

Reducing full-field training data to time series

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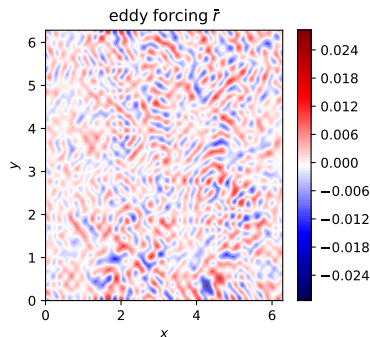
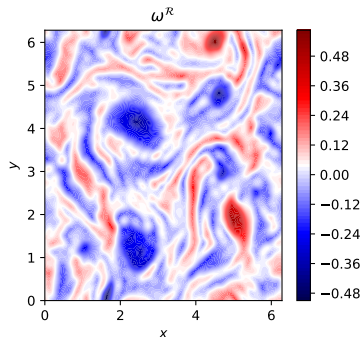
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1) Multiscale governing equations

- ▶ Forced dissipative vorticity eqs, coarse-grained to 64×64 grid
- ▶ Coarse graining: subgrid-scale (SGS) model now required

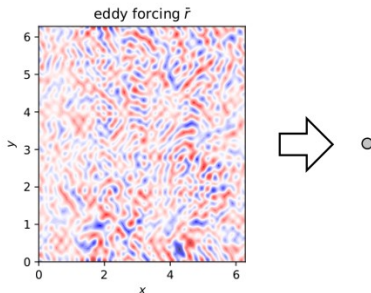
$$\frac{\partial \bar{\omega}}{\partial t} + \overline{J(\bar{\psi}, \bar{\omega})} = \nu \nabla^2 \bar{\omega} + \mu (F - \bar{\omega}) - \text{SGS model},$$
$$\nabla^2 \bar{\psi} = \bar{\omega}.$$



2) Goal

- ▶ Goal: predict **statistics of time series**, e.g. energy $E(t)$
- ▶ How: machine learning of a SGS surrogate, using data from 256×256 simulation
- ▶ Problem: exact SGS is **full-field** $\rightarrow$ complex surrogate
- ▶ Our goal is not full field: **compress ML training data**

New SGS model where **unclosed components are time series**



3) Assumptions

- Our new (reduced) SGS term has the following form:

$$\text{Reduced SGS} = \sum_{i=1}^d \tau_i(t) P_i(x, y, t),$$

- and must 'track' d time-dependent quantities of interest:

$$Q_i(t) = \left(\frac{1}{2\pi} \right)^2 \int_0^{2\pi} \int_0^{2\pi} q_i(\bar{\omega}, \bar{\psi}; x, y, t) \, dx dy, \quad i = 1, \dots, d.$$

such that $Q_i^{ref}(t) - Q_i(t)$ is small $\forall t$ in training period

4) Compute effect of assumptions

- Derive transport equation of the Q_i :

$$\frac{dQ_i}{dt} = \dots + \left(\frac{\partial q_i}{\partial \bar{\omega}}, \text{Reduced SGS} \right) = \dots + \sum_{j=1}^d \tau_j \left(\frac{\partial q_i}{\partial \omega}, P_j \right)$$

- Every Q_i has d SGS terms: let's remove $\sum_{j=1}^d$
- Simplify, make P_j orthogonal:

$$P_i := \frac{\partial q_i}{\partial \bar{\omega}} - \sum_{j=2}^d c_{i,j}(t) \frac{\partial q_j}{\partial \bar{\omega}}$$

$$\boxed{\left(\frac{\partial q_i}{\partial \bar{\omega}}, P_j \right) = 0 \text{ if } i \neq j} \quad \forall t$$

5) Extract τ_i from data

- ▶ Due to orthogonality, transport equation of the Q_i become:

$$\frac{dQ_i}{dt} = \dots + \left(\frac{\partial q_i}{\partial \bar{\omega}}, \text{Reduced SGS} \right) = \dots + \tau_i \left(\frac{\partial q_i}{\partial \omega}, P_i \right)$$

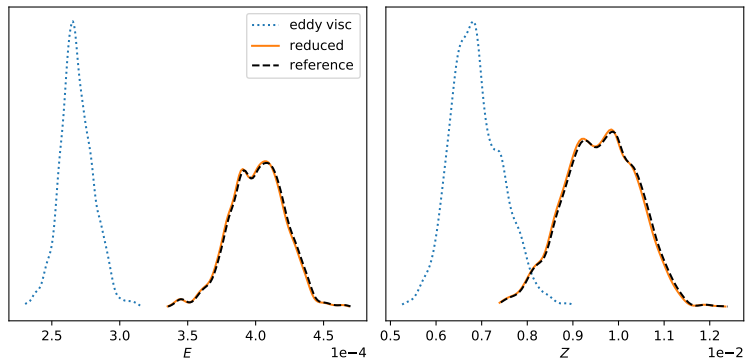
- ▶ Goal: $Q_i^{ref}(t) - Q_i(t) := \Delta Q_i$ is small $\forall t$ in training
- ▶ Simply equate SGS term to ΔQ_i :

$$\tau_i \left(\frac{\partial q_i}{\partial \omega}, P_i \right) = \Delta Q_i, \quad i = 1, \dots, d$$

- ▶ imposes linear relaxation towards reference

6) Example results

- ▶ Track reference energy E and enstrophy Z ($d = 2$)
→ $q_1 := \psi\omega/2$ and $q_2 = \omega\omega/2$



- ▶ Reduced SGS term yields the same E & Z pdf
- ▶ For every t , training data is reduced in size from 64^2 to 2