A formula for downscaling extreme sub-daily rainfall intensities

Benestad & Lutz, J; Dyrrdal, A; Haugen, JE; Parding, K; Dobler, A

Norwegian Meteorological

Sub-daily rainfall

Approximate formula for intensity-duration-frequency (IDF).

IDF for sites with daily rain gauge data but not hourly?

Perhaps very useful...



Simple formula for sub-daily rainfall intensity

$$x_L = lpha \mu ig(rac{L}{24} ig)^\zeta \ln[f_w au]$$

 α = *correction factor* - divergence from exponential distribution (~universal; ~ 1.2);

 μ = wet-day mean precipitation; f_w = wet-day frequency (site-specific parameters);

 ζ = scaling dependency (~0.4; **~universal? Fractal dimension**).

Benestad, R; Lutz, J; Dyrrdal, A; Haugen, JE; Parding, K; Dobler, A; "<u>Testing a simple formula for calculating approximate</u> <u>intensity-duration-frequency curves</u>", accepted, ERL, DOI: 10.1088/1748-9326/abd4ab

Test of scaling in time

Calibrating the formula: Dependency factor ζ from the slopes of the curves

IDF-curves exhibit an almost linear log-log relationships. Approximation



Rainfall: 10-yr return levels

Evaluation of the formula against independent data.

Is it a universally valid formula?

Is the scaling factor ζ (~0.4) universal or systematically affected by geographical factors in a predictable way?



L (hr)

L (hr)

Many sites: 24-hr rainfall

$$x_T = lpha \ \mu \ \ln(au f_w)$$



Ref: Benestad, et al. (2019), "A simple equation to study changes in rainfall statistics", Environ. Res. Lett. <u>https://doi.org/10.1088/1748-9326/ab2bb2</u>

The correction factor vs log(return period)



τ

Examine scaling in μ and f_w .

Almost linear log-log relationship between μ and timescale L. Slope = β .

Almost linear log relationship between f_w and timescale L.

Both μ and f_w scaling dependencies are expected to influence the scaling dependency in x_L .



The inspiration behind the formula

$$x_L = lpha \mu (L/24)^\zeta \ln[(f_w + \gamma \ln[L/24]) au]$$

The scaling factor is a sum of scaling dependencies in μ and α : $\zeta = \beta + 1 + a_0$.

Empirical relationships:

$$egin{aligned} &\ln(\mu/L) &= eta \ln(L/24) + \mu_0 \ &\ln(lpha) &= a_0 \ln(L/24) + lpha_0 \ &f_w &= \gamma \ln(L/24) + f_0 \end{aligned}$$

Some results from Andreas Dobler (NorCP)







Timesteps [h] 1, 2, 3, 6, 12 & 24 1998 – 2018 (21 yr) 3km Oslo - observed: β ~ -0.12 $v \sim +0.6$ (Benestad, et al, 2020, su to ERL)

- 0.9

- 0.88

0.86

0.84

0.82

Some results from Andreas Dobler (NorCP)





Timesteps [h] ¹⁄₄, ¹⁄₂, 1, 2, 3, 6, 12 & 24 1998 – 2018 (21 yr) 3km

0.86

0.84

0.82

A change in return level can be expressed in terms of changes in μ and f_{μ} :

$$\delta x_L = lpha ig(rac{L}{24} ig)^\zeta \ln(f_w au) \delta \mu + lpha \mu ig(rac{L}{24} ig)^\zeta rac{\delta f_w}{f_w}$$

The first term dominates over the latter according to:

$$\ln(f_w au)\delta\mu>\mu\;rac{\delta f_w}{f_w}\quad \ln(f_w au)rac{\delta\mu}{\mu}>rac{\delta f_w}{f_w}$$

where

$$f_w imes au=(n_w/n) imes(n au_{yrs})=n_w au_{yrs}$$

Typically, $n_w \sim 100$ for a year, and the return interval in years is $\tau_{vrs} \sim 1-200$.