A modified Coulomb failure seismicity model to study earthquake occurrence and frequency-magnitude distributions

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- Can we use a single model to explain all aspects of seismicity?
- What is the relationship between background seismicity & stress?
- What controls aftershock activity?
- What does the short-term absence of seismicity mean?
- Is the Gutenberg Richter relation valid for time-dependent loads?
Seismicity models

1) Forecast rate and magnitude of earthquakes in a region under specified stressing
2) In addition to the background rate the temporal response of stress steps must also be explained.

Existing models (not complete):
A) Linear Coulomb failure (1 parameter)

B) Rate & state frictional (Dieterich, 1981, 1994) (3 parameter)
Concept of earthquake nucleation at fault asperities

Snap shots of a growing, crack-type asperity

a) subcritical state:
asperity has a subcritical length and the stress intensity in the cohesion zone is below $S_0$

b) critical state, rupture begins:
asperity has grown to critical length and the stress intensity exceeds the strength $S_0$

from Dahm (submitted to GJI)
Implementation: concept

1. Random distribution of asperities at different stages of peak stress $\sigma_c$

2. Stress loading acting on volume $V$ affects each asperity equally

3. Asperities with $\sigma_c$ above strength $S_0$ are triggered and “removed” from distribution (points above red line)

Dahm (submitted to GJI)
Implementation: numerical scheme

Dahm (submitted to GJI)

Numerical scheme:

\( p \): probability to be triggered

(depends only on stress, not on \( t \))

\( \chi \): distribution of asperities

(function of stress and time \( t = i\Delta t \))

\( n \): number of EQ triggered in interval \( \Delta t \)

\[ t = t_0 : \quad p = H(\sigma - S_0), \quad \chi = \chi_0 V \cdot H(S_0 - \sigma) \]

\[ n = 0 \]

\[ t_{i+1} = t_i + \Delta t : \quad \sigma_c(t_{i+1}) = \sigma + \dot{\sigma}_c(t_i) \Delta t = \sigma + \Delta \sigma_c \]

\[ \chi^{(tmp)} = \chi_0 V \cdot H(S_0 - \sigma_c(t_{i+1})) = T^S(\Delta \sigma_c) \cdot \chi_i \]

\[ \Delta n = \int_{-\infty}^{\infty} \chi^{(tmp)}(\sigma) \cdot p(\sigma) d\sigma \]

\[ \chi_{i+1} = \chi^{(tmp)} \cdot (1 - p) \quad \text{(update distribution)}, \]

\[ p(\sigma) = H(\sigma - S_0) + H(S_0 - \sigma) e^{-(S_0 - \sigma)/\delta \sigma} \]
Positive stress step $\Delta \sigma$ to simulate aftershock distributions

Aftershock rate: $r(t) - r(\infty) \leq \chi_0 V \cdot \delta \dot{\sigma} \left[ e^{-\frac{\dot{\sigma} \Delta t}{\delta \dot{\sigma} \Delta t}} + \frac{\Delta \sigma_c}{\delta \sigma \Delta t} \delta(t) \right] \cdot \frac{\Delta t}{(\Delta t + t)} \quad \text{with } t \geq 0$
Application – global aftershock productivity and decay times

Earthquake productivity \((t-t_0 < 2.4h)\) as a function of the main shock magnitude

\[ r(t<2.4h)/r_{max} \]

\[ \sim \log \sigma_i^{3/2} + 1.9 \]

\[ K \sim 10^{0.83 M_{W}} \]

Aftershock decay rates over lapse time for different main shock intervals

\[ r(1/\text{days}) \]

\[ (t-t_0) \text{ (days)} \]

Data from Hainzl & Marsan (2008)

Dahm (submitted to GJI)
Background seismicity rate at subduction zones

- The background rate depends on 2 parameters
- Increase nonlinear with stressing
- Saturates if $e^{e^{-...}} = 0$

Background rate: $r_\infty = \chi_0 V \cdot \delta \dot{\sigma} \left[ 1 - e^{-\frac{\dot{\sigma}_c}{\delta \dot{\sigma}}} \right]$
The effect of $\Delta \sigma < 0$ is similar to RS:

- $r$ is reduced but $> 0$
- recovery starts immediately
- the larger $|\Delta \sigma|$ the longer the time needed to recover to $r_\infty$

Dahm (submitted to GJI)
Application KTB: 2 yrs of pumping & injection in 4 km depth

Data from J. Kummerow, see Haney et al. (2011)

\[ \chi_0 = 444 \text{ MPa}^{-1} \text{km}^{-3} \]
\[ \delta \sigma \Delta t = 0.25 \text{ MPa} \]
\[ r = 1 \text{ km} \]
\[ \Delta t = 1 \text{ d} \]
Weekly loading in a salt mine (Morsleben) from cement refilling from Becker et al. (2010)

thermally induced Coulomb stress

observed

from Becker et al. (2010)

Dahm (submitted to GJI)
What about frequency magnitude distributions?

The asperity nucleation model (e.g. Ohnaka, 2013) suggests that the seismic moment $M_0$ is controlled by the size $a_c$ of the asperity (or breakdown slip $\Delta u$). We assume a fractal distribution of asperities by Dahm (submitted to GJI):

$$\rho(a_c) = \frac{1}{a_0} \left( \frac{a_c}{a_0} \right)^{-2}$$

This leads to Gutenberg Richter scaling for steady state background seismicity as:

$$r_\Sigma(M_0, t = \infty) = \chi_0 V \cdot \left[ \frac{M_0}{M_r} \right]^{-2/3} \cdot f, \quad \text{with } f = \delta \dot{\sigma} \left[ 1 - e^{-\dot{\sigma}/\delta \dot{\sigma}} \right]$$

By analogy, we suggest a scaling of time dependent rates as:

$$\frac{M_0(t)}{M_r} = \left( \frac{\chi_0 V}{\Delta t} \right)^{3/2} \cdot \left[ r_\zeta(M_0, t) \right]^{-3/2} \cdot \left[ e^{-\dot{\sigma}^*(t)/\delta \dot{\sigma}} \right]^{3/2}$$

convolution with step response of duration $t^*$

Dahm (submitted to GJI)
Application to KTB seismicity

Predictions if $t^* > 0$

- $M_r = -5.3$, $t^* = 116$ days

Predicted f-M relations if a memory effect is considered ($t^* > 0$) or not considered ($t^* = 0$)

Traditional approach if $t^* = 0$

Measured f-M distributions at different times after the onset of seismicity (grayish)

data from J. Kummerow, see Haney et al. (2011)
### Parameter comparison

<table>
<thead>
<tr>
<th>Application</th>
<th>$M_c$</th>
<th>$V$</th>
<th>$\chi_0$</th>
<th>$\delta\sigma$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morsleben salt</td>
<td>-5</td>
<td>$2.25 \cdot 10^{-6}$</td>
<td>$0.9 \cdot 10^{12}$</td>
<td>110</td>
<td>Becker et al. (2010)</td>
</tr>
<tr>
<td>KTB pump-inject</td>
<td>-2.3</td>
<td>2.25</td>
<td>444</td>
<td>91</td>
<td>Haney et al. (2011)</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>Shapiro et al. (2006)</td>
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<tr>
<td>M6 aftershocks</td>
<td>4.5</td>
<td>$1 \cdot 10^3$</td>
<td>2.2</td>
<td>2</td>
<td>Hainzl &amp; Marsan (2008)</td>
</tr>
<tr>
<td>M7 aftershocks</td>
<td>4.5</td>
<td>$3 \cdot 10^4$</td>
<td>2.2</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
Summary

- The modified Coulomb failure model considers time-dependent nucleation.
- It has only 2 independent parameter to explain seismicity (but $\Delta t$!).
- The Omori-Utsu relation is explained, but intensity is $t$-dependent and $p=1$.
- A instantaneous peak of seismicity is predicted at the time of the mainshock.
- Stress shadow effects can be modelled.
- A small parameter range indicated for very different applications.