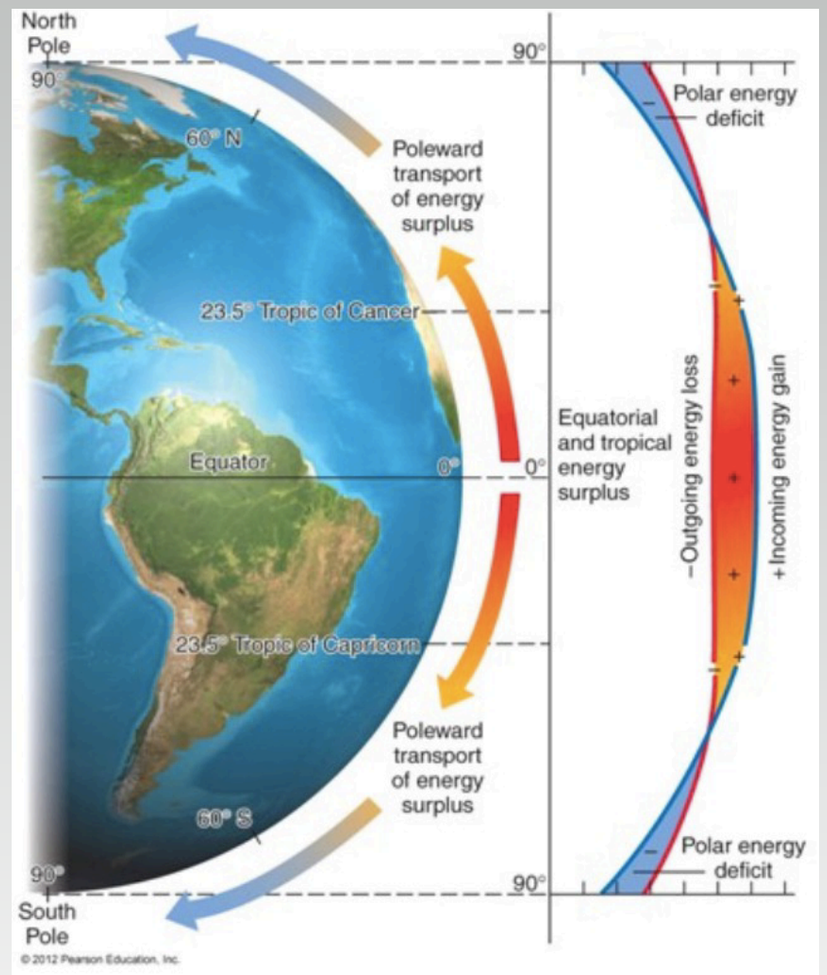
Beyond Forcing Scenarios: Predicting Climate Change through Response Operators in a Coupled General Circulation Model

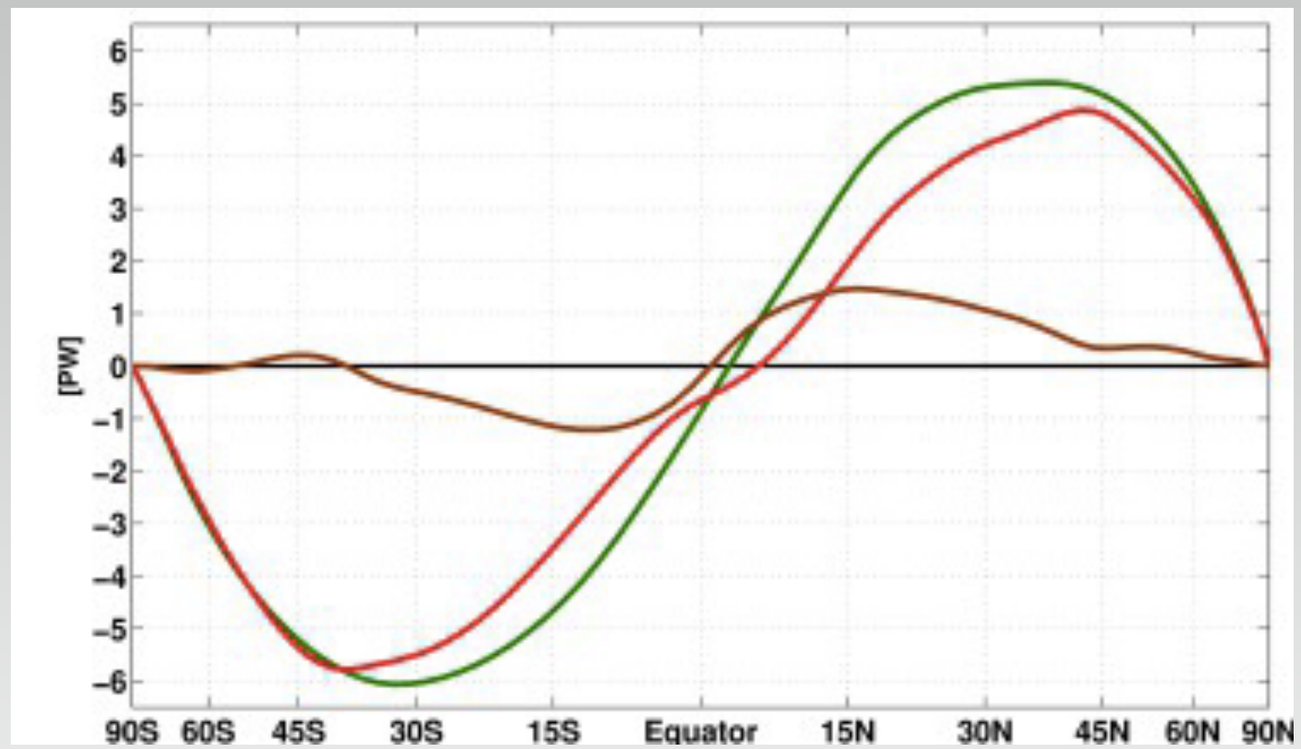
V. Lembo, V. Lucarini, F. Ragone

28th April 2021

Climate: a complex dynamical system with challenging features

- The climate is a non-equilibrium system whose evolution is driven by inhomogeneous absorption of solar radiation;



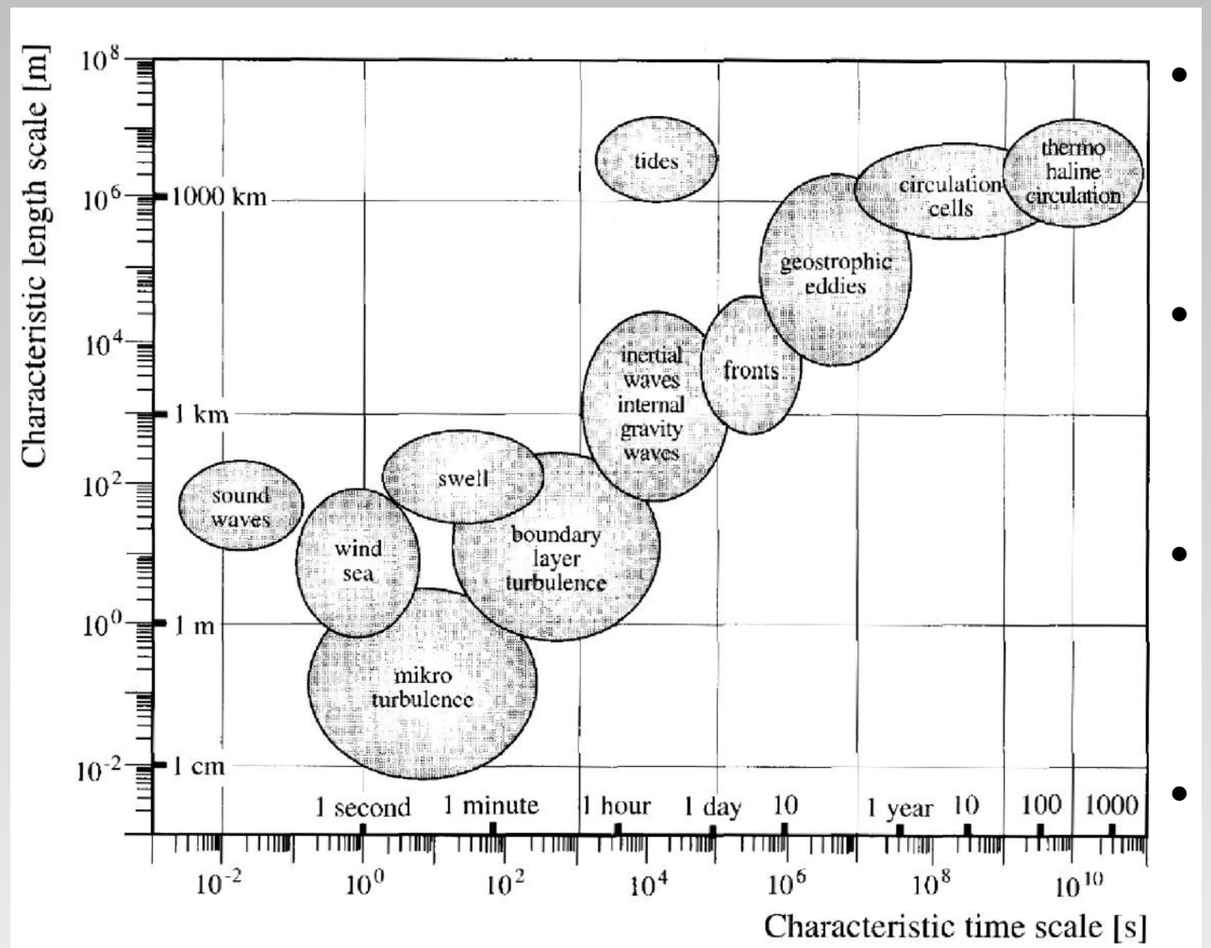


(Lembo et al. 2017)

Its steady state is not a thermodynamic equilibrium!

Climate: a complex dynamical system with challenging features

- The climate features variability on a vast range of spatial and temporal scales;



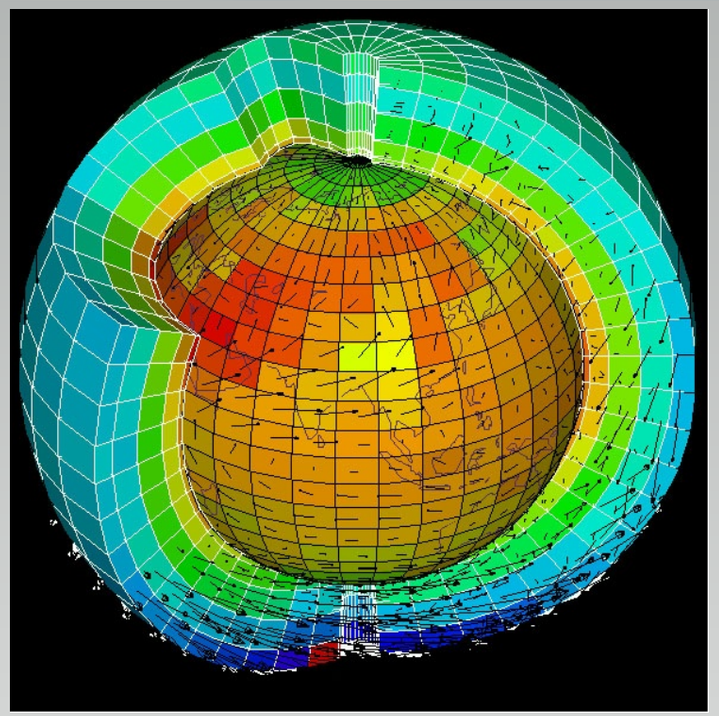
Burchard (2002)

Understanding the interplay between climate response and variability

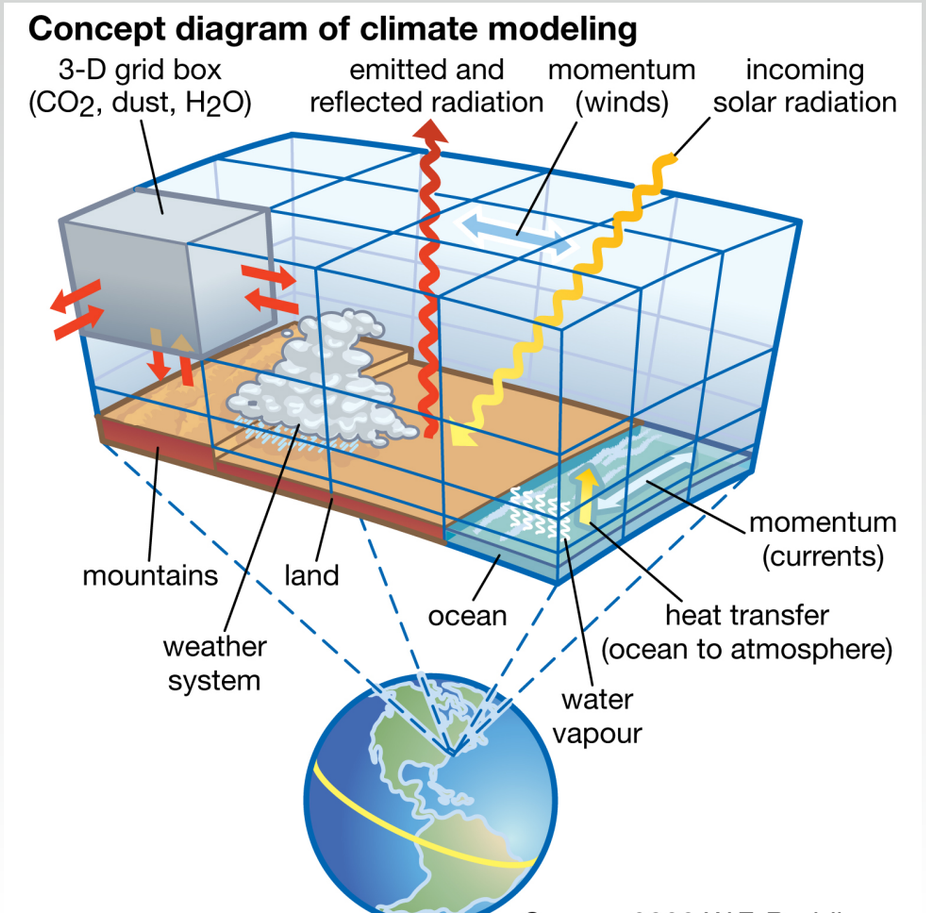
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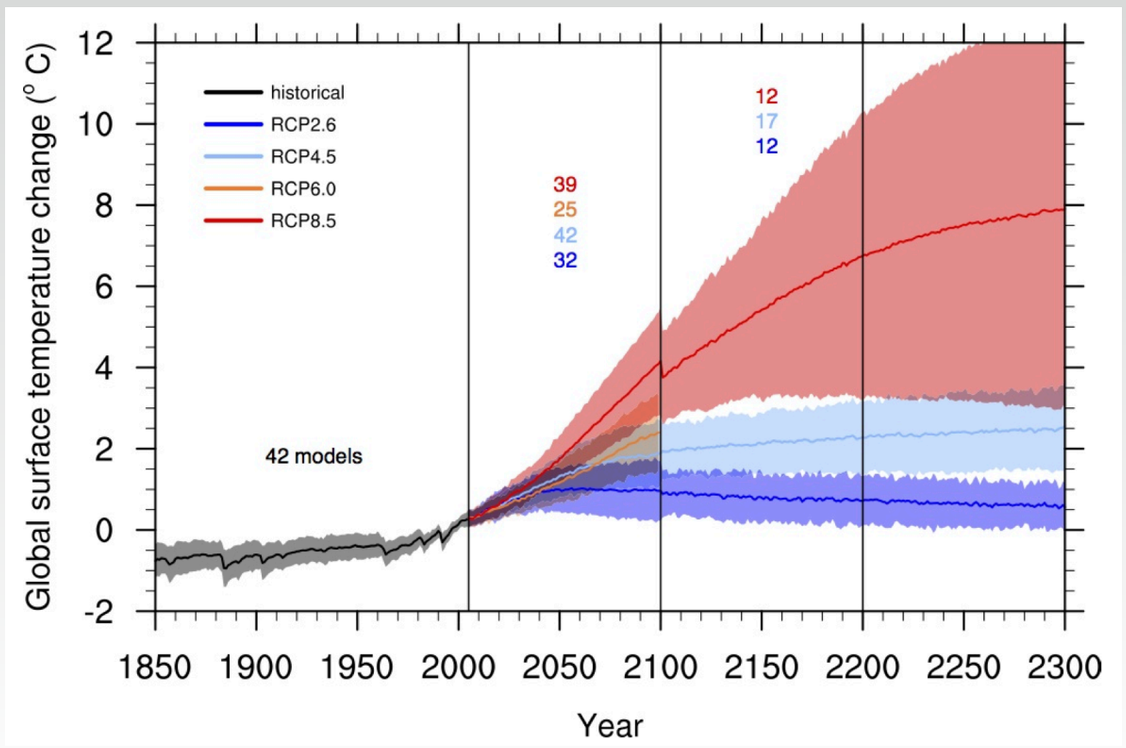
Investigating conditions for <u>smooth</u> vs. non-smooth response

Global Climate Models (GCM) are the most advanced tool for the study of climate evolution



- Ordinary differential equations are solved on a grid at every forecast step;
- Processes that are not explicitly solved in the equations (i.e. convection, radiative transfer, sub-grid scale turbulence) are parametrized;
- Boundary conditions are given to constrain the evolution (e.g. GHG concentrations);

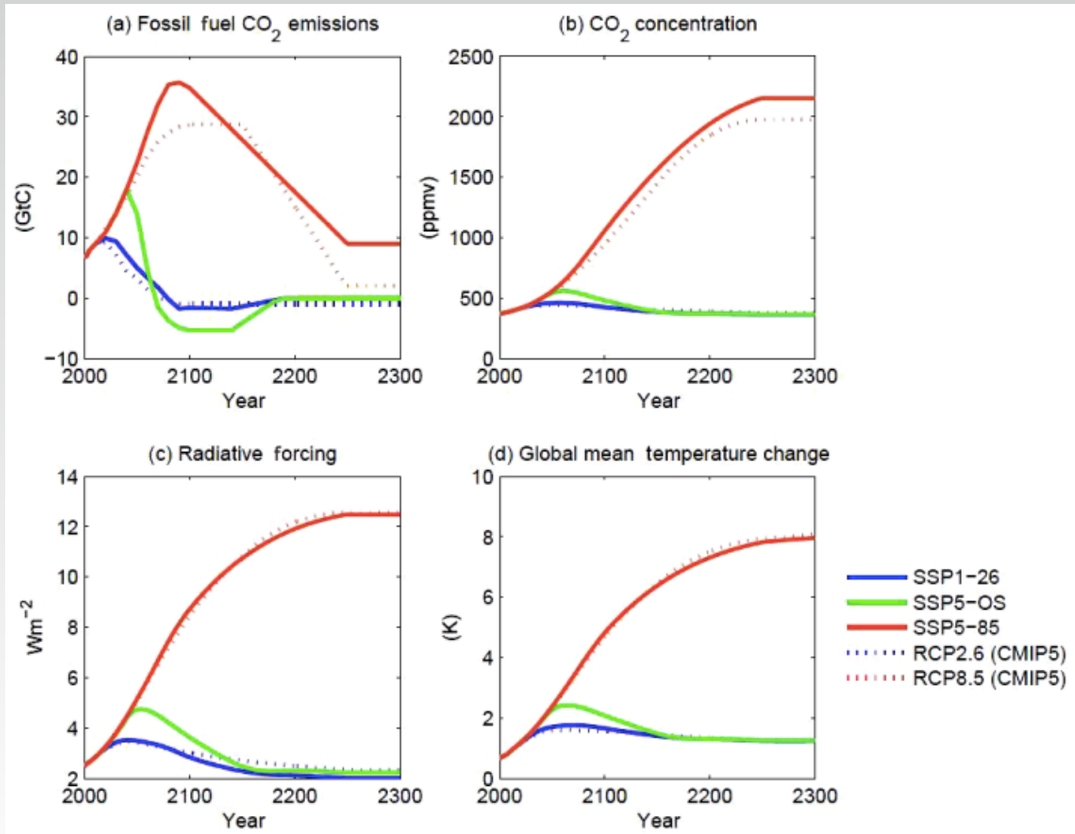




CMIP: a community effort to compare model simulations

Experiment short name	CMIP6 label	Experiment description	Forcing methods	Start year	End year	Minimum no. years per simulation	Major purpose
DECK experiments							
AMIP	*amip*	Observed SSTs and SICs prescribed	All; CO_2 concentration prescribed	1979	2014	36	Evaluation, variability
Pre-industrial control	*piControl* or *esm-piControl*	Coupled atmosphere–ocean pre-industrial control	CO_2 concentration prescribed or calculated	n/a	n/a	500	Evaluation, unforced variability
Abrupt quadrupling of CO_2 concentration	*abrupt-4×CO2*	CO_2 abruptly quadrupled and then held constant	CO_2 concentration prescribed	n/a	n/a	150	Climate sensitivity, feedback, fast responses
1 % yr^{-1} CO_2 concentration increase	*1pctCO2*	CO_2 prescribed to increase at 1 % yr^{-1}	CO_2 concentration prescribed	n/a	n/a	150	Climate sensitivity, feedback, idealized benchmark
CMIP6 historical simulation							
Past ~ 1.5 centuries	*historical* or *esm-hist*	Simulation of the recent past	All; CO_2 concentration prescribed or calculated	1850	2014	165	Evaluation

CMIP6 DECK experiments



CMIP6 future climate scenarios

(Eyring et al., 2016)

CMIP new phases -> exponential growth in computational resources!

Model	N. hours per sim. year	Energy per month	N. hours per month	Joule per sim. year
CM2.6 S	2.12×10^5	3.14×10^{12}	5.64×10^7	1.17×10^{10}
CM2.6 T	1.81×10^5	3.14×10^{12}	5.64×10^7	1.00×10^{10}
CM2.5 T	14 327	3.14×10^{12}	5.64×10^7	7.99×10^8
FLOR T	5844	3.14×10^{12}	$5.6 \times 10^7 4$	3.26×10^8
CM3 T	2974	3.14×10^{12}	5.64×10^7	1.66×10^8
ESM2G S	279	1.97×10^{12}	3.02×10^7	1.82×10^7
ESM2G T	235	1.97×10^{12}	3.02×10^7	1.53×10^7
CM4H T	7729	1.68×10^{12}	3.47×10^7	3.75×10^8
CM4L T	3277	1.68×10^{12}	3.47×10^7	1.59×10^8
ESM4L T	5340	1.68×10^{12}	3.47×10^7	2.59×10^8
ARPEGE5-NEMO T	5190	5.92×10^{12}	5.88×10^7	5.22×10^8
EC-Earth3.2 T	12 126	1.62×10^{12}	3.86×10^7	5.08×10^8
EC-Earth3.2 S	21 481	1.62×10^{12}	3.86×10^7	8.99×10^8
CESM1.2.2-NEMO T	59 100	9.00×10^{12}	6.76×10^7	7.87×10^9
MPI-ESM1 T	3363	1.30×10^{12}	2.65×10^7	1.64×10^8
NorESM1 S	1369	1.41×10^{12}	1.64×10^7	1.18×10^8
IPSL-CM6-LR S	2166	5.92×10^{12}	5.88×10^7	2.18×10^8
HadGEM3-GC2 T	6504	4.27×10^{12}	8.5×10^7	3.27×10^8

(Balaji et al., 2017)

Computational resources – CMIP6

CMIP3: 17 institutes (groups) and 25 models (40 TB)
- Total years simulated: 70,000
- Individual models simulated 500 to 8,400 years (mean: 2,800 yr)
- Individual groups simulated on average 70,000/17 = 4,100 yr

x50

CMIP5: 26 institutes (groups) and 60 models (2 PB)
- Total years simulated: 330,000 (estimate Oct 2014)
- Individual models simulated on average 330,000/60 = 5,500 yr
- Individual groups estimated on average 330,000/26 = 13,000 yr

→ **4.3 Mio files**

CMIP6: length of simulations similar to CMIP5, but higher resolution models, larger ensemble sizes, more diverse experiment structure

→ **Factor of 20: 36 PB in 86 Mio files**

→ **Factor of 50: 90 PB in 215 Mio files**

→ **Careful planning required to determine subset of CMIP6 model output to best serve WHOI community**

According to Taylor & Balaji (2015) and Denvil (2015)

(Ummenhofer, 2019)

Each model simulation hopefully comes with an ensemble of realizations, crucial to sample internal variability (Deser et al. 2012)

Can we do better?

- Models can be ranked according to their performances in many (less -> more) smart ways;
- Theoretical considerations about the nature of the climate attractor can help constraining the expected response to forcing;
- Forcing scenarios can be selected based on actual needs

—> actual simulations with a probe forcing contain information about the intrinsic response of the forced system;

- Predictability of the forced response should be tested against non-smooth effects of internal variability (e.g. resonances, tipping points, climate surprises);

A tentative algorithm



A tentative algorithm



Several options, e.g.:

- Find the autocovariance matrix of the unperturbed state and employ some form of the fluctuation-dissipation theorem (FDT; Kubo 1957; Leith 1975);

- Find the invariant measure of the unperturbed system and investigate its correction wrt. a reference state using the response theory (RT; Kubo 1966; Ruelle 1998; 1999; 2009);

A tentative algorithm

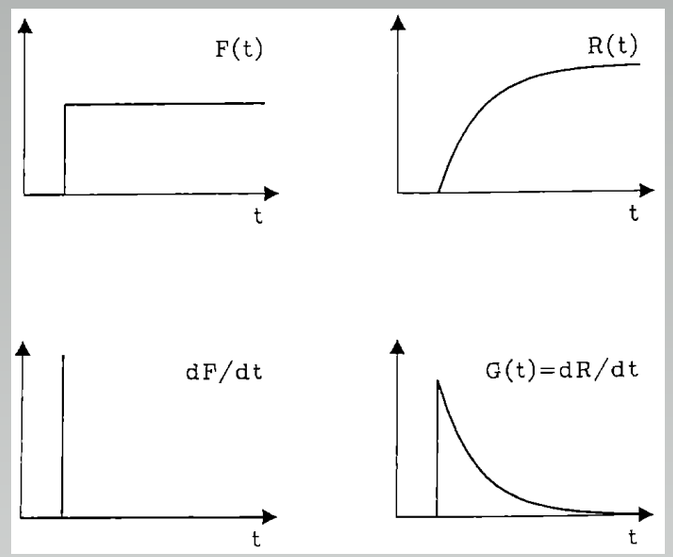


- The FDT lies on the assumption that the unperturbed system is in thermodynamic equilibrium;

-> in a perturbative approach, assumptions are made on the invariant measure (Gaussian (e.g. North et al. 1993), kernel-based (Cooper and Haynes 2011)) or a reduced phase space is used (Majda et al. 2009)

- Response Theory (RT) is a generalization of FDT for weakly perturbed systems, when the chaotic hypothesis holds;

The cold-start problem (Hasselmann et al., 1993)



Let us assume that an observable ϕ responds to an external forcing F, that has the shape of a step function at time t=0, through a simple linear feedback equation:

$$\frac{d\phi}{dt} = -\lambda\phi + \alpha F \qquad (1)$$

The transient linear response of the observable is written as:

$$\Phi(t) = \int_0^t \frac{dF}{d\sigma}(\sigma)R(t-\sigma)d\sigma = \int_0^t F(\sigma)G(t-\sigma)d\sigma \qquad (2)$$

Solving for the observable, we obtain the response function R as:

$$R(t) = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right) \qquad (3)$$

The cold-start problem (Hasselmann et al., 1993)

- For a generic non-degenerate linear system, the response can be represented by a linear combination of “normal modes” with complex eigenvalues $\mu_j = \lambda_j - i\omega_j$:

$$R(t) = \sum_j \frac{\alpha_j}{\mu_j}\left(1 - e^{-\mu_j t}\right) \qquad (4)$$

- The impulse function, or Green’s function, can be in this context defined as the time derivative of the response:

$$G(t) = \frac{dR}{dt} = \sum_j \alpha_j e^{-\mu_j t} \qquad (5)$$

The response theory

- It is the theoretically rigorous counterpart of the heuristically justified Hasselmann's approach;
- Let us consider a generic dynamical system:

$$\dot{x} = F(x) \qquad \text{where} \qquad F \in R^n \qquad \text{is smooth}$$

note that the condition is achieved for high-dimensional chaotic systems when the chaotic hypothesis is realized;

- Let us perturb the dynamics with a perturbation of the form $\Psi(x,t) = X(x)f(t)$. The expectation value of the observable ϕ is:

$$\langle \Phi_f(t) \rangle = \langle \Phi \rangle_0 + \sum_{n=1}^{\infty} \langle \Phi \rangle_f^{(n)}(t) \qquad (6)$$

where <> indicate the expectation value and $\Phi_f^{(n)}$ the nth order perturbation. The 1st order contribution reads:

$$\langle \Phi \rangle_f^{(1)}(t) = \int d\sigma_1 G_{\Phi}^{(1)}(\sigma_1) f(t - \sigma_1) \qquad (7)$$

The response theory

Remarks

- The Green's function is 0 for t<0, given the causality;
- The problem reduces to the impulse-response problem, when:

$$f(t) = kH(t) \quad \text{where} \quad H(t) = 0 \text{ for } t < 0 \text{ and } H(t) = 1 \text{ for } t \geq 0 \qquad (8)$$

<u>Key is retrieving the observable-dependent Green's function!</u>

N.B.: Unlike in Hasselmann et al. 1993, no assumption is here made on the nature of the relation between the observable and the forcing. We only restrict ourselves to the 1rd order perturbation.

Procedure for the retrieval of the Green's function

1. We pose ourselves in the comfortable situation of a step-like forcing (cfr. Eq. 8)

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2. We compute the Green's function from an ensemble of model realizations

3. We use the convolution as in Eq. 7 to predict the response to a transient forcing of the same amplitude

Experimental setup

- Model version: MPI-ESM-CR v1.2 - ECHAM6 (T31L31) + MPIOM (GR30L40);
- 2 ensembles: 20 members for each ensemble with same initial conditions;
- Integration period: 2000 years for the predictor, 1000 years for the predictand;
- Observables: 2-metre temperature (T2m), ocean heat uptake (OHU), AMOC at 26N and ACC;
- Forcings:

PREDICTOR

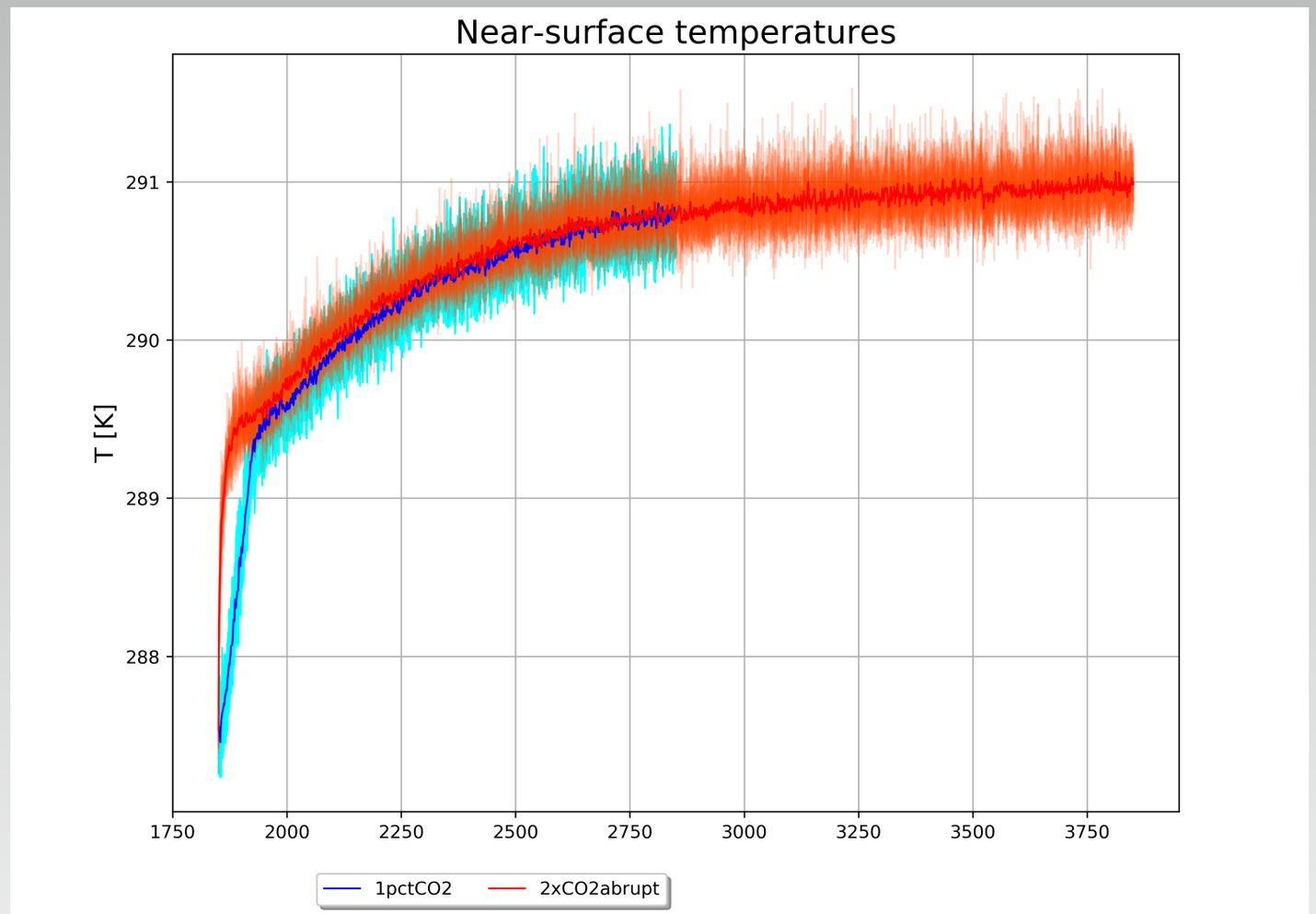
- A step increase in CO2 concentrations at time t=0 until 2x the preindustrial value (0-2000 yrs);

$$G_{\Phi}^{(1)}(t) = f_{2CO2}\frac{d}{dt}\Phi_{2CO2}^{(1)}(t)$$

PREDICTAND

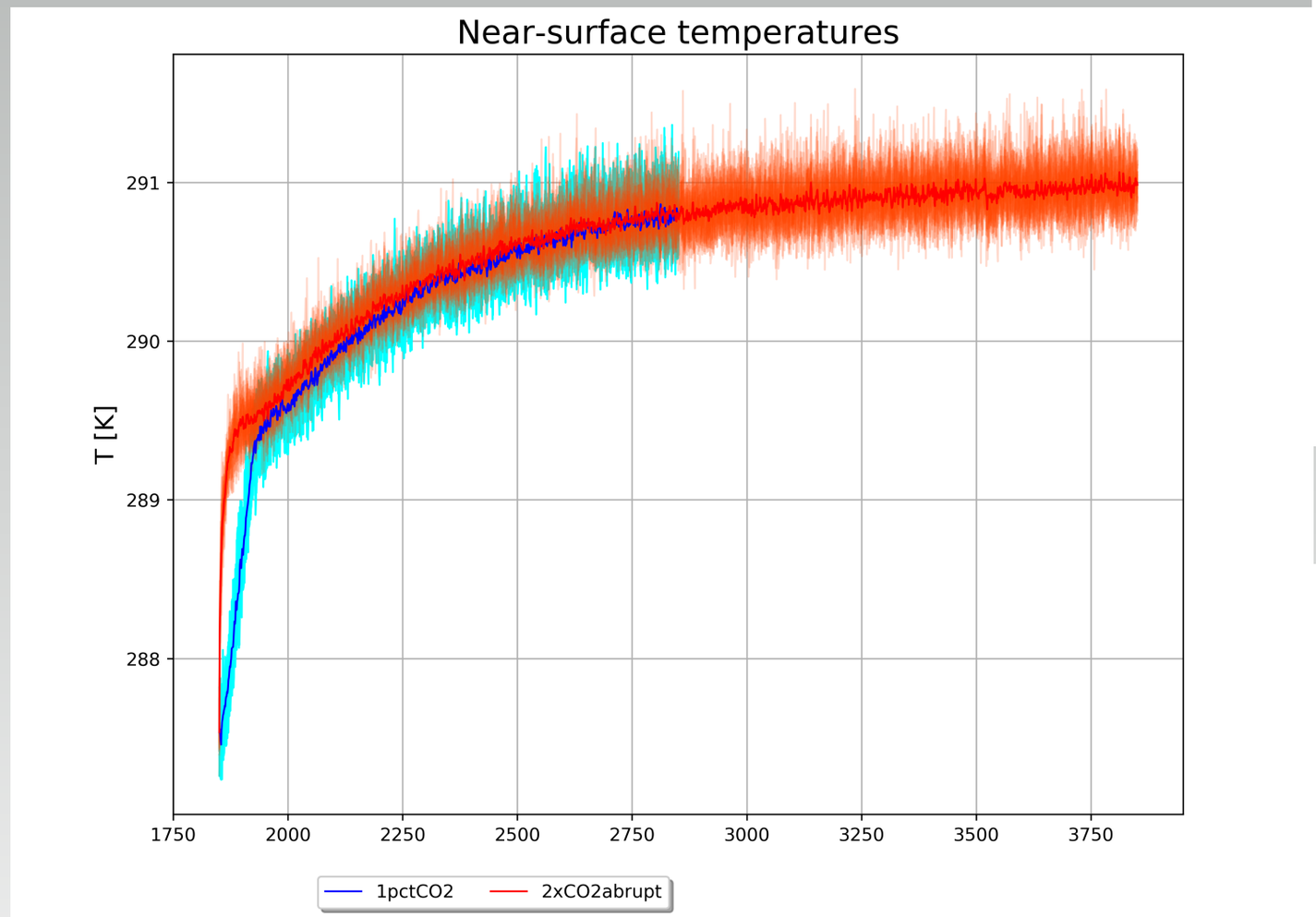
- A ramp function experiment:
  1. CO2 linear increase by 1% until doubling (0-70 yrs);
  2. Stationary CO2 with 2x the preindustrial value (70-1000 yrs);

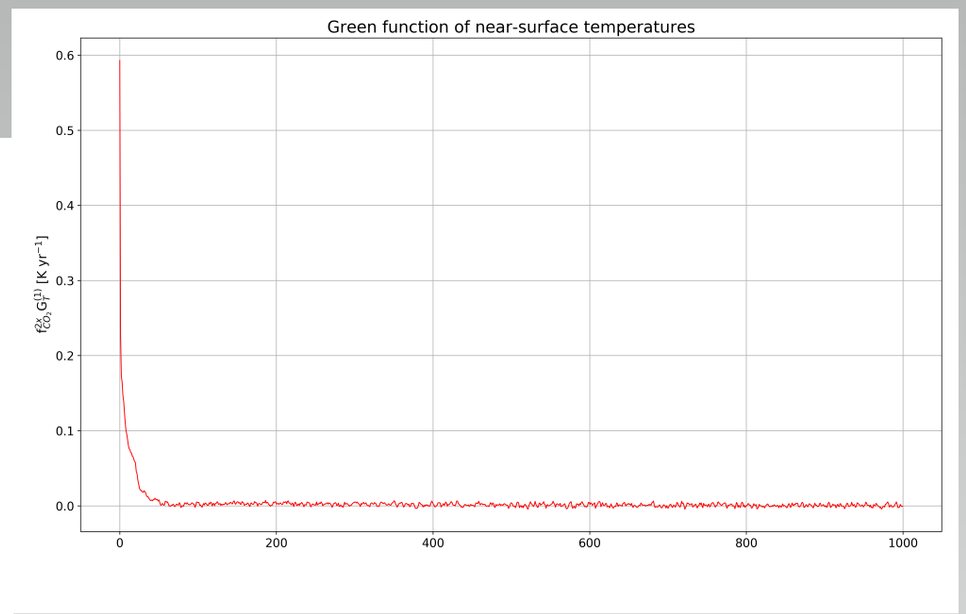
Prediction of 2-metres temperature

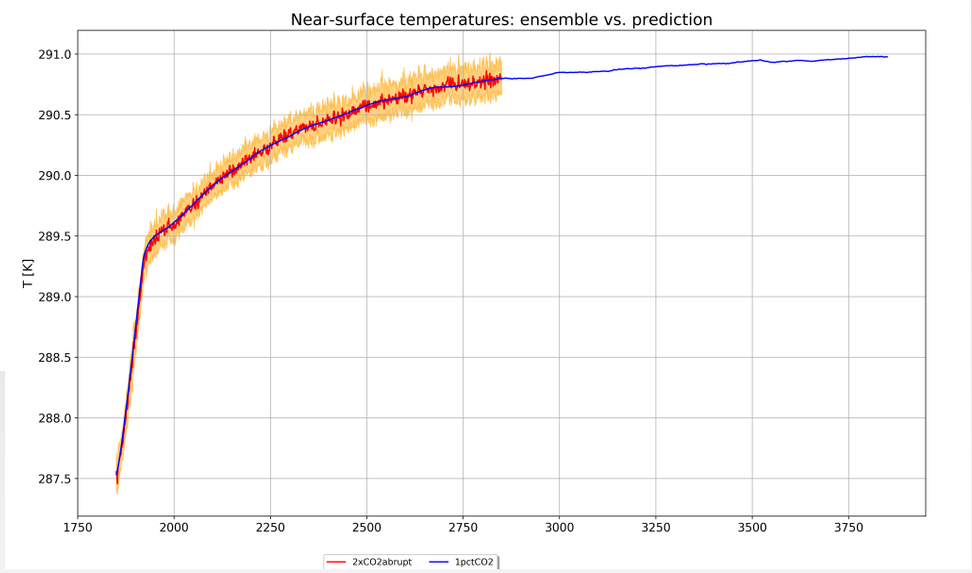


- Fast response different in the two scenarios;
- Same amount of heating at the end of the ramp CO2 increase;
- Slow response as exponential relaxation;

Prediction of 2-metres temperature

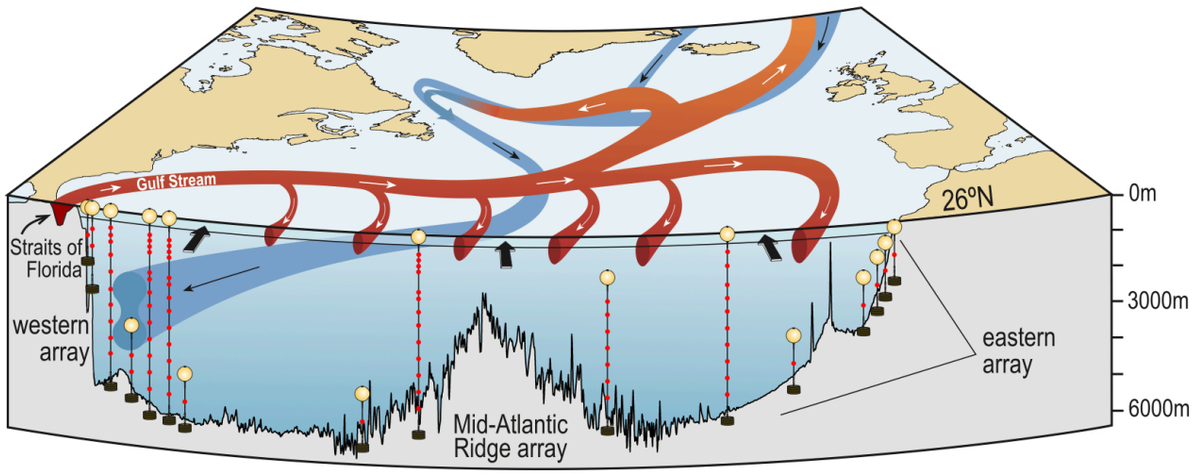




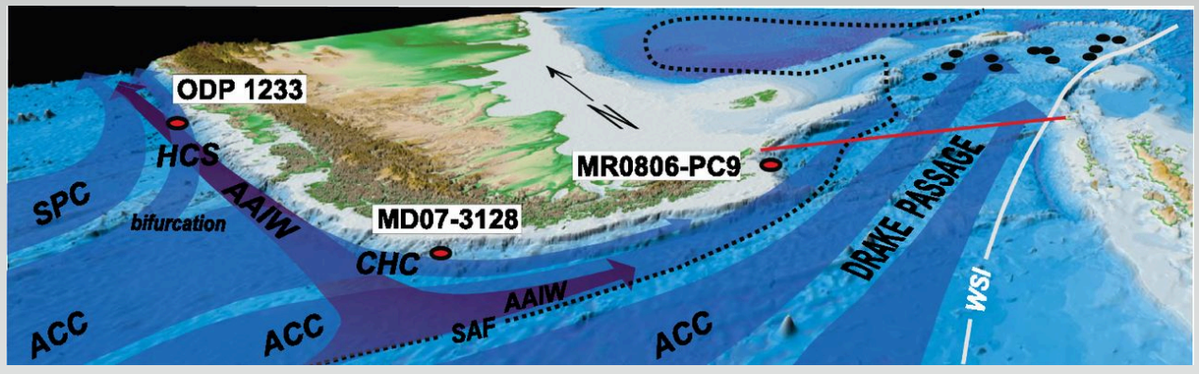


Prediction of oceanic large-scale circulation

Atlantic meridional overturning circulation (AMOC)

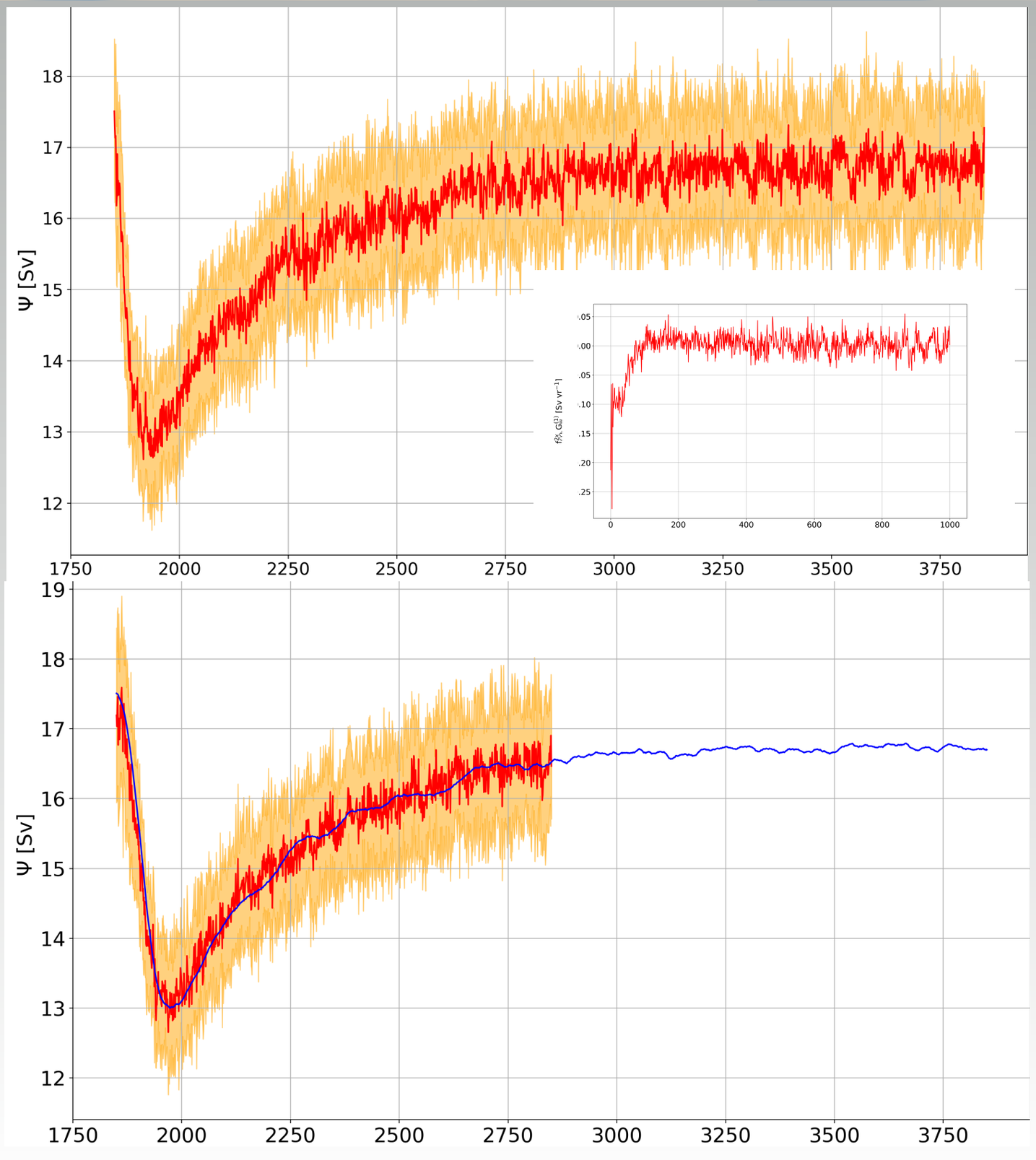


Antarctic circumpolar current (ACC)



AMOC at 26N

2xCO2 step forcing at t=0 (predictor)



1% CO2 increase (predictand)

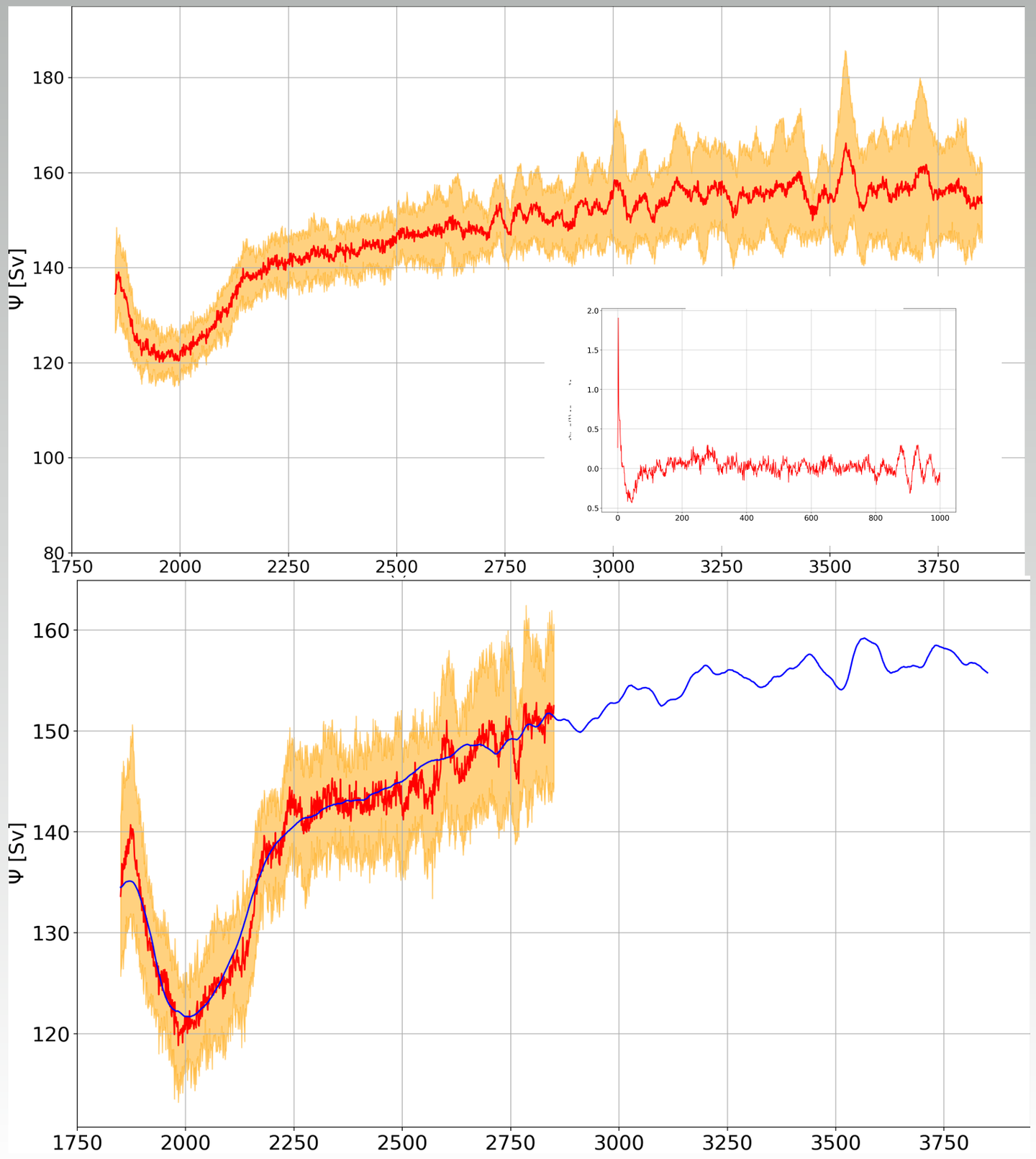
The Green function is shown in the inset for the first 1000 years

Red: ensemble mean evolution

Blue: predicted evolution with linear response

Prediction of ACC at Drake passage

2xCO2 step forcing at t=0 (predictor)



1% CO2 increase (predictand)

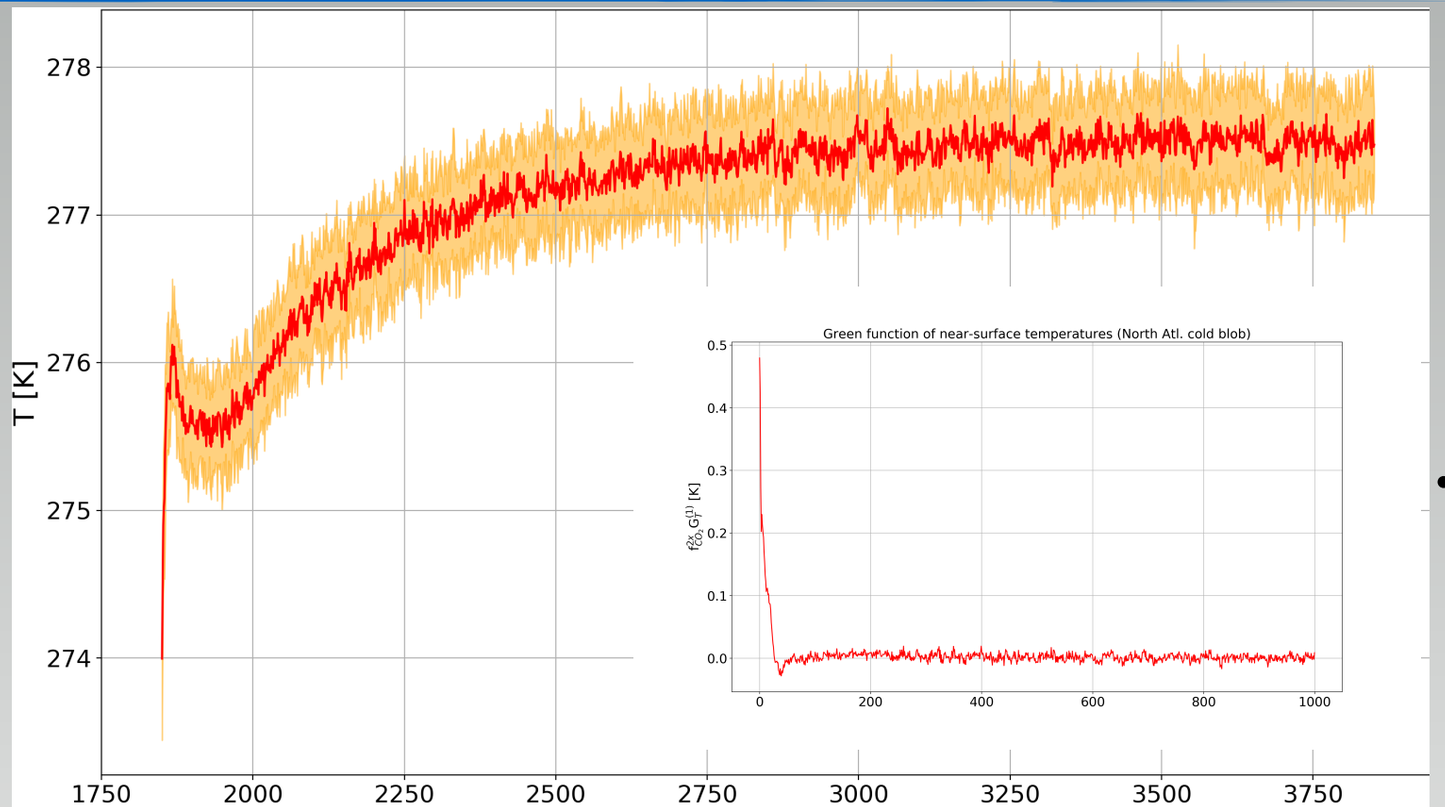
The Green function is shown in the inset for the first 1000 years

Red: ensemble mean evolution

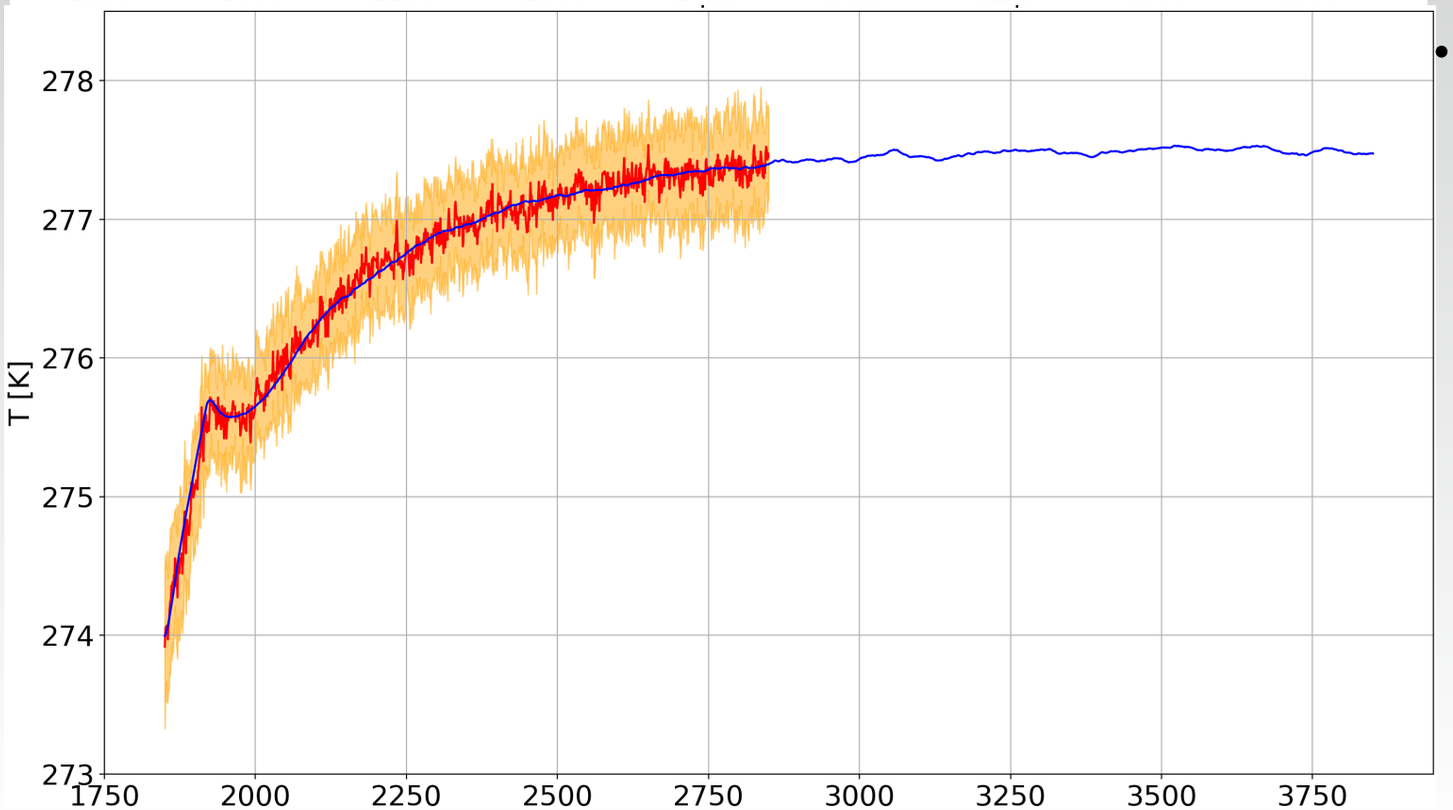
Blue: predicted evolution with linear response

Prediction of SSTs in the extratropical Northern Atlantic

2xCO2 step forcing at t=0 (predictor)



1% CO2 increase (predictand)



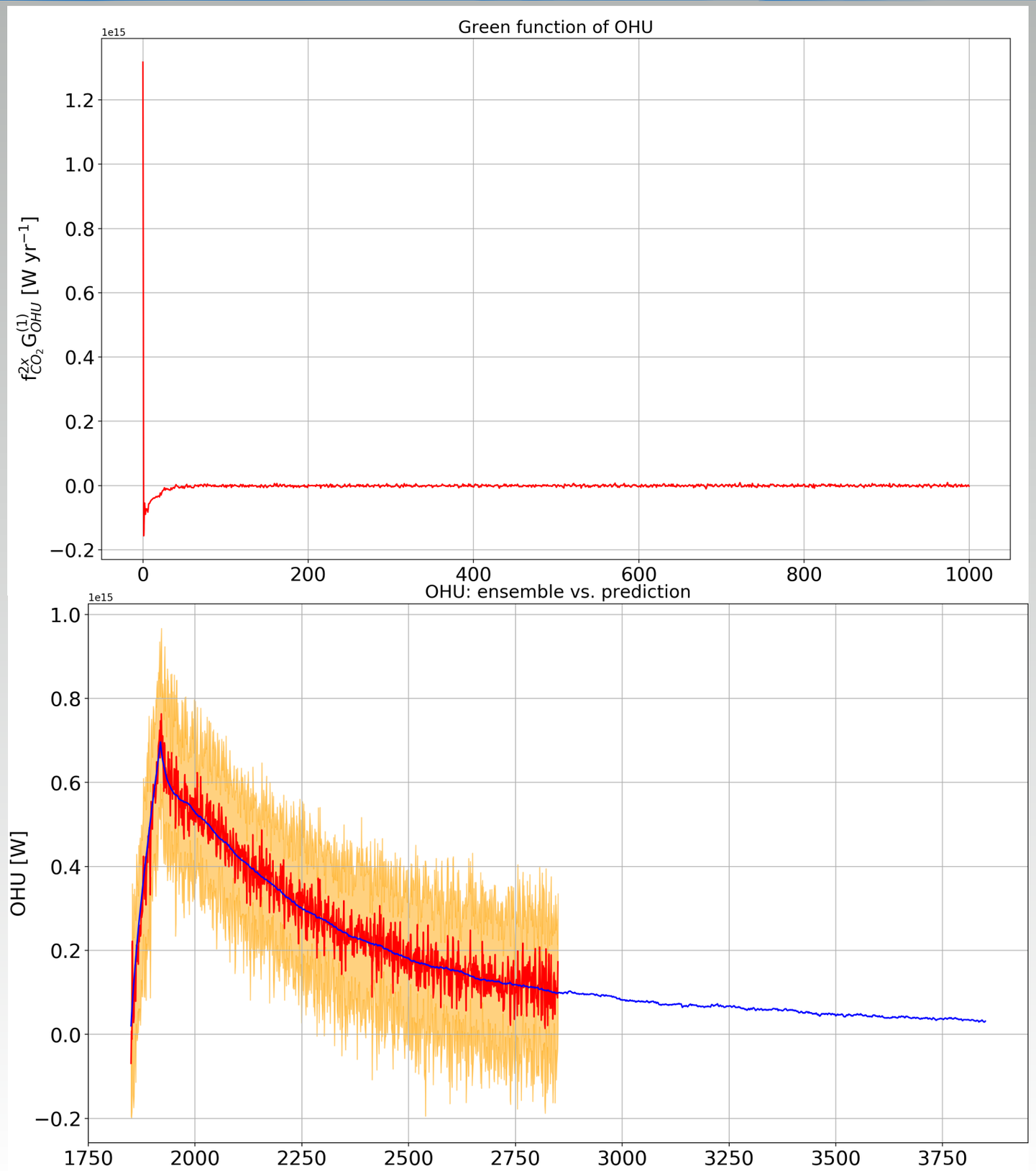
- The negative feedback triggered by the fast warming projects on the linear mode of the response;
- This suggests that the response of a global-scale quantity can be used as a predictor for the response of a local observable (cfr. Lucarini 2018)

Ocean Heat Uptake (OHU): A Green function with a Dirac's delta at t=0

Green function

$$\lim_{t\to 0}\langle\Phi\rangle_f(t) \neq 0$$

1% CO2 increase (predictand)



The OHU is representative of the TOA energy imbalance at these scales

The OHU anomaly is largest at t=0, as the step forcing drives the system out of balance

OHU tends to vanishing values as the system approaches statistically steady state

Conclusions

- We have applied an algorithm to retrieve the first order Green function for the response of a generic observable in a smoothly forced climate system;
- For suitable choices of the forcing (i.e. a step forcing), the retrieval of the Green function is straightforward;
- We demonstrated the strength of the algorithm predicting the ensemble evolution of key observables in a typical forcing scenario;
- The response of the overturning circulation is to a large extent skillfully predicted via the linear response;
- A key feature of the regional climate response, the North Atlantic cold blob, is also well predicted;
- The response theory is a valid alternative to running fully coupled climate models for various applications related to climate prediction;

References

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- *Ruelle, D., 1998, General linear response formula in statistical mechanics, and the fluctuation-dissipation theorem far from equilibrium. Phys. Lett. A 245 , 220–224, DOI: 10.1016S0375-9601(98)00419-8*

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Ruelle's response theory/2

- Perturbation : $\dot{x} = \tilde{F}(x,t) = F(x) + \varepsilon e(t) X(x)$
- Change in expectation value of a smooth Φ:

$$\langle \Phi \rangle (t) = \langle \Phi \rangle_0 + \sum_{k=1}^{\infty} \varepsilon^k \langle \Phi \rangle^{(k)} (t)$$

- Linear term: $\langle \Phi \rangle^{(1)} (t) = \int d\sigma \, G_\Phi^{(1)} (\sigma) e(t-\sigma)$
- Linear Green: $G_\Phi^{(1)} (t) = \int \rho_0 (dx) \Theta(t) X \cdot \nabla S^t \Phi$
- Linear suscept: $\chi_\Phi^{(1)} (\omega) = \int dt \exp[i\omega t] G_\Phi^{(1)} (t)$

obeys KK relations $\langle \Phi \rangle^{(1)} (\omega) = \chi_\Phi^{(1)} (\omega) e(\omega)$

Ruelle's response theory/3

$$G^{(1)}_{\Phi,X}(t) = \int dx \rho_0(x) \Theta(t) X(x) \cdot \nabla \left(\Pi^t \Phi(x) \right)$$

FDT ↓ ?

$$G^{(1)}_{\Phi,X}(t) = -\int dx \rho_0(x) \frac{\nabla \cdot \left(\rho_0(x) X(x) \right)}{\rho_0(x)} \Pi^t \Phi(x) = C\left[\sigma(x) \Phi(x(t)) \right]$$

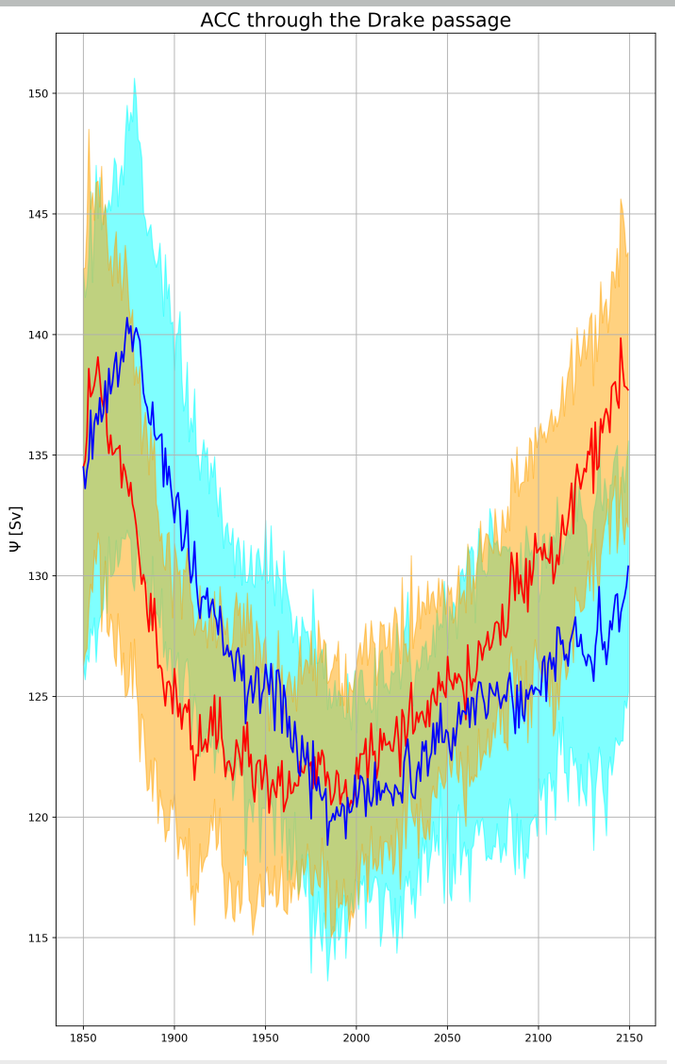
$$\sigma(x) = \sigma_A(x) + \sigma_B(x) = -\nabla \cdot X(x) - X(x) \cdot \nabla \log\left(\rho_0(x) \right)$$

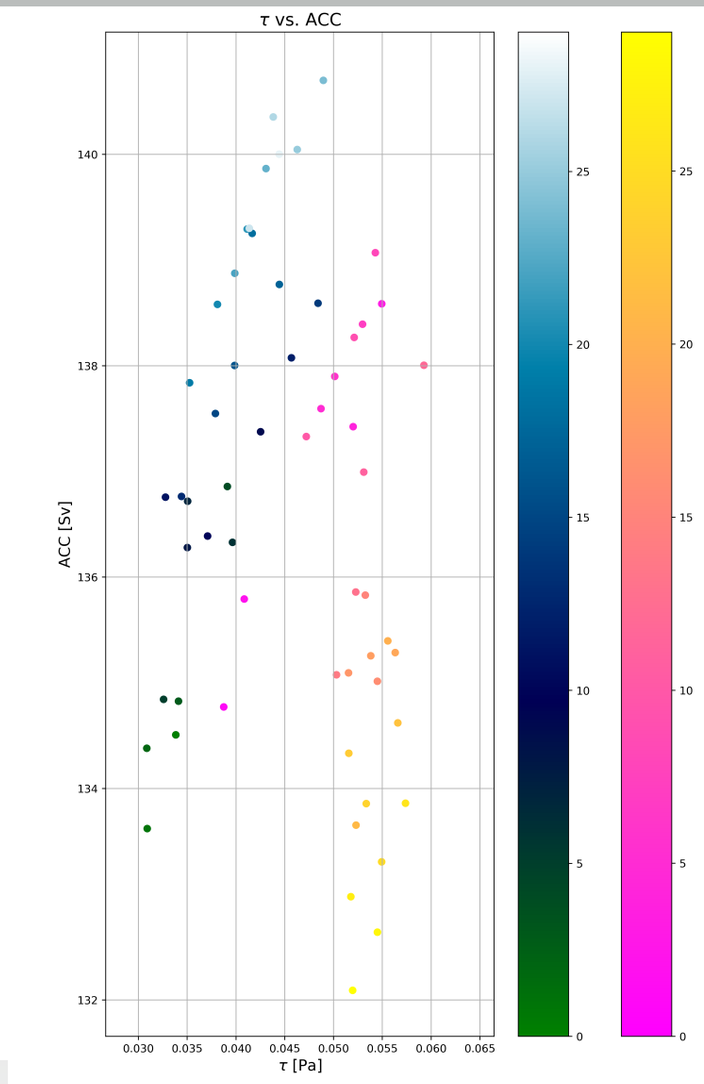
Note that the integral in the Green's function is equivalent (under the ergodic principle) to:

$$\int \mathcal{L}_0^t \rho(x) \Psi(x) \mathrm{d}x = \int \rho(x) S_0^t \Psi(x) \mathrm{d}x$$

with Lt the Perron-Frobenius or transfer operator and St the Koopman or composition operator

Failure of the RRT





- The linear response theory fails in predicting the fast response;
- The surface wind correlates with the ACC at the fast timescales (<30 yrs);
- The timescale of the forcing is critical, suggesting some kind of nonlinear interaction at these scales;