Data-driven Subgrid-scale Models for Fluid Dynamics

Yifei Guan, Ashesh Chattopadhyay, Adam Subel, Pedram Hassanzadeh
Department of Mechanical Engineering, Rice University

Layout

- Subgrid modeling of 2D decaying turbulence
    - Data-driven subgrid-scale models using CNN
    - SGS forward transfer and backscattering analysis
    - Transfer learning generalize the network to work for higher Re and different grid resolution

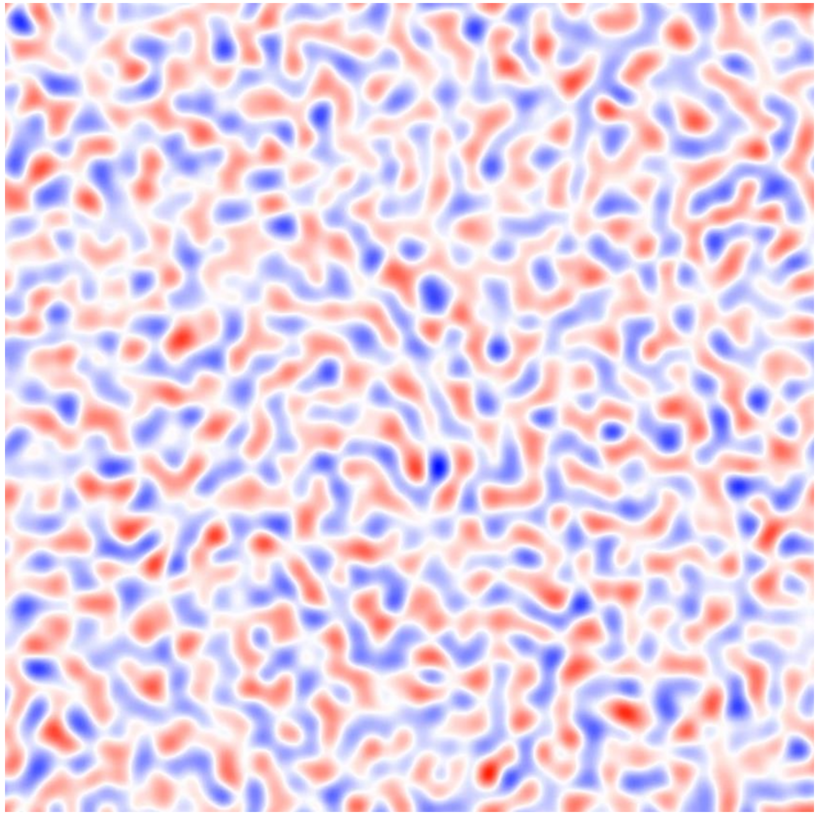
2D Decaying Turbulence

Domain: $[LX, LY] = [2\pi, 2\pi]$

Grid size: $[NX, NY] = [2048, 2048]$

LES grid size: $[NX, NY] = [256, 256]$

Re=8000 -> 128000



Numerical scheme:
Fourier pseudo-spectral methods
2nd order Adams-Bashforth for convection term
Crank-Nicolson for diffusion term

Filtering

Governing equations:

$$\frac{\partial \omega}{\partial t} + J\left(\omega,\varphi\right) = \frac{1}{\mathrm{Re}}\nabla^2\omega$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial \varphi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \varphi}{\partial x}\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

ω – vorticity

φ – stream function

Spatial filtering:

$$\bar{U}\left(x\right) = \int_{-\infty}^{\infty} G(r)U\left(x-r\right)dr$$

Gaussian filter:

$$G(r) = \left(\frac{6}{\pi\Delta^2}\right)^{1/2}\exp\left(-\frac{6r^2}{\Delta^2}\right)$$

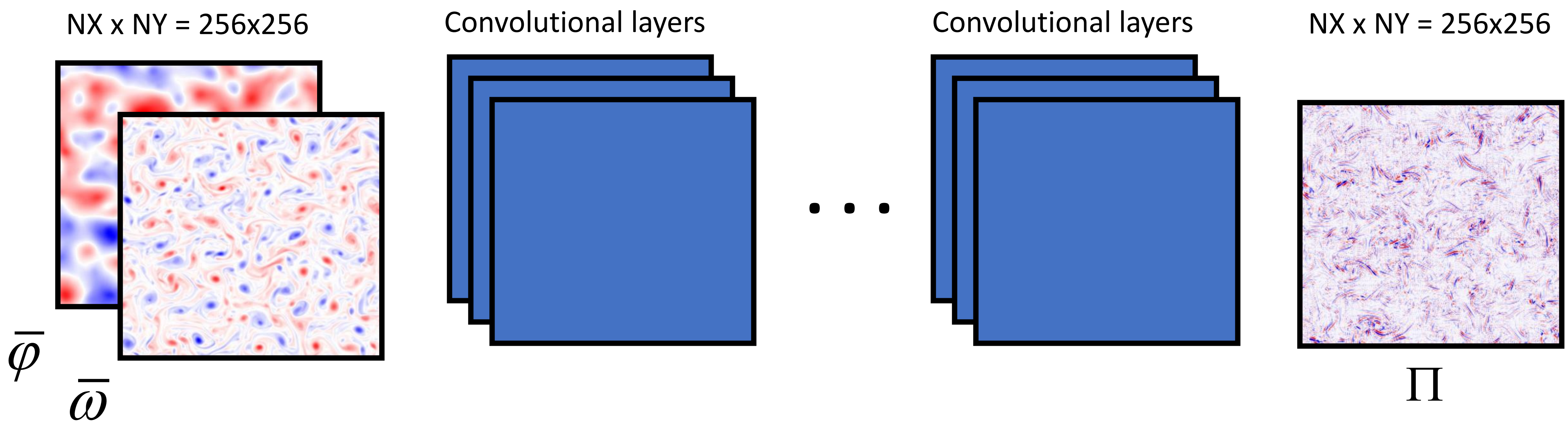
Filtered equation:

$$\overline{\frac{\partial \omega}{\partial t}} + \overline{\frac{\partial \varphi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \varphi}{\partial x}\frac{\partial \omega}{\partial y}} = \frac{1}{\mathrm{Re}}\left(\overline{\frac{\partial^2 \omega}{\partial x^2}} + \overline{\frac{\partial^2 \omega}{\partial y^2}}\right)$$

$$\frac{\partial \bar{\omega}}{\partial t} + \frac{\partial \bar{\varphi}}{\partial y}\frac{\partial \bar{\omega}}{\partial x} - \frac{\partial \bar{\varphi}}{\partial x}\frac{\partial \bar{\omega}}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \bar{\omega}}{\partial x^2} + \frac{\partial^2 \bar{\omega}}{\partial y^2}\right) + \underbrace{\left(\frac{\partial \bar{\varphi}}{\partial y}\frac{\partial \bar{\omega}}{\partial x} - \frac{\partial \bar{\varphi}}{\partial x}\frac{\partial \bar{\omega}}{\partial y} - \overline{\frac{\partial \varphi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \varphi}{\partial x}\frac{\partial \omega}{\partial y}}\right)}_{\Pi}$$

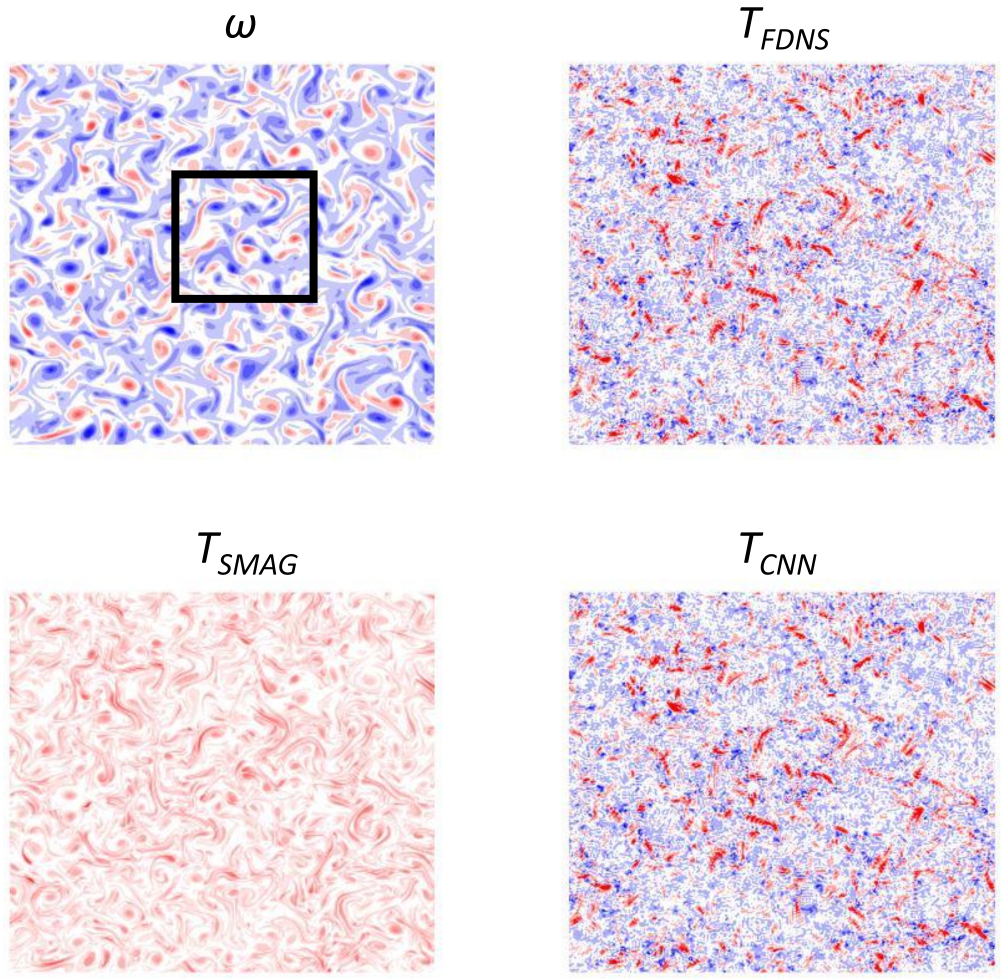
DNS data-driven parameterization

Fully convolutional neural networks

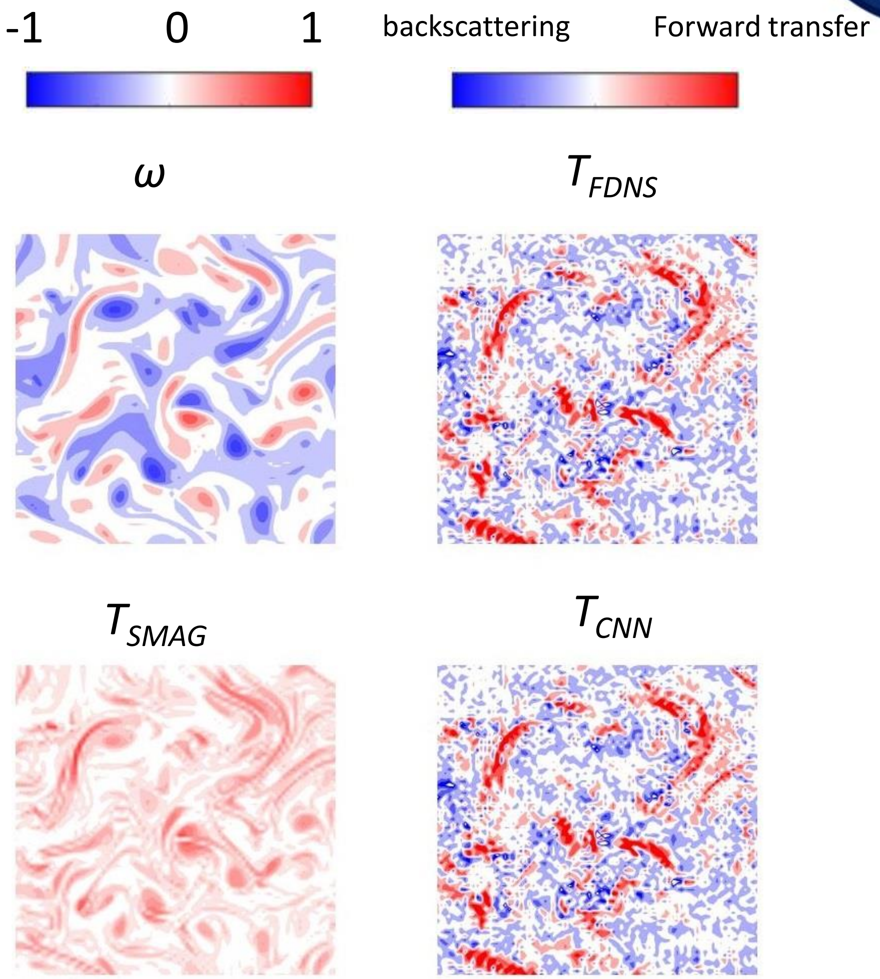


Guan, Yifei, Ashesh Chattopadhyay, Adam Subel, and Pedram Hassanzadeh. "Stable a posteriori LES of 2D turbulence using convolutional neural networks: Backscattering analysis and generalization to higher Re via transfer learning." *arXiv preprint arXiv:2102.11400* (2021).

Subgrid-scale transfer $T = sign(\nabla^2 \omega)\Pi$

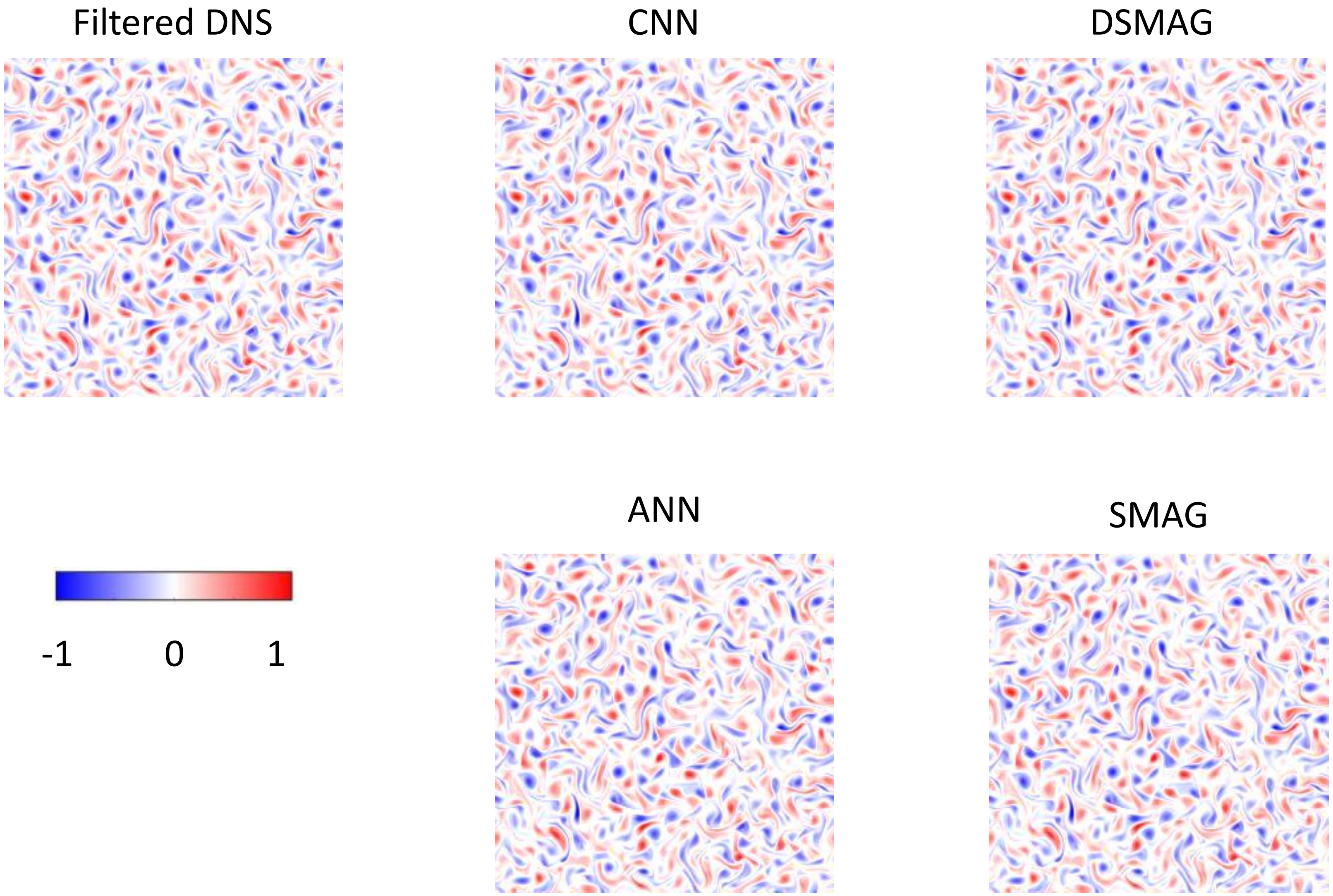


(a) Vorticity and subgrid scale energy transfer T for filrered DNS (FDNS), physics-based Smagorinsky scheme (SMAG), and convolutional neural network (CNN).

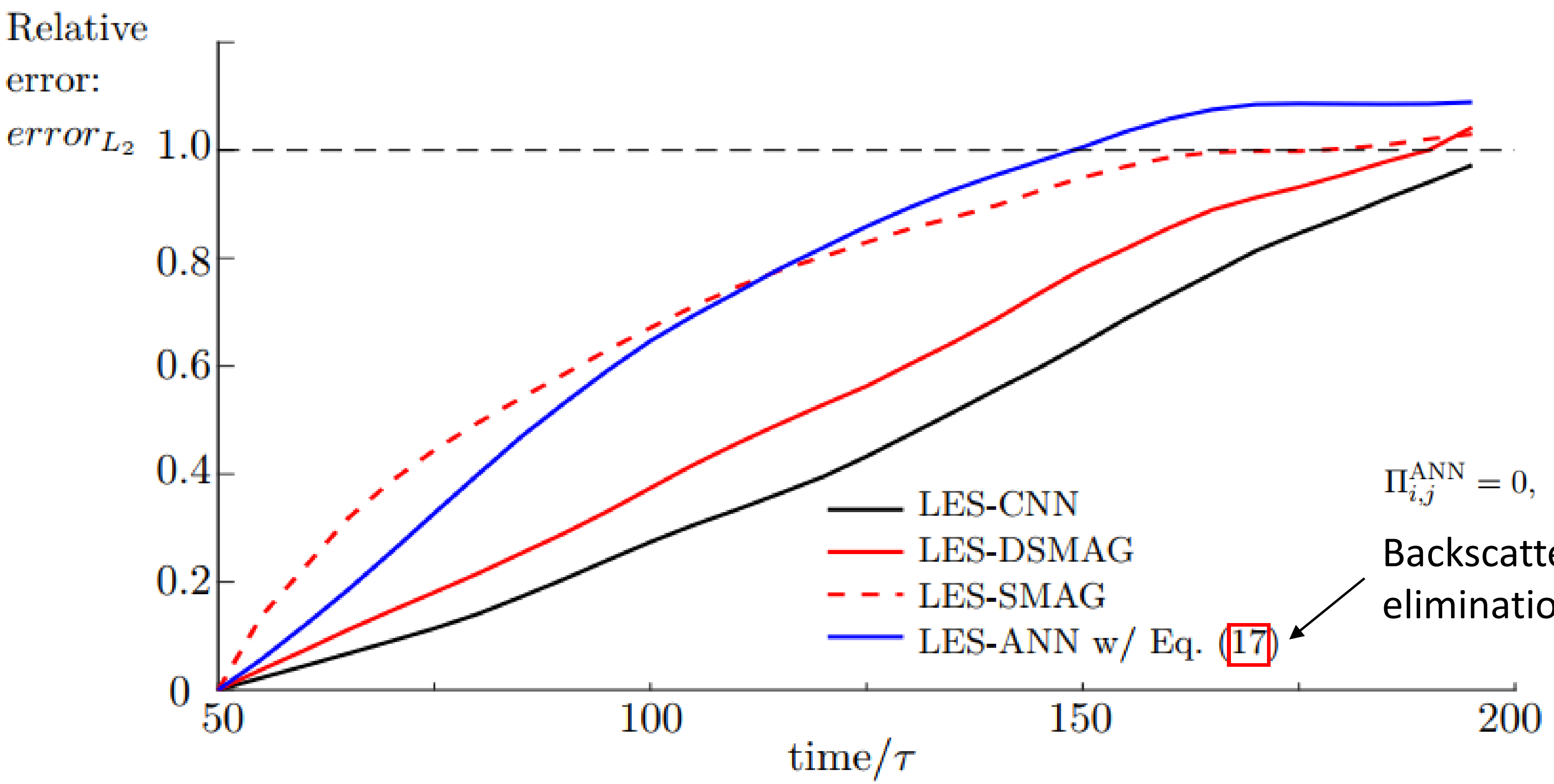


(b) Section in the black square of (a)

Online prediction – vorticity field



Tested on Re = 32k

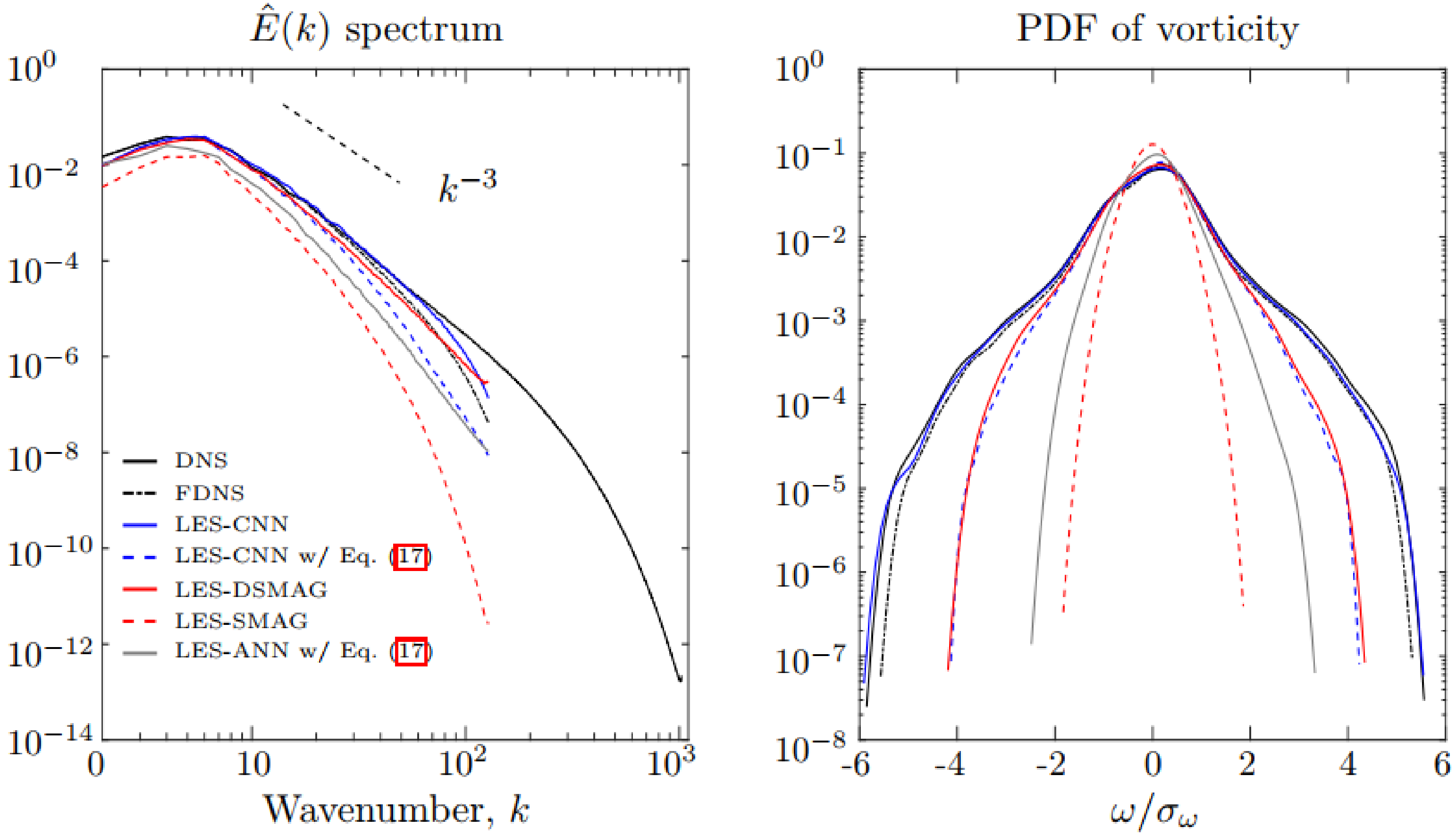


$\Pi_{i,j}^{\mathrm{ANN}} = 0, \ \forall \ T_{i,j} < 0,$

Backscattering elimination

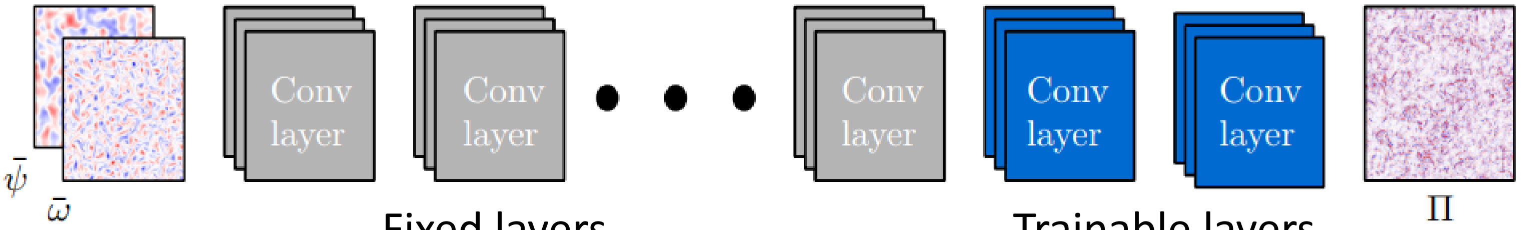
The short-term prediction is evaluated by the relative l2 norm error of the vorticity. The CNN has smaller errors than physics-based methods and the local ANN model.

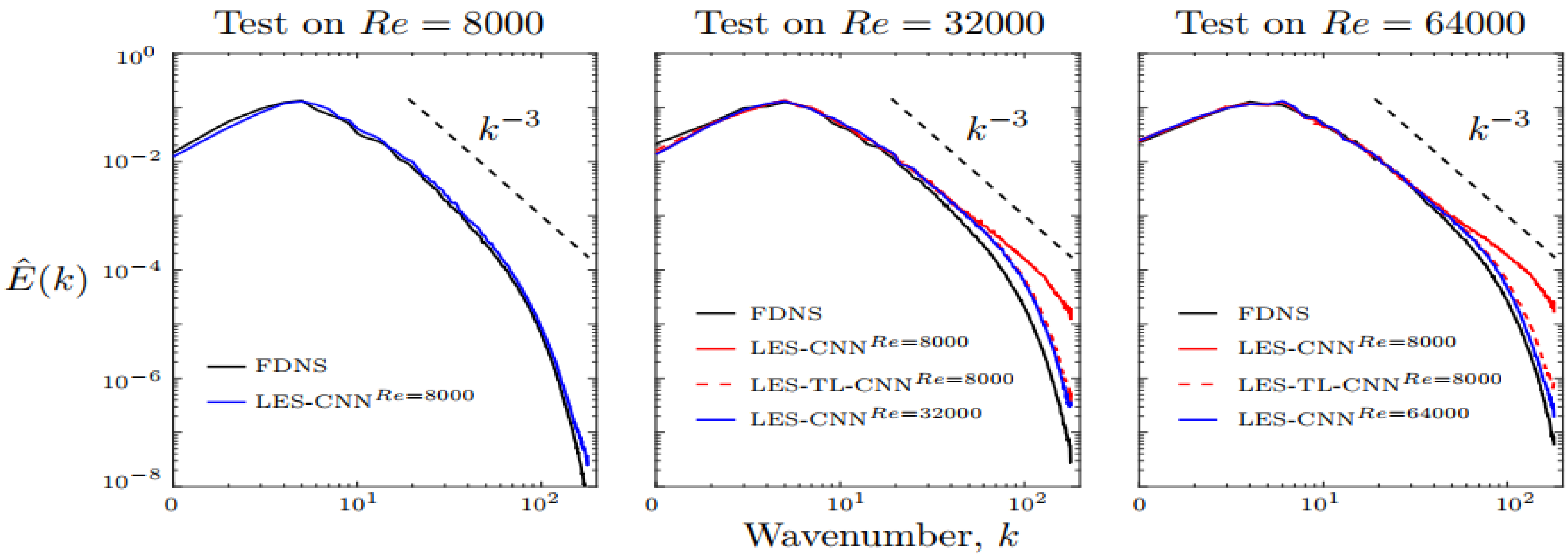
Online tests on Re = 32000



The statistics of long-term prediction are evaluated by the turbulent kinetic energy. The CNN better predicts the statistics than physics-based models. Transfer learning helps the CNN trained on Re = 8k works at Re = 32k. Cancellation of the backscattering makes the spectrum deviate from the -3 law due to the excessive energy dissipation.

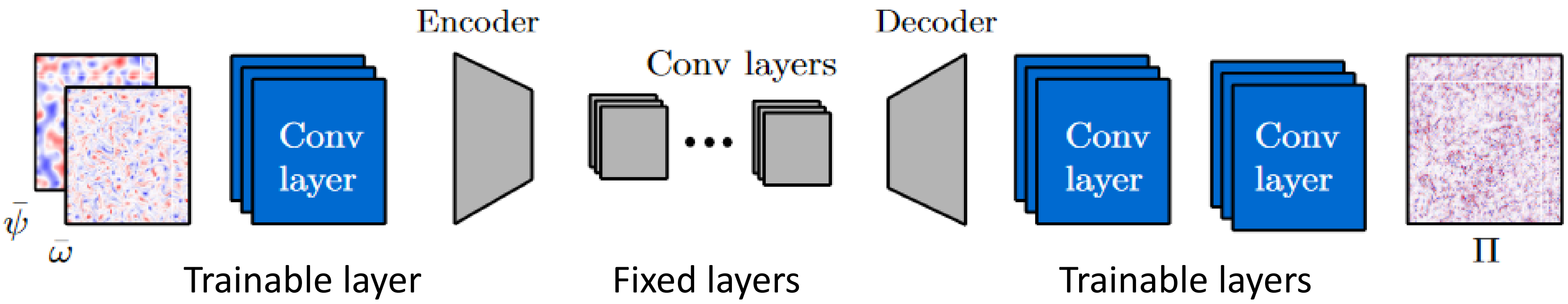
Generalization to higher Re via transfer learning

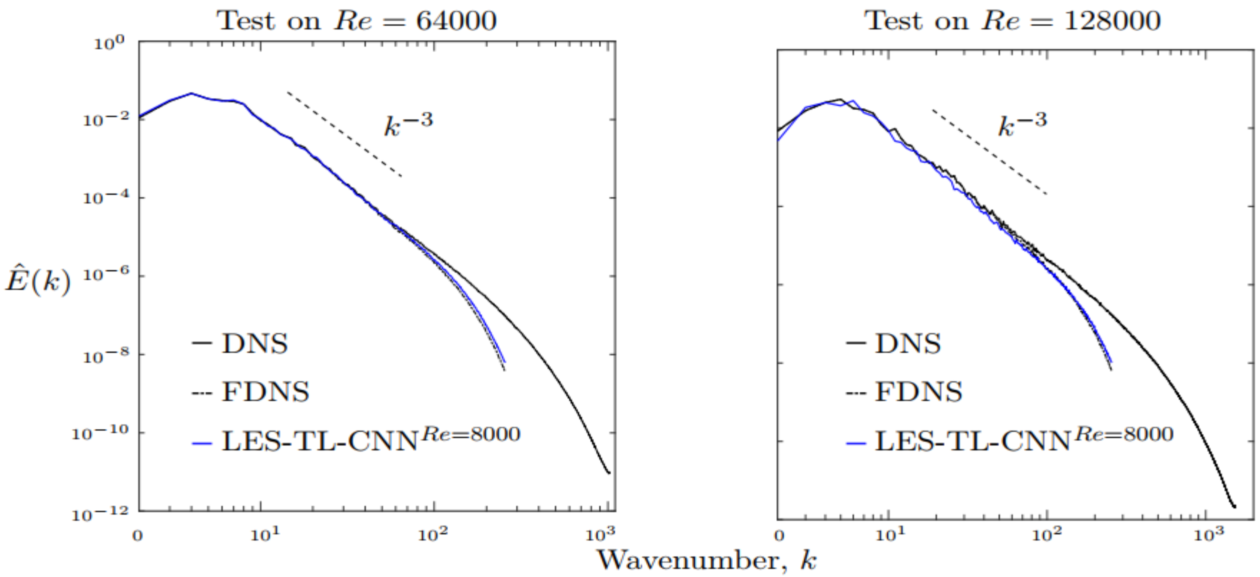




Guan, Yifei, Ashesh Chattopadhyay, Adam Subel, and Pedram Hassanzadeh. "Stable a posteriori LES of 2D turbulence using convolutional neural networks: Backscattering analysis and generalization to higher Re via transfer learning." *arXiv preprint arXiv:2102.11400* (2021).

Generalization to high Re and higher resolution via transfer learning





References:

1. Chattopadhyay, Ashesh, Adam Subel, and Pedram Hassanzadeh. "Data-Driven Super-Parameterization Using Deep Learning: Experimentation With Multiscale Lorenz 96 Systems and Transfer Learning." Journal of Advances in Modeling Earth Systems 12, no. 11 (2020): e2020MS002084.

2. Subel, Adam, Ashesh Chattopadhyay, Yifei Guan, and Pedram Hassanzadeh. "Data-driven subgrid-scale modeling of forced Burgers turbulence using deep learning with generalization to higher Reynolds numbers via transfer learning." Physics of Fluids 33, no. 3 (2021): 031702.

3. Guan, Yifei, Ashesh Chattopadhyay, Adam Subel, and Pedram Hassanzadeh. "Stable a posteriori LES of 2D turbulence using convolutional neural networks: Backscattering analysis and generalization to higher Re via transfer learning." *arXiv preprint arXiv:2102.11400* (2021).