

# IMPACT OF LOW-DEGREE GRAVITY FIELD ESTIMATION IN THE SLR DATA PROCESSING OF SPHERICAL SATELLITES AT AIUB

Linda Geisser<sup>1</sup>, Ulrich Meyer<sup>1</sup>, Daniel Arnold<sup>1</sup>, Adrian Jäggi<sup>1</sup>, Daniela Thaller<sup>2</sup>

<sup>1</sup>Astronomical Institute, University of Bern, Switzerland

<sup>2</sup>Federal Agency for Cartography and Geodesy, Frankfurt am Main, Germany

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# OUTLINE

- Introduction
  - Contribution of SLR to determine Earth's gravity field models
  - Parametrization
- Analysis
  - Compare estimated gravity field coefficients and geodetic parameters for
    - LAGEOS+ETALON-solutions
    - LAGEOS+ETALON+LARES-solutions
- Summary and Outlook

# CONTRIBUTION OF SLR TO DETERMINE EARTH'S GRAVITY FIELD MODELS

## Outline:

- **Introduction**

- Contribution of SLR

- Parametrization

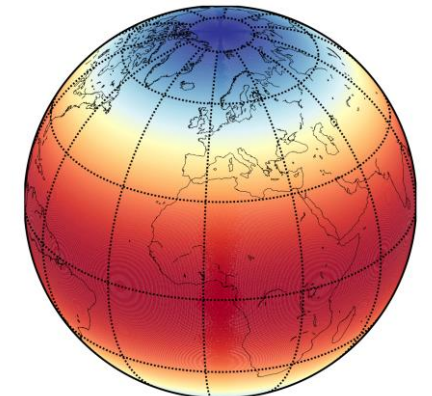
- **Analysis**

- **Summary and Outlook**

- SLR geodetic technique is based on runtime measurements of short laser pulses (e.g.  $\lambda$ : 532/1064 nm) from a station to a satellite with retroreflectors and back to the station
- In addition to the geodetic parameters, SLR is also used to determine the very long wavelengths of Earth's gravity field, especially the zonal harmonic coefficient  $C_{20}$  (describing the oblateness of the Earth)



Zonal harmonic coefficient  $C_{20}$

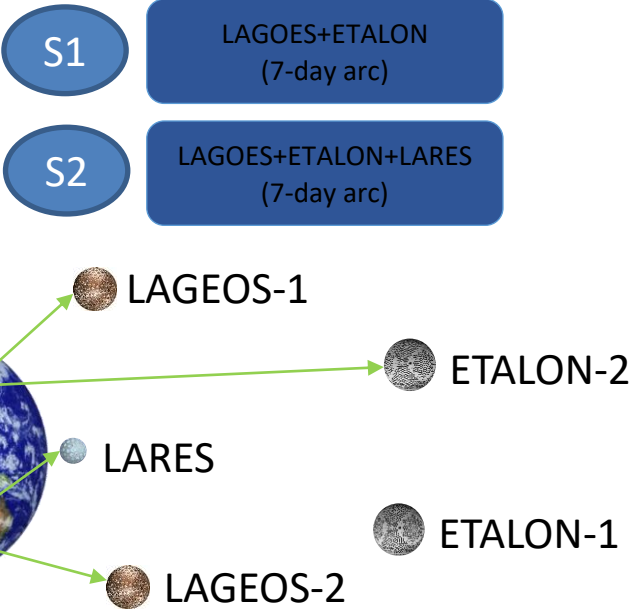


# PARAMETRIZATION

## Outline:

- **Introduction**
  - Contribution of SLR
  - **Parametrization**
- **Analysis**
- **Summary and Outlook**

| Satellites                              | LAGEOS-1/2<br>ETALON-1/2                                    | LARES                          |
|-----------------------------------------|-------------------------------------------------------------|--------------------------------|
| Parametrization                         |                                                             |                                |
| Osculating elements                     | $a, e, i, \Omega, \omega, u_0$<br>1 set per 7 days          |                                |
| Constant and once-per-rev accelerations | $S_0, S_S, S_C, (W_S, W_C)$<br>1 set per 7 days             |                                |
| Pseudo-stochastic pulses                | No pulses                                                   | In along-track (twice per day) |
| Earth rotation parameters               | $X_p, Y_p, UT1 - UTC$<br>Piecewise-linear                   |                                |
| Geocenter coordinates                   | 1 set per 7 days                                            |                                |
| Station coordinates                     | 1 set per 7 days<br>NNT/NNR                                 |                                |
| Range biases                            | 1 set per 7 days for<br>selected stations      all stations |                                |
| Gravity field coefficients              | 1 set per 7 days<br>$C_{20}$ only<br>Up to d/o 4            |                                |



**REMARK:**  
The once-per-revolution periodic accelerations in out-of-plane  $W_S$  is strongly correlated with  $C_{20}$ .

# BACKGROUND MODELS

## Outline:

### • Introduction

- Contribution of SLR

### – Parametrization

### • Analysis

### • Summary and Outlook

| MODELS                   | DESCRIPTION                                                                                                                                                                                                 |
|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Length of arc            | 7 days                                                                                                                                                                                                      |
| Data editing             | 2.5 sigma editing, Maximum overall sigma: 25 mm, Minimum normal points per week (10: LAGEOS, 1: ETALON, 3: LARES)                                                                                           |
| Satellite center of mass | Station- and Satellite-specific                                                                                                                                                                             |
| Troposphere delay        | Mendes-Pavlis (Mendes and Pavlis, 2004)                                                                                                                                                                     |
| Cut-off angle            | 3 degrees, no elevation-dependent weighting                                                                                                                                                                 |
| Relativity (orbit)       | Schwarzschild term, Lense-Thirring effect, de Sitter effect are all applied: IERS Conventions 2010 (Petit and Luzum, 2010)                                                                                  |
| Third body               | Earth's Moon, Sun, Venus, Mars, Jupiter (JPL DE405 ephemeris) (Standish, 1998)                                                                                                                              |
| Subdaily pole model      | DESAI: IERS Conventions 2010 (Petit and Luzum, 2010)                                                                                                                                                        |
| Tidal forces             | Solid earth tide model: IERS Conventions 2010 (Petit and Luzum, 2010)<br>Pole tide model : IERS Conventions 2010 (Petit and Luzum, 2010)<br>Ocean pole tides: IERS Conventions 2010 (Petit and Luzum, 2010) |
| Nutation model           | IAU2000 (Mathews et al., 2002)                                                                                                                                                                              |
| Loading corrections      | Ocean tidal loading: FES2014b (Lyard et al., 2020)<br>Ocean non-tidal loading: not applied<br>Atmospheric tidal loading: not applied<br>Atmospheric non-tidal loading: not applied                          |

| MODELS                       | DESCRIPTION                                                                                                                                                                                                         |
|------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Solar radiation pressure     | Radiation pressure: applied with radiation pressure coefficient $C_R = (1.13: \text{LAGEOS}, 1.2: \text{ETALON}, 1.07: \text{LARES})$<br>Earth's albedo reflectivity: applied<br>Earth's albedo emissivity: applied |
| Numerical integration        | Interval: (120 seconds: LAGEOS and ETALON, 60 seconds: LARES)<br>polynomial degree: 12<br>collocation method (Beutler et al., 2005)                                                                                 |
| Earth orientation parameters | BULLETIN_A from IERS                                                                                                                                                                                                |
| Reference frame              | SLRF2014                                                                                                                                                                                                            |
| A priori range biases        | Provided with SLRF 2014                                                                                                                                                                                             |
| A priori core stations       | Provided with SLRF 2014                                                                                                                                                                                             |
| Earth gravity field          | GGM05S. (Ries et al., 2016)<br>Time-variations of degree 2 and zonal coefficients as provided by the ILRS                                                                                                           |
| Earth potential degree       | 90                                                                                                                                                                                                                  |
| Ocean tide model             | FES2014b (Lyard et al., 2020)                                                                                                                                                                                       |
| De-aliasing Products         | Atmosphere+Ocean RL06 (Dobslaw et al., 2017)                                                                                                                                                                        |
| Airdrag model for LARES      | MSIS-00 with oxygen (Picone et al., 2002)                                                                                                                                                                           |

# GRAVITY FIELD COEFFICIENTS

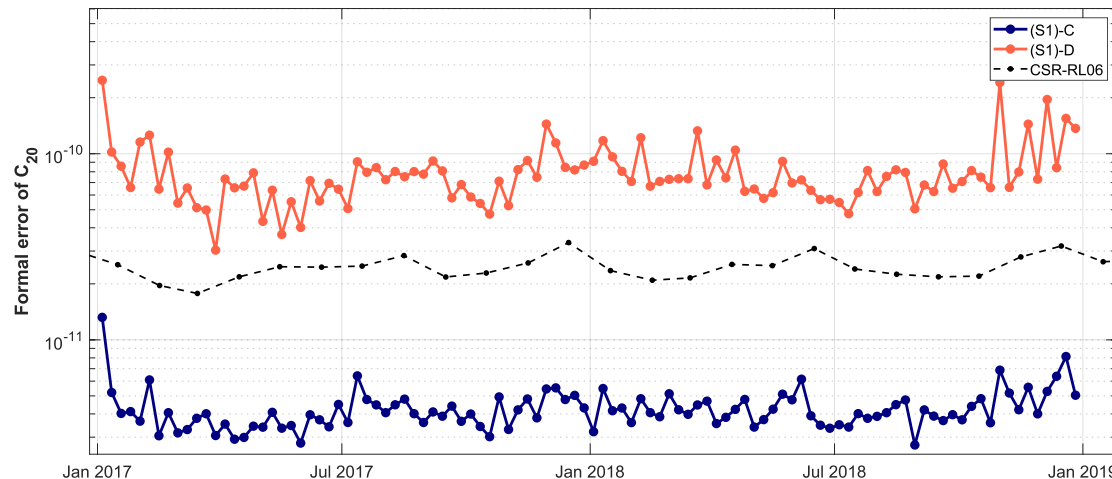
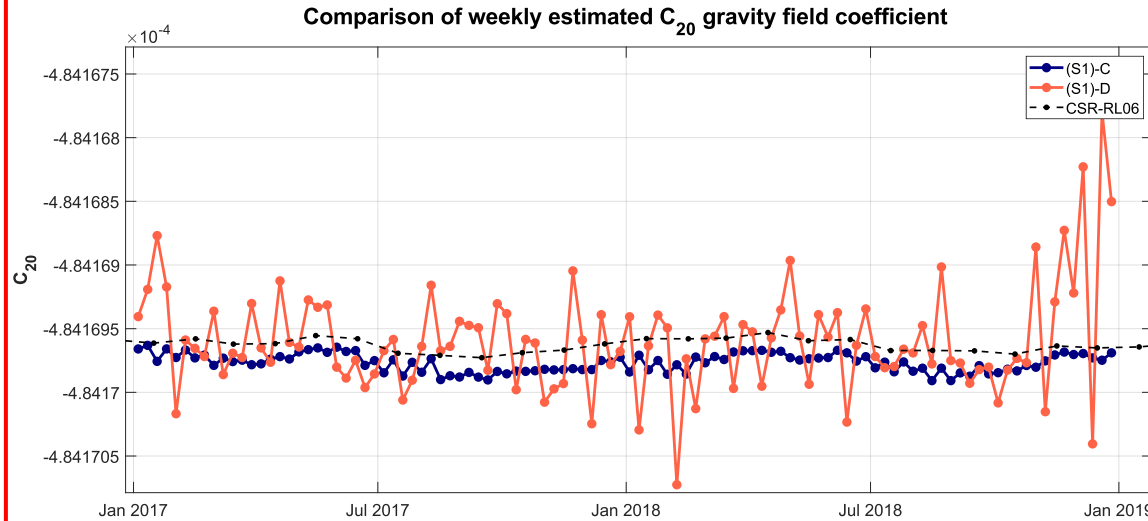
## Outline:

- Introduction
- Analysis  
LAGEOS+ETALON
  - Gravity field coefficients
- Summary and Outlook

## Abbreviations:

(S1): LAGEOS+ETALON

- A: OPR-W, no grav
- B: no OPR-W, no grav
- C: no OPR-W,  $C_{20}$
- D: no OPR-W, d/o 4

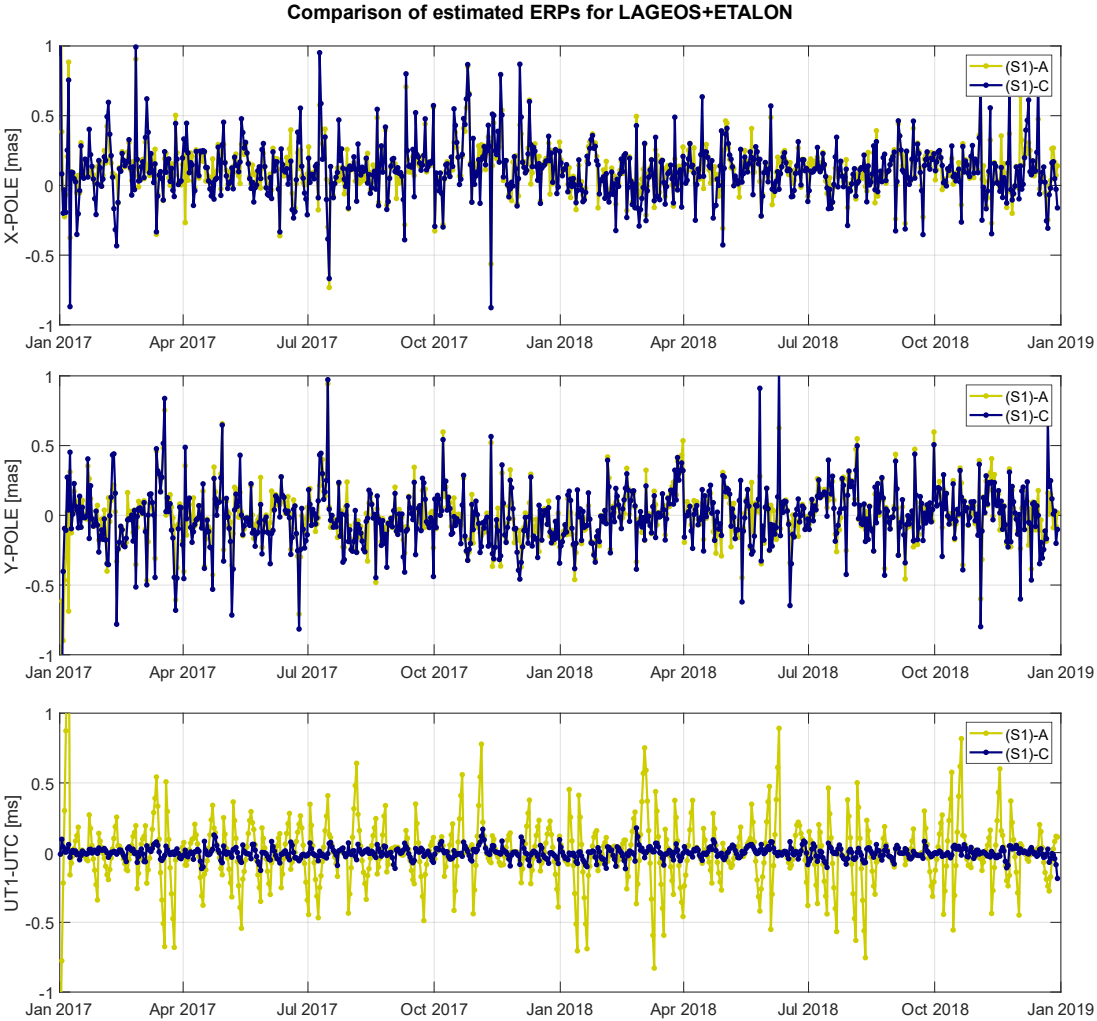


- If only  $C_{20}$  is estimated, the time series shows an offset of  $1.4e-10$  with respect to the CSR reference series
- $C_{20}$  becomes unstable, if gravity field coefficients up to d/o 4 are estimated

# COMPARISON OF EARTH ROTATION PARAMETERS

- Outline:**
- Introduction
  - **Analysis**  
**LAGEOS+ETALON**
    - ERPs
  - Summary and Outlook

- Abbreviations:**
- (S1): LAGEOS+ETALON**
- A: OPR-W, no grav
  - B: no OPR-W, no grav
  - C: no OPR-W, C20
  - D: no OPR-W, d/o 4



| Solution | X pole [ $\mu$ as] |       | Y pole [ $\mu$ as] |       | UT1-UTC [ $\mu$ s] |      |
|----------|--------------------|-------|--------------------|-------|--------------------|------|
|          | Bias               | WRMS  | Bias               | WRMS  | Bias               | WRMS |
| (S1)-A   | 106.9              | 151.1 | -6.6               | 131.5 | -2.2               | 85.9 |
| (S1)-B   | 135.1              | 256.6 | -44.7              | 228.0 | -2.0               | 31.3 |
| (S1)-C   | 98.1               | 163.7 | -13.6              | 152.8 | -2.1               | 30.6 |
| (S1)-D   | 78.7               | 174.0 | 7.4                | 174.2 | 7.5                | 49.0 |

- At least  $C_{20}$  has to be estimated, if the once-per-rev accelerations in out-of-plane are not estimated
- The estimation of gravity field coefficients reduces the WRMS of UT1-UTC

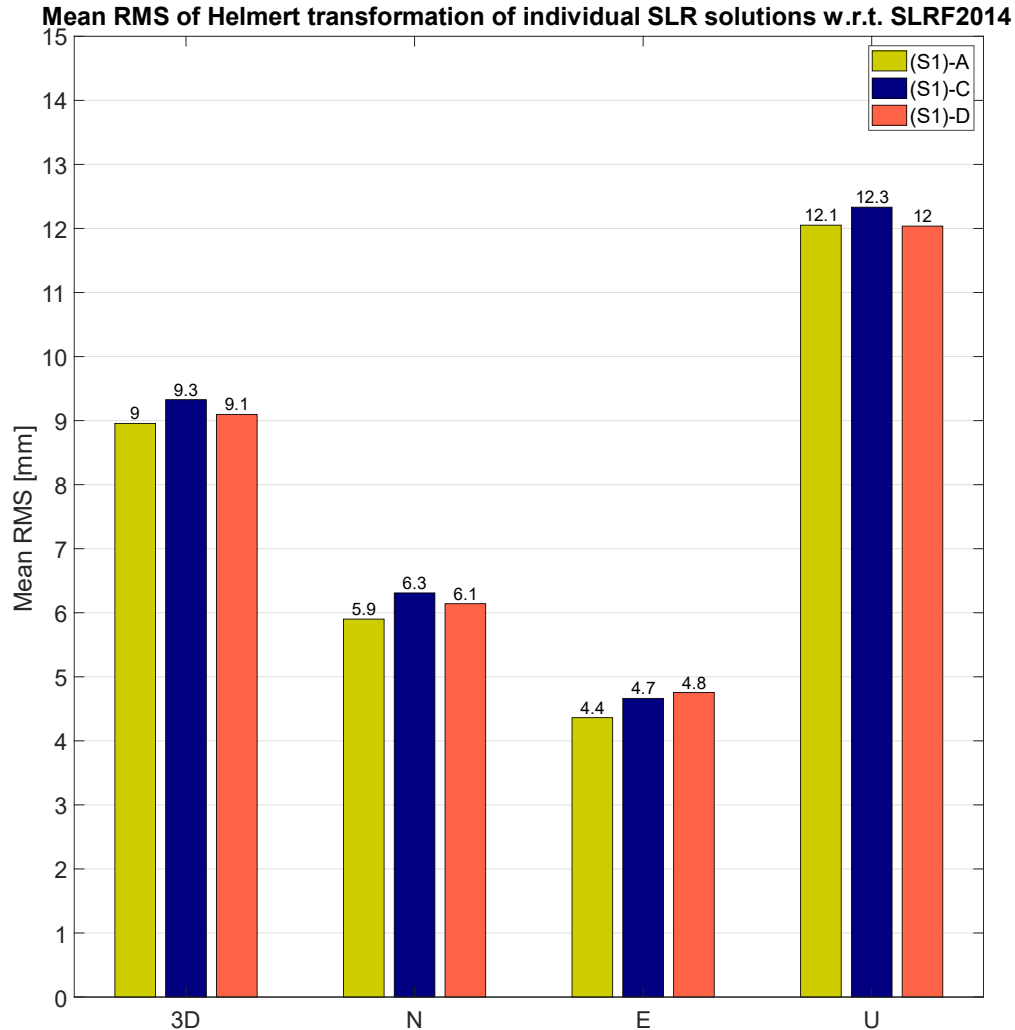
# COMPARISON OF HELMERT TRANSFORMATION

## Outline:

- Introduction
- **Analysis**  
**LAGEOS+ETALON**
  - Station coordinates
- Summary and Outlook

## Abbreviations:

- (S1): LAGEOS+ETALON
- A: OPR-W, no grav
  - B: no OPR-W, no grav
  - C: no OPR-W, C20
  - D: no OPR-W, d/o 4



- The RMS of Helmert transformations slightly increases, if gravity field coefficients are co-estimated

# GRAVITY FIELD COEFFICIENTS

## Outline:

- Introduction
- **Analysis**  
**LAGEOS+ETALON**  
**+LARES**
  - Gravity field coefficients
- Summary and Outlook

## Abbreviations:

**(S1): LAGEOS+ETALON**  
-C: no OPR-W,  $C_{20}$

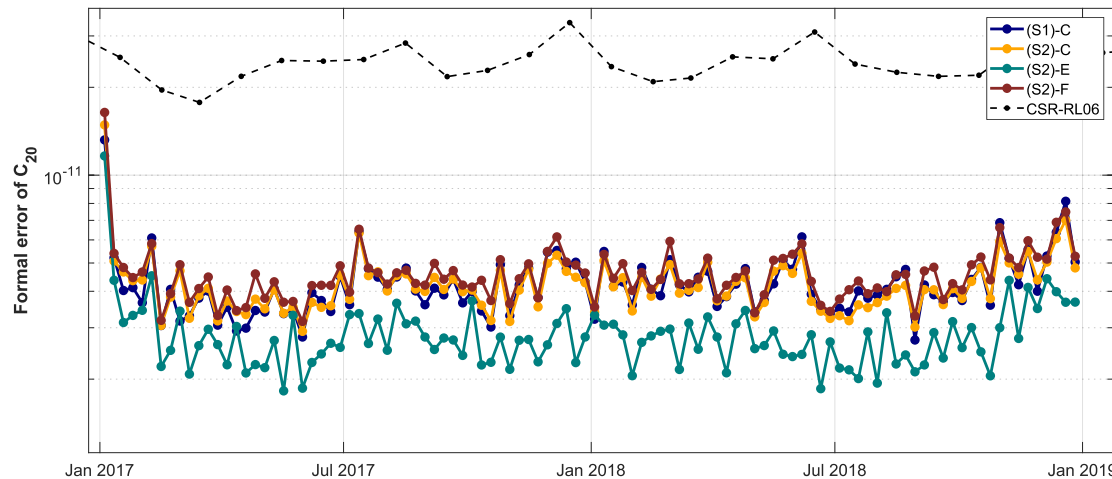
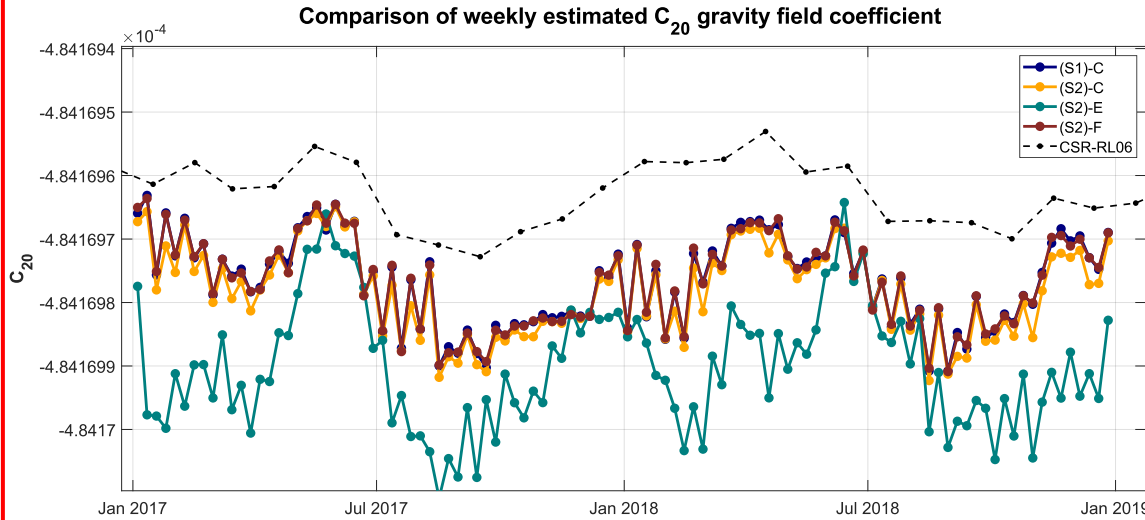
**(S2): LAGEOS+ETALON**  
+LARES

-C: no OPR-W,  $C_{20}$ ,  
pulses in W

-D: no OPR-W, d/o 4,  
pulses in W

-E: no OPR-W,  $C_{20}$

-F: OPR-W,  $C_{20}$



- To get a meaningful  $C_{20}$  estimate, the once-per-revolution accelerations or a set of stochastic pulses in out-of-plane have to be applied for LARES

# CORRELATION BETWEEN $W_S$ AND $C_{20}$

## Outline:

- Introduction
- **Analysis**  
**LAGEOS+ETALON**  
**+LARES**
  - Gravity field coefficients
- Summary and Outlook

## Abbreviations:

**(S1): LAGEOS+ETALON**

-C: no OPR-W,  $C_{20}$

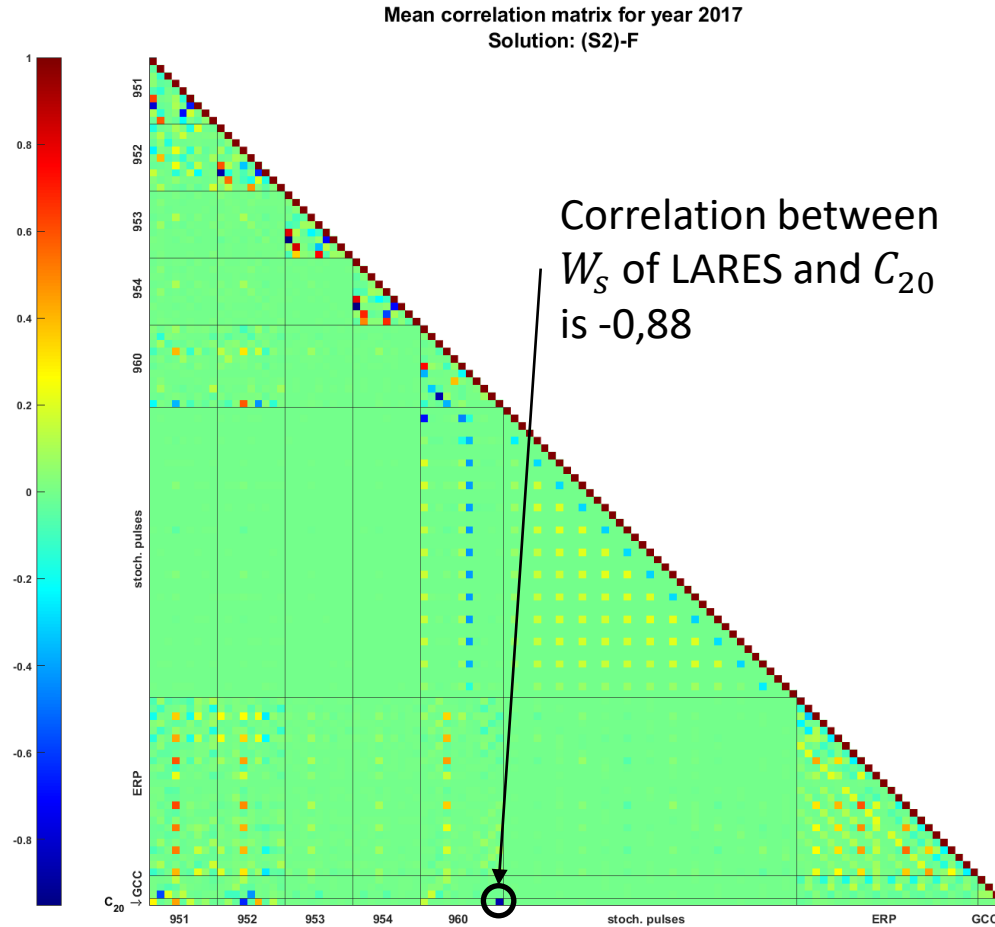
**(S2): LAGEOS+ETALON**  
**+LARES**

-C: no OPR-W,  $C_{20}$ ,  
pulses in W

-D: no OPR-W, d/o 4,  
pulses in W

-E: no OPR-W,  $C_{20}$

-F: OPR-W,  $C_{20}$



- For each Satellite
  - With OPR-W (11 parameters):  
A, E, I, NODE, PER, U0, Y0, YC, YS,  $W_C, W_S$
  - Without OPR-W (9 parameters):  
A, E, I, NODE, PER, U0, Y0, YC, YS
- Earth rotation parameters PWL (24 parameters)
  1. Day UT1-UTC, XPOLE, YPOLE
  2. Day UT1-UTC, XPOLE, YPOLE
  - ...
  8. Day UT1-UTC, XPOLE, YPOLE
- Geocenter coordinates (3 parameters)
- Geopotential coefficient  $C_{20}$

# CORRELATION BETWEEN $W_s$ AND $C_{20}$

## Outline:

- Introduction
- Analysis  
LAGEOS+ETALON  
+LARES
  - Gravity field coefficients
- Summary and Outlook

## Abbreviations:

(S1): LAGEOS+ETALON

-C: no OPR-W, C20

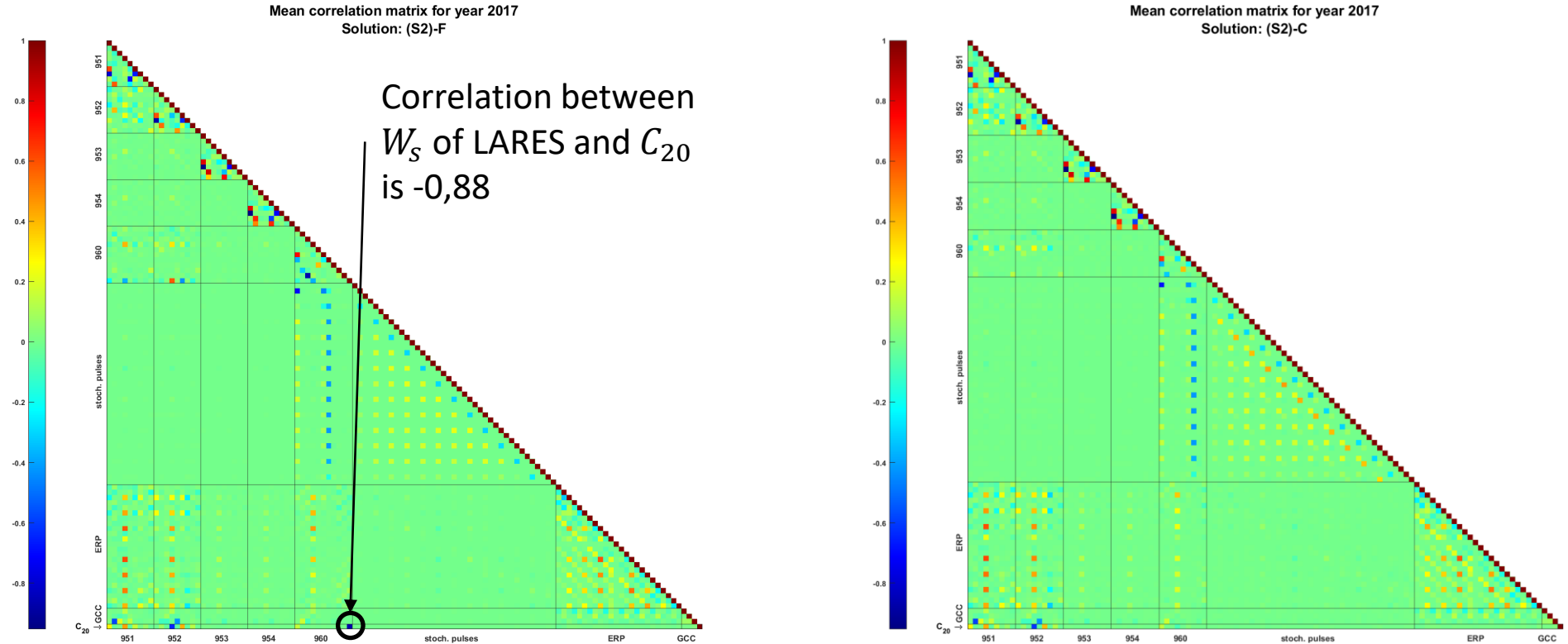
(S2): LAGEOS+ETALON  
+LARES

-C: no OPR-W, C20,  
pulses in W

-D: no OPR-W, d/o 4,  
pulses in W

-E: no OPR-W, C20

-F: OPR-W, C20



- That LARES will also contribute to the solution of  $C_{20}$ , it is better to set up stochastic pulses in out-of-plane than the once-per-revolution acceleration  $W_s$

# GRAVITY FIELD COEFFICIENTS

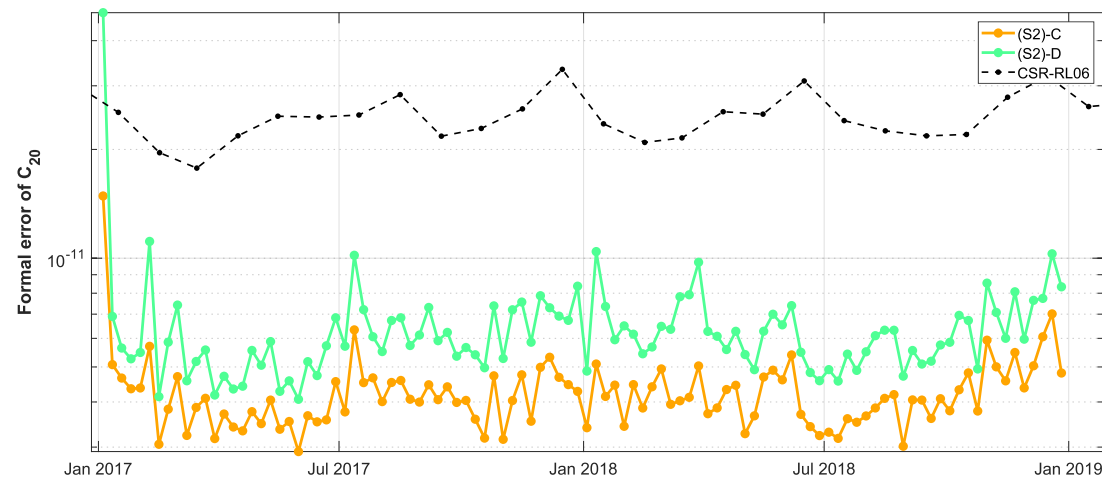
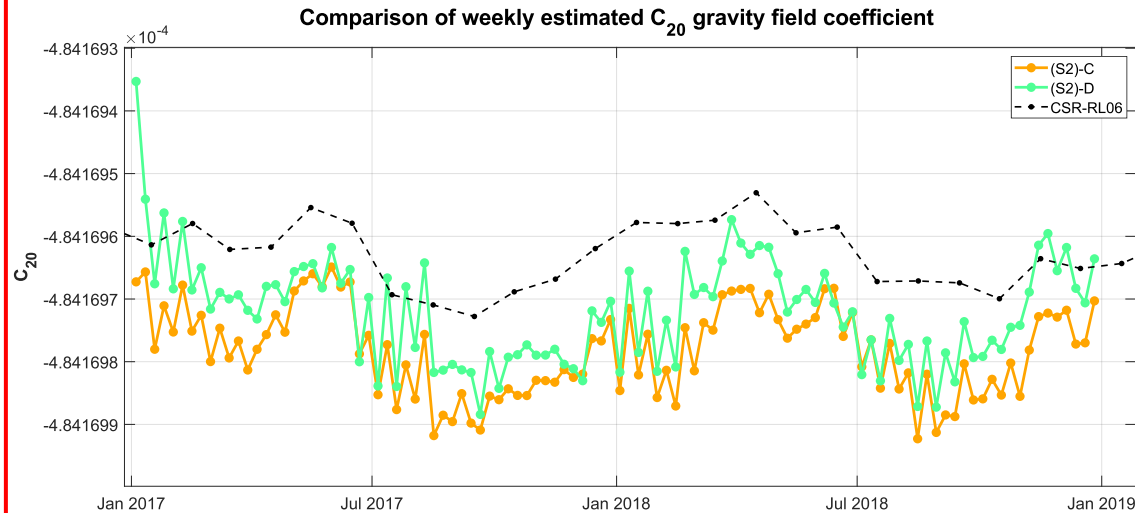
## Outline:

- Introduction
- Analysis  
LAGEOS+ETALON  
+LARES
  - Gravity field coefficients
- Summary and Outlook

## Abbreviations:

(S2): LAGEOS+ETALON  
+LARES

- C: no OPR-W, C20, pulses in W
- D: no OPR-W, d/o 4, pulses in W
- E: no OPR-W, C20
- F: OPR-W, C20



- If gravity field coefficients up to d/o 4 are estimated, the offset to the CSR Reference series becomes smaller with  $9.2e-11$

# GRAVITY FIELD COEFFICIENTS

## Outline:

- Introduction
- Analysis  
LAGEOS+ETALON  
+LARES
  - Gravity field coefficients
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+LARES

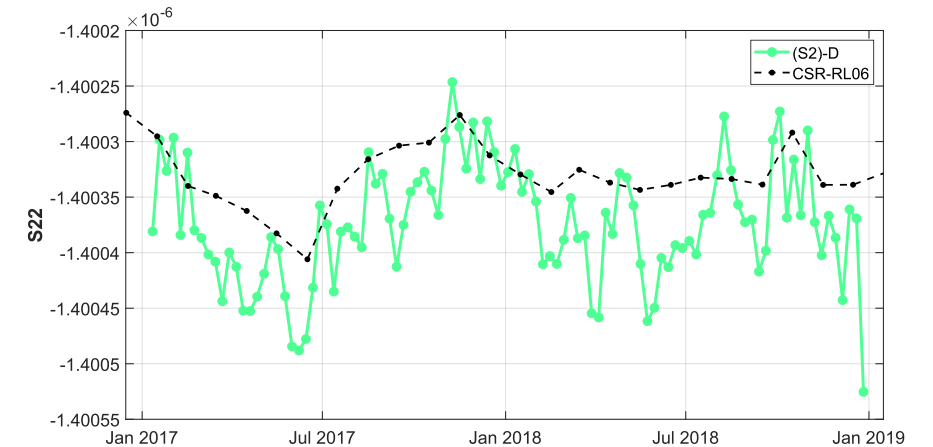
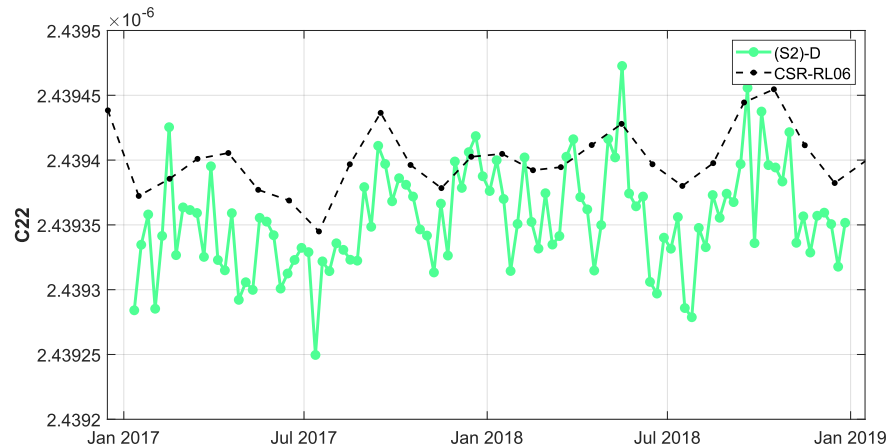
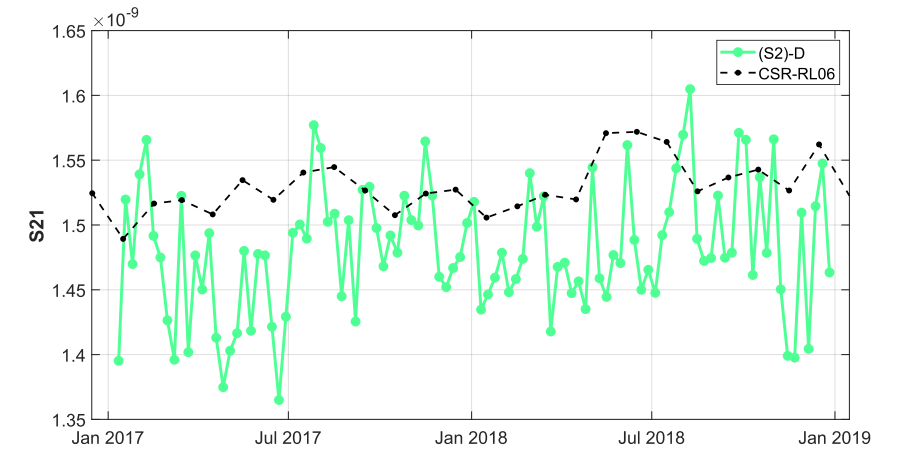
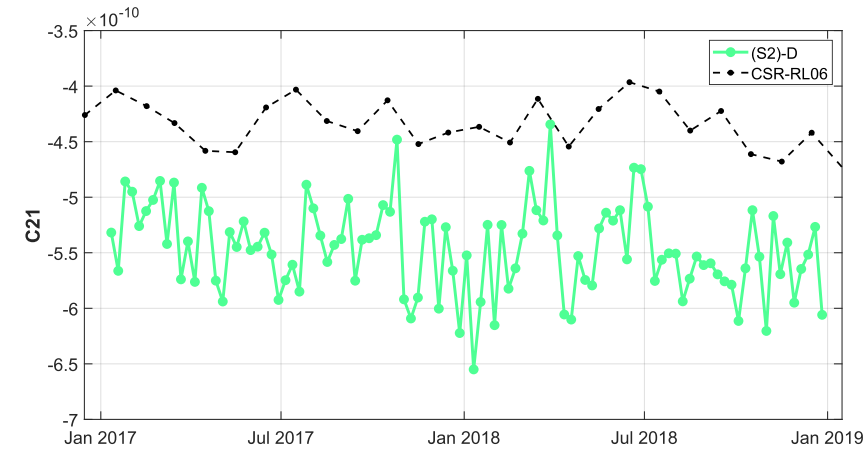
-C: no OPR-W, C20,  
pulses in W

-D: no OPR-W, d/o 4,  
pulses in W

-E: no OPR-W, C20

-F: OPR-W, C20

Comparison of weekly estimated gravity field coefficients



# COMPARISON OF EARTH ROTATION PARAMETERS

- Outline:**
- Introduction
  - **Analysis**  
**LAGEOS+ETALON**  
**+LARES**  
– ERPs
  - **Summary and Outlook**

- Abbreviations:**
- (S1): LAGEOS+ETALON**  
-C: no OPR-W, C20
- (S2): LAGEOS+ETALON +LARES**  
-C: no OPR-W, C20, pulses in W  
-D: no OPR-W, d/o 4, pulses in W  
-E: no OPR-W, C20  
-F: OPR-W, C20



| Solution | X pole [ $\mu as$ ] |       | Y pole [ $\mu as$ ] |       | UT1-UTC [ $\mu s$ ] |      |
|----------|---------------------|-------|---------------------|-------|---------------------|------|
|          | Bias                | WRMS  | Bias                | WRMS  | Bias                | WRMS |
| (S1)-A   | 106.9               | 151.1 | -6.6                | 131.5 | -2.2                | 85.9 |
| (S2)-C   | -84.5               | 164.6 | -17.3               | 166.9 | -1.6                | 30.5 |
| (S2)-D   | 62.0                | 142.3 | -9.5                | 140.5 | 1.7                 | 28.3 |
| (S2)-F   | -75.8               | 175.9 | -15.9               | 162.3 | -2.1                | 30.8 |

- If LARES is included, the estimation of the gravity field coefficients up to d/o 4 reduces the biases as well as the WRMS of all Earth rotation parameters

# COMPARISON OF HELMERT TRANSFORMATION

## Outline:

- Introduction
- **Analysis**  
**LAGEOS+ETALON**  
**+LARES**
  - Station coordinates
- Summary and Outlook

## Abbreviations:

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**-A:** OPR-W, no grav

**(S2): LAGEOS+ETALON**

**+LARES**

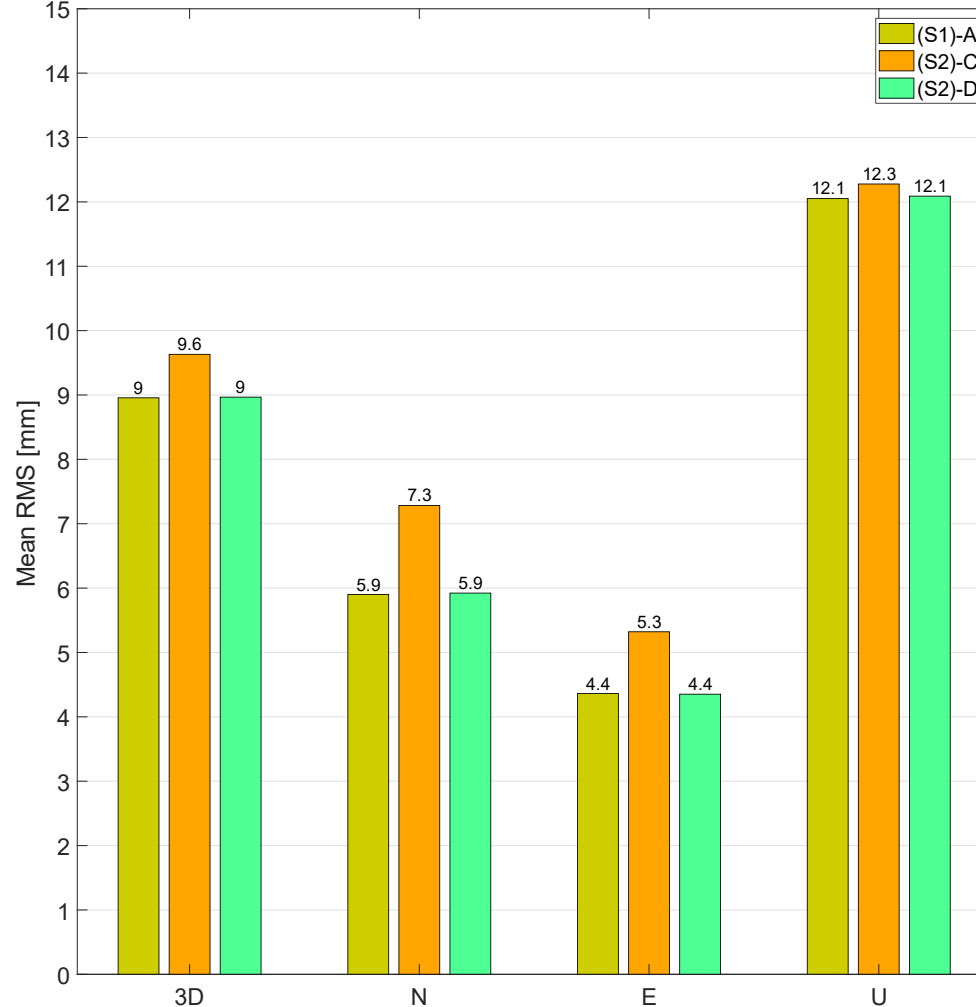
**-C:** no OPR-W, C20, pulses in W

**-D:** no OPR-W, d/o 4, pulses in W

**-E:** no OPR-W, C20

**-F:** OPR-W, C20

Mean RMS of Helmert transformation of individual SLR solutions w.r.t. SLRF2014



- The RMS of Helmert transformations can be decreased, if gravity field coefficients up to d/o 4 are estimated

# SUMMARY & OUTLOOK

## Outline:

- Introduction
- Analysis
- **Summary and Outlook**

## Summary:

- To get meaningful gravity field coefficients, LARES has to be parametrized with stochastic pulses
- We are able to co-estimate low-degree gravity field coefficients up to d/o 4, if LARES is included

## Outlook:

- Study SLR solutions where for LARES the constant and once-per-revolution accelerations are set up daily
- Extend the SLR processing to also include other geodetic satellites as Starlette, Stella and AJISAI with the aim to further stabilize the solution

# REFERENCES

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