Revisiting the crevasse-depth calving law with a 2D elastic model

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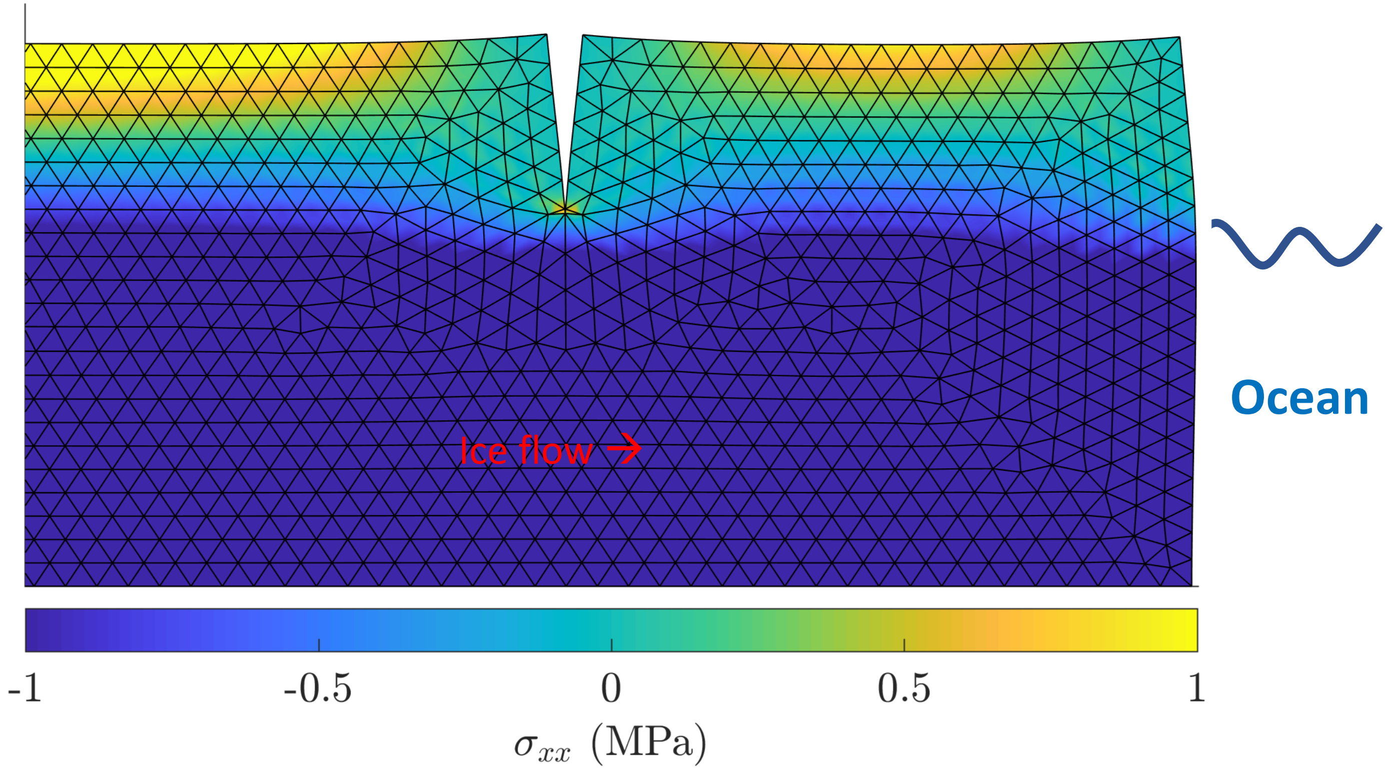
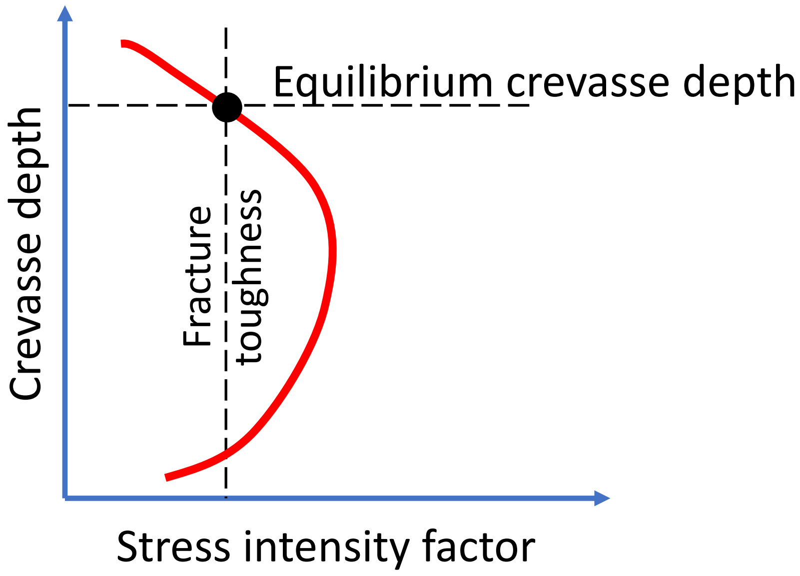




We investigate the depth of surface and basal crevasses near the front of tidewater glaciers by using a 2D elastic/stress intensity factors framework, thus accounting for the feedback of crevassing on the stress field

We predict substantially deeper crevasses than the Nye zero-stress model on which the crevasse-depth calving law is based

Motivation

The “crevasse-depth” law for calving from tidewater glaciers has been very widely used in glaciology (e.g. *Benn 2007, Nick 2010, Nick 2013, Choi 2018, Amaral 2020*)

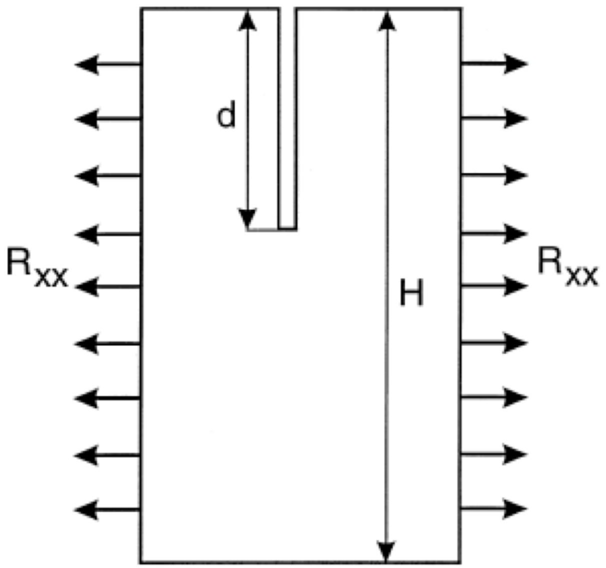
The law usually applies the Nye zero-stress assumption (*Nye 1955*) to calculate crevasse depth; that is, a crevasse penetrates to the depth where the sum of the deviatoric stress (opening) and any water pressure (opening) balance the cryostatic pressure (closing)

This approach does not take account of the feedback of crevassing on the stress field

Linear elastic fracture mechanics can be used to take account of this feedback (e.g. *van der Veen 1998*), but the weighting functions often used assume idealised stress distributions and are not always appropriate for the boundary conditions of tidewater glaciers (*Jimenez & Duddu, 2018*)

Here we perform 2D elastic modelling of a tidewater glacier terminus and apply the displacement correlation method to provide a fully-general estimate of the stress intensity factor associated with a crevasse

By this method we estimate crevasse depth while taking full account of the feedback of crevassing on the stress field



van der Veen 1998

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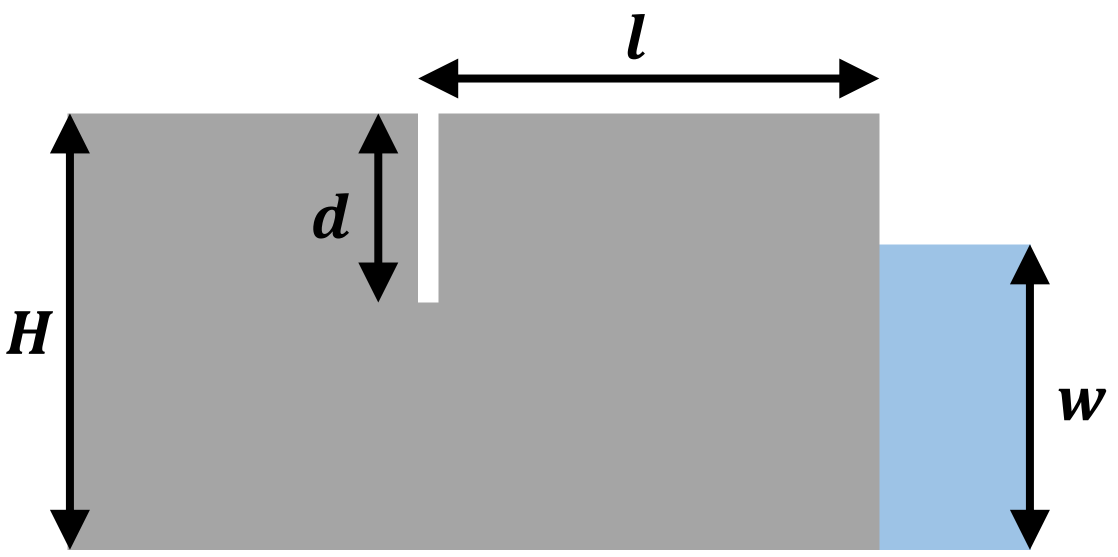
2D elastic model of a tidewater glacier



Equations (see e.g. *Sergienko 2010, J. Geophys. Res.*)
Momentum balance

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} - \rho_i g = 0$$

Relate strain ε to displacements u by

$$\varepsilon = \frac{1}{2}(\nabla u + \nabla u^T)$$

Relate stress to strain by Hooke's Law

$$\varepsilon_{xx} = \frac{1+\nu}{E}[(1-\nu)\sigma_{xx} - \nu\sigma_{zz}]$$
$$\varepsilon_{zz} = \frac{1+\nu}{E}[(1-\nu)\sigma_{zz} - \nu\sigma_{xx}]$$
$$\varepsilon_{xz} = \frac{1+\nu}{E}\sigma_{xz}$$

Parameters
Ice density ρ_i = 917 kg/m^3
Water density ρ_w = 1030 kg/m^3
Gravity g = 9.81 m/s^2
Young's modulus E = 10 GPa
Poisson's ratio ν = 0.5 (incompressibility)

Simulations
We consider three ice thicknesses H = 100, 500, 900 m, three distances of crevasse from front l = 0.25H, H, 3H and 6 water depths $\frac{\rho_w w}{\rho_i H}$ between 0.5 and 1

We impose an explicit crevasse (either dry surface or water-filled basal) of width 1 m and consider 11 depths $\frac{d}{H}$ between 0.01 and 0.99

Total simulations 3 x 3 x 6 x 11 $\sim$ 600 for each of surface and basal crevasses
Finite element solution in MATLAB

Boundary conditions
All boundaries stress free except for submerged portion of front where we apply water pressure
'Roller' boundary conditions at left and bottom (zero displacement normal to boundary)

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Estimating crevasse depth

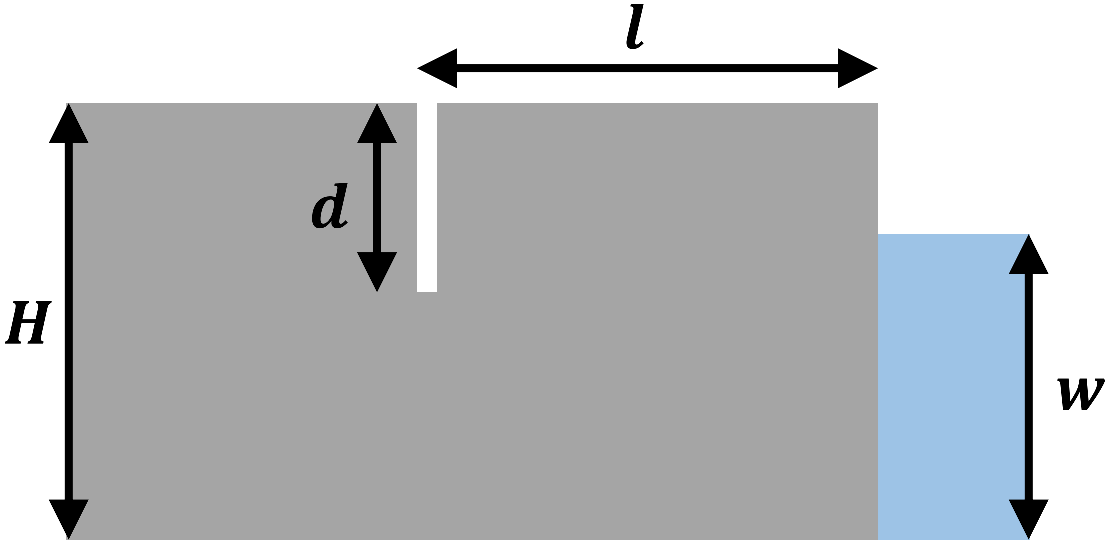
Consider here only mode I (tensile failure) crevassing

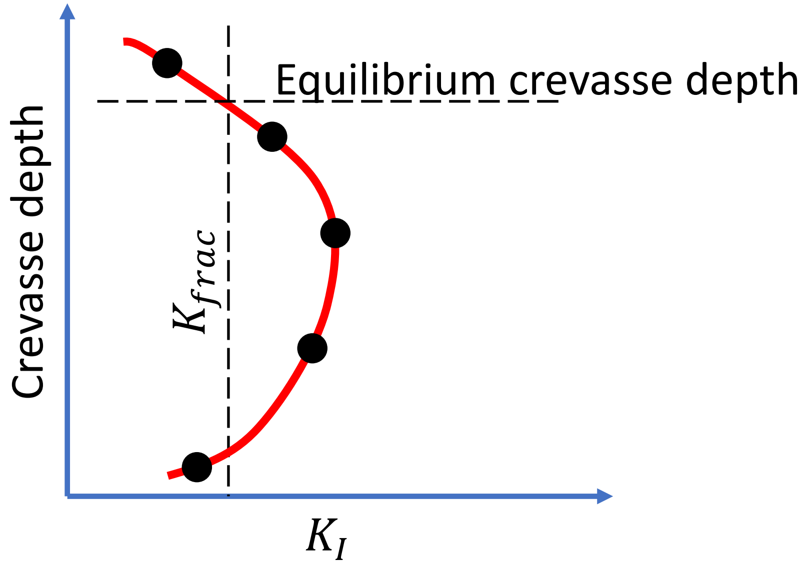
For each simulation, calculate a stress intensity factor K_I associated with the crevasse using the displacement correlation method (e.g. *Jimenez & Duddu, 2018, J. Glac.*)

The equilibrium crevasse depth is defined as the deepest crevasse having a stress intensity factor that exceeds the fracture toughness of ice: $K_I > K_{frac}$ where K_{frac} = 0.25 MPa m$^{1/2}$

If the stress intensity factor does not exceed the fracture toughness for any crevasse depth we define the crevasse depth as 0

This gives an equilibrium crevasse depth for each value of H, l and w. We compare with the current crevasse-depth calving law in which surface and basal crevasse depth are calculated using the Nye zero-stress assumption





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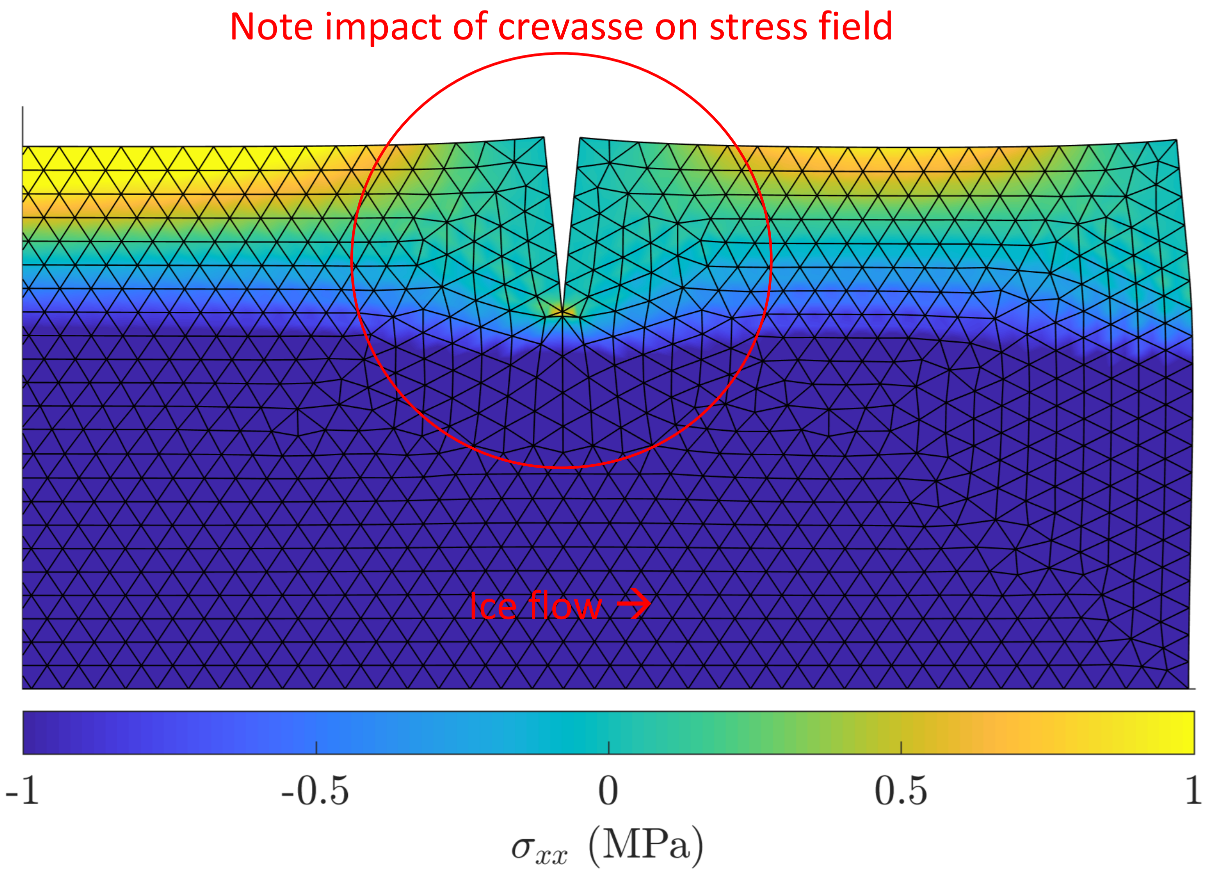
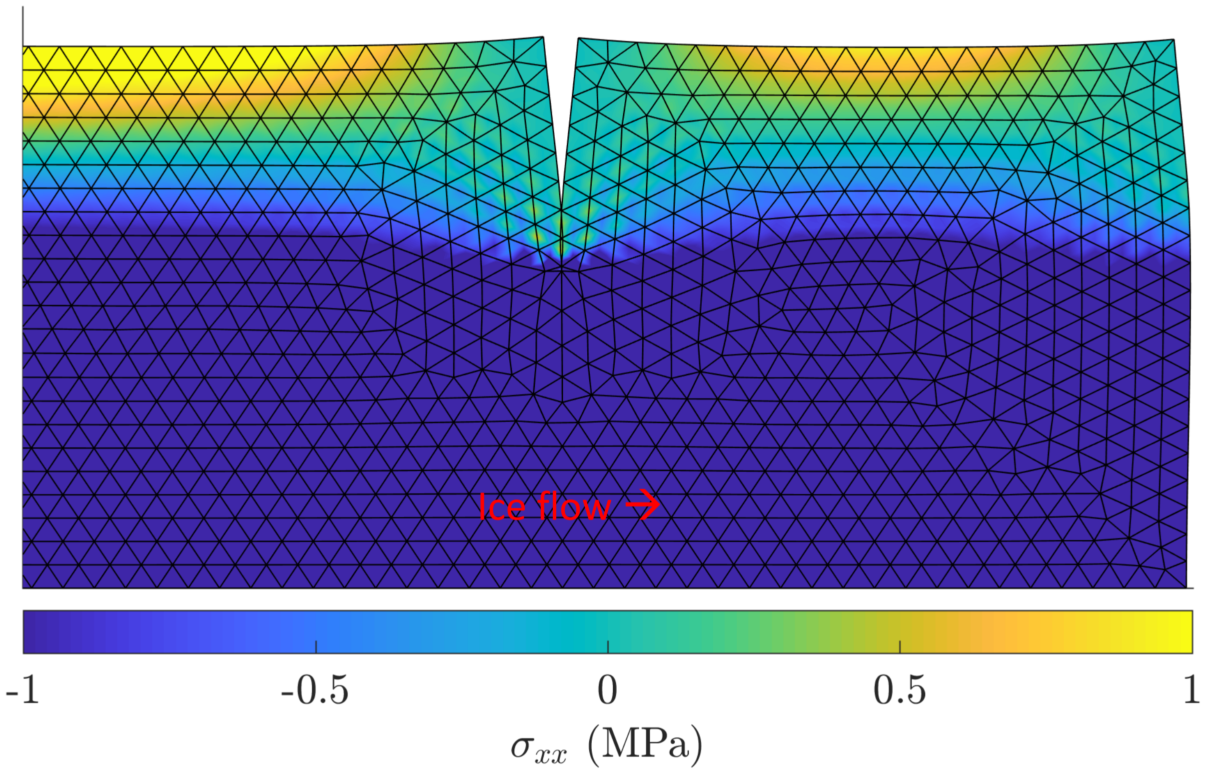
Example: H = 900 m, l = H, $\frac{\rho_w w}{\rho_i H}$ = 0.8





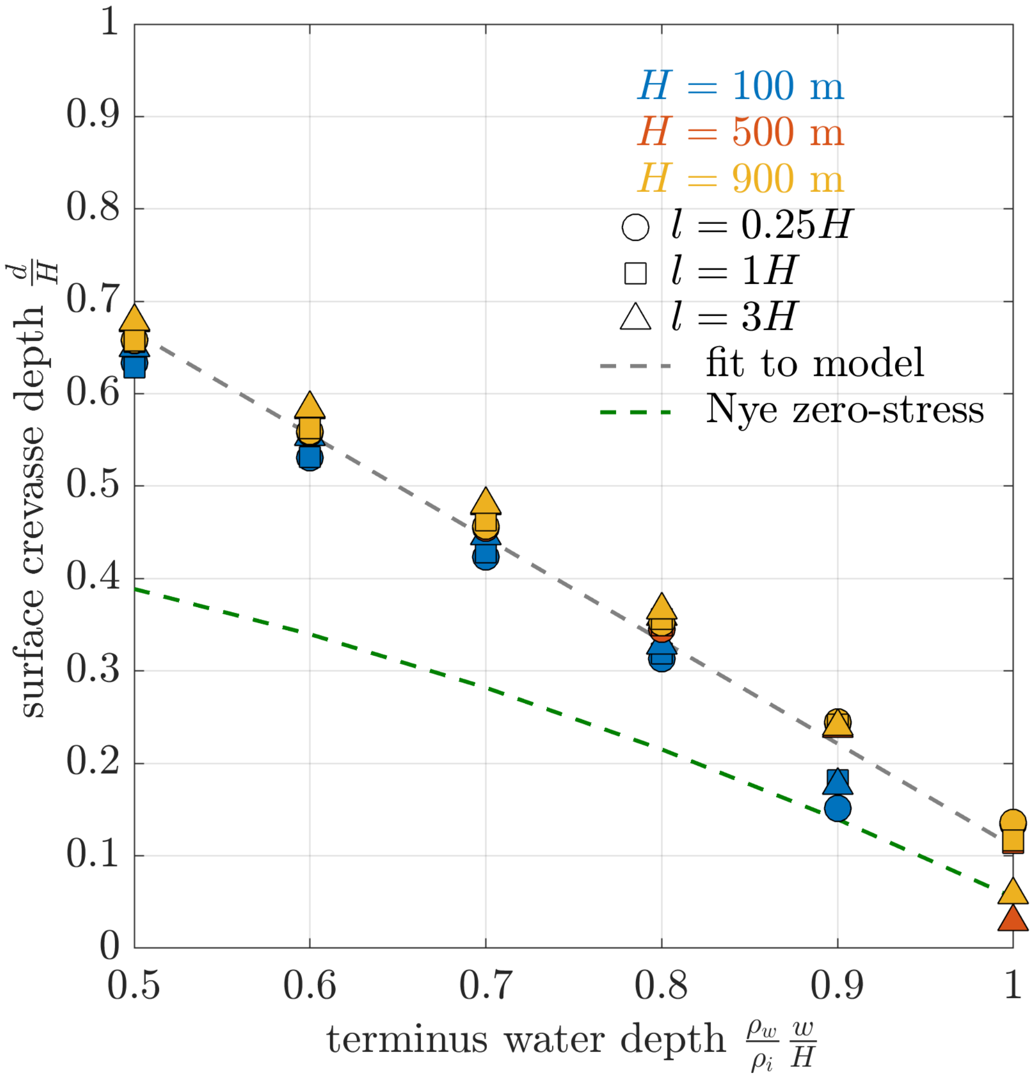
$\frac{d}{H}$ = 0.31, K_I = 6

Since $K_I > K_{frac}$, a crevasse of this depth would open and get deeper

$\frac{d}{H}$ = 0.41 , K_I = -10

Since $K_I < K_{frac}$, a crevasse of this depth would close and get shallower

Type-I dry surface crevasse results



Results are nearly independent of ice thickness

Crevasse depth not strongly sensitive to distance to front (note no basal friction, so this is testing the importance of non-hydrostatic stresses near front). Possible exception for glaciers at flotation

In general, crevasses are deeper than Nye zero-stress model (positive feedback of crevassing on crevasse depth)

For glaciers close to flotation (like tidewater glaciers in Greenland), dry crevassing is <30% of the ice thickness – unlikely to alone result in calving

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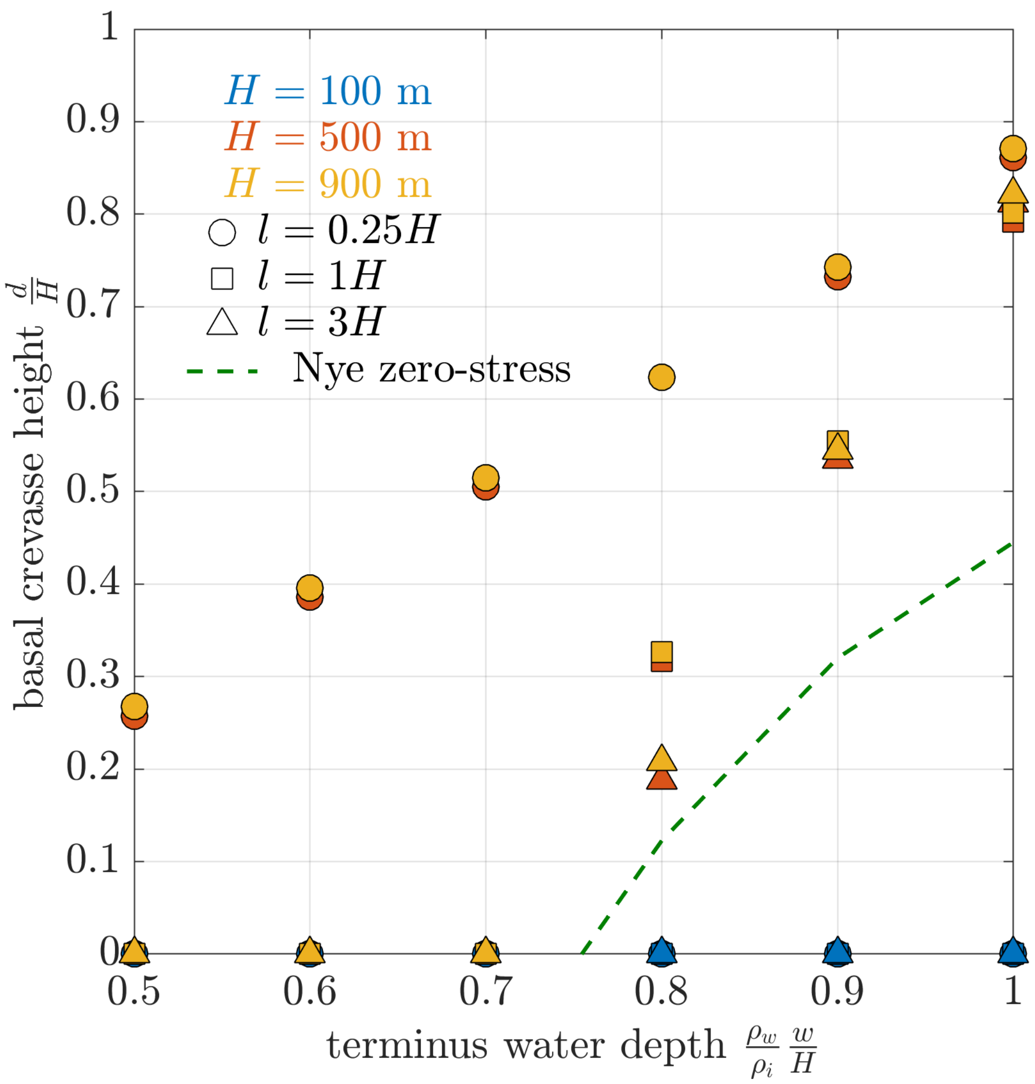
Type-I basal crevasse results



No basal crevasses for thin ice. For thick ice there is little sensitivity to ice thickness

Strong sensitivity to proximity to front – for thick ice being closer to the front means a larger basal crevasse. Note there is no basal friction so this effect arises from non-hydrostatic stresses near the front.

For thick ice, basal crevasse height increases with terminus water depth due to greater water pressure in crevasses

Overall, little resemblance to Nye zero-stress model. For thick ice and deep water, basal crevasse height is much greater than predicted by Nye zero-stress model

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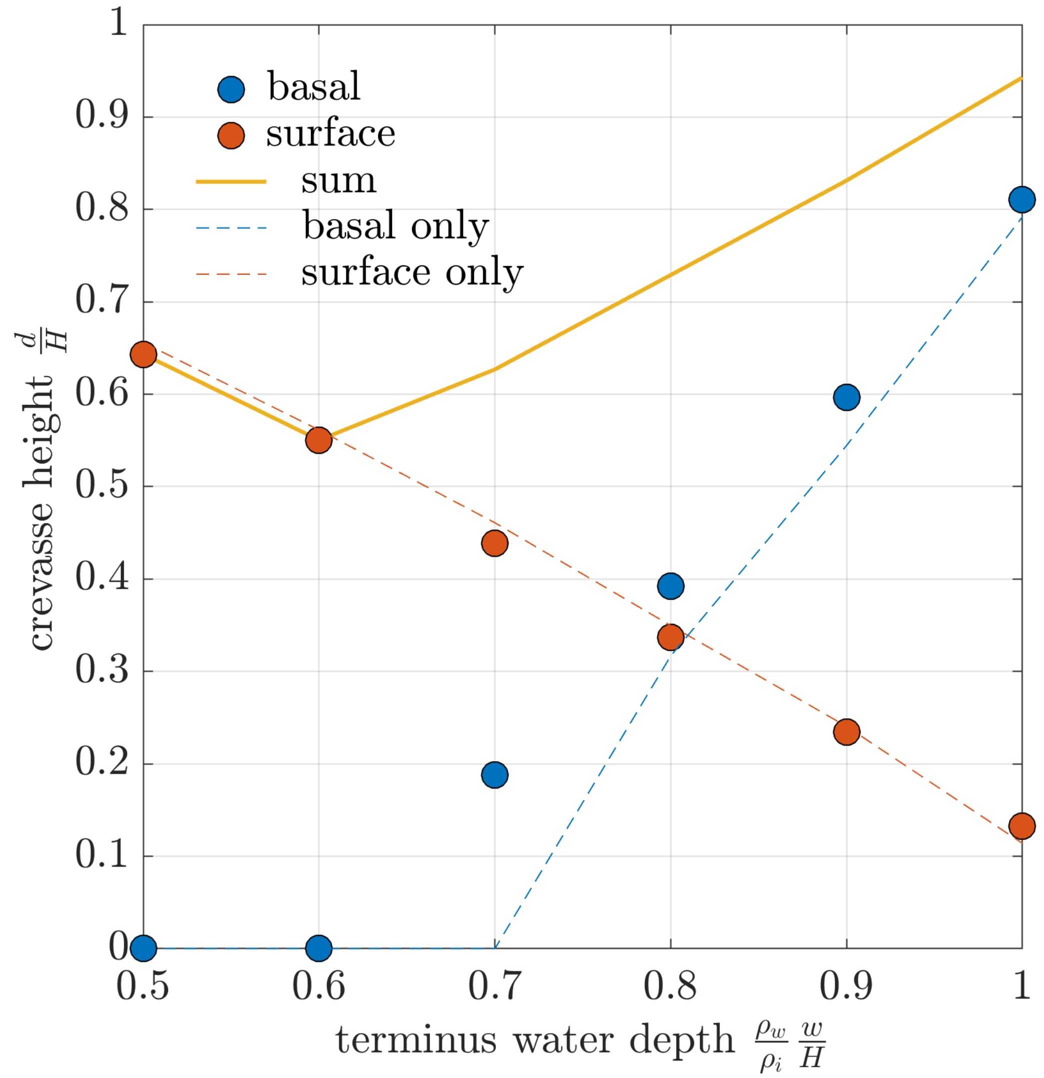
Type-I both surface and basal crevasse results (just for H = 500 m, l = H)

[i.e. propagate both surface and basal crevasses until $K_I = K_{frac}$ for both]



The presence of basal crevasses has little impact on surface crevassing

The presence of surface crevassing increases basal crevassing for some water depths

It may be possible to treat surface and basal crevasses independently, but need to check other ice thicknesses and crevasse proximity to front

The sum of basal and surface crevassing does not penetrate full ice thickness in this case for any water depth (but may do if the crevasses were closer to the front)

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Summary

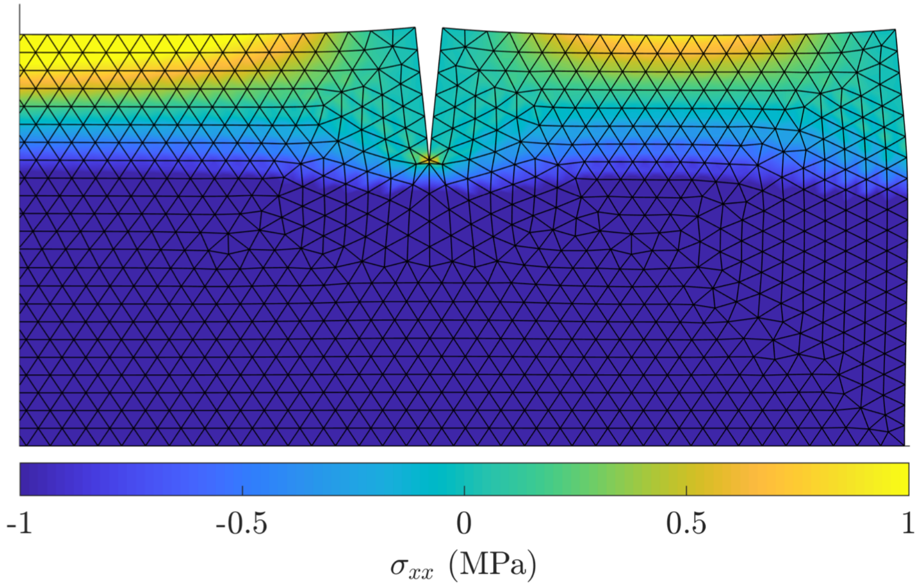
We use a 2D elastic model to estimate equilibrium crevasse depths in a fully general stress field, taking account of the feedback of crevassing on this stress field

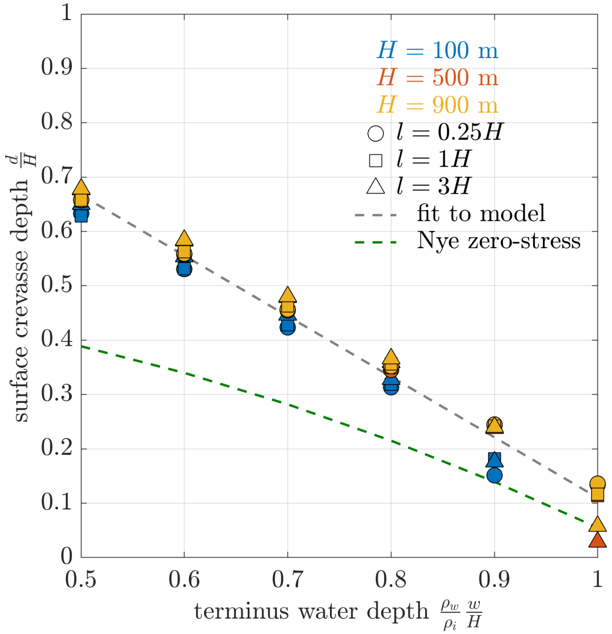
In general, the resulting crevasses are deeper than would be obtained from the Nye zero-stress model (i.e. there is a positive feedback of crevassing on crevasse depth)

As a result glaciers may calve at lower deviatoric stresses than suggested by the common crevasse-depth law (which often puts water in surface crevasses to achieve deep enough crevasses)

With more work and understanding it may prove possible to parameterise these results (e.g. ‘fit to model’ line on right) and propose a revised crevasse-depth law

Caveats: elastic rheology, flowline nature, have only shown results for a single crevasse





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