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19 - 30 April 2021

Towards Backus-Gilbert style inversions for mantle anisotropy using normal mode data

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Why use the SOLA method?

Over the past decades, global tomographic models have exploited variants of the damped least-squares (DLS) linear inversion approach. However, DLS-based models can have **amplitudes locally biased** towards lower or higher values, especially in regions of poor data illumination. This can potentially cause physical misinterpretations of tomographic images. Moreover, the computation of the full DLS-generalised inverse - required to infer both the model resolution and its covariance - is **prohibitive** for large-scale problems.

The SOLA (*Subtractive Optimised Locally Averages*) method (*Zaroli, 2006*)

- is based on a **Backus–Gilbert** (B-G) approach
- allows to produce quantitative **unbiased** tomographic images
- provides the estimated model together with the full appraisal associated to this model, composed of **uncertainties** and **resolution**
- retains all the advantages of the B-G approach but is **computationally more efficient** and versatile.

Zaroli, C. (2016). Geophys. J. Int.

DLS vs. SOLA (1)

What are we looking for?

DLS

Estimates of the (damped) model parameters themselves in specific locations

SOLA

A linear combination of the true model parameters → their average with uncertainties

How do we remove the non-uniqueness of the solution?

DLS

Introducing *ad hoc,* subjective regularisation constraints on the model itself (norm damping or smoothing)

↓

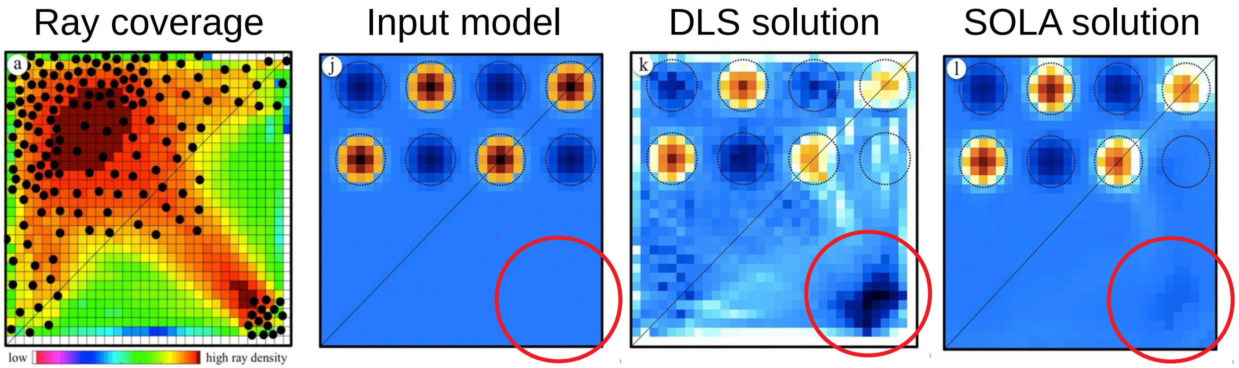
Amplitudes can be biased

SOLA

Through the process of averaging: a unique combination of the parameter can be found even if the parameters are not uniquely defined

↓

Amplitudes are constrained to be unbiased



Toy problem: in regions with isolated receivers (bottom right corner), DLS-amplitudes are biased (dark blue spot in the red circle, not present in the input model or in the SOLA solution). From *Zaroli et al., 2017*

Zaroli, C. et al. (2017). Geophys. Res. Lett.

DLS vs. SOLA (2)

How do we formulate the minimisation problem?

DLS

Aims to minimise the data misfit and the model complexity

SOLA

Aims to minimise the difference between target and resolving kernels (linked to the model resolution) and model uncertainties

Key idea of SOLA: we specify an *a priori* target form for each resolving kernel.
In this way, we add *a priori* information on the model resolution, but not on the model itself!

Do we always have info about uncertainties and resolution?

DLS

NO, the full generalised inverse is often not computed for large-scale problems

SOLA

YES, it is inherent to the method

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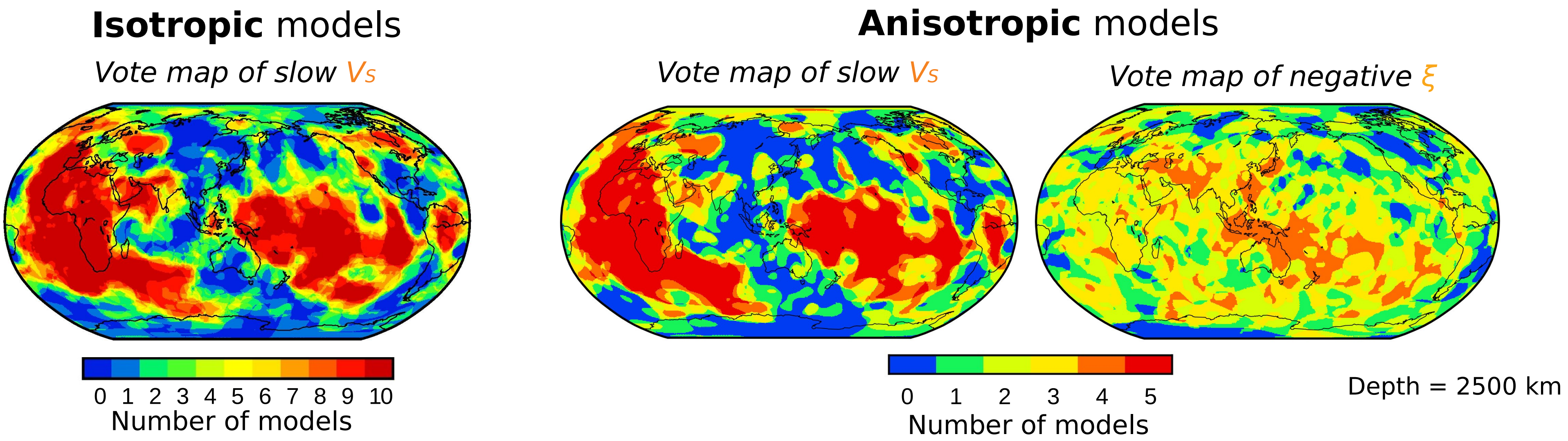
Why study seismic anisotropy?

Seismic anisotropy can provide us information about mantle flow, and therefore the thermal state, composition and mechanical properties of the mantle.

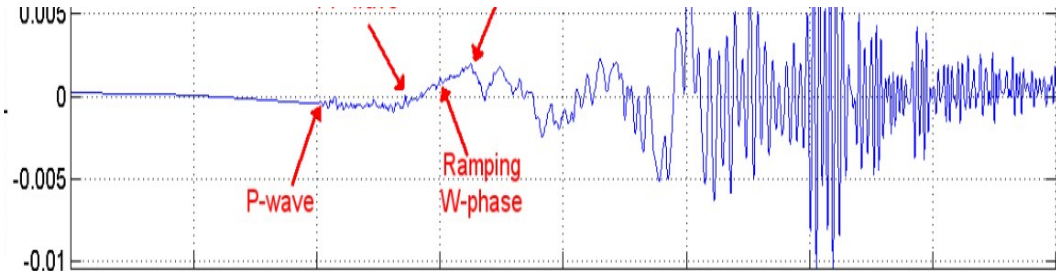


While isotropic models are consistent at least on long wavelengths (as evidenced in the vote maps above), agreement on anisotropy is still lacking, particularly in the deep mantle.

The lack of consensus and uncertainty information makes the interpretation of seismic anisotropy difficult!

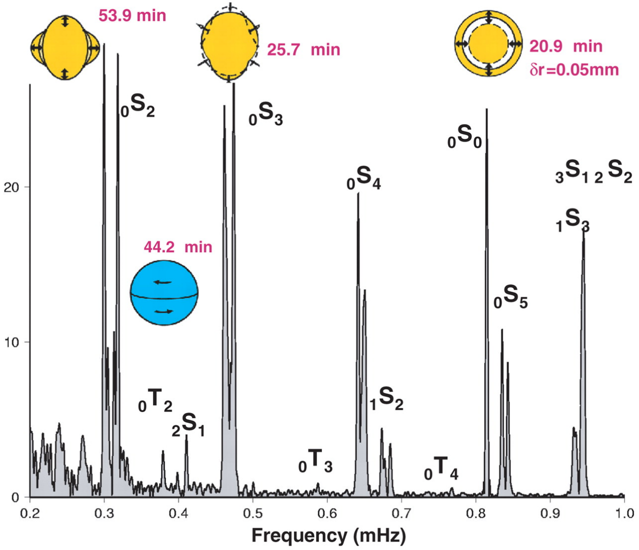
What are normal modes and why use them?

Normal modes are the long-period (~3000-100 s) standing waves of the Earth observed after very large earthquakes ($M_w \geq 7.4$). They can be observed as peaks at discrete frequencies of a Fourier transformed seismogram.



Displacement seismogram of the 2004 Sumatra earthquake.
From *Lockwood and Kanamori, 2006*

Fourier transform



Amplitude spectrum from the 2004 Sumatra earthquake.
From *Park et al., 2005*

Normal modes have been chosen because they provide **global coverage** and are directly **sensitive to V_S and V_P anisotropy**, whose interpretation provides information about mantle flow.

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Park, J., et al. (2005). Science
Lockwood, O. G., & Kanamori, H. (2006). Geochem, Geophys, Geosys

SOLA and normal modes: 3D noise

- Normal modes are sensitive to multiple parameters simultaneously
- With SOLA, we treat the additional sensitivity of the parameters not of interest as noise – called *3D noise (Masters et al., 2003)*, instead of including scaled sensitivity kernels
- We can make an estimation of the 3D noise by computing the splitting function coefficients from models with only V_P, V_S, ρ or topography at internal boundaries perturbations
- For example, the total 3D noise when resolving for e.g. V_S will be given by the contribution from V_P, ρ and topography
- These uncertainties are added to the data uncertainties in the inversion
- The 3D noise characterisation is vital, since the inversion is largely driven by errors.

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Masters, G., & Gubbins, D. (2003). Phys. Earth Planet. Inter.

First inversions set-up

For simplicity, we start by inverting for isotropic velocities.

The SOLA method requires the user to define:

- Sensitivity matrix, which contains the sensitivity kernels of the normal modes

  → sensitivity kernels discretised in 52 layers
- Data vector, which contains the splitting function coefficients and their uncertainties (and, in our case, the 3D noise)

  → same dataset as SP12RTS (*Koelemeijer at al., 2016:* 143 spheroidal normal modes)
- A vector that contains the thickness of the layers of the model parameterisation

  → model space parameterised in 52 layers
- Target kernels

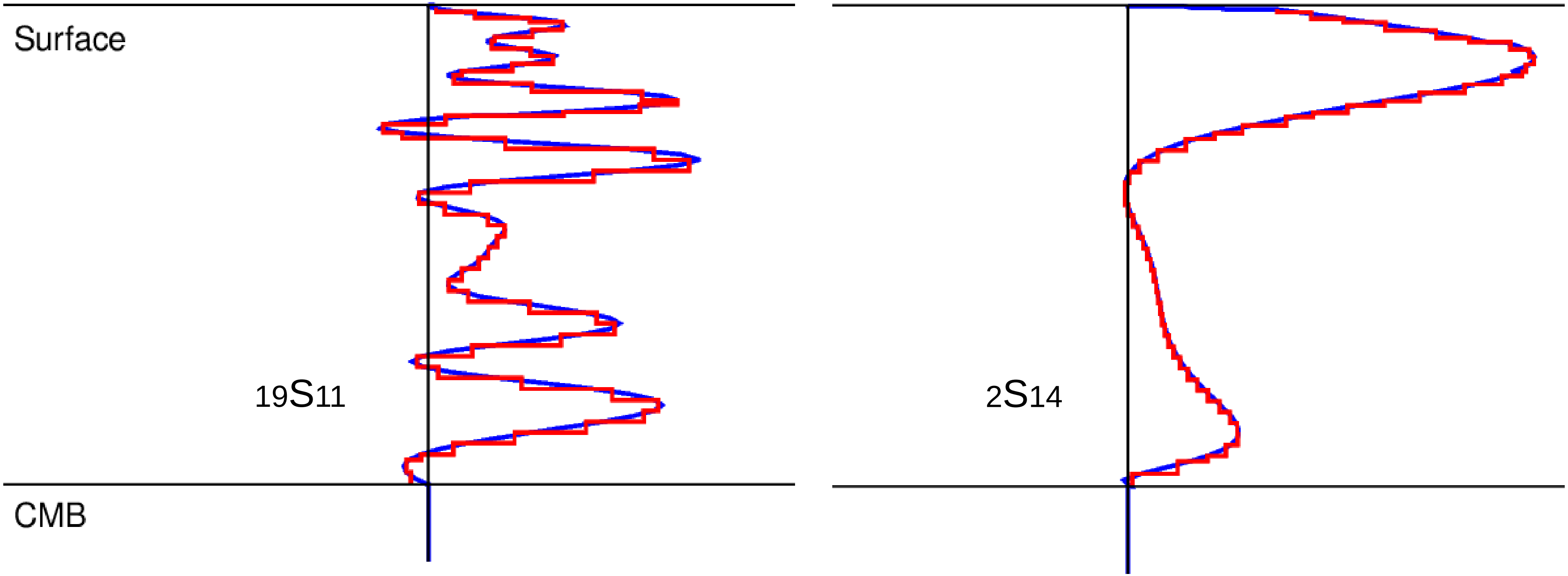
  → 52 wide and overlapping target kernels

We first invert for Vs, as it has the largest amplitudes in the Earth.

Koelemeijer, P. et al. (2016). Geophys. J. Int.

Isotropic inversions set-up (1)

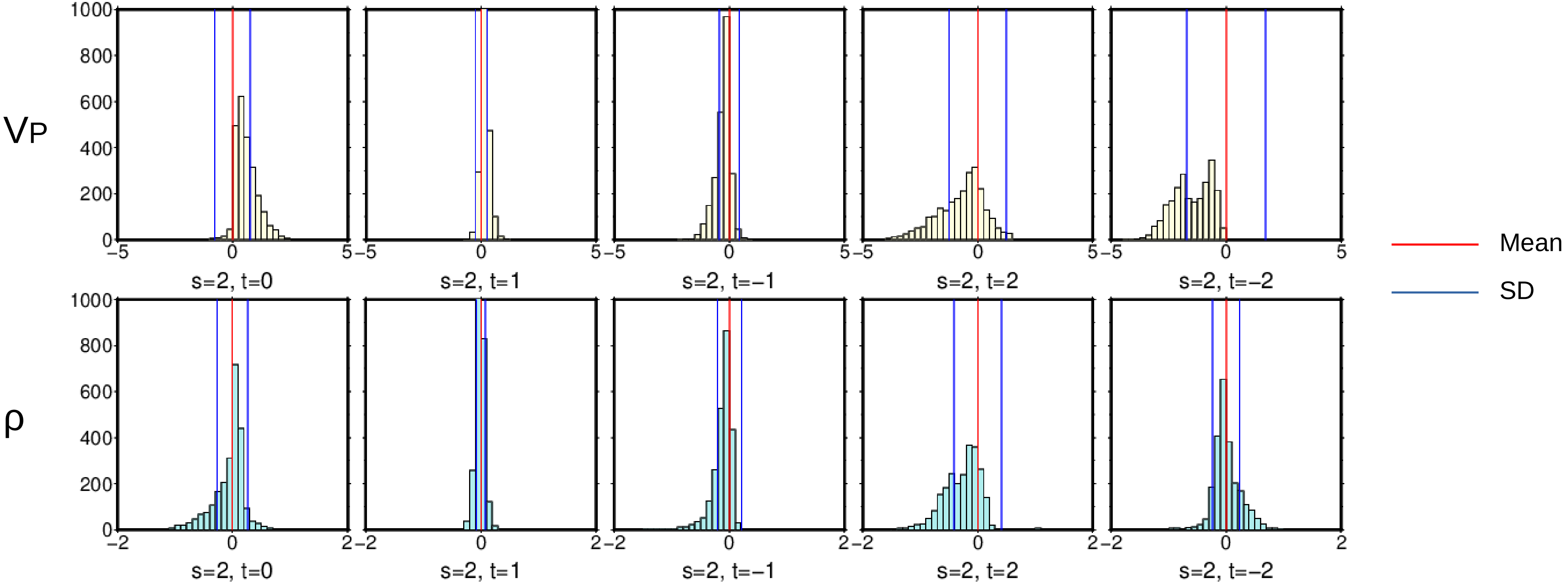
Sensitivity kernels



Example of two continuous V_S sensitivity kernels (blue) and V_S sensitivity kernels discretised in 52 layers (red) for modes $_{19}S_{11}$ (left) and $_{2}S_{14}$ (right).

Isotropic inversions set-up (2)

3D noise

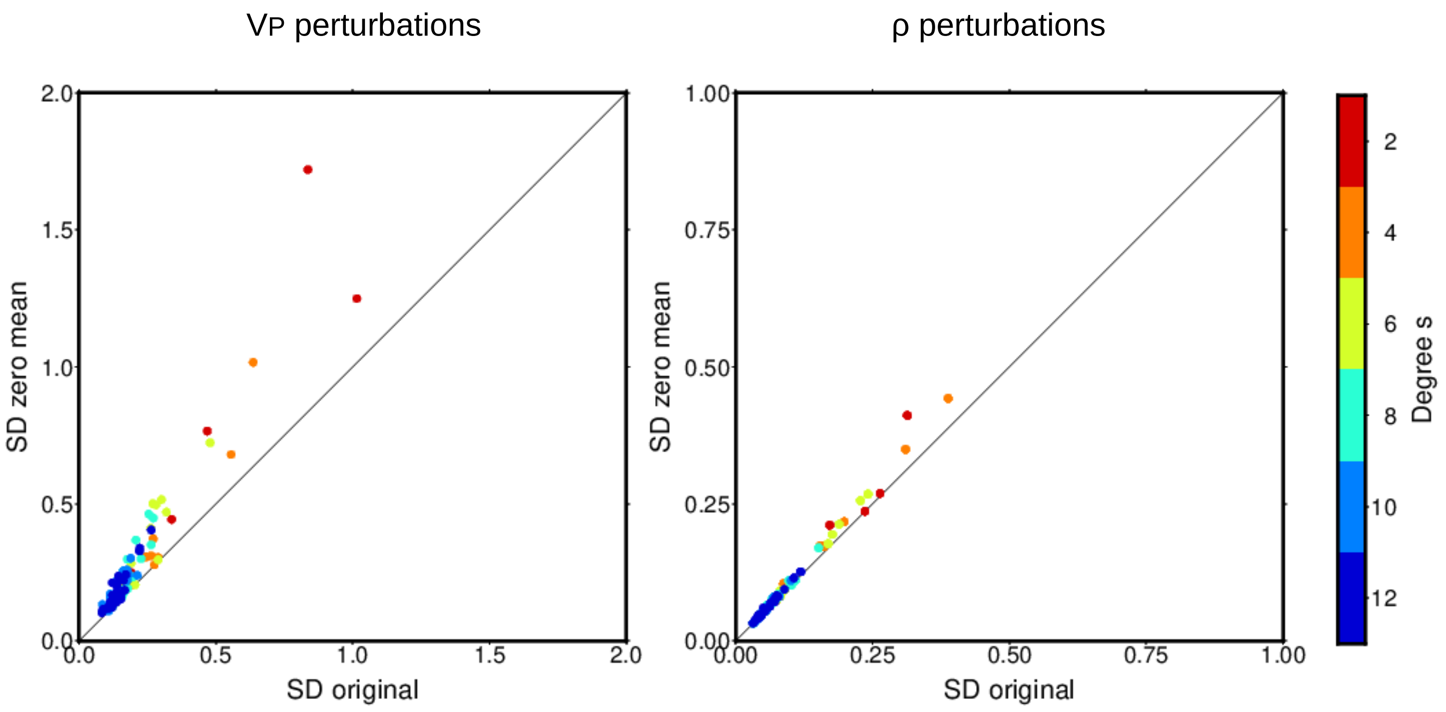


Quantification of 3D noise for VP and ρ computed using the splitting function predictions for 16 existing models, each time only including VP or ρ.

For each degree *s* and order *t* (*s=2* shown in the figure above), we evaluate the standard deviation (SD) of the distribution, imposing the mean to be zero.

Isotropic inversions set-up (2)

3D noise



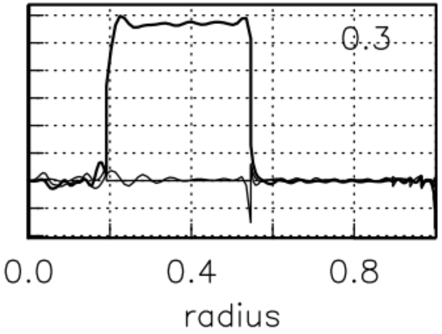
Comparison of 3D noise estimated for VP and ρ imposing either a zero mean ("SD zero mean", y-axis) or not ("SD original", x-axis).

SD with zero mean is always higher, or equal, to the original SD, meaning we do not underestimate the 3D noise by imposing a zero mean.

Isotropic inversions set-up (3)

Target kernels

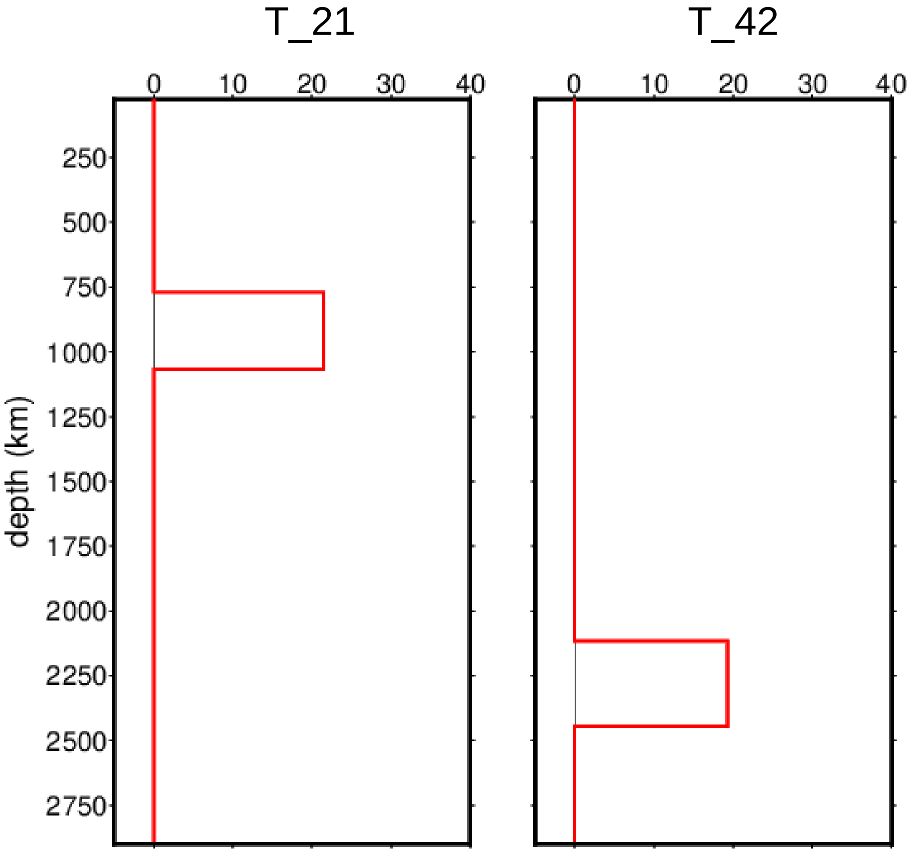
Similar to *Masters et al. (2003),* we aim to obtain resolving kernels in the shape of a boxcar.



An example of boxcar resolving kernel.
From *Masters et al., 2003*

Therefore, we build boxcar target kernels (two examples in the figure on the right).

Their amplitudes are normalised by their respective thickness.



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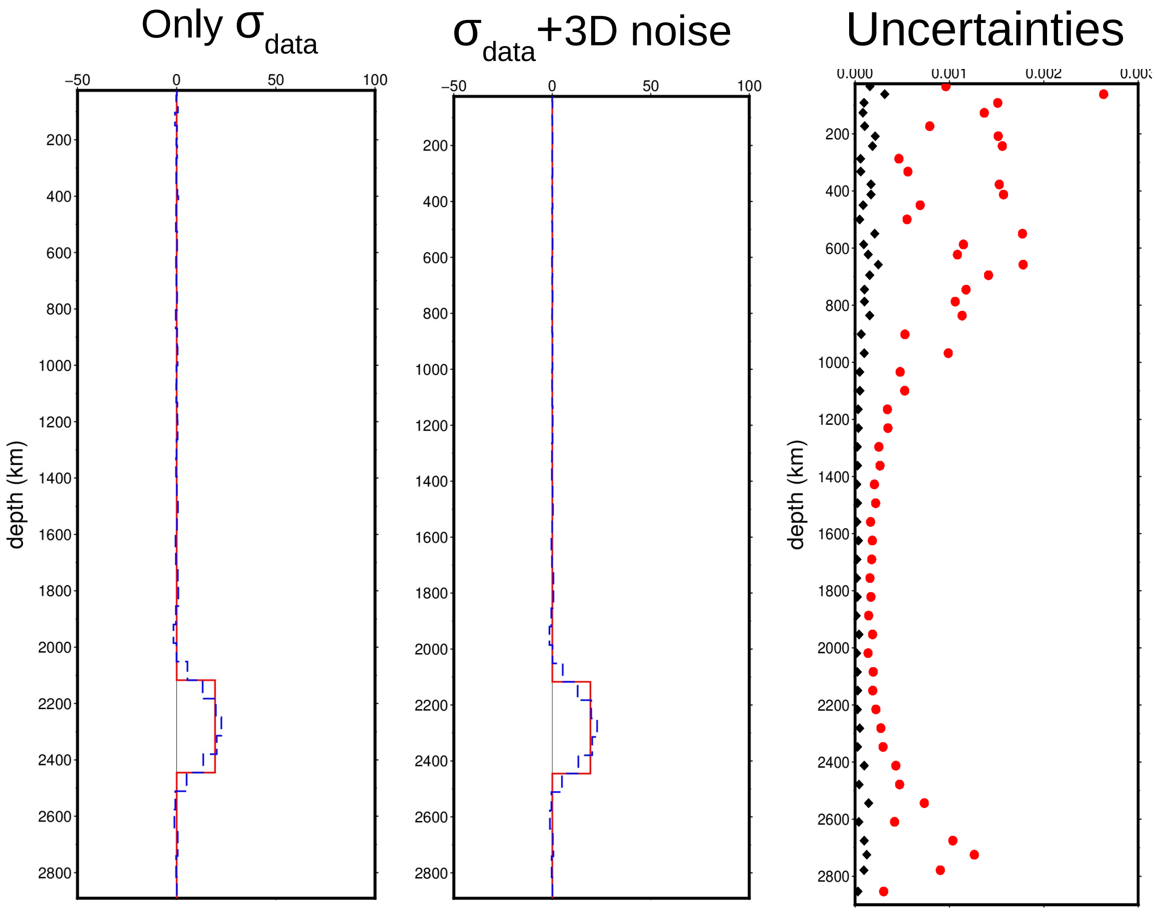
Masters, G., and Gubbins, D. (2003). Phys. Earth Planet. Inter.

Isotropic inversions preliminary results

Resolving kernels and uncertainties

The resolving kernels (i.e. resolution) are unaffected by the addition of 3D noise but uncertainties increase substantially.



Target kernels
Resolving kernels

With 3D noise
Without 3D noise

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Summary

- Anisotropy can provide us with valuable information about the mantle flow, but anisotropic tomography models are not consistent → difficult interpretation!

- The SOLA method allows us to obtain unbiased tomography images, provided with uncertainty and resolution information

- We aim to develop a new anisotropy model based on normal mode data using SOLA. Normal modes are sensitive to VS and VP anisotropy, and provide global coverage

- 3D noise characterisation is a vital step: errors largely drive the inversions

- Our preliminary results are able to produce resolving kernels similar to the target kernels, with 3D noise having a big effect on model uncertainties and not on model resolution.