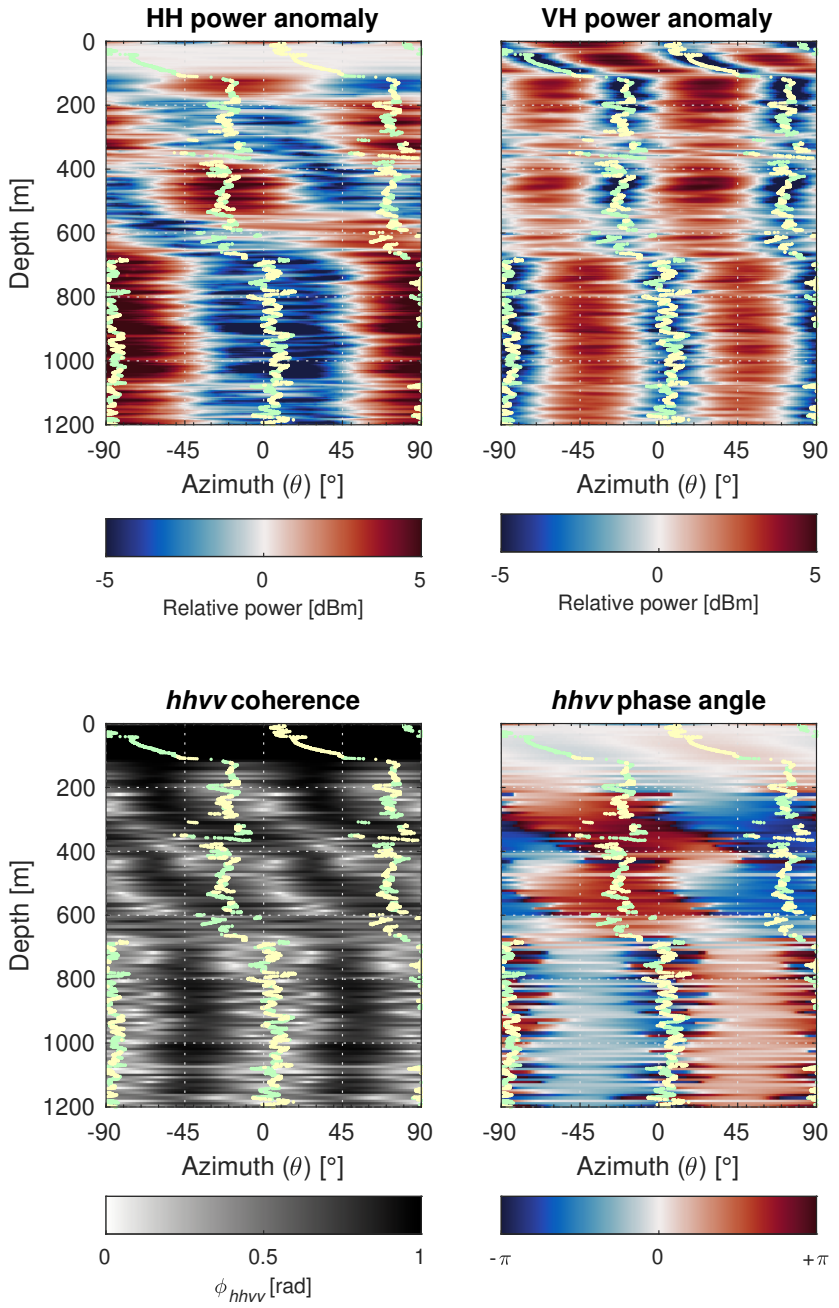
❄ Shown here are PRELIMINARY RESULTS.

❄ Observations (here, made using a quad-pol ApRES) are always more complicated than theory.

❄ At the point of maximum shear, graphical output is difficult to analyse, but the depth of the co-polarised nodes at multiples of 100 m, with increasing periodicity through depth, suggests high fabric strength.

❄ The cause of unconstrained eigenvector orientations at 200-500 m is unknown, but we believe it to indicate physical properties relating to its shear state.





❄ Shown here are PRELIMINARY RESULTS.

❄ Observations (here, made using a quad-pol ApRES) are always more complicated than theory.

❄ Inside the shear margin in fast flow, we observe complex fabric separated in three vertical sections.

❄ The top section (<100 m) shows rapid in-column rotation consistent with previous results (Jordan et al. *in review*). The rate of rotation is thought to increase with amount of shear strain.

❄ Abrupt jump dislocations in fabric orientation between the middle (100-650 m) and lower (>650 m) sections is puzzling, but could indicate different properties between the two sections.

Still to come…

- Forward modelling to deduce theoretical fabric response to varying shear scenarios
- Combination of (non-polarimetric) airborne and (polarimetric) ground-based observations to parameterise ice flow enhancement
- Relate possible flow regimes that could give rise to observed fabric through anisotropic modelling
- Use all outputs in conjunction with TIME and ITGC to investigate the historical stability of the Eastern Shear Margin

Thanks for viewing! I hope you found this work as exciting as I did!

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❄ Preliminary results show complex fabric that is likely to be a combination of one or more of the proposed mechanisms. It will be a challenge to disentangle these mechanisms to reveal the stories lying behind the observations.

❄ Although there are physical limitations on how far radar can resolve fabric observations, our work is part of a larger project (TIME, within the ITGC) that tackles the question of shear margin stability using a variety of observational and modelling methods. TIME hopes to reveal the stability of the Thwaites Glacier Eastern Shear Margin and its role in potential rapid retreat of the West Antarctic Ice Sheet.