Solutal Convection in Layered Sorbing Porous Media

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Problem Statement

- We study convective instability in the vertically layered porous media saturated with mixture. The mixture consists of a carrier fluid and solid nanoparticles. The nanoparticles are considered as solute within the continuous approach.
- The porous media are two horizontal sublayers with different permeabilities.
- The solute concentration is maximal near the upper boundary and is zero near the lower boundary of the superposed sublayers. Thus, one has suitable conditions for the onset of solutal convection in the gravitational field.
Equations for solutal convection

The Mobile-Immobile Media model describes mixture transport and immobilization [1, 2]

\[ \rho = \rho_0 \left[ 1 + \beta_C \left( C - C_0 \right) \right] \]

The Boussinesq approximation is used in the equations for convection in each of the sublayers.

Equations in the \( i \)-th porous sublayer (\( i = 1, 2 \)):

1. The equation for mixture transport

\[ \frac{V_f}{K(\phi_i)} U_i = -\frac{1}{\rho_0} \nabla P_i - \beta_C g \left( C_i - C_0 \right) \gamma \]

2. The mass balance equation for particles

\[ \frac{\partial}{\partial t} \left[ \phi_i \left( C_i + S_i \right) \right] + \left( U_i \nabla \right) C_i = \text{div} \left( D_f \phi_i \nabla C_i \right) \]

3. The mass balance equation for fluid

\[ \frac{\partial (\rho_0 \phi_i)}{\partial t} + \text{div} (\rho_0 U_i) = 0 \]

4. Kinetic equation for mixture redistribution between the mobile and immobile particles

\[ \frac{\partial S_i}{\partial t} = \alpha \left( K_d C_i - S_i \right) \]

5. The porosity variation due to immobilization

\[ \phi_i = \phi_{0i} \left[ 1 - \beta_\phi \left( S_i - S_0 \right) \right] \]

6. The solutal pore shrinkage coefficient

\[ K(\phi_i) = \frac{D_p^2 \phi_i^3}{180 (1 - \phi_i)^2} \]

\[ \beta_\phi = -\frac{1}{\phi_0} \left( \frac{\partial \phi}{\partial S} \right) > 0 \]


Boundary conditions

Impermeability conditions are fulfilled and concentrations are kept constant at the upper and lower solid boundaries, respectively:

\[ z = H : \quad U_{2z} = 0, \quad C_2 = C_0 + \Delta C \]

\[ z = 0 : \quad U_{1z} = 0, \quad C_1 = C_0 \]

Continuity conditions for normal velocities, normal stresses, concentrations and solutal transfer rates are set at the interface between the sublayers [1]:

\[ z = \delta H : \quad U_{1z} = U_{2z}, \quad P_1 = P_2, \quad C_1 = C_2, \quad \phi_1 D_f \frac{\partial C_1}{\partial z} = \phi_2 D_f \frac{\partial C_2}{\partial z}. \]

### Dimensionless parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_r = \phi_{02}/\phi_{01}$</td>
<td>porosity ratio in uncontaminated sublayers</td>
</tr>
<tr>
<td>$K_r = K(\phi_{02})/K(\phi_{01})$</td>
<td>permeability ratio in uncontaminated sublayers</td>
</tr>
<tr>
<td>$\phi_{01} = 0.4$</td>
<td>porosity of the lower sublayer</td>
</tr>
<tr>
<td>$\delta$</td>
<td>interface parameter</td>
</tr>
<tr>
<td>$P_\phi = \beta_\phi \Delta S$</td>
<td>solutal pore shrinkage coefficient</td>
</tr>
<tr>
<td>$\phi_{0m} = \delta \phi_{01} + [1 - \delta] \phi_{02}$</td>
<td>average porosity of the uncontaminated sublayers</td>
</tr>
<tr>
<td>$K(\phi_{0m}) = \delta K(\phi_{01}) + [1 - \delta] K(\phi_{02})$</td>
<td>average permeability of the uncontaminated sublayers</td>
</tr>
<tr>
<td>$R_m = \frac{g \beta_c \Delta C H K(\phi_{0m})}{\nu_f D_f \phi_{0m}}$</td>
<td>the solutal Rayleigh-Darcy number defined through the average porosity and permeability of sublayers</td>
</tr>
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</table>
Convection in an uncontaminated layered porous medium

Thin bottom sublayer ($\delta < 0.5$)

Convection in an uncontaminated layered porous medium

Thick bottom sublayer ($\delta > 0.5$)

Convection in layered sorbing porous medium

Thick bottom sublayer ($\delta = 0.7$)

- A decrease in porosity due to sorption delays the solutal convection onset.
- An abrupt change in convection flows - from the local to large-scale ones.

Porosity variation

$$\phi_i = \phi_{0i} \left[ 1 - P_\phi \Theta_i \right]$$

Permeability variation

$$K(\phi) = K(\phi_{0i}) \left\{ 1 - \frac{(3 - \phi_{0i})}{(1 - \phi_{0i})} P_\phi \Theta_i(z) \right\}$$
Conclusions

- The solutal convection onset in sorbing media of two porous sublayers with different permeabilities has been studied.

- For the uncontaminated layered media, local convective flows are in more permeable sublayer and the large-scale convection ones are in both sublayers. They are similar to the thermal convective flows considered by McKibbin and O'Sullivan (1980).

- In sorbing layered porous media, the type of convective flows strongly depends on the solutal pore shrinkage coefficient.

- Even a small increase in its value can produce a large variation of flows from the local to large-scale convection.

- A decrease in porosity due to sorption delays the solutal convection onset.
Thank you for your attention!