Can ETAS-Models Predict Earthquake Doublets?

Testing Anisotropic and Restricted Spatial Kernels in Japan

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① Motivation, Research Questions and Data

Earthquake sequences add substantial hazard beyond the solely declustered perspective of common probabilistic seismic hazard analysis. The financial losses produced by sequences of several, similarly strong events are of particular interest to the risk assessment of governments and in the insurance industry.

Definition of Earthquake Doublets

A suitable term for strong event clusters is given by so-called earthquake *doublets* (Felzer, 2004). However, there is no consistent specification of this term in the literature.

In our work, we define an earthquake doublet as a pair of earthquakes with no more than 0.4 units of magnitude difference, occurring at most 365 days apart from each other and within a distance of 2.5 rupture length estimates.

Main Drivers of Doublet Occurrences

- triggering of direct and secondary aftershocks in a sequence
- independent seismicity in the same time-space window
- magnitude size distribution of triggered events(*)

(*) In our study, we neglected this driver by assuming identically distributed magnitudes of both triggered and independent events.

Research Questions

- How well does the widely used *Epidemic Type Aftershock Sequence (ETAS)* model predict occurrence frequencies of earthquake doublets?
- Can we obtain more realistic predictions by ETAS model variants?
- Do typical global catalog scale measures, such as log-likelihood or Akaike's Information Criterion (AIC), reflect the goodness of model fits for strong event clusters?

Utilized Earthquake Catalogs

We tested our models on an earthquake catalog for Japan:

- time window: 1/7/1997 - 31/10/2020
- space window: latitude 28-44°N, longitude 129-144°E
- complete from $M_w = 4.0$
- from *National Research Institute for Earth Science and Disaster Resilience (NIED)* (Kubo, 2002)

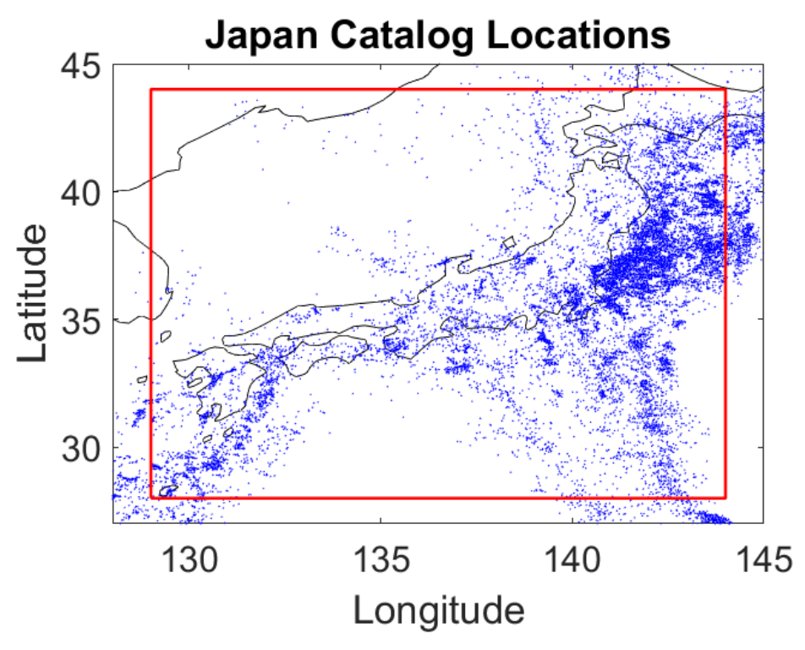

Figure 1: *Blue: event locations; Red: Spatial polygon*

② ETAS Model Variants

We use the ETAS model as described in Jalilian (2019):

$$\lambda(t,x,y|H_t) = \mu\, h(x,y) + \sum_{i:t_i<t} \kappa_{A,\alpha}(m_i)\; g_{c,p}(t-t_i)\; f_{D,\gamma,q}(x,y,i) \qquad (1)$$

with parameters $\mu, A, \alpha, c, p, D, \gamma > 0,\; q > 1$ and

- $h(x,y)$: spatial probability density function (pdf) of the background seismicity.
- $\kappa_{A,\alpha}(m) = A\,\exp(\alpha(m-M_c))$: expected number of *direct* aftershocks triggered by an event with magnitude m, given cut-off magnitude M_c.
- $g_{c,p}(t-t_i)$: Omori-Utsu law for the decay of aftershock rates with increasing after-event time $t-t_i$
- $f_{D,\gamma,q}(x,y,i)$: Spatial trigger function that models the decay of aftershock rates depending on the distance of (x,y) to the triggering event i

Modification I: Spatial Kernel

We test the conventional isotropic design and a generalized anisotropic kernel

$$f_{D,\gamma,q}(x,y,i) := \begin{cases} \frac{q-1}{D\exp(\gamma(m_i-M_c))}\left(1+\frac{\pi r_i(x,y)^2}{D\exp(\gamma(m_i-M_c))}\right)^{-q} & \text{isotropic (iso),} \\ \frac{q-1}{D\exp(\gamma(m_i-M_c))}\left(1+\frac{2\;l(m_i)\;r_i(x,y)+\pi r_i(x,y)^2}{D\exp(\gamma(m_i-M_c))}\right)^{-q} & \text{anisotropic (aniso),} \end{cases} \qquad (2)$$

with

- $l(m_i)$: Estimated rupture length depending on magnitude m_i
- $r_i(x,y)$: Point-to-point distance of (x,y) to epicenter location of event i in isotropic model; Point-to-line distance of (x,y) to estimated rupture line of event i in anisotropic model

Modification II: Spatial Restriction

We test the spatial restriction of the spatial kernels (2) to a distance of 2.5 rupture lengths, i.e.

$$\tilde{f}_{D,\gamma,q}(x,y,i) = \begin{cases} f_{D,\gamma,q}(x,y,i) & \text{if } r_i(x,y) \le 2.5\, l(m_i) \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

and re-normalize (3) to a pdf.

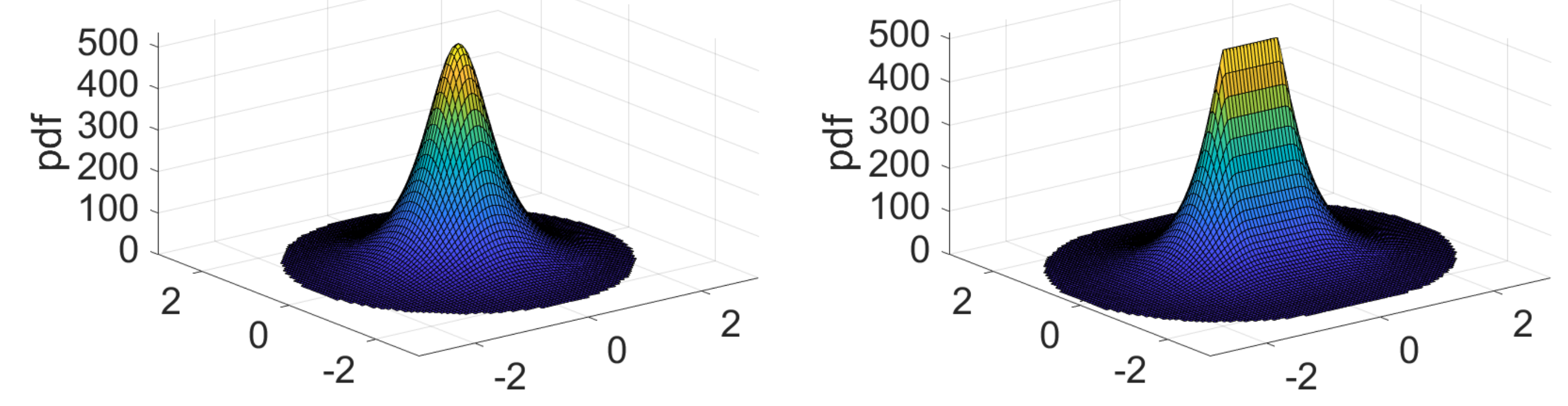

Figure 2: *PDFs of an isotropic (left) and anisotropic (right) spatial kernel with restriction (3) for exemplary magnitude* $M_w = 5.0$.

③ Results

Models Under Comparison

M_0: unrestricted iso; M_1: unrestricted aniso; M_2: restricted iso; M_3: restricted aniso

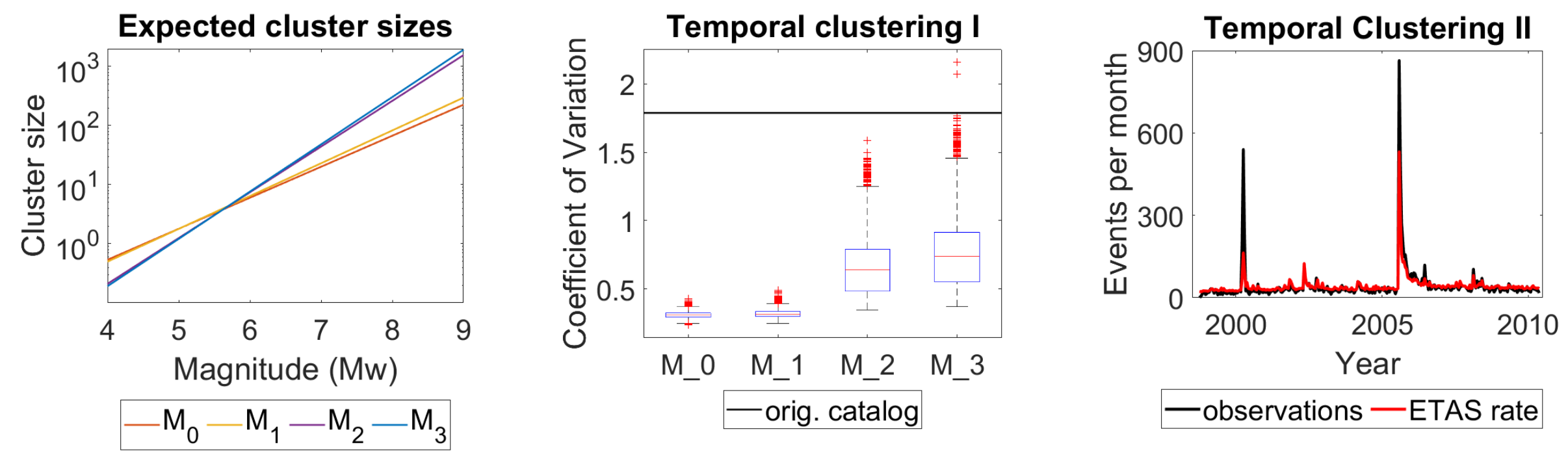

Figure 3: *Expected cluster sizes (left); Coefficients of variation of simulated monthly event occurrences vs observation (center); Monthly event occurrences observed vs integrated ETAS rate of model* M_3 *(right)*

⇒ Restricted models lead to larger cluster sizes for $M_w \geq 5.5$

⇒ All models, unrestricted more than restricted, smooth out spatio-temporal occurrences

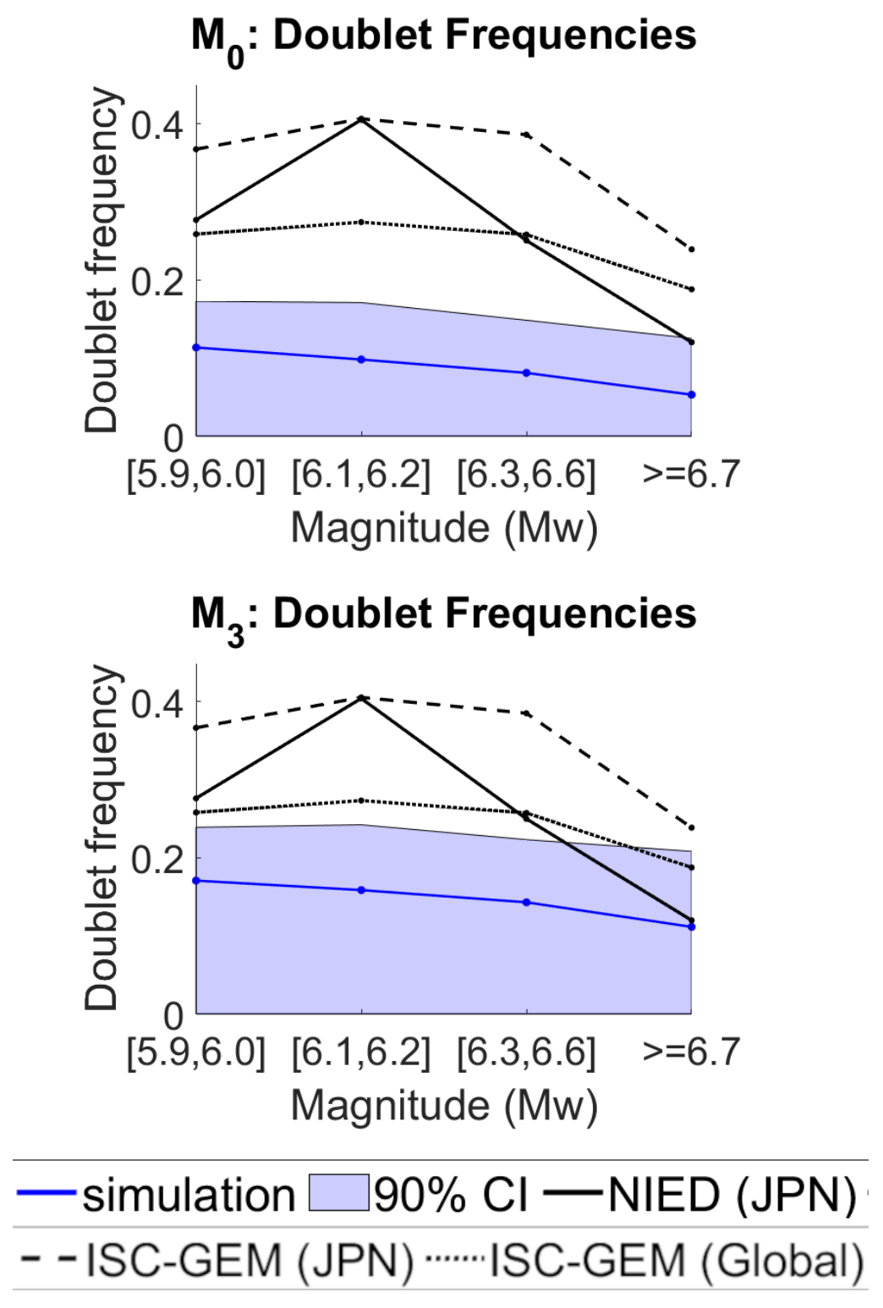

Figure 4: *Doublet simulations with models* M_0 *(top) and* M_3 *(below) vs observations*

Earthquake Doublet Frequencies

We benchmarked the simulated doublet rates (fig. 4) against observations in

1. the NIED catalog for Japan (JPN),
2. a regional JPN-extract from the ISC-GEM catalog
3. the entire, global ISC-GEM catalog (Di Giacomo, 2018)

⇒ Restricted models, here M_3, lead to larger and more realistic doublet rate predictions

⇒ Doublet rates in Japan seem to be structurally larger than globally

M_0	M_1	M_2	M_3
-21063	-18626	-22684	-19814

Table 1: Log-likelihood values

⇒ **Log-likelihood** favors unrestricted models (table 1).

④ Conclusion & Outlook

⇒ The conventional ETAS model poorly represents earthquake doublets with $M_w \geq 5.9$.

⇒ The spatial restriction (3) highly improves doublet rate predictions, by shifting aftershock productivity from smaller to larger events and increasing spatio-temporal clustering.

⇒ Log-likelihood or AIC are no adequate tools to measure goodness of fit for strong event clusters since they are dominated by the majority of weak events.

Future Research

We plan to investigate the potential impact of trigger-dependent magnitude size distributions on earthquake doublets. Furthermore, we want to analyze the influence of geophysical information, e.g. strain rates and heat flow, on the aftershock productivity.

References

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