

Heteroskedasticity between GNSS time-series repeatabilities and noise magnitudes

Huseyin Duman^{1*} and D. Ugur Sanli¹

¹Yildiz Technical University, Civil Engineering Faculty, Geomatic Engineering, Istanbul, Turkey (hduman@yildiz.edu.tr; usanli@yildiz.edu.tr)

#vEGU General Assembly [G1.3]
19-30 April 2021



Outline

Introduction

Data and Methods

Results

- Hypothesis Test

- Heteroskedastic Form

- Numerical Example

Conclusions

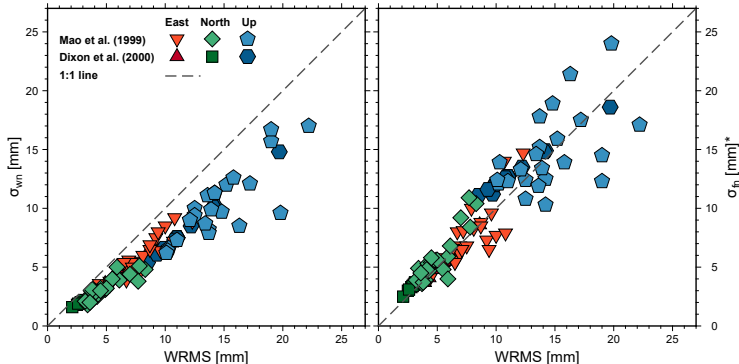
References

Introduction

- ▶ Possible to state that there is a linear relationship between time-series repeatability and noise magnitudes.
- ▶ That linearity was build upon homoskedastic assumption, and frequently used in the studies considering stochastic properties especially for episodic time-series even not as continuous time-series.
- ▶ However, we provided a form of heteroskedastic structure for that linear relationship.

Data and Methods

- ▶ Using data for series repeatability (i.e., WRMS) and noise magnitudes from
 - ▶ Table 6 of Mao et al. (1999)
 - ▶ Table 3 of Dixon et al. (2000)



*The amplitudes had been computed using Zhang et al. (1997) flicker matrix.

- ▶ Established the following hypotheses to check whether both flicker and white noise versus WRMS are heteroskedastic or not

H_0 : *Homoskedastic*

vs.

H_A : *Heteroskedastic*

- ▶ Both Breusch-Pagan test and Goldfeld-Quandt test are used in regression analysis to determine which hypothesis is valid.

Results

Hypothesis Test

- ▶ For both tests, 95% confidence interval is adopted.

the p-value, which is a probability measure, for test statistics.

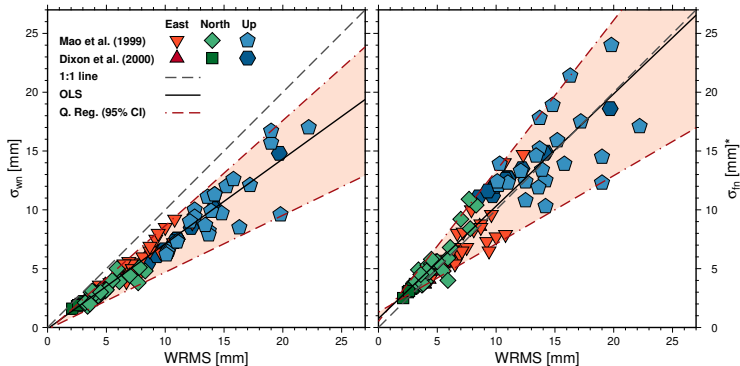
	Breusch-Pagan	Goldfeld-Quandt
White Noise	3.31E-08	1.20E-15
Flicker Noise	2.83E-13	1.36E-14

- ▶ All p-values almost equal to **zero**, which means we fail to accept the null hypothesis (i.e., H_0 : Homoskedastic). Therefore, we can accept alternative hypothesis (i.e., H_A : Heteroskedastic) with 95% confidence.

Results

Heteroskedastic Form

- ▶ As a results of hypothesis test, the linear relationship is heteroskedastic.



*The amplitudes had been computed using Zhang et al. (1997) flicker matrix.

Results

Heteroskedastic Form

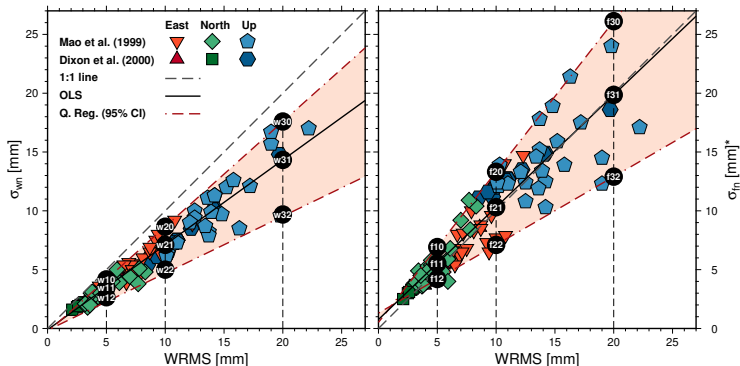
- ▶ Heteroskedastic form limits estimated through quantile regression method, that is, the limit equations from quantile regression for 0.975^{th} and 0.025^{th} quantiles with 95% confidence interval.
- ▶ Which means that noise magnitudes corresponding to any WRMS value should be pointed somewhere with a user-defined confidence interval (e.g., 95% for our example) between the quantile regression equations.

Results

Numerical Example

- ▶ To test how rate uncertainties we set affect campaign examples
 - ▶ 3 successive days
 - ▶ 5 years in total

WRMS		σ_{wn}	σ_{fn}
[mm]		[mm]	[mm.yr ^{-0.25}]
5	vs.	w1?	f1?
10		w2?	f2?
20		w3?	f3?
?: 0, 1, and 2 for 0.975th quantile, OLS, 0.025th quantile regression, respectively			



*The amplitudes had been computed using Zhang et al. (1997) flicker matrix.

Results

Numerical Example

► Rate uncertainty

$$\sigma_r = \sigma_0 \sqrt{q_{trend}}$$

where σ_0 stands for apriori variance, accepted as 1, while q_{trend} is the corresponding diagonal element of cofactor matrix, **Q**, which is of the form

$$\mathbf{Q} = (\mathbf{A}^T \cdot (\sigma_{wn}^2 \cdot \mathbf{I} + \sigma_{fn}^2 \cdot \mathbf{J})^{-1} \cdot \mathbf{A})^{-1}$$

where also **A** stands for the design matrix, **I** is identity matrix, and **J** is the flicker matrix from Williams (2003).

Results

Numerical Example

Case	WRMS [mm]	σ_{wn} [mm]	σ_{fn} [mm]	$\sigma_{fn}^*1.744$ [mm.yr ^{-0.25}]	σ_r [mm.yr ⁻¹]
10	5.0	4.17	6.92	12.07	1.45
11	5.0	3.50	5.54	9.66	1.18
12	5.0	2.66	4.21	7.33	0.89
20	10.0	8.65	13.32	23.23	2.85
21	10.0	7.11	10.31	17.98	2.25
22	10.0	5.00	7.11	12.39	1.56
30	20.0	17.59	26.12	45.56	5.67
31	20.0	14.33	19.86	34.64	4.41
32	20.0	9.69	12.91	22.51	2.91

* Flicker Noise amplitudes were scaled by 1.744
with reference to Williams (2003)

- ▶ $\frac{\sigma_{r,0.975}}{\sigma_{r,0.025}}$ is varying from 1.6 to 2 for 5 through 20 mm WRMS, respectively.
- ▶ Ratio between quantile rate uncertainties and $\sigma_{r,OLS} \cong 1.2$ to 1.5

Conclusions

- ▶ It could be clearly concluded that the linear relation between time-series repeatabilities and noise magnitudes (i.e., white and flicker noise) is heteroskedastic.
- ▶ Rate uncertainties corresponding only to a WRMS could be underestimated, for instance, it is up to 60% for 5 mm, and even capable of reaching up to 100% for 20 mm WRMS.
- ▶ It seems that this underestimation would increase to higher percentages as WRMS increases.
- ▶ Once compared with the ordinary least-squares, these percentages range from 20% to 50%, 5 mm through 20 mm WRMS, respectively.

References

- ▶ Dixon, T. H., Miller, M., Farina, F., Wang, H., Johnson, D. (2000). Present-day motion of the Sierra Nevada block and some tectonic implications for the Basin and Range province, North American Cordillera. *Tectonics*, 19(1), 1-24.
- ▶ Mao, A., Harrison, C. G., Dixon, T. H. (1999). Noise in GPS coordinate time series. *Journal of Geophysical Research: Solid Earth*, 104(B2), 2797-2816.
- ▶ Williams, S. D. P. (2003). The effect of coloured noise on the uncertainties of rates estimated from geodetic time series. *Journal of Geodesy*, 76(9-10), 483-494.