

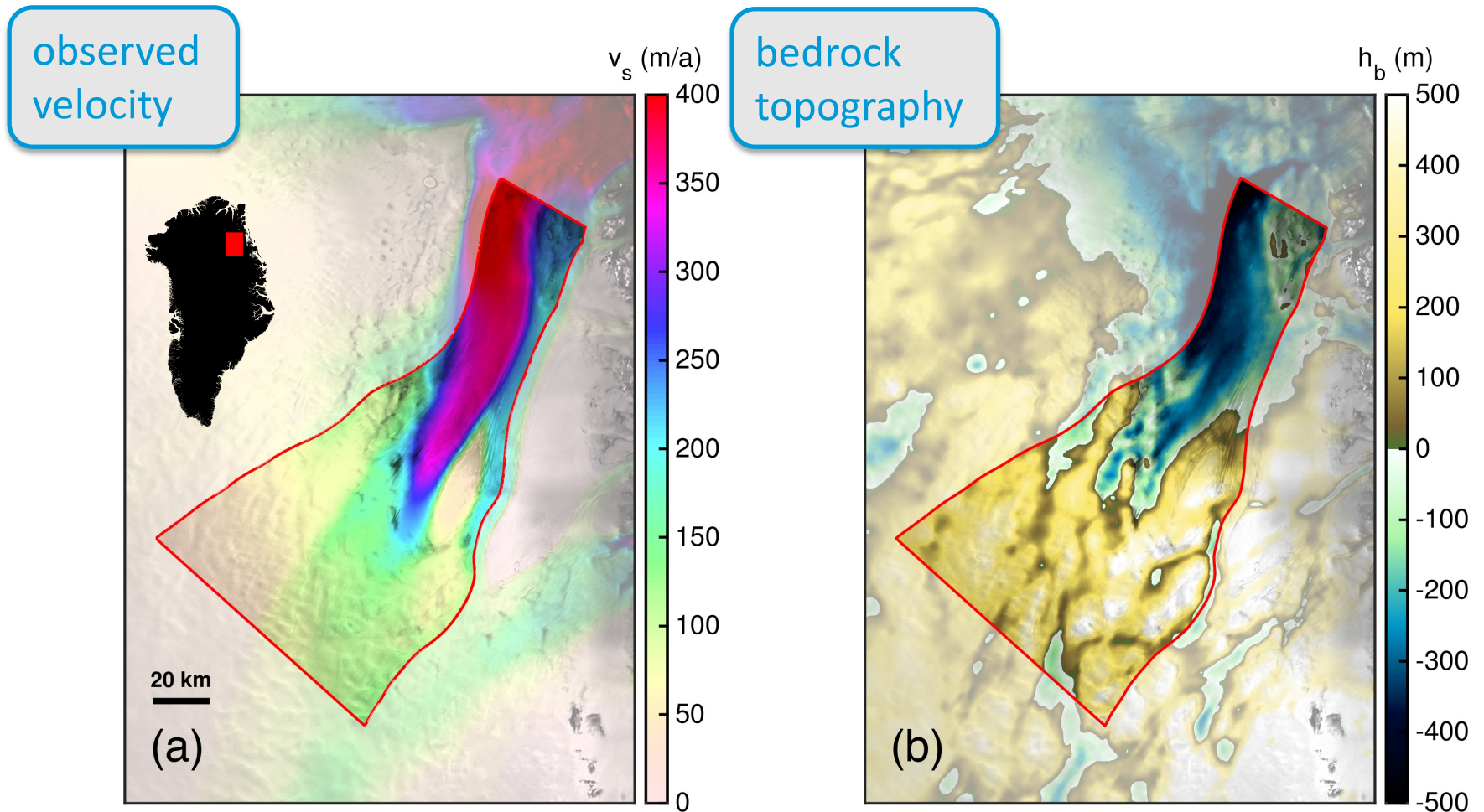
Consistent comparison of full-Stokes and higher-order approximation in the central North East Greenland Ice Stream



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Full-Stokes vs Blatter-Pattyn (Rückamp et al. 2021, TC, in prep)



MEaSURE (Joughin et. al. 2016,2018)

BedMachine (Morlighem et al., 2017)

A consistent framework

Full-Stokes

$$\begin{aligned}\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} &= 0 \\ \frac{\partial}{\partial x} \left(2\eta \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_x}{\partial z} + \eta \frac{\partial v_z}{\partial x} \right) - \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial}{\partial x} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial y} \left(2\eta \frac{\partial v_y}{\partial y} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_y}{\partial z} + \eta \frac{\partial v_z}{\partial y} \right) - \frac{\partial p}{\partial y} &= 0 \\ \frac{\partial}{\partial x} \left(\eta \frac{\partial v_x}{\partial z} + \eta \frac{\partial v_z}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial v_y}{\partial z} + \eta \frac{\partial v_z}{\partial y} \right) + \frac{\partial}{\partial z} \left(2\eta \frac{\partial v_z}{\partial z} \right) - \frac{\partial p}{\partial z} &= \rho_i g\end{aligned}$$

Blatter-Pattyn

$$\begin{aligned}\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} &= 0 \\ \frac{\partial}{\partial x} \left(2\eta \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_x}{\partial z} + \eta \frac{\partial v_z}{\partial x} \right) - \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial}{\partial x} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial y} \left(2\eta \frac{\partial v_y}{\partial y} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_y}{\partial z} + \eta \frac{\partial v_z}{\partial y} \right) - \frac{\partial p}{\partial y} &= 0 \\ \frac{\partial}{\partial x} \left(\eta \frac{\partial v_x}{\partial z} + \eta \frac{\partial v_z}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial v_y}{\partial z} + \eta \frac{\partial v_z}{\partial y} \right) + \frac{\partial}{\partial z} \left(2\eta \frac{\partial v_z}{\partial z} \right) - \frac{\partial p}{\partial z} &= \rho_i g\end{aligned}$$

Consistency:

- one single ISM
- avoid mesh effects
- avoid parameterisation effects
- eliminate numerical/implementation issues

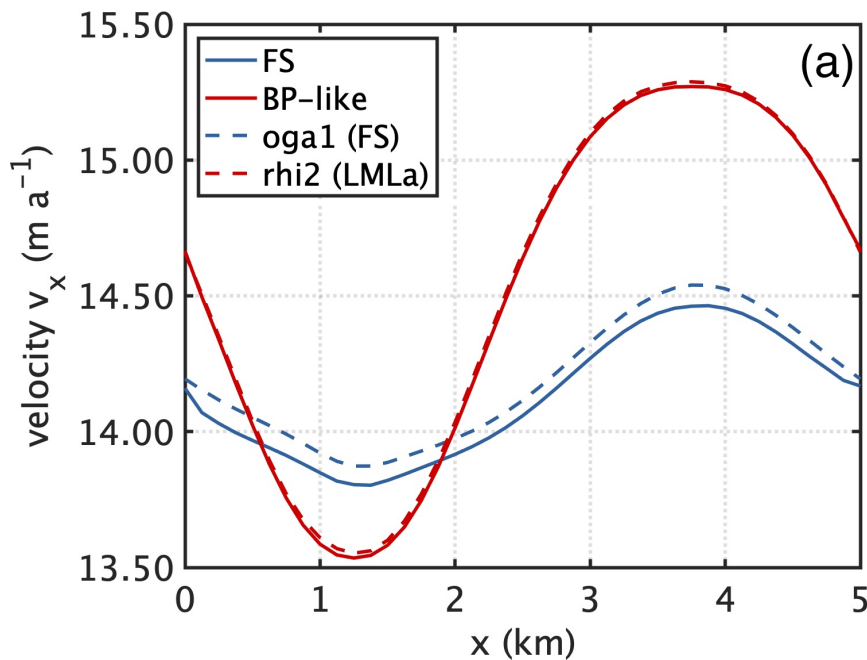
Blatter-Pattyn

$$\begin{aligned}\frac{\partial}{\partial x} \left(4\eta \frac{\partial v_x}{\partial x} + 2\eta \frac{\partial v_y}{\partial y} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_x}{\partial z} \right) &= \rho_i g \frac{\partial h_s}{\partial x} \\ \frac{\partial}{\partial x} \left(\eta \frac{\partial v_x}{\partial y} + \eta \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial y} \left(4\eta \frac{\partial v_y}{\partial y} + 2\eta \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial v_y}{\partial z} \right) &= \rho_i g \frac{\partial h_s}{\partial y} \\ v_z &= v_z|_{z=h_b} - \int_{h_b}^z \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dz'\end{aligned}$$

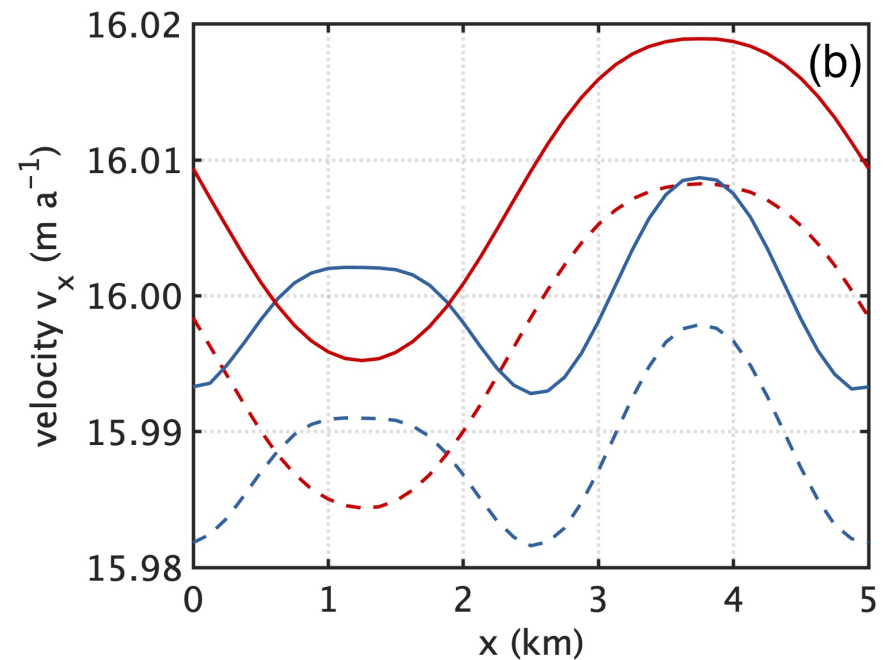
$$d_e = \left(\frac{1}{2} \left(\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right) + \frac{1}{4} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 \right)^{1/2}$$

Verification with ISMIP-HOM ([Pattyn et. al., 2008](#))

Exp. A 5km



Exp. C 5km



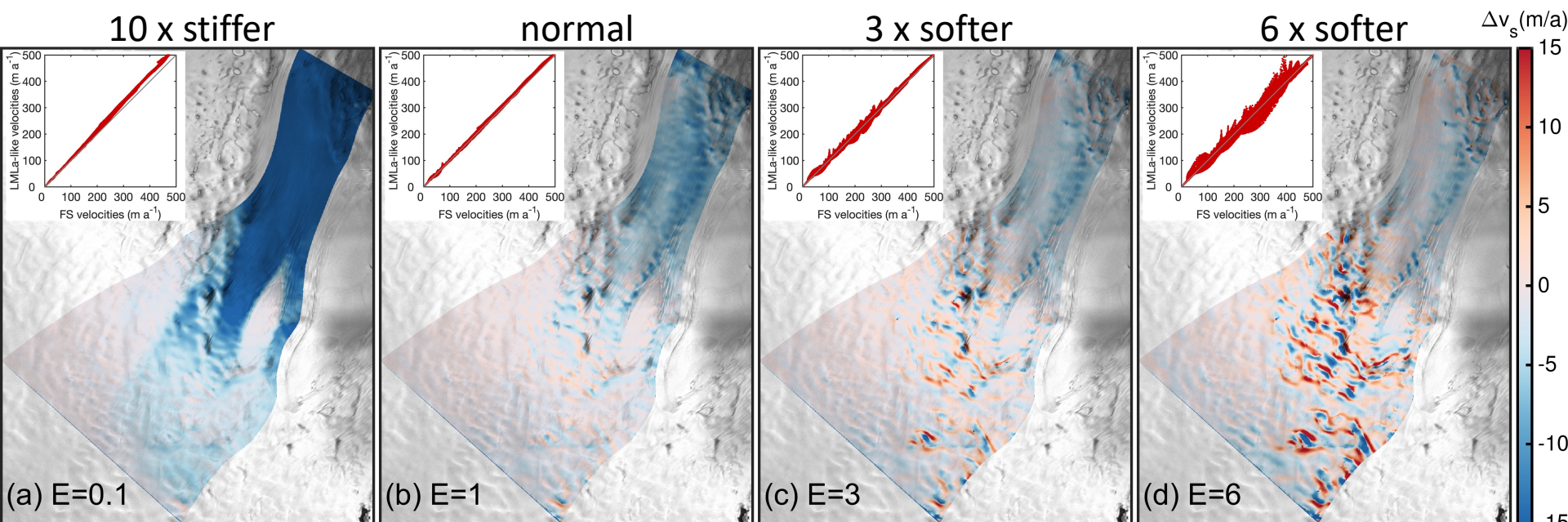
Consumed computational resources

resolution (km)	#elements	#DOF	lin-Solver	#non-lin iterations	time (minutes)	RAM (GB)	MPI OMP
12.8 [†]	1960	5280	MUMPS	11	0.11	1.87	1 36
6.4 [†]	8560	20680	MUMPS	10	0.48	2.24	1 36
3.2 [†]	32700	75548	MUMPS	9	1.72	4.03	1 36
1.6 [†]	128530	289960	MUMPS	9	2.99	4.80	4 36
0.8 [†]	512640	1142108	MUMPS	9	15.22	12.29	4 36
0.4 [†]	2066130	4573976	DDSchwarz*	10	86.50	28.91	24 18
0.2 [†]	8220400	18141816	DDSchwarz*	10	248.75	46.95	24 9
0.15 [†]	14556150	32099408	DDSchwarz*	11	320.75	47.15	56 6
0.1 [‡]	32864970	72416740	DDSchwarz*	11	708.11	359.60	30 48

- 11 vertical layers
- solved diagnostically
- coarse = MUMPS
- fine = domain decomposition solver with additive Schwarz
- P1P1 + GLS-stabilization
- friction coefficient k^2 from ISSM inversion
- basal sliding: $\tau_b = u_b k^2 N_{\text{eff}}$



Surface velocity differences FS-BP

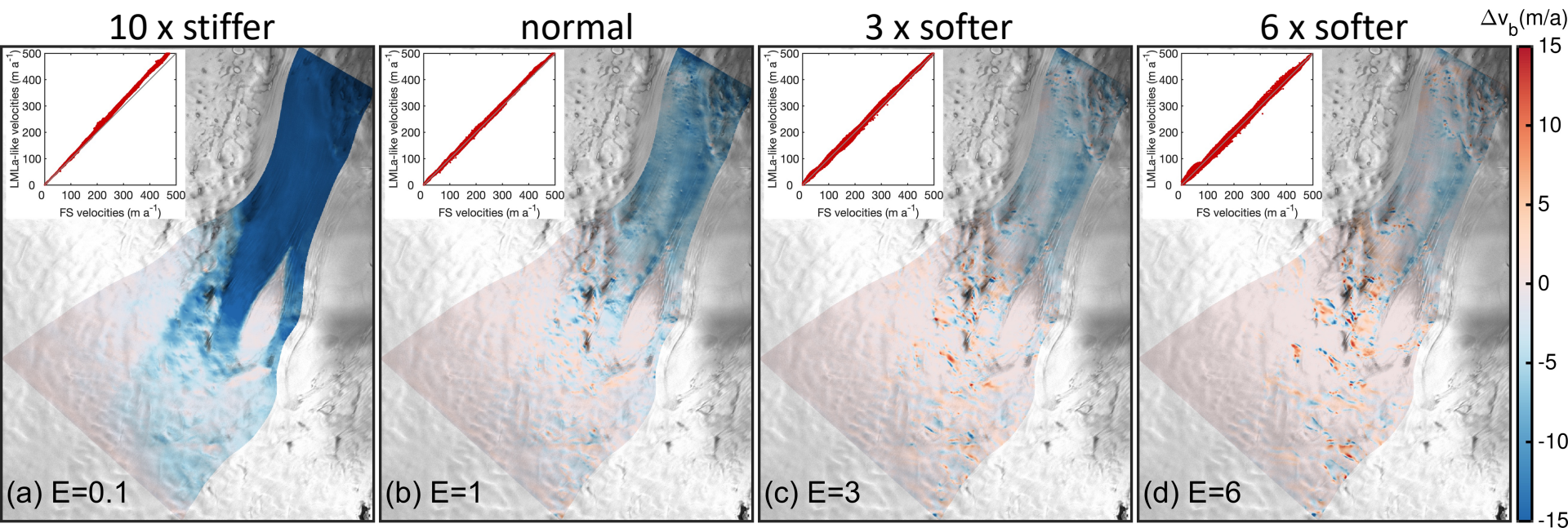


$$\Delta v_{\max} = -43 \text{ m/a}$$

$$\Delta v = \pm 80 \text{ m/a}$$

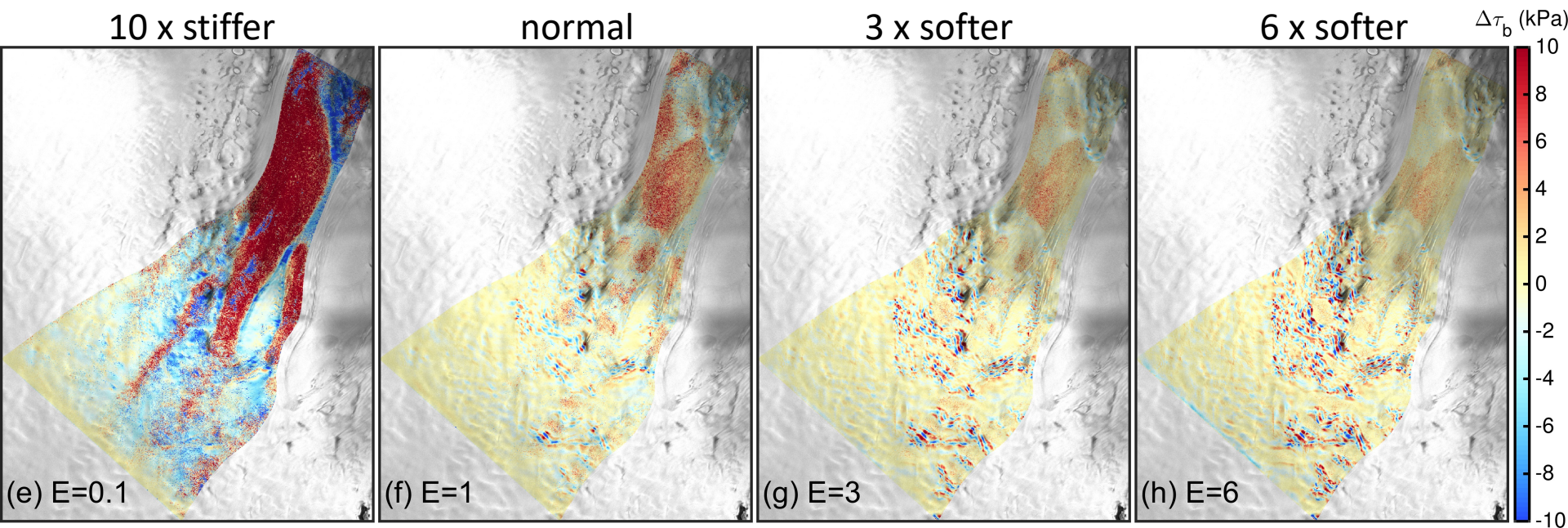
100m resolution

Basal velocity differences FS-BP



100m resolution

Basal shear stress FS-BP



100m resolution

Vertically integrated viscosity FS-BP

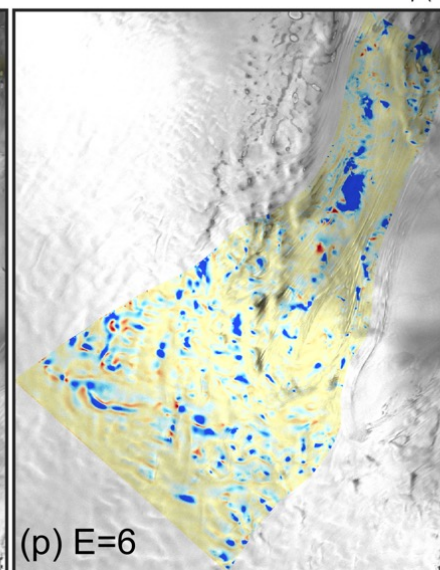
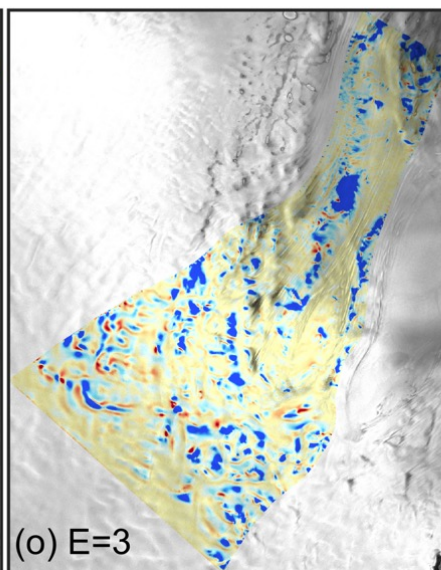
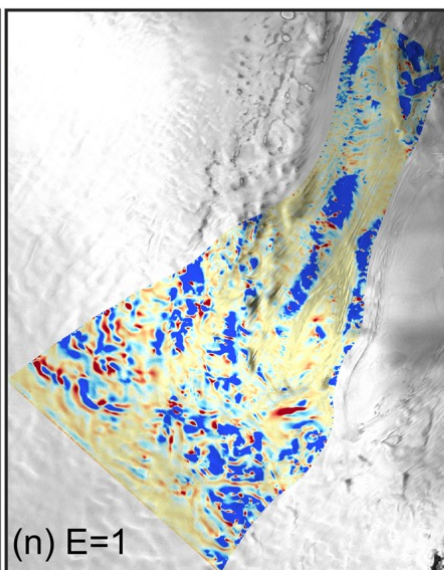
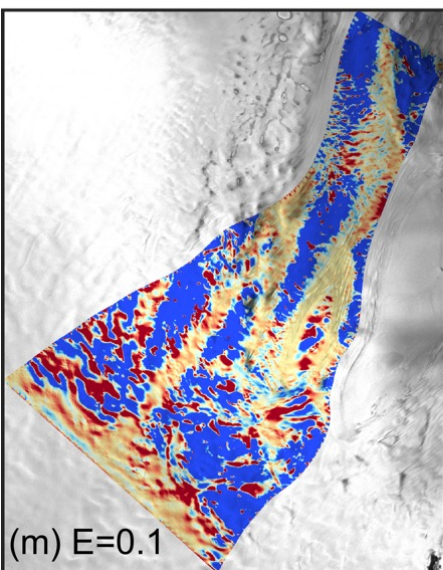
10 x stiffer

normal

3 x softer

6 x softer

$\Delta\bar{\eta}$ (Pas $\times 10^{14}$)



100m resolution

Vertically integrated viscosity FS-BP

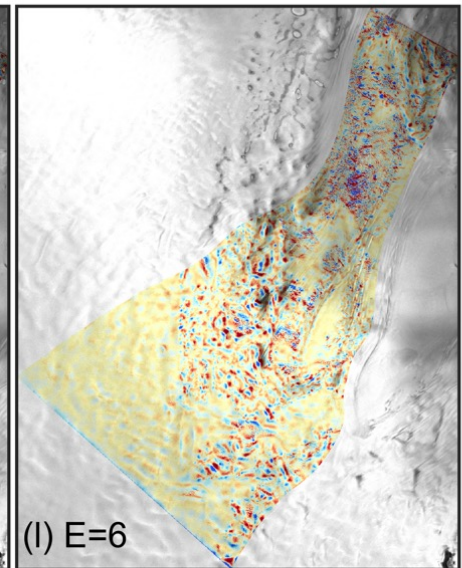
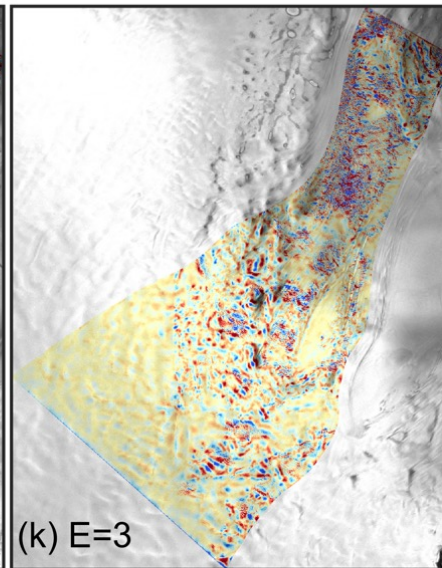
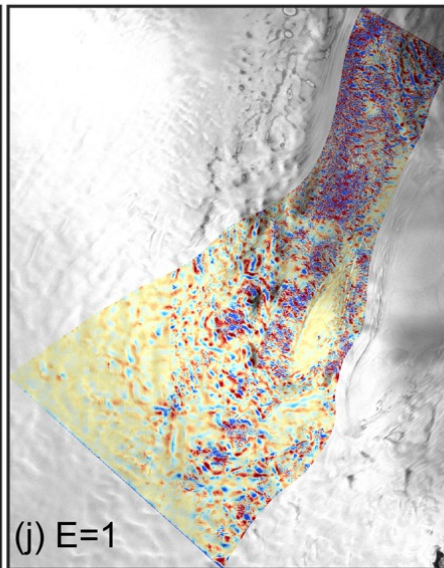
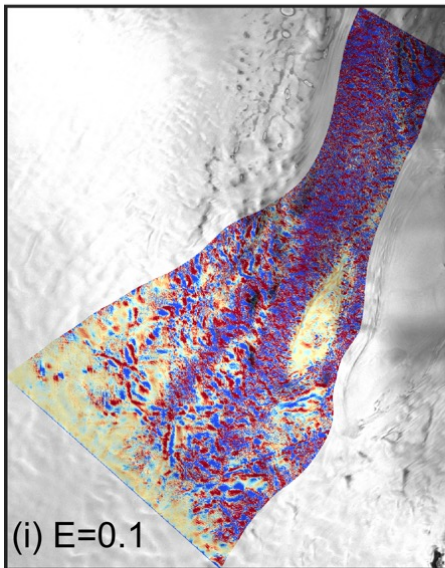
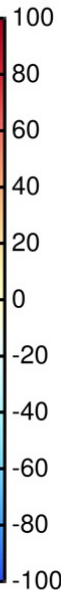
10 x stiffer

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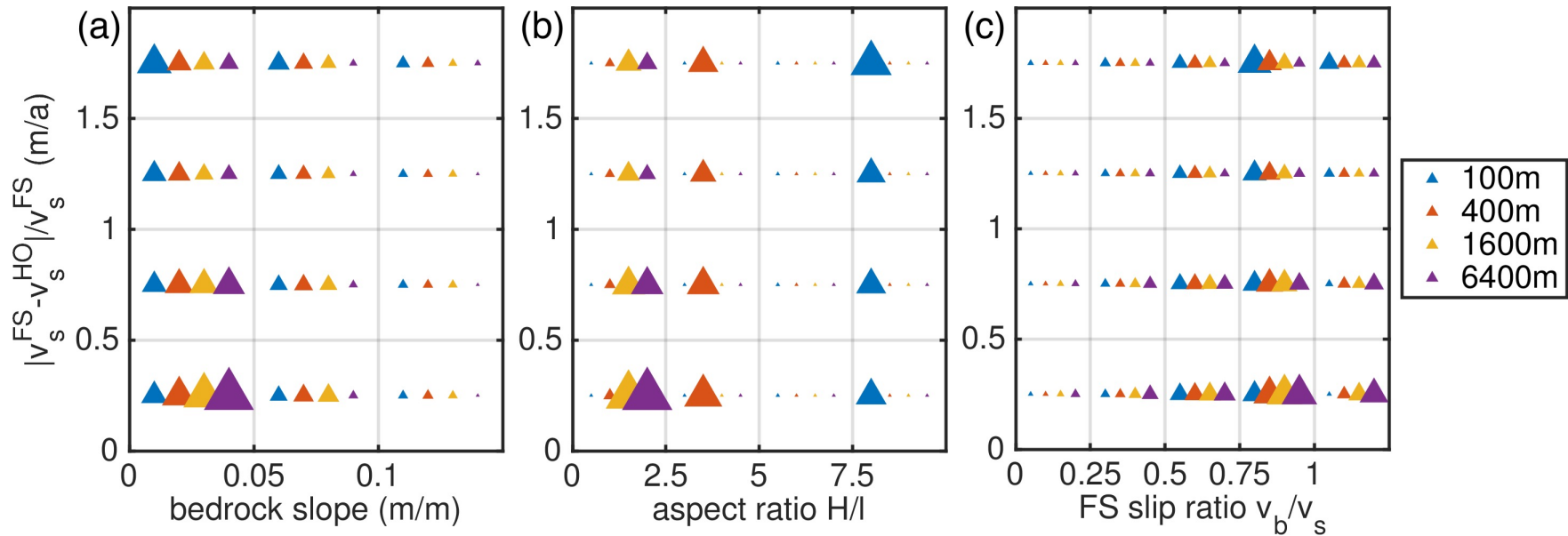
6 x softer

ΔP_i (kPa)



100m resolution

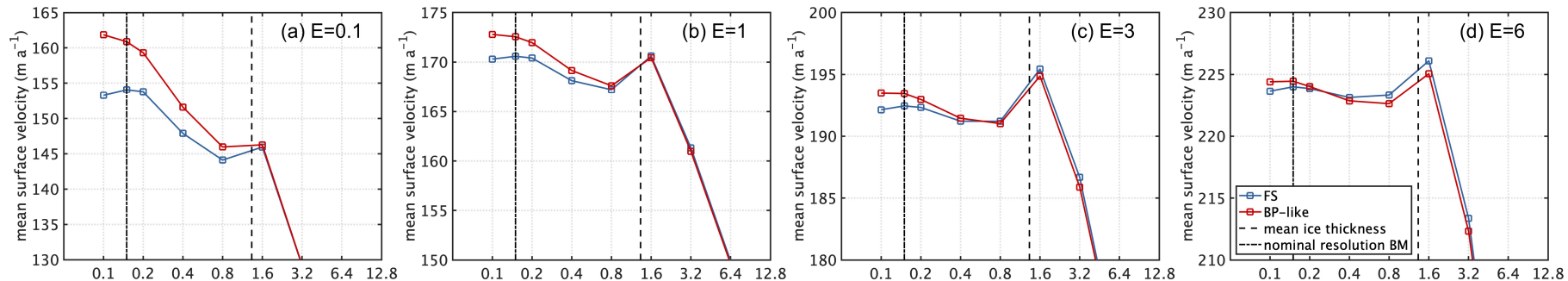
2D dimensional histogram



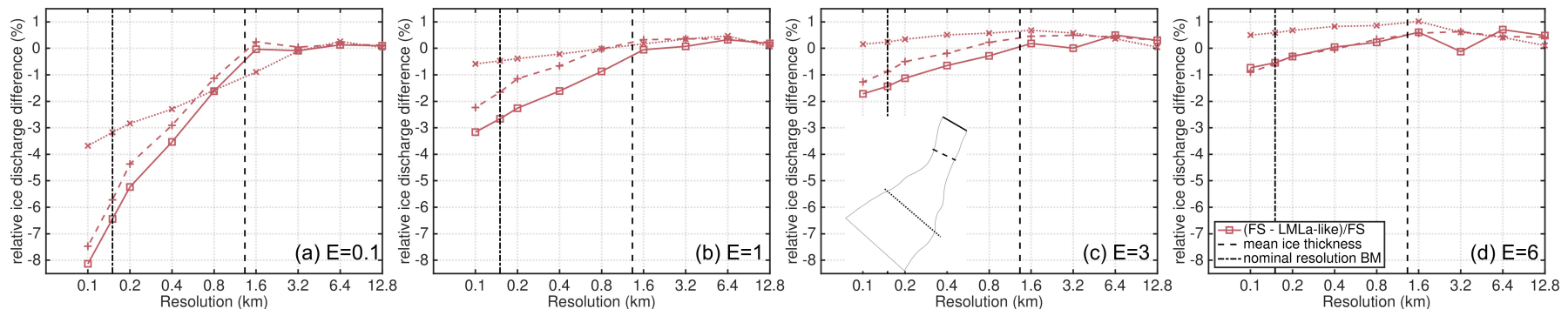
Counts of grid nodes per relative velocity error bin and per (a) bedrock slope bin, (b) local aspect ratio bin, and (c) FS slip ratio bin. The size of the triangles is normalized by the total number of horizontal grid nodes of each mesh.

Integrative view

Spatially averaged surface velocity



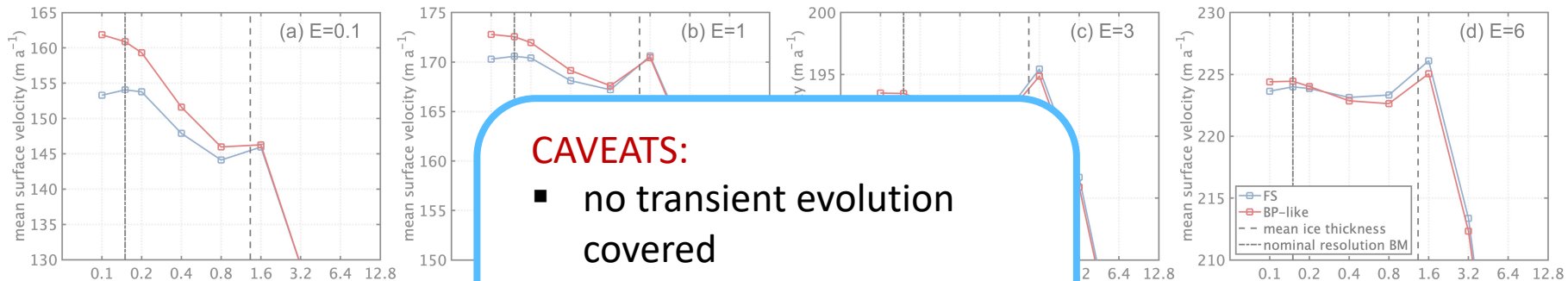
Ice discharge at flux gates



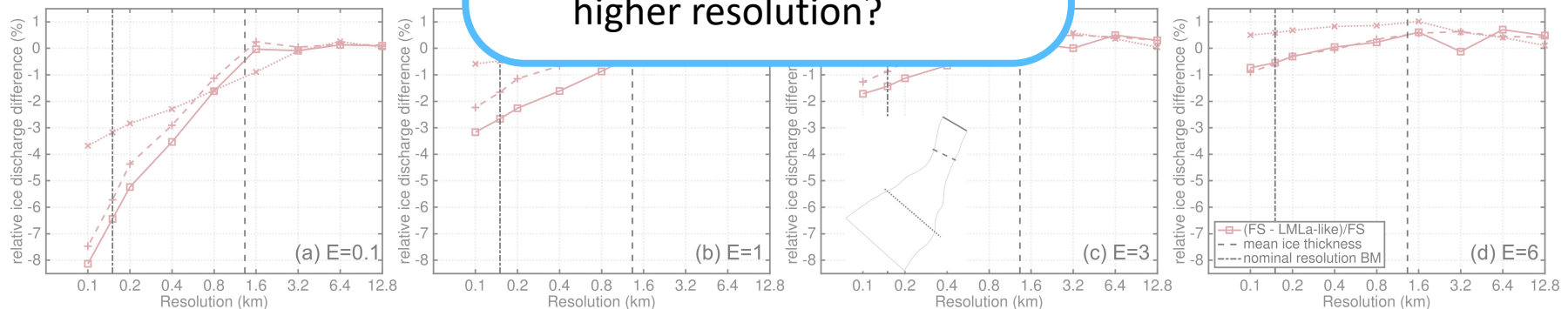
➤ If you can afford really high resolution, BP is overestimating ice discharge by 3%

Integrative view

Spatially averaged surface velocity



Ice discharge at flux gate



➤ If you can afford really high resolution, BP is overestimating ice discharge by 3%