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Ice or rock matrix?
Improved quantitative imaging of Alpine permafrost evolution through time-lapse petrophysical joint inversion

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Permafrost

Ground at or below 0 °C during at least two consecutive years

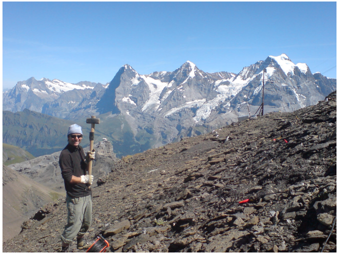
- Permafrost is a sensitive **climate change indicator** and expected to gradually **degrade**
- **Hazard potential:** loss of slope stability, release of organic carbon, effect on hydrological cycle
- **Quantitative knowledge** of the **permafrost composition** is required for
    - process understanding and simulation of permafrost systems
    - physically-based assessment of the hazard potential

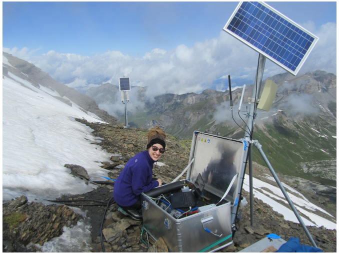
↯ **Ground ice content is difficult to quantify**

- ground temperature monitoring is insufficient

Potential of Geophysical Monitoring

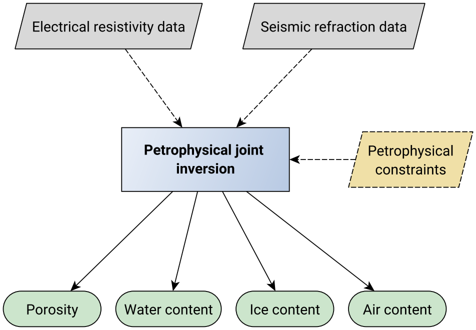
- Geophysical imaging allows **non-invasive** estimates of ground ice distribution
- **Seismic and electrical methods** (RST and ERT) are well suited
  - ice has significantly higher P-wave velocity and higher electrical resistivity compared to unfrozen water
- Quantitative **information of single-method approaches is ambiguous**
- **Seismic and electrical methods have complementary sensitivities**, which can be exploited in a joint inversion framework





Petrophysical Joint Inversion (PJI)

One simultaneous inversion of multiple geophysical data sets, which relies on statistical or physically based relationships between petrophysical and geophysical properties.



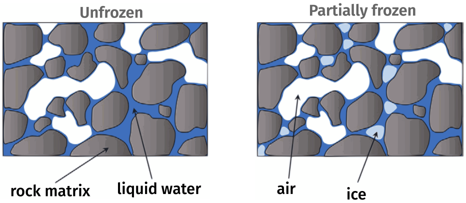
Wagner et al. (2019)

PJI by Wagner et al. (2019):

- simultaneous inversion of **seismic travel times and apparent resistivities**
- **petrophysical relations** and **volume conservation** are honored during inversion
- petrophysical basis: **four-phase model (4PM)** by Hauck et al. (2011)

1. Permafrost soils consist of the rock matrix and a pore-filling mixture of water, ice and air:

$$f_r + f_w + f_i + f_a = 1 \quad \text{with} \quad 0 \leq f_r, f_w, f_i, f_a \leq 1$$



Mohammed et al. (2018)

Legend

s: seismic slowness [s m^{-1}]

ρ: electrical resistivity [Ω m]

Volumetric fractions:

f_r: rock matrix content

f_w: water content

f_i: ice content

f_a: air content

Medium velocities:

v_r: rock velocity [m s^{-1}]

v_w: water velocity [m s^{-1}]

v_i: ice velocity [m s^{-1}]

v_a: air velocity [m s^{-1}]

Archie parameters:

ρ_w: water resistivity [Ω m]

m: cementation exponent

n: saturation exponent

1. Permafrost soils consist of the rock matrix and a pore-filling mixture of water, ice and air:

$$f_{\mathrm{r}} + f_{\mathrm{w}} + f_{\mathrm{i}} + f_{\mathrm{a}} = 1 \quad \text{with} \quad 0 \leq f_{\mathrm{r}}, f_{\mathrm{w}}, f_{\mathrm{i}}, f_{\mathrm{a}} \leq 1$$

2. Seismic slowness is described by a time-averaging equation:

$$s = \frac{1}{v} = \frac{f_{\mathrm{w}}}{v_{\mathrm{w}}} + \frac{f_{\mathrm{i}}}{v_{\mathrm{i}}} + \frac{f_{\mathrm{a}}}{v_{\mathrm{a}}} + \frac{f_{\mathrm{r}}}{v_{\mathrm{r}}}$$

3. Electrical resistivity is dominated by the liquid water content:

$$\rho = \rho_{\mathrm{w}} (1 - f_{\mathrm{r}})^{-m} \left(\frac{f_{\mathrm{w}}}{1 - f_{\mathrm{r}}} \right)^{-n}$$

Legend

s: seismic slowness [s m^{-1}]

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Archie parameters:

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n: saturation exponent

Data Space

$$\mathbf{d} = \left[\mathbf{t}, \quad \log(\rho_\mathrm{a})\right]^T$$

Model Space

$$\mathbf{p} = \left[\mathbf{f}_\mathrm{w}, \quad \mathbf{f}_\mathrm{i}, \quad \mathbf{f}_\mathrm{a}, \quad \mathbf{f}_\mathrm{r}\right]^T$$

During inversion, a transformed model vector is used such that $m_j^k = \log(p_j^k) - \log(1 - p_j^k)$.

Sensitivities

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{t}}{\partial \mathbf{f}_\mathrm{w}} & \frac{\partial \mathbf{t}}{\partial \mathbf{f}_\mathrm{i}} & \frac{\partial \mathbf{t}}{\partial \mathbf{f}_\mathrm{a}} & \frac{\partial \mathbf{t}}{\partial \mathbf{f}_\mathrm{r}} \\ \frac{\partial \log(\rho_\mathrm{a})}{\partial \mathbf{f}_\mathrm{w}} & \frac{\partial \log(\rho_\mathrm{a})}{\partial \mathbf{f}_\mathrm{i}} & \frac{\partial \log(\rho_\mathrm{a})}{\partial \mathbf{f}_\mathrm{a}} & \frac{\partial \log(\rho_\mathrm{a})}{\partial \mathbf{f}_\mathrm{r}} \end{bmatrix}$$

Objective Function

$$\Psi = \underbrace{||\mathbf{W}_\mathrm{d}(\mathbf{d} - \mathcal{F}(\mathbf{m}))||_2}_{\text{data misfit}} + \alpha\underbrace{||\mathbf{W}_\mathrm{m}\mathbf{m}||_2}_{\text{spatial smoothness constraint}} + \beta\underbrace{||\mathbf{W}_\mathrm{p}^\mathrm{sum}\mathbf{p} - \mathbf{1}||_2}_{\text{volume conservation constraint}} + \gamma\underbrace{||\mathbf{W}_\mathrm{p}(\mathbf{p} - \mathbf{p}_\mathrm{o})||_2}_{\text{damping towards reference model}} \rightarrow \mathbf{min}$$

Legend

Geophysical data:

t: seismic travel time [s]

ρ_a: apparent resistivity [Ω m]

Volumetric fractions:

f_r: rock matrix content $(1 - \Phi)$

f_w: water content

f_i: ice content

f_a: air content

Model transformation:

j: model cell index

k: pore filling index

Inversion:

$\mathcal{F}$: joint forward operator

$\mathbf{W}_\mathrm{d}$: data weighting matrix

$\mathbf{W}_\mathrm{m}$,$\mathbf{W}_\mathrm{p}^\mathrm{sum}$,$\mathbf{W}_\mathrm{p}$: model weighting matrices

α,β,γ: regularization param.

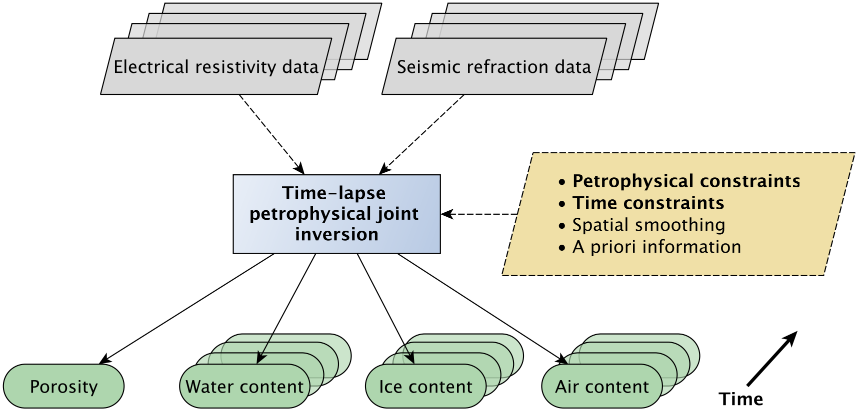
Petrophysical joint inversion...

- outperforms post-inversion transformation of conventional tomograms
  (by honoring petrophysics and volume conservation during parameter estimation)
- is not able to overcome the inherent **petrophysical ambiguities between ice and rock matrix**
- artificially introduces implausibly high changes in porosity in a time-lapse context

Approach: extend the PJI along the time axis $\longrightarrow$ **time-lapse petrophysical joint inversion (TLPJI)**

- assumption: **constant porosity**
- increases ratio of data and parameters, reduces degrees of freedom
- more dynamic perspective: **temporal evolution** of permafrost systems

To do: extend equations to the time dimension, add a regularization in time


Electrical resistivity data
Seismic refraction data
Time-lapse petrophysical joint inversion
• Petrophysical constraints
• Time constraints
• Spatial smoothing
• A priori information
Porosity
Water content
Ice content
Air content
Time

Implementation

Data Space

$$\mathbf{d} = \left[\mathbf{t}_{T_1}, \mathbf{t}_{T_2}, \cdots, \mathbf{t}_{T_N}, \log\left(\rho_{\mathrm{a},T_1}\right), \log\left(\rho_{\mathrm{a},T_2}\right), \cdots, \log\left(\rho_{\mathrm{a},T_N}\right)\right]^T$$

Model Space

$$\mathbf{p} = \left[\mathbf{f}_{\mathrm{w},T_1}, \cdots, \mathbf{f}_{\mathrm{w},T_N}, \mathbf{f}_{\mathrm{i},T_1}, \cdots, \mathbf{f}_{\mathrm{i},T_N}, \mathbf{f}_{\mathrm{a},T_1}, \cdots, \mathbf{f}_{\mathrm{a},T_N}, \mathbf{f}_{\mathrm{r},T_1}, \cdots, \mathbf{f}_{\mathrm{r},T_N}\right]^T$$

Transformed model vector: $m_j^k = \log\left(p_j^k\right) - \log\left(1 - p_j^k\right)$.

Sensitivities

$$\mathbf{J} = \begin{bmatrix}
\frac{\partial \mathbf{t}_{T_1}}{\partial \mathbf{f}_\mathrm{w}} & & & \frac{\partial \mathbf{t}_{T_1}}{\partial \mathbf{f}_\mathrm{i}} & & & \frac{\partial \mathbf{t}_{T_1}}{\partial \mathbf{f}_\mathrm{a}} & & & \frac{\partial \mathbf{t}_{T_1}}{\partial \mathbf{f}_\mathrm{r}} & & \\
& \ddots & & & \ddots & & & \ddots & & & \ddots & \\
& & \frac{\partial \mathbf{t}_{T_N}}{\partial \mathbf{f}_\mathrm{w}} & & & \frac{\partial \mathbf{t}_{T_N}}{\partial \mathbf{f}_\mathrm{i}} & & & \frac{\partial \mathbf{t}_{T_N}}{\partial \mathbf{f}_\mathrm{a}} & & & \frac{\partial \mathbf{t}_{T_N}}{\partial \mathbf{f}_\mathrm{r}} \\
\frac{\partial \log(\rho_{\mathrm{a},T_1})}{\partial \mathbf{f}_\mathrm{w}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_1})}{\partial \mathbf{f}_\mathrm{i}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_1})}{\partial \mathbf{f}_\mathrm{a}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_1})}{\partial \mathbf{f}_\mathrm{r}} & & \\
& \ddots & & & \ddots & & & \ddots & & & \ddots & \\
& & \frac{\partial \log(\rho_{\mathrm{a},T_N})}{\partial \mathbf{f}_\mathrm{w}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_N})}{\partial \mathbf{f}_\mathrm{i}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_N})}{\partial \mathbf{f}_\mathrm{a}} & & & \frac{\partial \log(\rho_{\mathrm{a},T_N})}{\partial \mathbf{f}_\mathrm{r}}
\end{bmatrix}$$

Legend

Geophysical data:

t: seismic travel time [s]

ρ_a: apparent resistivity [Ω m]

Volumetric fractions:

f_r: rock matrix content $(1 - \Phi)$

f_w: water content

f_i: ice content

f_a: air content

Time-lapse inversion:

T_i: time step i

Objective Function

$$\Psi = \Psi_\mathrm{d} + \Psi_\mathrm{m} + \Psi_\mathrm{sum} + \Psi_\mathrm{prior} + \Psi_\mathrm{T}$$

$$= \underbrace{\|\mathbf{W}_\mathrm{d}(\mathbf{d} - \mathcal{F}(\mathbf{m}))\|_2}_{\text{data misfit}} + \underbrace{\alpha\|\mathbf{W}_\mathrm{m}\mathbf{m}\|_2}_{\text{spatial smoothness constraint}} + \underbrace{\beta\|\mathbf{W}_\mathrm{p}^\mathrm{sum}\mathbf{p} - \mathbf{1}\|_2}_{\text{volume conservation constraint}} + \underbrace{\gamma\|\mathbf{W}_\mathrm{p}(\mathbf{p} - \mathbf{p}_\mathrm{o})\|_2}_{\text{damping towards reference model}} + \underbrace{\frac{N_\mathrm{T}}{N_\mathrm{T} - 1}\delta\|\mathbf{W}_\mathrm{T}\mathbf{m}\|_2}_{\text{temporal smoothness constraint}} \rightarrow \mathbf{min}$$

The additional **temporal regularization term**...

- ensures smooth changes from one time step to another
- needs to be scaled with respect to the other terms of the objective function

Legend

Inversion:

$\mathcal{F}$: joint forward operator

$\mathbf{W}_\mathrm{d}$: data weighting matrix

$\mathbf{W}_\mathrm{m}$,$\mathbf{W}_\mathrm{p}^\mathrm{sum}$,$\mathbf{W}_\mathrm{p}$,$\mathbf{W}_\mathrm{T}$: model weighting matrices

α,β,γ,δ: regularization param.

N_T: number of time steps

Objective Function

$$\Psi = \Psi_\mathrm{d} + \Psi_\mathrm{m} + \Psi_\mathrm{sum} + \Psi_\mathrm{prior} + \Psi_\mathrm{T}$$

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The inversion algorithm **favors updated models that...**

- fit the data (Ψ_d)
- are smooth in space (Ψ_m)
- fulfill the volume conservation constraint (Ψ_sum)
- are close to a predefined reference model (optional, Ψ_prior)
- are smooth in time (Ψ_T)

Legend

Inversion:

$\mathcal{F}$: joint forward operator

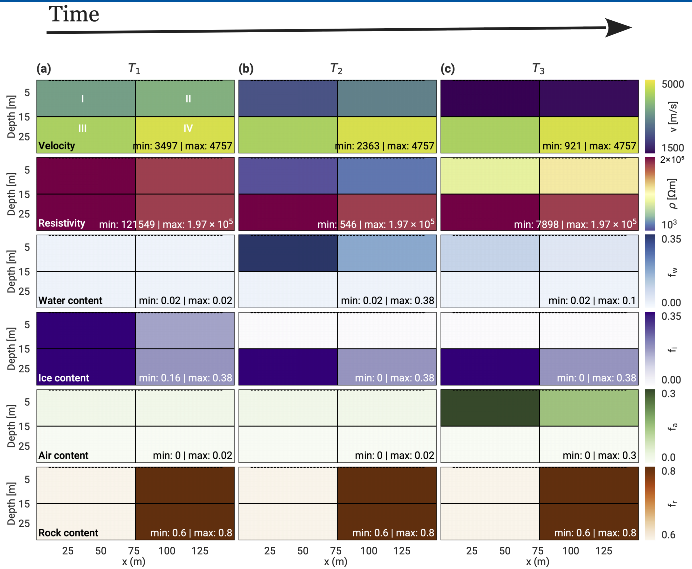
$\mathbf{W}_\mathrm{d}$: data weighting matrix

$\mathbf{W}_\mathrm{m}$,$\mathbf{W}_\mathrm{p}^\mathrm{sum}$,$\mathbf{W}_\mathrm{p}$,$\mathbf{W}_\mathrm{T}$: model weighting matrices

α,β,γ,δ: regularization param.

N_T: number of time steps

Model Setup



Three stages:

T_1 – frozen

T_2 – after thawing

T_3 – after drainage

Lateral porosity contrast

Forward modeling:

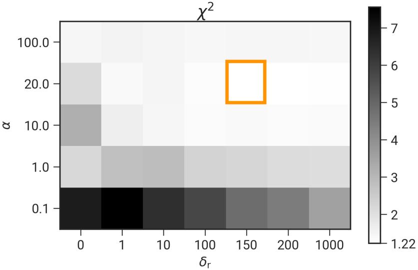
53 geophones/electrodes spaced by 2.5 m

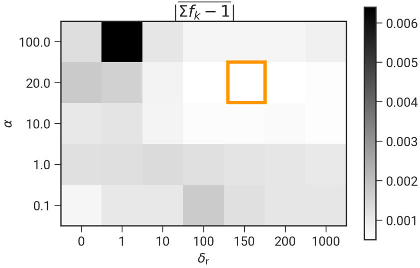
Parameter	Function	Value
α	Spatial smoothing	?
$\mathrm{zWeight}$	Vertical anisotropic smoothing	1
β	Volume conservation constraint	10000
γ	Damping towards reference model	0
δ_{r}	Temporal smoothing of f_{r}	?
$\delta_{\mathrm{w,i,a}}$	Temporal smoothing of f_{w}, f_{i}, f_{a}	0

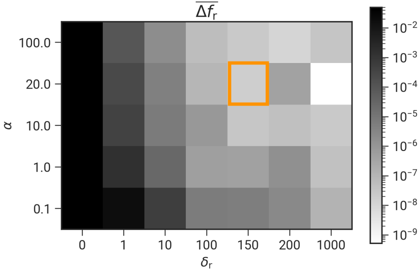
Three criteria:

- **Data misfit** (χ^2): Does the model still fit the data within the error bounds?
- **Average deviation of the sum of all fractions from one** $\left(\overline{\left|\Sigma f_k - 1\right|}\right)$: Is the volume conservation constraint still fulfilled?
- **Average change in rock content over time** $\left(\overline{\Delta f_{\mathrm{r}}}\right)$: Is δ_{r} chosen high enough to ensure that the porosity is constant over time?

Choice of Regularization Parameters



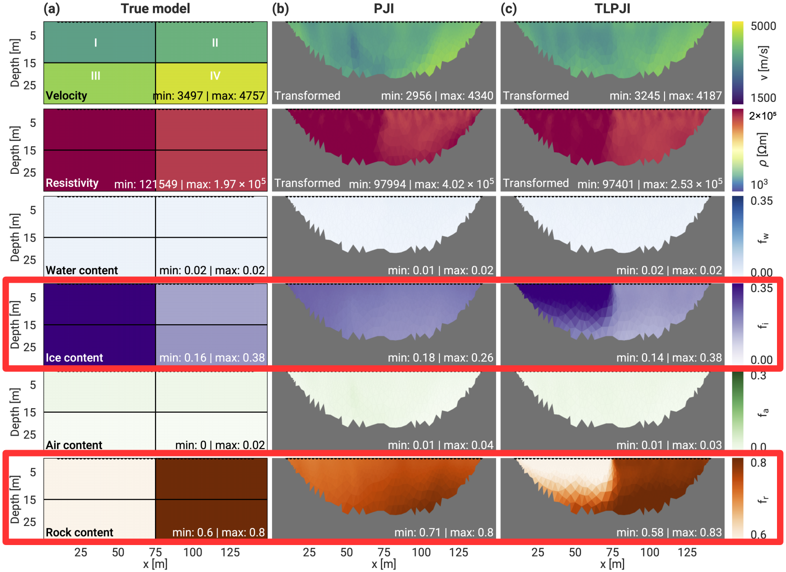




$\Rightarrow \alpha = 20$

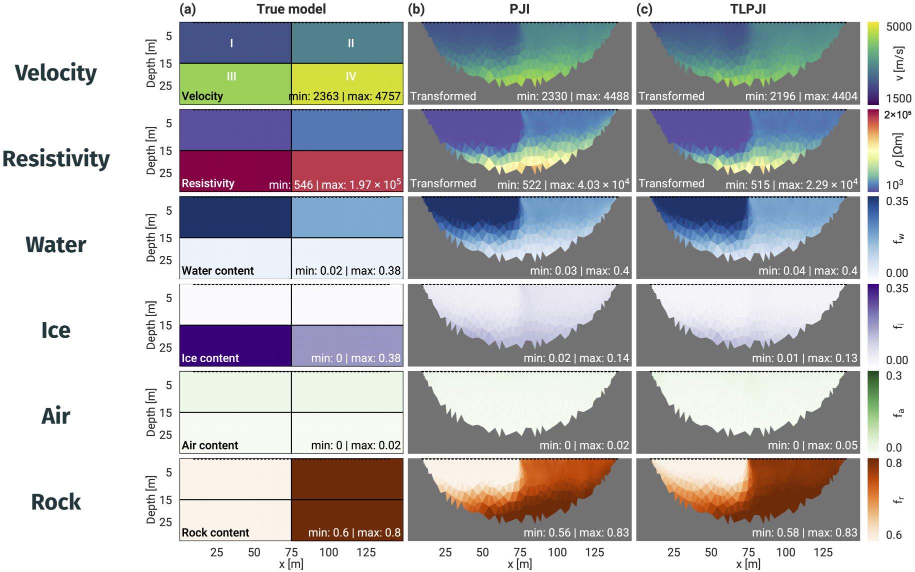
$\delta_r = 150$

PJI and TLPJI Results for Time Step 1

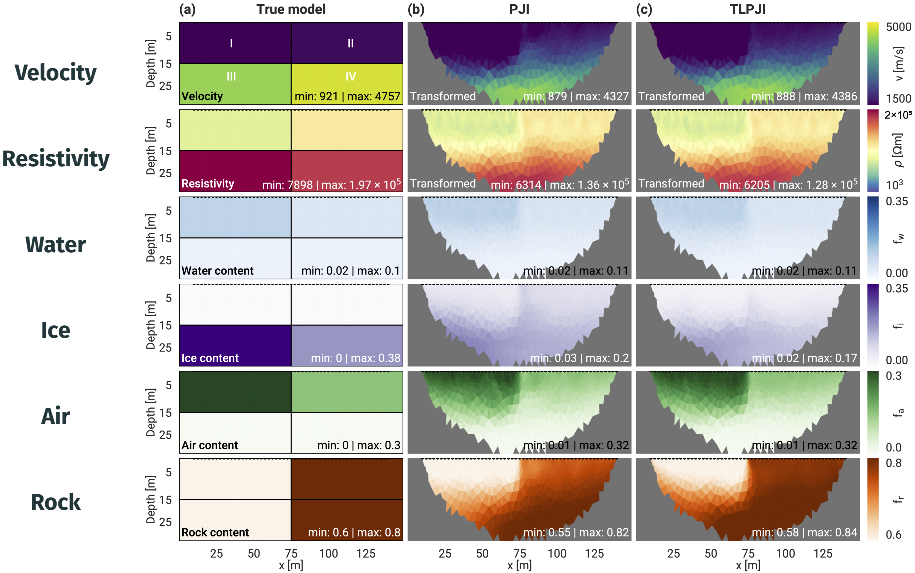


improved ice-rock discriminability

PJI and TLPJI Results for Time Step 2



PJI and TLPJI Results for Time Step 3



Conclusions

A time-lapse petrophysical joint inversion of electrical and seismic data was developed and...

- defines the subsurface as a **space-time model** and merges all time steps into a single least-squares minimization problem of a global cost function
- adds a **temporal regularization** term to the objective function (that offers different possibilities of temporal smoothing, e.g. **Active Time Constraints**)
- allows to image the **quantitative evolution** of the different permafrost constituents (liquid water, ice, air)
- considerably **improves the ice-rock discriminability** of the conventional PJI (by postulating a temporally constant porosity)
- was applied to a **synthetic three-stage thawing scenario** and succeeded in differentiating between ice and rock (contrary to the PJI)
- was applied to **field data from Schilthorn** (Swiss Alps) and detected significant **permafrost degradation** between 2008 and 2017 (not shown here)

Future Work

Future research could include...

- a more comprehensive **multiphysical characterization** of permafrost sites (by combining the TLPJI with e.g. moisture, temperature and porosity data)
- **advanced petrophysical formulations** and **site dependent petrophysical input parameters**
- **spatially variable petrophysical parameters** throughout the model domain
- **addition of another freeze-thaw sensitive geophysical data** set to the TLPJI concept (e.g. IP, SP)
- quantification of **time-lapse data errors** and adjusted **data weighting**
- **technical improvements** (e.g. further parallelization)
- further **exploration of the implemented Active Time Constraints**

Thank you for your interest!

Please contact me if you have any questions, feedback, or further input!

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