

Study of Internal Stresses of Mining Massif within the Framework of Elastic Layered Block Models with Inclusions of the Hierarchical Structure of L-th Rank



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Abstract

In recent years, new models of continuum mechanics, generalizing the classical theory of elasticity, have been intensively developed. These models are used to describe composite and statistically heterogeneous media, new structural materials, as well as for complex arrays in mine conditions. The paper presents an algorithm for the propagation of acoustic waves in the framework of active monitoring of elastic layered block media with inclusions of the hierarchical type of L-th rank. Relations for internal stresses and deformations for each hierarchical rank are obtained, which constitute the nonlocal theory of elasticity. The essential differences between the nonlocal theory of elasticity and the classical one and the connection between them are investigated. A characteristic feature of the theory of media with a hierarchical structure is the presence of scale parameters in explicit or implicit form, i.e. explicit or implicit non locality of the theory. This work focuses on the study of the effects of non locality and internal degrees of freedom, reflected in internal stresses, which are not described by the classical theory of elasticity and which can be potential precursors of the development of a catastrophic process in a mountain range.

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INTRODUCTION

In recent years, new physical and mathematical models of material media have been intensively developed, which can be considered as far-reaching generalizations of the classical theories of elasticity [1-4]. The science of plasticity and strength of solids is undergoing a paradigm shift. For a long time, the description of plastic deformation and fracture of solids has been developed in the framework of linear approximations of continuum mechanics (macro scale level) and physics of deformation defects in a loaded solid (micro scale level). However, in recent decades, it has become obvious that a deformable solid is a multilevel hierarchically organized system that should be described within the framework of nonlinear mechanics and non equilibrium thermodynamics [5]. The fundamental problems arising when applying the second law of thermodynamics to the analysis of systems at the macroscopic and microscopic levels are considered. It is shown that the non-equilibrium state of the system can cause the appearance of order in it and that irreversible processes can lead to the emergence of a new type of dynamic states of matter, called "dissipative structures" [2].

[1] V.E.Panin, V.A.Likhachev, Yu.V.Grinyaev Structural levels of deformation of solids. Novosibirsk,Russia:Nauka.(1985) 254p.

[2] I.Prigogine Introduction to the thermodynamics of irreversible processes. Time, structure and fluctuations. Nobel Lecture in Chemistry 1977. Mjscow, Russia: Scientific Publishing Center "Regular and Chaotic Dynamics". (2001)123-155.

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[4] E.I.Shemyakin, G.L.Fisenko, M.V.Kurlenya, V.N.Oparin et al. Effect of zonal disintegration of rocks around underground holes. DAN USSR.(1986), vol. 289, no. 5. 830-832.

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Kunin's work [3] is devoted to a relatively narrow issue: the study of models of elastic media with a microstructure. Historically, one of the first models of an elastic medium that cannot be described within the framework of the classical theory of elasticity is the Cosserat continuum (1909). However, for a long time the memoir of E. and F. Cosserat remained unnoticed, and only starting from about 1958-60. generalized models of the Cosserat continuum began to develop intensively: the theory of oriented media, asymmetric, moment, multipolar, micro morphic, etc. theory of elasticity (for brevity, we will call them moment theories). To date, several hundred papers on this topic have been published, and their number continues to grow rapidly. Let us try to classify quite schematically various nonlocal theories of elastic media. A characteristic feature of all such theories is their explicit or implicit non locality. The latter, in turn, is manifested in the fact that the theory contains parameters that have the dimension of length. These scale parameters can have different physical meanings: for example, the distance between particles in discrete structures, the grain or cell size, the characteristic correlation radius or long-range forces, etc. But we will always assume that the scale parameters are small compared to the characteristic size of the body. A distinction should be made between cases of strong and weak non locality.

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If the “resolution” of the model is of the order of the scale parameter, i.e. within the framework of the corresponding theory, it is physically permissible to consider wavelengths commensurate with the scale parameter, then we will call the theory non local, or strongly non local (if we wish to emphasize this) .In such models, one can consider the elements of the medium on the order of the scale parameter, but, as a rule, distances, much smaller than the scale parameter have no physical meaning. In non local models, the propagation speed of waves depends on their wavelength; therefore, the term "medium with spatial dispersion" is also often used. We emphasize that non locality or spatial dispersion can have different origins. They can be caused by the microstructure of the medium (in particular, by the discreteness of the micro model) or by approximate consideration of such parameters as the thickness of a rod or plate. Accordingly, we can talk about the physical or geometric nature of non locality. Only effective elastic modules “know” about the structure of the original micro model, but it is, of course, impossible to extract this information from them. Hence it follows that an explicit account of the effects of the microstructure and, in particular, of the internal degrees of freedom is possible only with the simultaneous account of non locality, i.e. a consistent theory of elastic media with a microstructure must necessarily be nonlocal [3].

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In recent years, much attention has been paid to the study of the spatial stress-strain state of the rock mass. These studies are being conducted both with the aim of the practical development of great depths using deep mines. So it is with the aim of studying stresses in the earth's crust. The main attention is paid to experimental methods of observation of deformations and displacements in the rock massif in the vicinity of workings and stress estimates. The development of instrumental methods requires a thorough penetration of physical concepts into mining sciences. The discovered effect of zonal disintegration of rocks in the vicinity of underground workings at great depths is one of the natural states of a rock massif under the influence of gravitational and tectonic forces [4]. The work [6, see bibliography] describes in detail the problem of determining residual stresses by experimental methods, the nature of the occurrence and preservation of residual stresses in rocks, metal structures and multicomponent materials. When measuring stresses in a rock massif by unloading methods, the total result of deformations from removal is recorded residual stresses and stresses of external forces. However, in order to predict the possible destruction of the massif, it becomes necessary to separate the stresses of the massif into stresses of external forces and residual internal stresses.

[6]. K.T.Tazhibaev, A.S.Tashmamatov Residual stresses in rocks and a method for their determination. I.Ts "Technik" Bishkek, Kyrgyzstan. (2014).126p.

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. In this case, due to the fact that the structure of the array has a layered block form with internal inclusions of a hierarchical type, which are located none locally, it is necessary to be able to more accurately determine the possible source of internal destruction, which entails destruction according to the domino principle. In this paper, a 2D algorithm for determining internal stresses has been developed within the framework of acoustic monitoring of a layered block elastic medium with elastic hierarchical inclusions of the L-th rank.

II. 2D ALGORITHM FOR DETERMINING INTERNAL STRESSES WITHIN THE FRAMEWORK OF ACOUSTIC MONITORING OF A LAYERED BLOCK ELASTIC MEDIUM WITH ELASTIC HIERARCHICAL INCLUSIONS OF THE L-TH RANK

A. Using Longitudinal Wave

In [7], an algorithm for modeling sound diffraction by a two-dimensional elastic homogeneous inclusion located in the J-th layer of an N-layer medium was written.

[7] O.A.Hachay, A.Yu. Khachay On the integration of seismic and electromagnetic active methods for mapping and monitoring the state of two-dimensional heterogeneities in an N-layer medium. Bulletin of SURGU, series "Computer technologies, control, radio electronics". (2011), no. 2 (219). 49-56.

$$\begin{aligned}
 & \frac{(k_{1ji}^2 - k_{1j}^2)}{2f} \iint_{S_c} \{ (M) G_{Sp,j}(M, M^0) d\ddagger_M + \frac{\dots ja}{\dots ji} \{^0(M^0) - \\
 & - \frac{(\dots ja - \dots ji)}{\dots ji 2f} \oint_C G_{Sp,j} \frac{\partial \{ }{\partial n} dc = \{ (M^0), M^0 \in S_c \\
 & \frac{\dots ji (k_{1ji}^2 - k_{1j}^2)}{\dots (M^0) 2f} \iint_{S_c} \{ (M) G_{Sp,j}(M, M^0) d\ddagger_M + \{^0(M^0) - \\
 & - \frac{(\dots ja - \dots ji)}{\dots (M^0) 2f} \oint_C G_{Sp,j} \frac{\partial \{ }{\partial n} dc = \{ (M^0), M^0 \notin S_c
 \end{aligned}
 \tag{1}$$

where $G_{Sp,j}(M, M^0)$ is the function of the source of the seismic field, the boundary value problem for which was formulated in [7], $k_{1ji}^2 = \check{S}^2(\dots ji / \} ji)$ is the wave number for the longitudinal wave, in the above expression, the index ji denotes the belonging of the properties of the medium inside the heterogeneity, ja - outside the heterogeneity, $\}$ is the Lamé constant, \dots is the density of the medium, \check{S} - circular frequency, $\vec{u} = grad\{$ - displacement vector, $\{^0$ - potential of a normal seismic field in a layered medium in the absence of heterogeneity:, $\}$ - contour of heterogeneity, S_c - area of heterogeneity. In [8], an algorithm for modeling sound diffraction on a two-dimensional elastic inclusion of a hierarchical structure of rank l ($l = 1 \dots L$), located in the J-th layer of an N-layer medium, is written out.

A. Using Longitudinal Wave

$$\begin{aligned}
 & \frac{(k_{1jil}^2 - k_{1j}^2)}{2f} \iint_{S_{Cl}} \{\}_l(M) G_{Sp,j}(M, M^0) d\ddagger_M + \frac{\dots ja}{\dots jil} \{\}_{l-1}^0(M^0) - \\
 & - \frac{(\dots ja - \dots jil)}{\dots jil 2f} \oint_{Cl} G_{Sp,j} \frac{\partial \{\}_l}{\partial n} dc = \{\}_l(M^0), M^0 \in S_{Cl} \\
 & \frac{\dots jil (k_{1jil}^2 - k_{1j}^2)}{\dots (M^0) 2f} \iint_{S_{Cl}} \{\}_l(M) G_{Sp,j}(M, M^0) d\ddagger_M + \{\}_{l-1}^0(M^0) - \\
 & - \frac{(\dots ja - \dots jil)}{\dots (M^0) 2f} \oint_{Cl} G_{Sp,j} \frac{\partial \{\}_l}{\partial n} dc = \{\}_l(M^0), M^0 \notin S_{Cl}
 \end{aligned}
 \tag{2}$$

where $G_{Sp,j}(M, M^0)$ is the function of the source of the seismic field, it coincides with the function of expression (1), $k_{2jil}^2 = \check{S}^2(\dots jil / \sim jil)$ is the wave number for the longitudinal wave, in the above expression, the index jil denotes the belonging of the properties of the medium inside the inhomogeneity of the l -th rank, ja ($l-1$) - outside the inhomogeneity, $l = 1 \dots L$ is the number of the hierarchical level, $\bar{u}_l = grad\{\}_l, \{\}_l^0$ - is the potential of a normal seismic field in a layered medium in the absence of heterogeneity of the previous rank, if $l = 2 \dots L$, if $l = 1$, $\{\}_l^0 = \{\}^0$, ja ($l-1$) = ja . The system of equations (2) represents an iterative process of solving first the internal problem, then the external problem with the corresponding rank of heterogeneity. We will assume that the physical properties for the heterogeneity of each rank are homogeneous within the occupied area. To model and determine the values of internal stresses, it is necessary to know the values of the physical parameters, as well as the geometric parameters of the contours and areas for the nested heterogeneities. Let $L = 3$.

A. Using Longitudinal Wave

E - Young's modulus, ν - Poisson's ratio, u_{yz} - deformation tensor, τ_{yz} - stress tensor [9].

$l=1;$

$$\begin{aligned}
 & \frac{(k_{1ji(l=1)}^2 - k_{1j(j-S_{C(l=1)})}^2)}{2f} \iint_{S_{C(j-S_{C(l=1)})}} \{_{(l=1)}(M) G_{Sp,j}(M, M^0) d\tau_M + \\
 & + \frac{\dots ja(j-S_{C(l=1)})}{\dots ji(l=1)} \{_{(j)}^0(M^0) - \frac{(\dots ja(j-S_{C(l=1)}) - \dots ji(l=1))}{\dots ji(l=1)} 2f} \times \\
 & \times \iint_{C(j-S_{C(l=1)})} G_{Sp,j} \frac{\partial \xi_l}{\partial n} dc = \{_{(l=1)}(M^0), M^0 \in S_{C(j-S_{C(l=1)})}
 \end{aligned} \quad (3)$$

$$u_{y,l=1}(M^0 \in S_{C(j-S_{C(l=1)})}) = \frac{\partial}{\partial y} (\{ (M^0 \in S_{C(j-S_{C(l=1)})}) \});$$

$$u_{z,l=1}(M^0 \in S_{C(j-S_{C(l=1)})}) = \frac{\partial}{\partial z} (\{ (M^0 \in S_{C(j-S_{C(l=1)})}) \})$$

$$u_{yz,Sc(j-S_{C(l=1)})} = \frac{1}{2} \left(\frac{\partial u_{y,l=1}}{\partial z} + \frac{\partial u_{z,l=1}}{\partial y} \right), \quad k_{1ji(l=1)}^2 = \check{S}^2(\dots ji(l=1) / \dots ji(l=1))$$

$$\tau_{yz,Sc(j-S_{C(l=1)})} = \frac{E_{Sc,l=1}}{1 + \nu_{Sc,l=1}} u_{yz,Sc(j-S_{C(l=1)})}$$

A. Using Longitudinal Wave

$$\begin{aligned}
 & ; \frac{\dots jil (k_{1ji(l=1)}^2 - k_{1j(j-S_C(l=1))}^2)}{\dots (M^0_{j(j-S_C(l=1))}) 2f} \iint_{S_C(j-S_C(l=1))} \{_{l=1}(M) G_{Sp,j}(M, M^0) d\ddagger_M + \\
 & ; \\
 & ; \\
 & + \{^0_{(j)}(M^0) - \frac{(\dots ja_{(j-S_C(l=1))} - \dots ji(l=1))}{\dots (M^0_{j(j-S_C(l=1))}) 2f} \times \\
 & \times \int_{(\mathcal{G}(l=1))} G_{Sp,j} \frac{\partial \{_{(l=1)} dc}{\partial n} = \{_{(l=1)}(M^0_{j(j-S_C(l=1))}), M^0_{j(j-S_C(l=1))} \quad (5)
 \end{aligned}$$

(8) $l=2$;

$$\begin{aligned}
 & \frac{(k_{1ji(l=2)}^2 - k_{1j(S_C(l=1)-S_C(l=2))}^2)}{2f} \times \\
 & \times \iint_{S_C(S_C(l=1)-S_C(l=2))} \{_{(l=2)}(M) G_{Sp,j}(M, M^0) d\ddagger_M + \\
 & + \frac{\dots ja(S_C(l=1)-S_C(l=2))}{\dots ji(l=2)} \{^0(M^0) - \frac{(\dots ja(S_C(l=1)-S_C(l=2)) - \dots ji(l=2))}{\dots ji(l=2) 2f} \times \\
 & \times \int_{C(S_C(l=1)-S_C(l=2))} G_{Sp,j} \frac{\partial \{_{l=2} dc}{\partial n} = \{_{(l=2)}(M^0), \\
 & M^0 \in S_{C(S_C(l=1)-S_C(l=2))}, \{^0(M^0) = \{_{(l=1)} \quad (6)
 \end{aligned}$$

$$u_{y,l=2}(M^0 \in S_{C(S_C(l=1)-S_C(l=2))}) = \frac{\partial}{\partial y} (\{ (M^0 \in S_{C(S_C(l=1)-S_C(l=2))}) \});$$

$$u_{z,l=2}(M^0 \in S_{C,l=2}) = \frac{\partial}{\partial z} (\{ (M^0 \in S_{C(S_C(l=1)-S_C(l=2))}) \})$$

$$u_{yz,S_C(S_C(l=1)-S_C(l=2))} = \frac{1}{2} \left(\frac{\partial u_{y,l=2}}{\partial z} + \frac{\partial u_{z,l=2}}{\partial y} \right),$$

$$\dagger_{yz,S_C(S_C(l=1)-S_C(l=2))} = \frac{E_{S_C(S_C(l=1)-S_C(l=2))}}{1 + \dagger_{S_C(S_C(l=1)-S_C(l=2))}} u_{yz,S_C(S_C(l=1)-S_C(l=2))} \quad (7)$$

$$k_{1ji(l=2)}^2 = \check{S}^2(\dots ji(l=2) / \}_{ji(l=2)})$$

$$\frac{\dots ji(l=2) (k_{1ji(l=2)}^2 - k_{1j(S_C(l=1)-S_C(l=2))}^2)}{\dots (M^0_{j(S_C(l=1)-S_C(l=2))}) 2f} \times$$

$$\times \iint_{S_C(S_C(l=1)-S_C(l=2))} \{_{l=2}(M) G_{Sp,j}(M, M^0) d\ddagger_M + \{^0(M^0) -$$

$$- \frac{(\dots ja((S_C(l=1)-S_C(l=2)) - \dots ji(l=2))}{\dots (M^0_{j(S_C(l=1)-S_C(l=2))}) 2f} \times$$

$$\times \int_{C(S_C(l=1)-S_C(l=2))} G_{Sp,j} \frac{\partial \{_{(l=2)} dc}{\partial n} =$$

$$= \{_{(l=2)}(M^0_{j(S_C(l=1)-S_C(l=2))}), M^0_{j(S_C(l=1)-S_C(l=2))},$$

$$\{^0(M^0) = \{_{(l=1)} \quad (8)$$

A. Using Longitudinal Wave

$$\begin{aligned}
 & l=3=L \\
 (9) \quad & \frac{(k_{1ji(l=3)}^2 - k_{1j(S_C(l=2)-S_C(l=3))}^2)}{2f} \times \\
 & \times \iint_{S_{C(S_C(l=2)-S_C(l=3))}} \{_{(l=3)}(M) G_{Sp,j}(M, M^0) d\ddagger_M + \\
 & + \frac{\dots ja(S_C(l=2)-S_C(l=3))}{\dots ji(l=3)} \{^0(M^0) - \frac{(\dots ja(S_C(l=2)-S_C(l=3)) - \dots ji(l=3))}{\dots ji(l=3)} 2f} \times \\
 & \times \int_{C(S_C(l=2)-S_C(l=3))} G_{Sp,j} \frac{\partial \{_{(l=3)} dc}{\partial n} = \{_{(l=3)}(M^0), \\
 & M^0 \in S_{C(l=3)}, \{^0(M^0) = \{_{(l=2)} \\
 & \dots jil(k_{1ji(l=3)}^2 - k_{1j(S_C(l=2)-S_C(l=3))}^2) \times \\
 & \dots (M^0_{j(S_C(l=2)-S_C(l=3))}) 2f \times \\
 & \times \iint_{S_{C(S_C(l=2)-S_C(l=3))}} \{_{l=3}(M) G_{Sp,j}(M, M^0) d\ddagger_M + \\
 & + \{^0(M^0) - \frac{(\dots ja((S_C(l=2)-S_C(l=3)) - \dots ji(l=3))}{\dots (M^0_{j(S_C(l=2)-S_C(l=3))}) 2f} \times \\
 & \times \int_{C(S_C(l=2)-S_C(l=3))} G_{Sp,j} \frac{\partial \{_{(l=3)} dc}{\partial n} = \{_{(l=3)}(M^0_{j(S_C(l=2)-S_C(l=3))}), \\
 & M^0_{j(S_C(l=2)-S_C(l=3))}, \{^0(M^0) = \{_{(l=2)} \\
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 u_{y,l=3}(M^0 \in S_{C(S_C(l=2)-S_C(l=3))}) &= \frac{\partial}{\partial y} (\{ (M^0 \in S_{C(S_C(l=2)-S_C(l=3))}) \}); \\
 u_{z,l=3}(M^0 \in S_{C(S_C(l=2)-S_C(l=3))}) &= \frac{\partial}{\partial z} (\{ (M^0 \in S_{C(S_C(l=2)-S_C(l=3))}) \}) \\
 u_{yz,Sc(S_C(l=2)-S_C(l=3))} &= \frac{1}{2} \left(\frac{\partial u_{y,l=3}}{\partial z} + \frac{\partial u_{z,l=3}}{\partial y} \right), \\
 \dagger_{yz,Sc(S_C(l=2)-S_C(l=3))} &= \frac{E_{Sc,l=3}}{1 + \dagger_{Sc,l=3}} u_{yz,Sc(S_C(l=2)-S_C(l=3))}
 \end{aligned} \tag{10}$$

$$k_{1ji(l=3)}^2 = \check{S}^2(\dots ji(l=3) / \} ji(l=3))$$

Conclusions

If, when moving to the next hierarchical level, the two-dimensionality axis does not change, but only the cross-sectional geometries of nested structures change, then the iterative process refers to modeling the displacement vector when moving from the previous hierarchical level to the next level. Within each hierarchical level, an integro-differential equation and an integro-differential representation are calculated using equations (3,5; 6,8; 9,11). If at some hierarchical level the structure of a local heterogeneity splits into several heterogeneities, then the double and contour integrals in expressions (3,5; 6,8; 9,11) are taken over all heterogeneities. In this algorithm, the case is considered when the physical properties of heterogeneities of the same level are the same, only the boundaries of the regions differ, and the centers of the hierarchical regions are shifted relative to each other. In this case, internal stresses are associated with the effect of shear in the rock massif. Thanks to the use of a model of a layered block medium with hierarchical inclusions, it is possible with the help of acoustic monitoring to determine the position of the highest values of internal stresses, to determine the type of stresses that have arisen and, with less effort, to implement the method of unloading the rock mass. If it is necessary to conduct short-term predictive monitoring of geodynamic areas and determine more accurately the position of the impending earthquake, as borehole active acoustic observations, they must be adjusted to a layered block model with hierarchical inclusions, and the values of the tensor of internal hierarchical stresses should be used as the observed monitoring parameter.

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