

# Improving the Sensitivity Levels Generated From Hypothesis Testing By Combining VLBI with GNSS Data

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# Sensitivity analysis

- ❖ The sensitivity level provides information on the detection capability of observing stations based on undetectable gross errors in a geodetic network solution.
- ❖ Sensitivity is the minimum value of the undetectable gross errors by hypothesis testing.
- ❖ The location of the station in the network and the total weight of its observations contribute to the sensitivity levels.

# Sensitivity analysis for combining VLBI with GNSS data

❖ The mathematical model:

$$\begin{bmatrix} N_{VLBI} & 0 \\ 0 & N_{GNSS} \end{bmatrix} \begin{bmatrix} x_{VLBI} \\ x_{GNSS} \end{bmatrix} = \begin{bmatrix} n_{VLBI} \\ n_{GNSS} \end{bmatrix}$$

$$(N_{VLBI} + N_{GNSS}) \cdot \hat{x} - (n_{VLBI} + n_{GNSS}) = 0$$

❖ The weighting factor:

$$w_{GNSS} = \frac{na_{GNSS}}{na_{VLBI}} \cdot \frac{(trace(N_{GNSS}) + trace(N_{VLBI}))/2}{trace(N_{GNSS})}$$

$$w_{VLBI} = \frac{(trace(N_{GNSS}) + trace(N_{VLBI}))/2}{trace(N_{VLBI})}$$

❖ Re-weighted mathematical model:

$$(w_{VLBI} \cdot N_{VLBI} + w_{GNSS} \cdot N_{GNSS}) \cdot \hat{x} - (w_{VLBI} \cdot n_{VLBI} + w_{GNSS} \cdot n_{GNSS}) = 0$$

$$N_w \cdot \hat{x} - n_w = 0$$

# Sensitivity analysis for combining VLBI with GNSS data

- ❖ The linearized form of the functional model at each co-location site:

$$l = L - L_0 = \Delta X^{ij} - \Delta X_0^{ij}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{tie} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{VLBI} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GNSS} \pm \begin{bmatrix} \sigma_{\Delta x} \\ \sigma_{\Delta y} \\ \sigma_{\Delta z} \end{bmatrix}$$

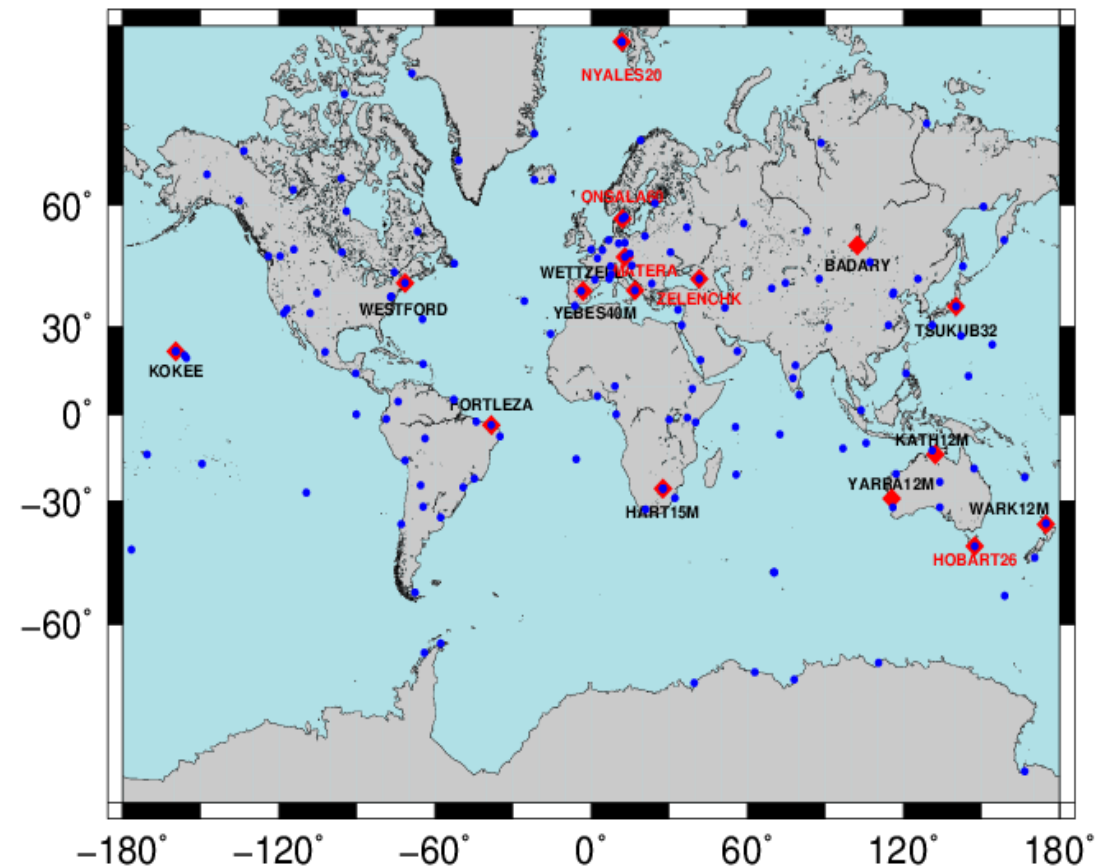
$$(N_w + N_{local\ tie})x - (n_w + n_{local\ tie}) = 0$$

- ❖ Sensitivity level of a station:

$$\|d\|_{min} = \frac{\delta_0 m_0^{total}}{\sqrt{\lambda_{max}}}$$

$$m_0^{total} = \sqrt{\frac{(v^T P v)_{VLBI} + (v^T P v)_{GNSS}}{f_{VLBI} + f_{GNSS}}}$$

# VLBI and GNSS stations using for combination



Blue dot: GNSS stations

Red/black diamond: VLBI stations

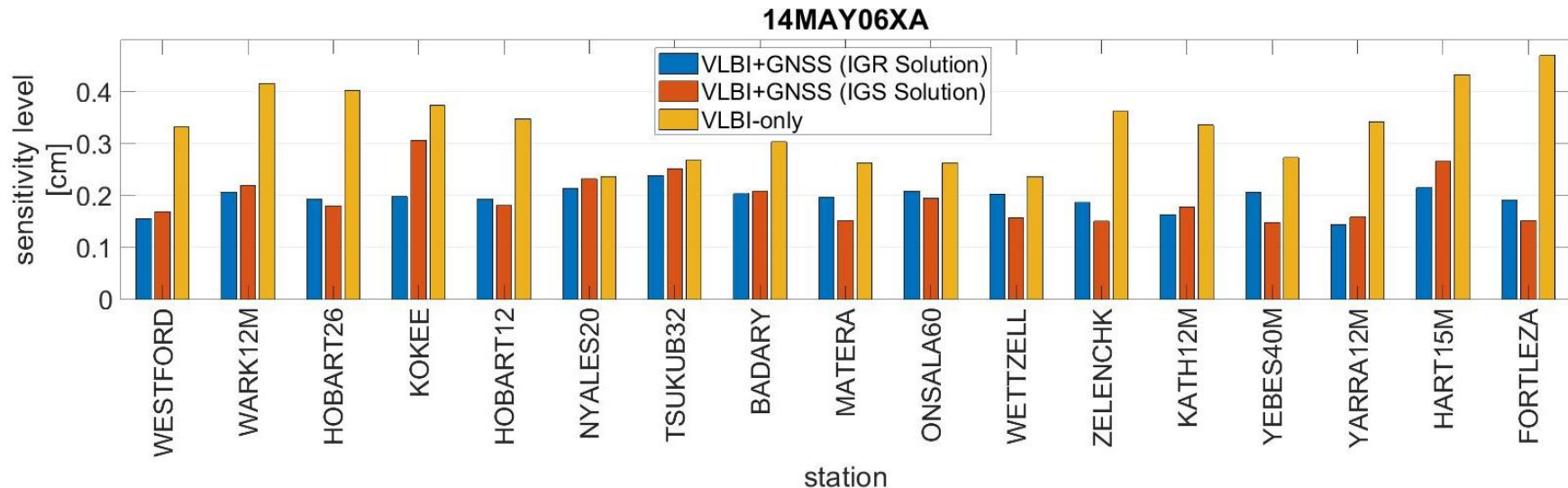
Red letter: Co-location sites

- ❖ 153 GNSS stations (ITRF2014) evaluated with EPOS
- ❖ 17 VLBI stations (ITRF2014-ICRF3) evaluated with VieVS@GFZ
- ❖ 5 co-located stations (ITRF2014)

## Co-location sites:

- ❖ ONSA-ONSALA60
- ❖ NYA1-NYALES20
- ❖ ZECK-ZELENCHK
- ❖ MATE-MATERA
- ❖ HOB2-HOBART26

# Sensitivity levels of the VLBI antennas after the combination vs. before the combination in the CONT14 session



IGS Solution → IGS Final Products

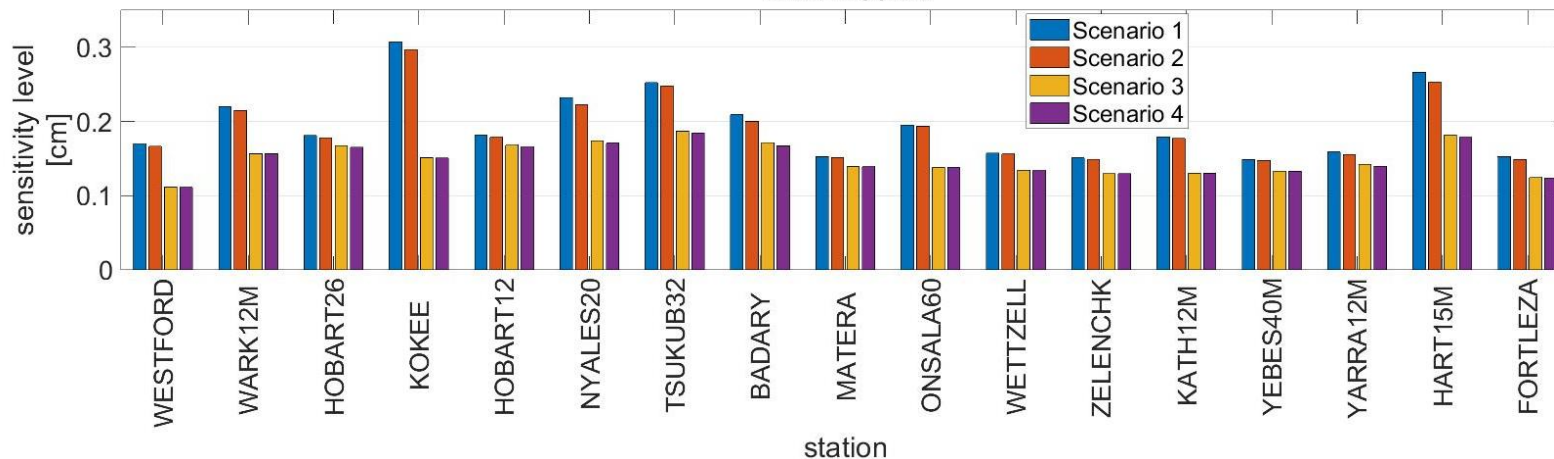
IGR Solution → IGS Rapid Products

# Effects of selected co-location sites on sensitivity levels in the CONT14 Session

- ❖ Effects of selected co-location sites on sensitivity levels are investigated using four scenarios.

| Scenario 1                        | Scenario 2                        | Scenario 3                        | Scenario 4                        |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| ONSA-ONSALA60<br>(max. $\sigma$ ) | ONSA-ONSALA60<br>(min. $\sigma$ ) | ONSA-ONSALA60<br>(max. $\sigma$ ) | ONSA-ONSALA60<br>(min. $\sigma$ ) |
| NYA1-NYALES20                     | NYA1-NYALES20                     | NYA1-NYALES20                     | NYA1-NYALES20                     |
| ZECK-ZELENCHK                     | ZECK-ZELENCHK                     | TSKB-TSUKUB32                     | TSKB-TSUKUB32                     |
| MATE-MATERA                       | MATE-MATERA                       | MATE-MATERA                       | MATE-MATERA                       |
| HOB2-HOBART26                     | HOB2-HOBART26                     | HOB2-HOBART26                     | HOB2-HOBART26                     |

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# Conclusions

- ❖ To increase the detection capability of the VLBI stations based on sensitivity, the selection of the co-location sites is very important.
- ❖ If co-location sites having minimum standard deviations are selected for the combination, the better sensitivity levels will be obtained.
- ❖ The smaller the sensitivity levels, the better the identification of the gross errors and the better the detection capability.

# Thank you!

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# References

- ❖ Altamimi, Z., P. Rebischung, L. Metivier, and X. Collilieux (2016), ITRF2014: A new release of the International Terrestrial Reference Frame modeling nonlinear station motions, *J. Geophys. Res. Solid Earth*, 121, doi:10.1002/2016JB01309.
- ❖ Böhm, J.; Böhm, S.; Nilsson, T.; Pany, A.; Plank, L.; Spicakova, H.; Teke, K.; Schuh, H. The New Vienna VLBI Software VieVS. In: Kenyon S., Pacino M., Marti U. (eds) *Geodesy for Planet Earth*. International Association of Geodesy Symposia, **2012**, 136. Springer, Berlin, Heidelberg. doi:[10.1007/978-3-642-20338-1\\_126](https://doi.org/10.1007/978-3-642-20338-1_126).
- ❖ Even-Tzur, G. Datum Definition and Its Influence on the Sensitivity of Geodetic Monitoring Networks. In *Proceedings of the 12th FIG Symposium, Baden, Austria, 22–24 May 2006*.
- ❖ Gendt G., Dick G., & Söhne W. (1997). GFZ Analysis Center of IGS - Annual Report. In J. F. Zumberge, D. E. Fulton, & R. E. Neilan (Eds.), *IGS 1996 Annual Report* (pp. 169–181). Pasadena, CA: JPL, Caltech.
- ❖ Glaser, S., König, R., Neumayer, K. H., Nilsson, Tobias, Heinkelmann, R., Flechtner, F., Schuh, H., (2018), On the impact of local ties on the datum realization of global terrestrial reference frames, *Journal of Geodesy*, 93,5, 655–667.
- ❖ Glaser, S., Fritsche, F., Sosnica, K., Rodriguez-Solano, C. J., Wang, K., Dach, R., Hugentobler, U., Rothacher, M., Reinhard, D., (2015), A consistent combination of GNSS and SLR with minimum constraints, *Journal of Geodesy*, 89, 1165–1180.
- ❖ Hsu R.; Hsiao K. Pre-Computing the Sensitivity of a GPS station for crustal deformation monitoring, *Journal of the Chinese Institute of Engineers*. **2002**, 25(6), 715–722. <https://www.tandfonline.com/doi/abs/10.1080/02533839.2002.9670745>
- ❖ Konak, H.; Küreç Nehbit, P.; İnce, C. D. Determination of experimental sensitivity capacities against crustal movements of dense-geodetic networks: a real GPS network application, *Arabian Journal of Geosciences*. **2019**, 12, 596. <https://link.springer.com/article/10.1007/s12517-019-4738-9>
- ❖ Küreç, P., Konak, H., (2014), A priori sensitivity analysis for densification GPS networks and their capacities of crustal deformation monitoring: A real GPS network application. *Nat. Hazards Earth Syst. Sci.*, 14, 1299–1308.
- ❖ Männel B., Brandt A., Bradke M., Sakic P., Brack A., & Nischan T. (2020) Status of IGS Reprocessing Activities at GFZ. In: *International Association of Geodesy Symposia*. Springer, Berlin, Heidelberg. [https://doi.org/10.1007/1345\\_2020\\_98](https://doi.org/10.1007/1345_2020_98).
- ❖ Nilsson, T.; Soja, B.; Karbon, M.; Heinkelmann, R.; Schuh, H. Application of Kalman filtering in VLBI data analysis. *Earth Planets Space*, **2015**, 67(1),1–9. <https://link.springer.com/article/10.1186/s40623-015-0307-y>
- ❖ Nothnagel, A.; Artz, T.; Behrend, D.; Malkin, Z. International VLBI service for Geodesy and Astrometry – Delivering high-quality products and embarking on observations of the next generation. *Journal of Geodesy*. **2017**, 91(7), 711–721. <https://link.springer.com/article/10.1007/s00190-016-0950-5>
- ❖ Schuh, H.; Behrend, D. VLBI: A fascinating technique for geodesy and astrometry, *Journal of Geodynamic*. 2012, 61, 68–80. <https://doi.org/10.1016/j.jog.2012.07.007>.