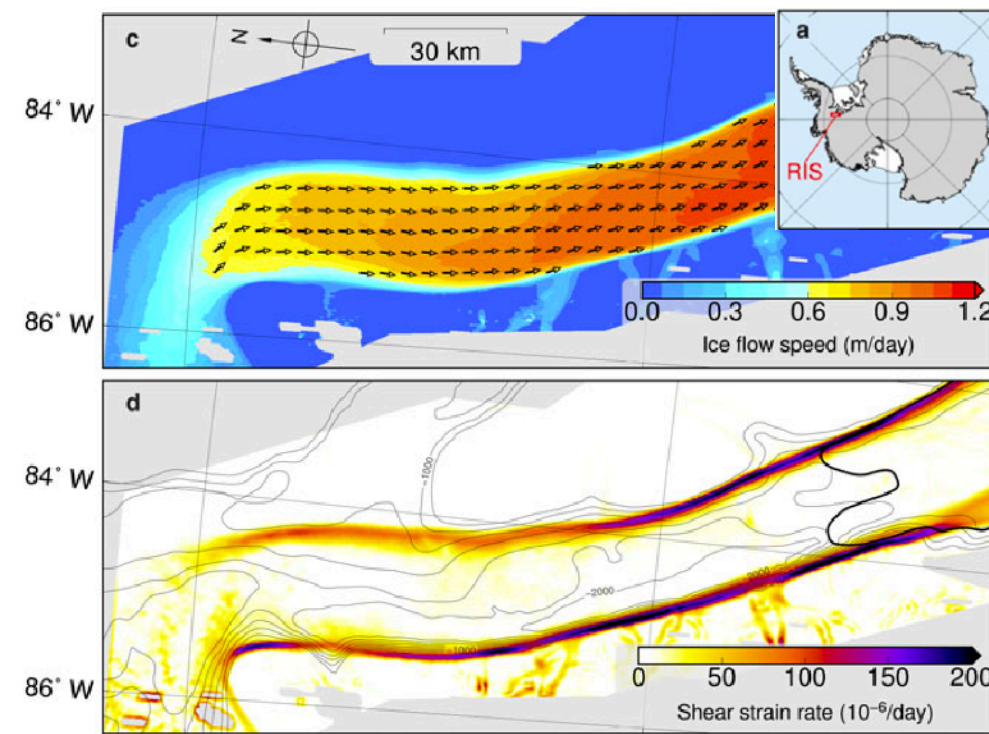


Ice flow localisation - fast vs no flow

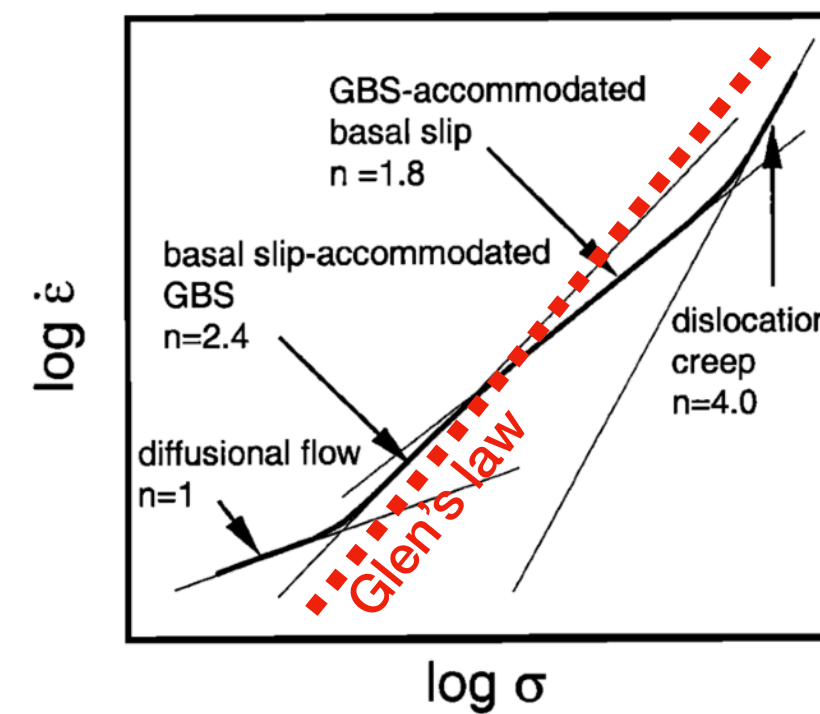
Complex 3D patterns



(Rutford Ice Stream) Minchew et al., 2018

Laboratory data on ice deformation

Controlled 0D-1D configurations



Modified from Goldsby and Kohlstedt, 2001

linear viscous

$$\dot{\epsilon} = c\sigma$$

non-linear viscous

$$\dot{\epsilon} = c\sigma^n$$

$$\dot{\epsilon} = cd^m\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

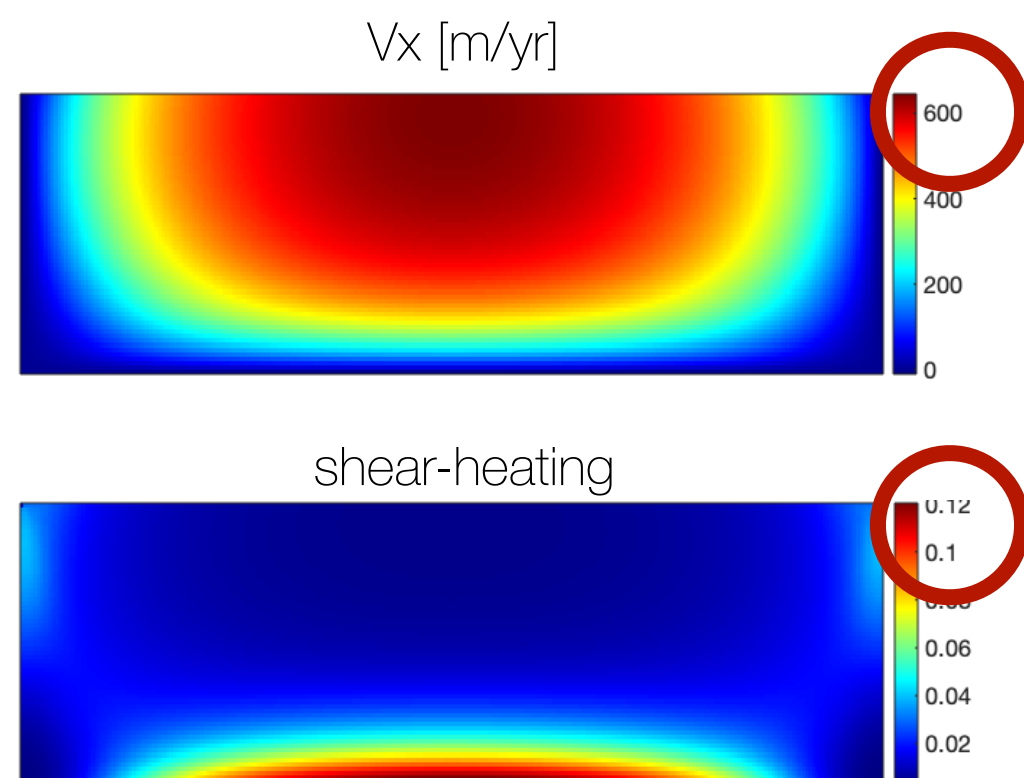
Mathematical / computer models

Implement consistent 0D-3D configurations

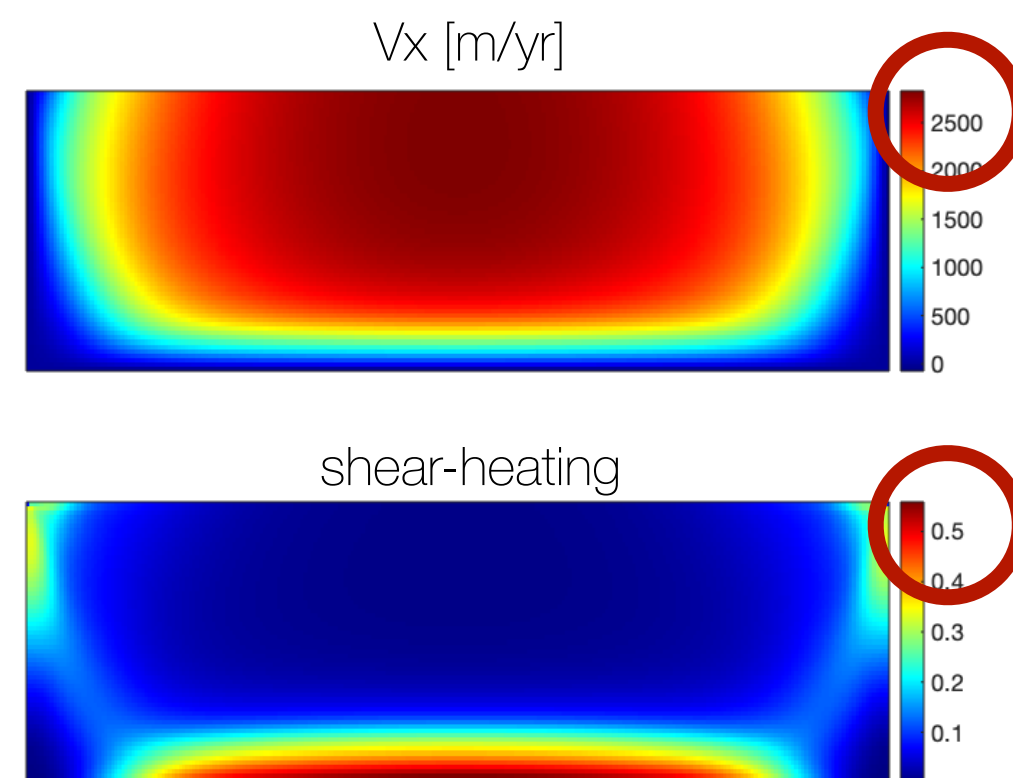
$$\tau_{II} = 2\eta\dot{\epsilon}_{II}$$

$$\tau_{II} = \sqrt{1/2\tau_{ij}\tau_{ij}} \quad \dot{\epsilon}_{II} = \sqrt{1/2\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$$

$$\eta = f(\tau_{II}, n, m, d, T)$$

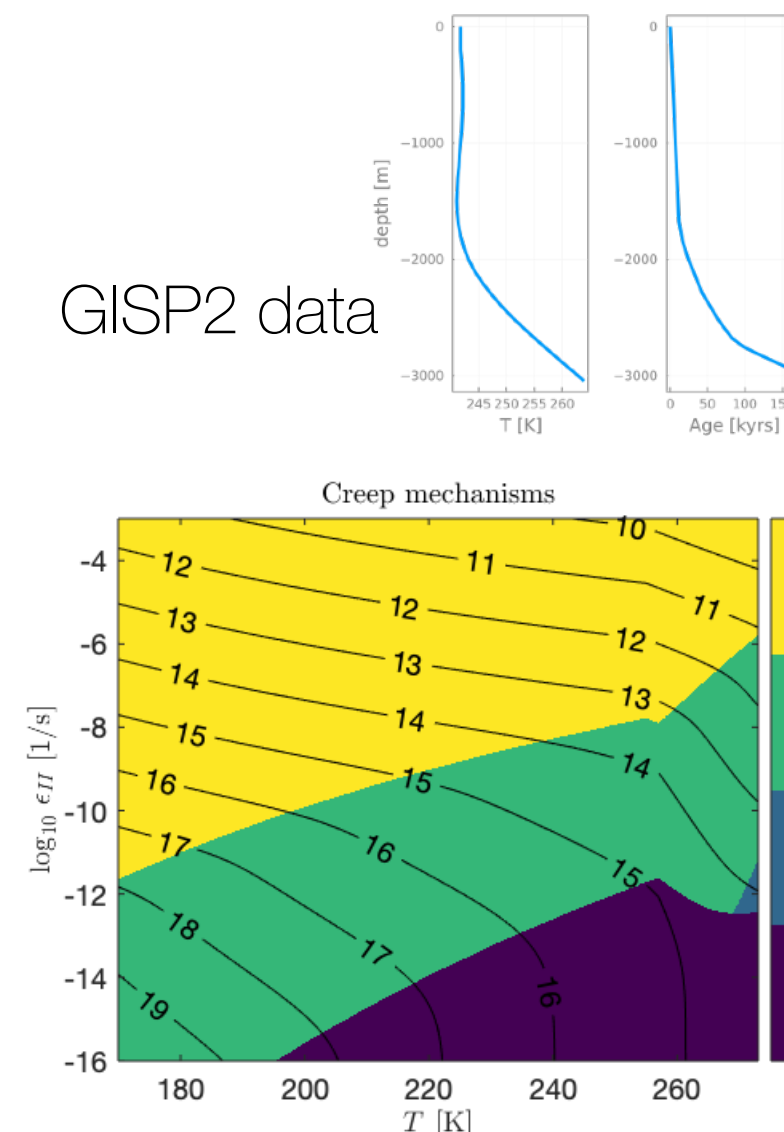


Glen's flow law

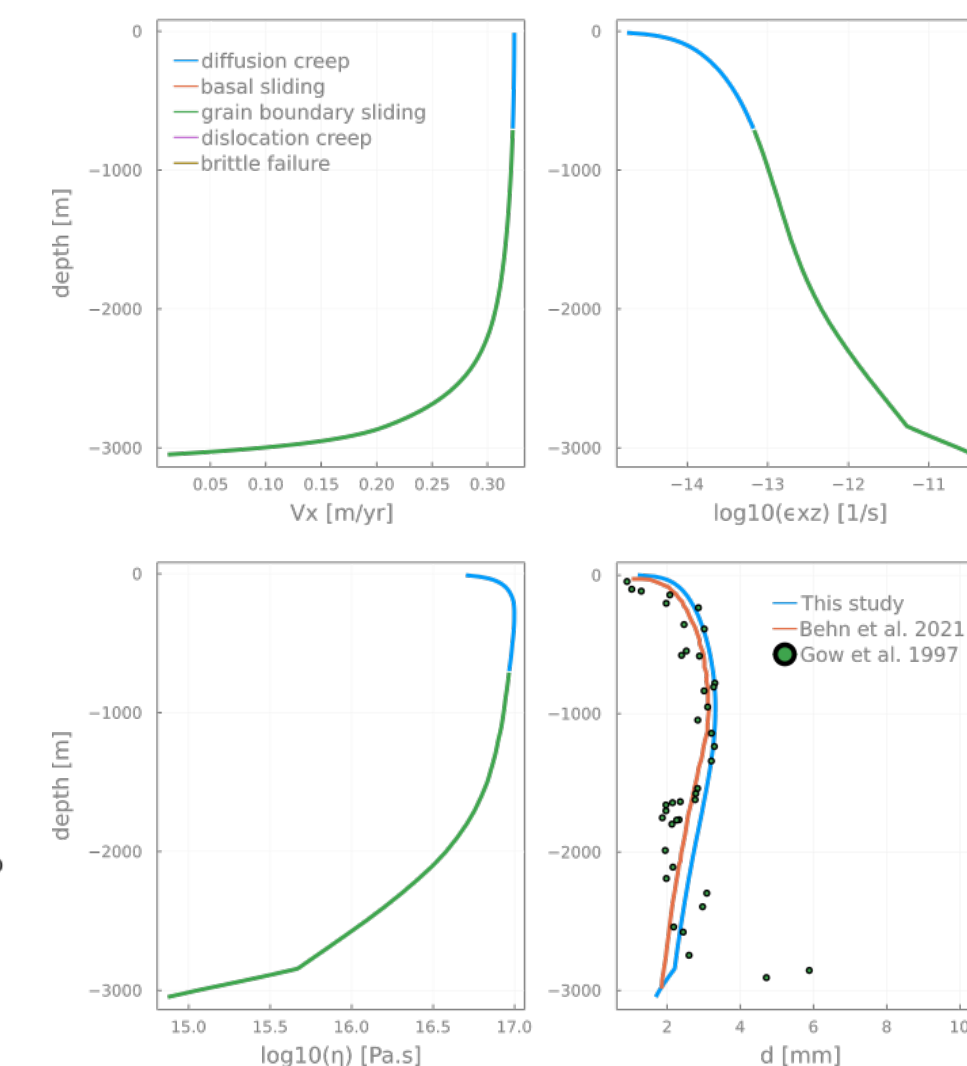


Goldsby's flow law

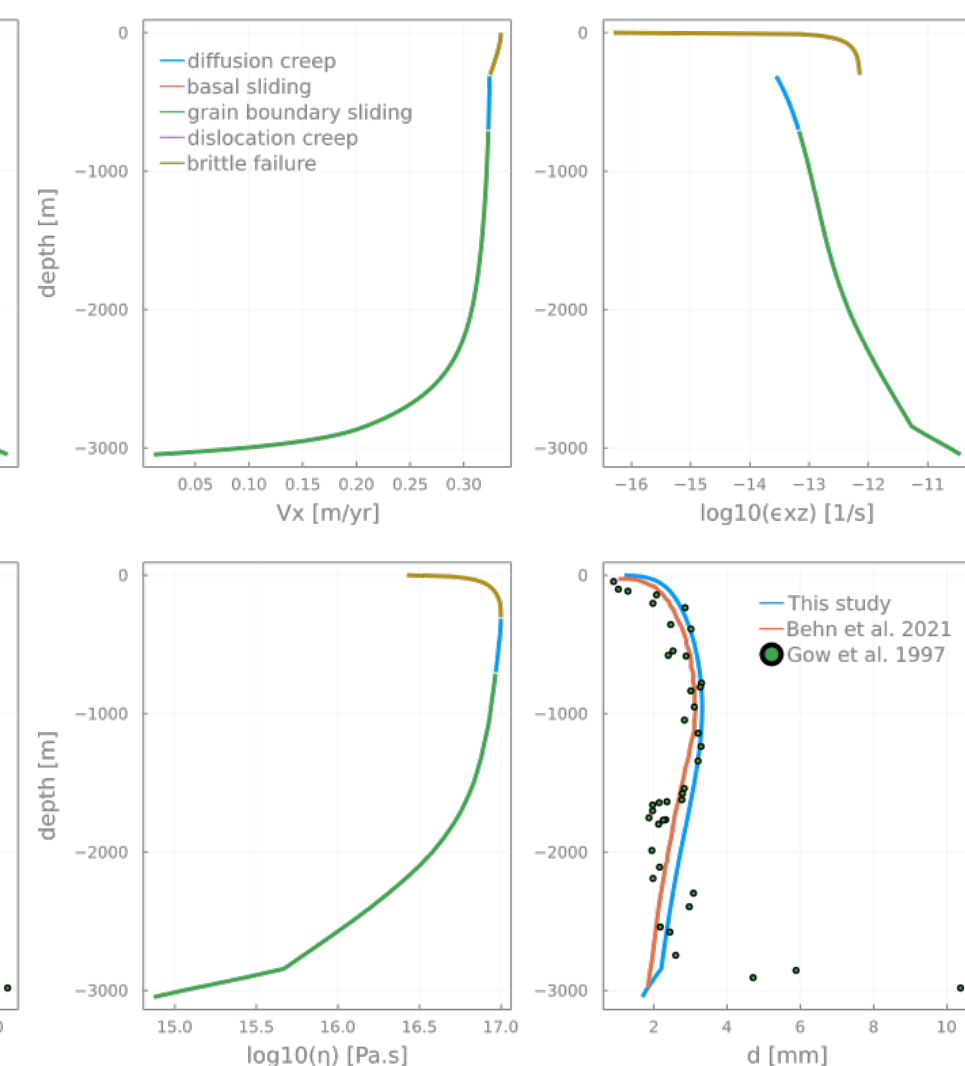
GISP2 data



Goldsby v-e



Goldsby v-e-p



Some details



Ice flow localisation - fast versus no flow

Ice rheology: how ice deforms due to an applied stress

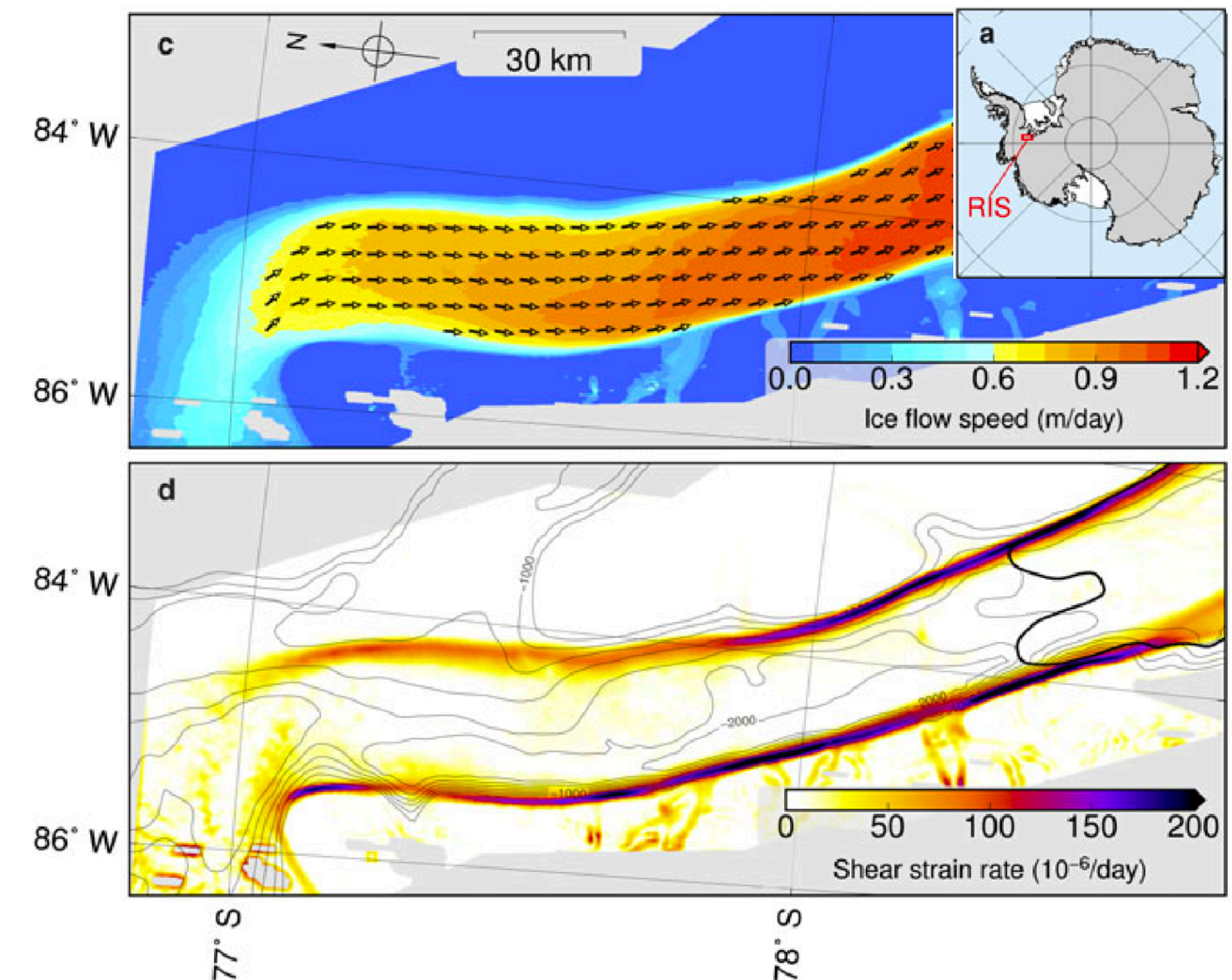
Ice rheology:

- 1) Viscous -> main mechanism(s) on long time scales
- 2) Elastic -> on short time scales
- 3) Plastic - brittle -> on short time scales

Different mechanisms may be (simultaneously) active depending on e.g. the temperature, strain-rate — stress, pressure, yield strength, ... conditions.

Ice rheology is non-linear (non-Newtonian) which can lead to spontaneous localisation of ice flow (e.g. ice streams).

Ice velocities are significantly impacted by rheology of the ice.



(Rutford Ice Stream) Minchew et al., 2018

Laboratory data on ice deformation

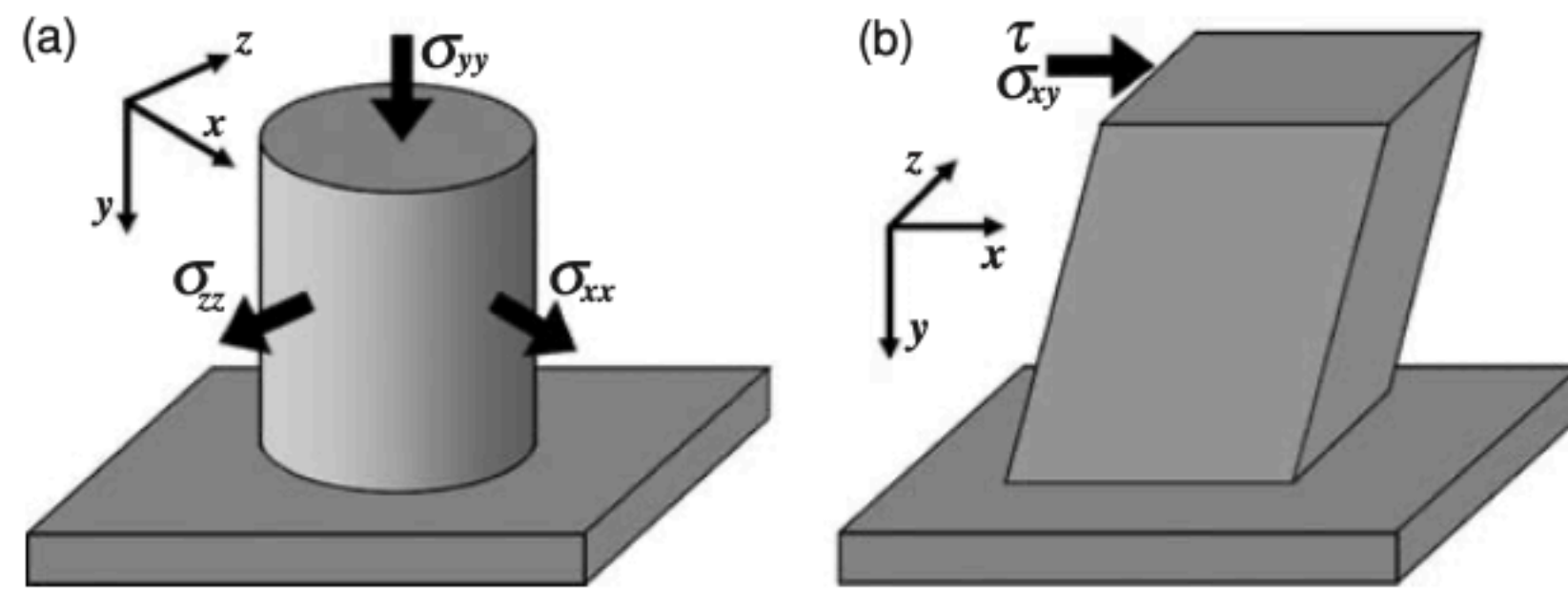


Fig. 6.1 Schematic geometrical relations for an axial compression (a) and a simple shear (b) experiment.

Gerya - Numerical Geodynamic Modelling

Laboratory measurement on rheology are performed to obtain the stress - strain-rate relation called flow-law.

linear viscous

$$\dot{\epsilon} = c\sigma$$

non-linear viscous

$$\dot{\epsilon} = c\sigma^n$$

$$\dot{\epsilon} = cd^m\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

The usual rheological model for non-linear viscous ice flow is the Glen law (power-law exponent $n \sim 3$, temperature dependence in the pre-factor - Arrhenius law).

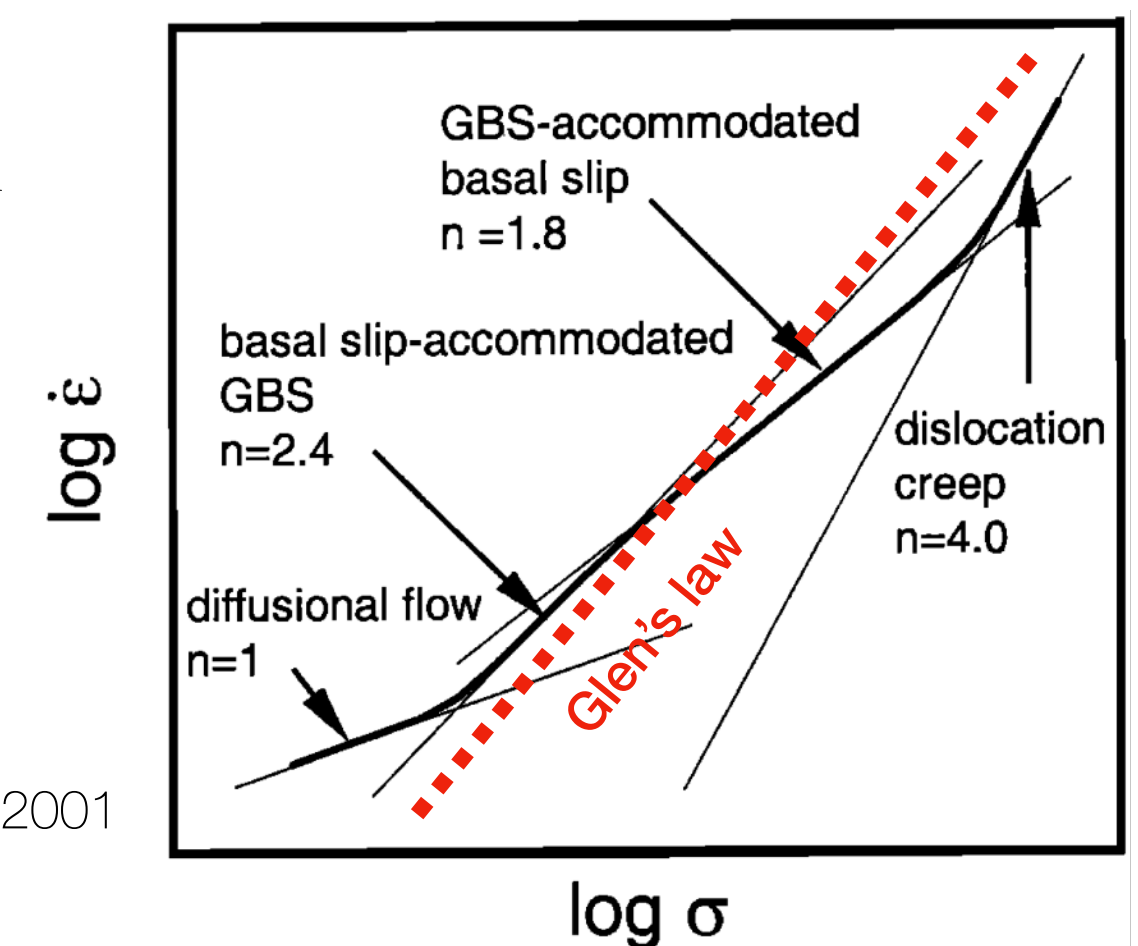
$$\dot{\epsilon} = c_0 \exp\left(-\frac{Q}{RT}\right) \sigma^n$$

Alternative flow laws like the one proposed by Goldsby and Kohlstedt (2001) combines 4 different creep mechanisms and capture more accurately strain rates at low and high stress. It also includes grain size dependence -> ice fabric evolution.

$$\dot{\epsilon} = \dot{\epsilon}_{\text{diff}} + \left(\dot{\epsilon}_{\text{basal}}^{-1} + \dot{\epsilon}_{\text{gbs}}^{-1}\right)^{-1} + \dot{\epsilon}_{\text{disl}}$$

$$\dot{\epsilon} = cd^m\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

Modified from Goldsby and Kohlstedt, 2001



Mathematical / computer models

Laboratory experiments are performed in test apparatus in a confined configuration 0D or 1D. Natural settings are 3-dimensional and stress - strain rates are tensors with components in all spatial dimensions.

$$\tau_{II} = \sqrt{1/2 \tau_{ij} \tau_{ij}}$$

$$\dot{\epsilon}_{II} = \sqrt{1/2 \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}}$$

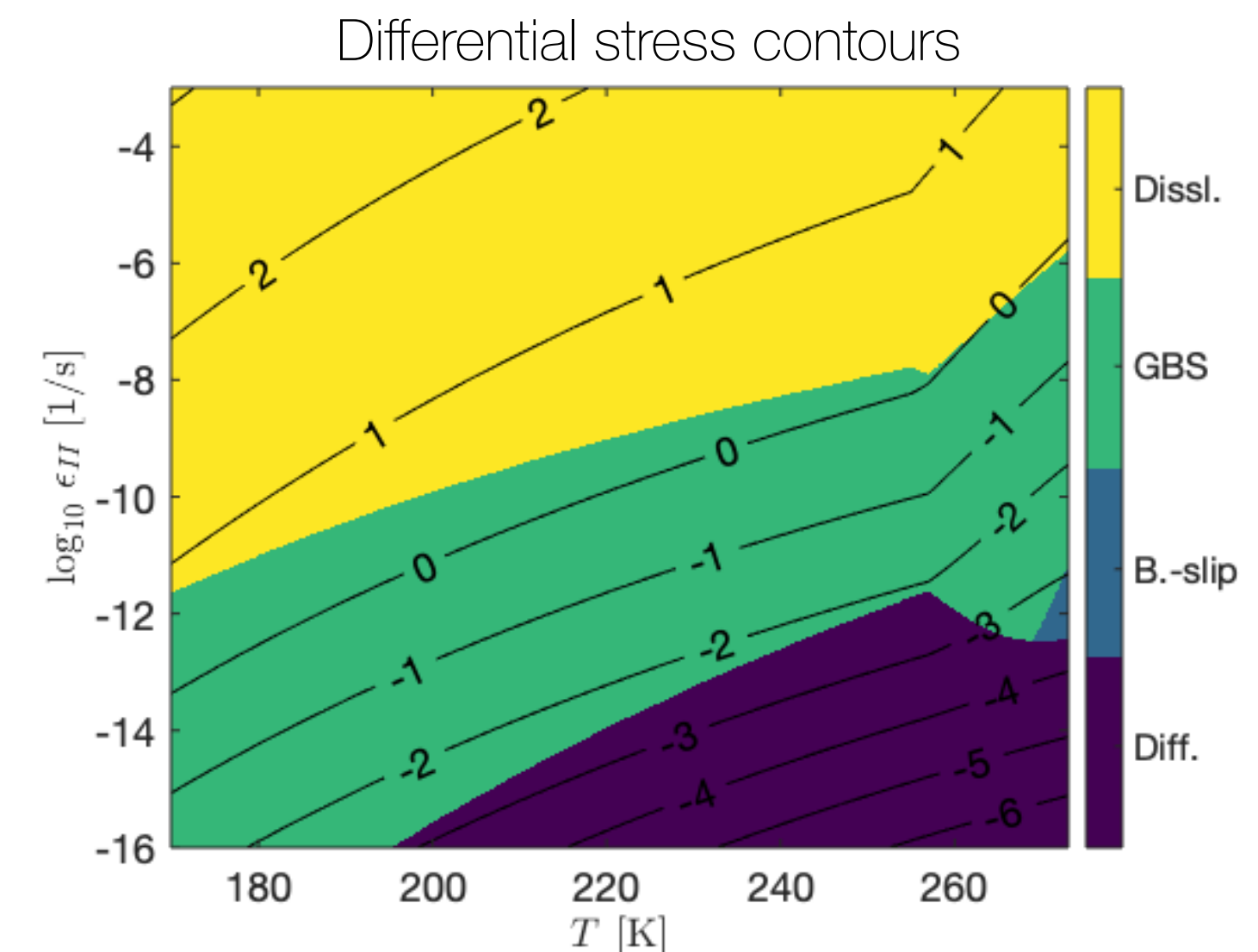
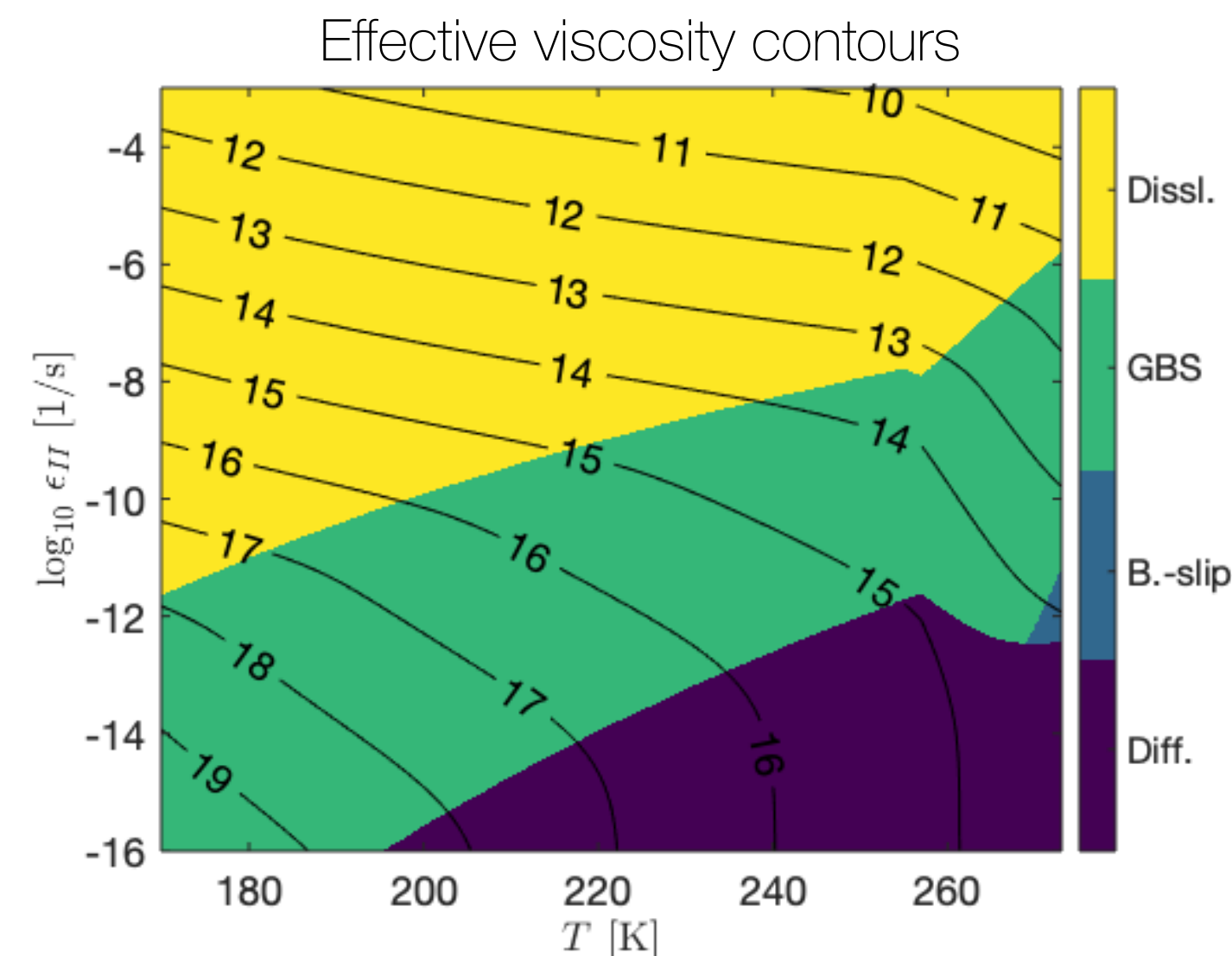
To use the experimentally parametrised rheological relations in (multi-dimension) models one needs to reformulate those in terms of effective viscosity as function of stress or strain-rate invariants to simulate the mechanics of ice deformation and predict strain-rates, pressure and temperature in a consistent way.

$$\tau_{II} = 2\eta\dot{\epsilon}_{II}$$

$$\eta = f(\tau_{II}, n, m, d, T)$$

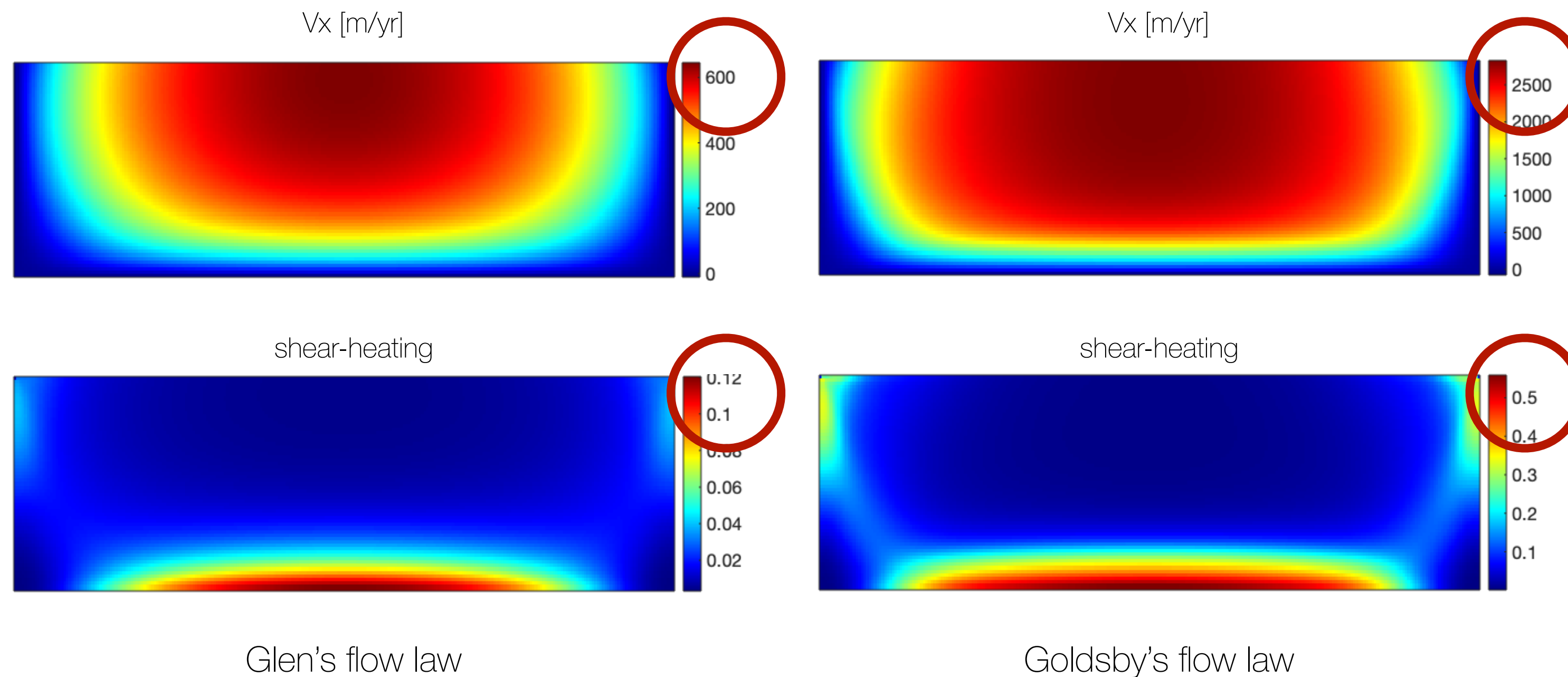
Using Goldsby rheology, it is possible to combine several mechanisms and predict which mechanism is active for a given strain-rate and temperature.

However, here grain size is fixed at e.g. 1mm

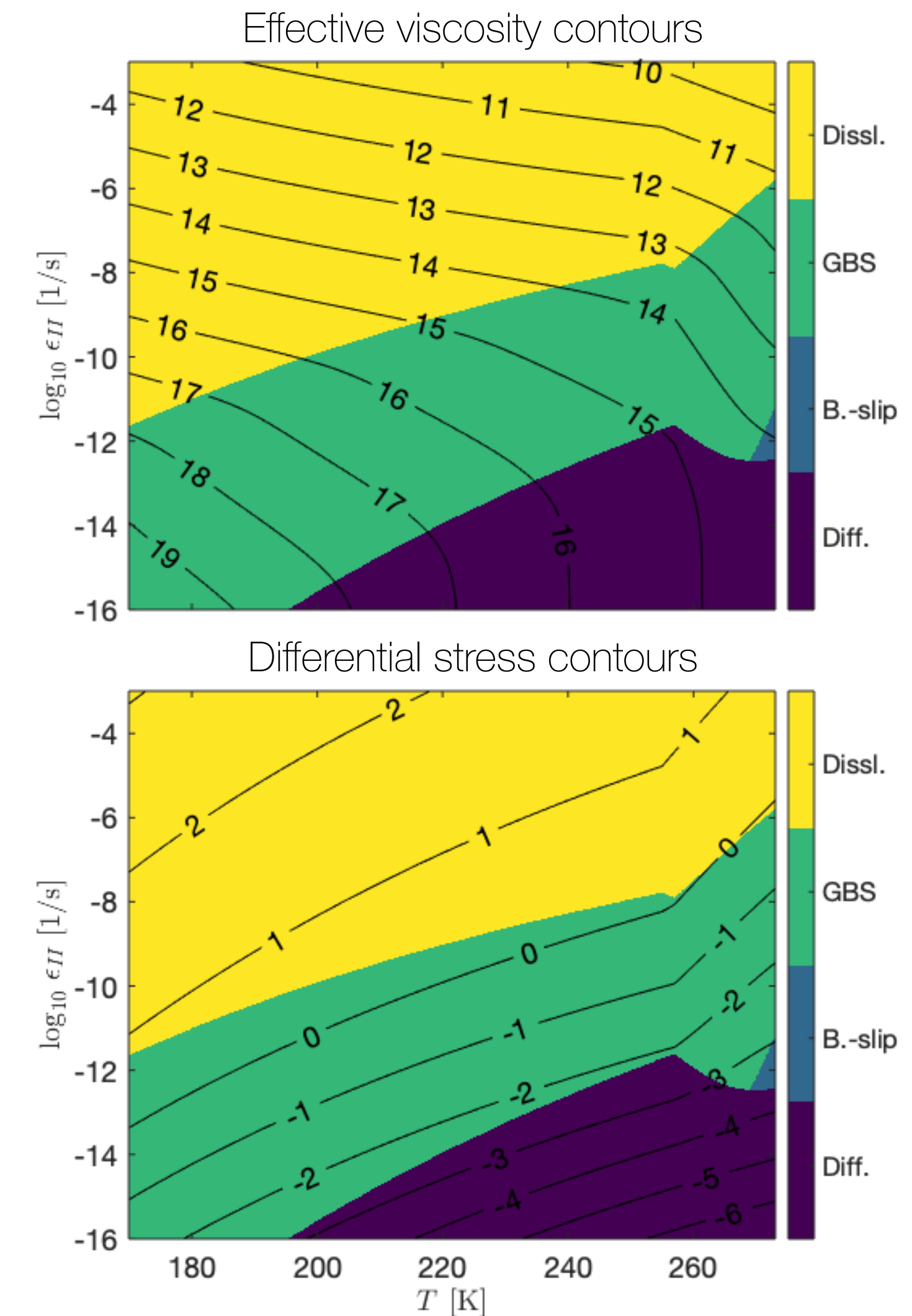


Mathematical / computer models

Already with fixed grain size, Goldsby rheology may lead to enhanced localisation resulting in larger surface velocities and more internal deformation.



But about having grain size evolution included as well ?



Grain size evolution - GSE

Including grain-size evolution (GSE) in complex rheological models (Goldsby rheology)

$$\frac{d}{dt}d = d_{\text{gg}} + d_{\text{red}}$$

Grain growth (Arrhenius relation)

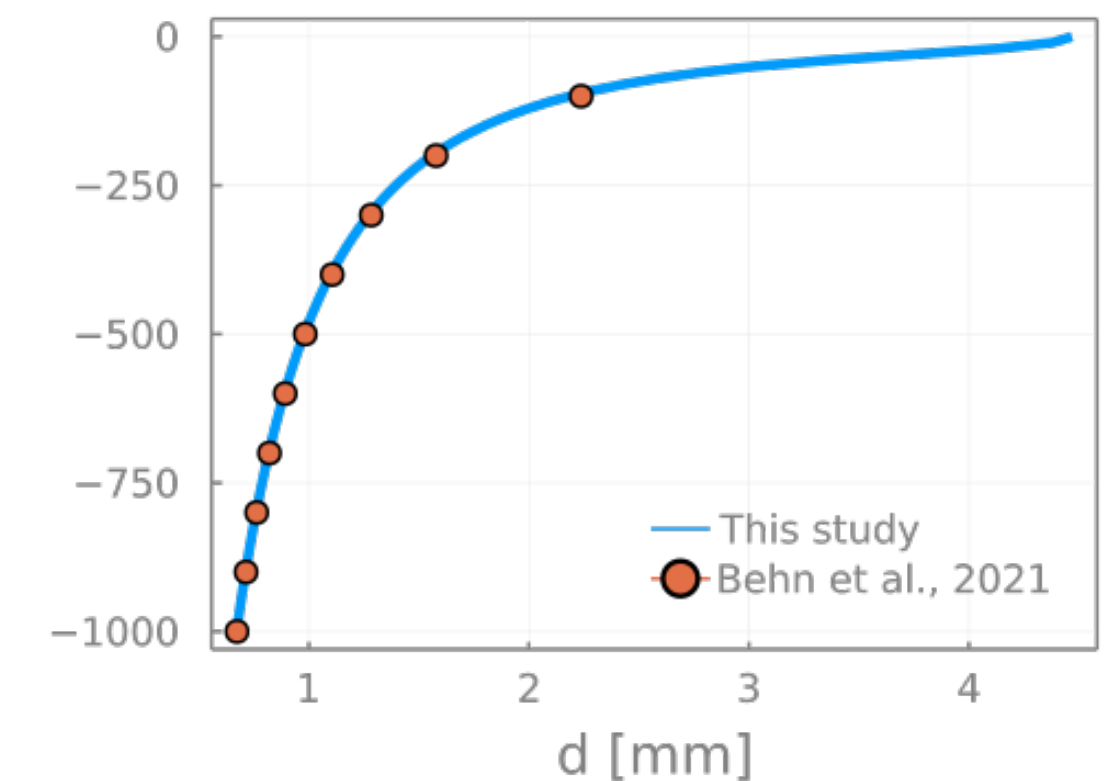
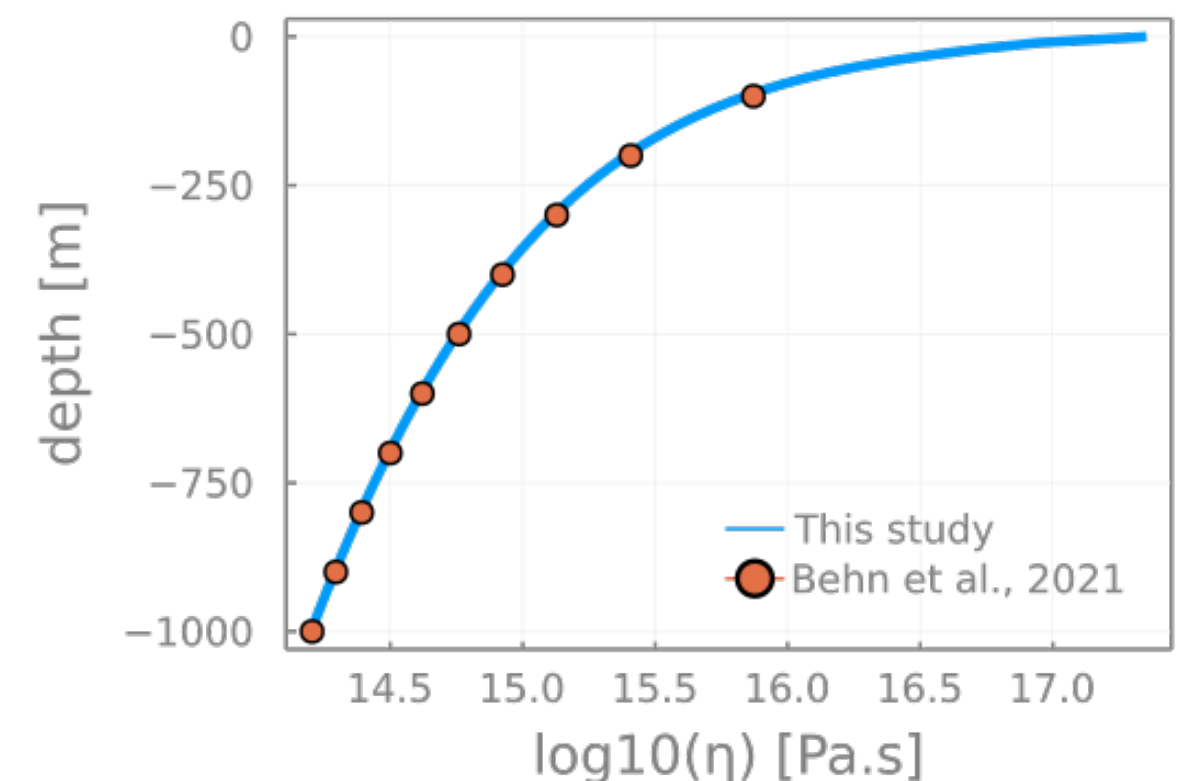
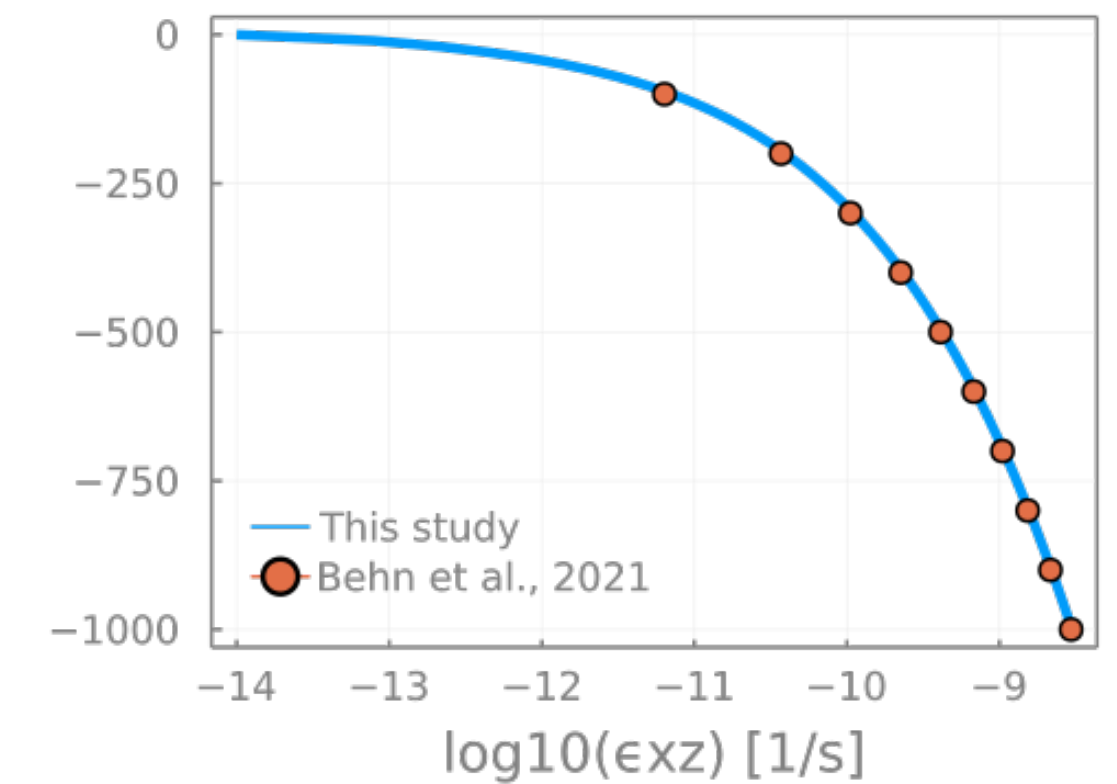
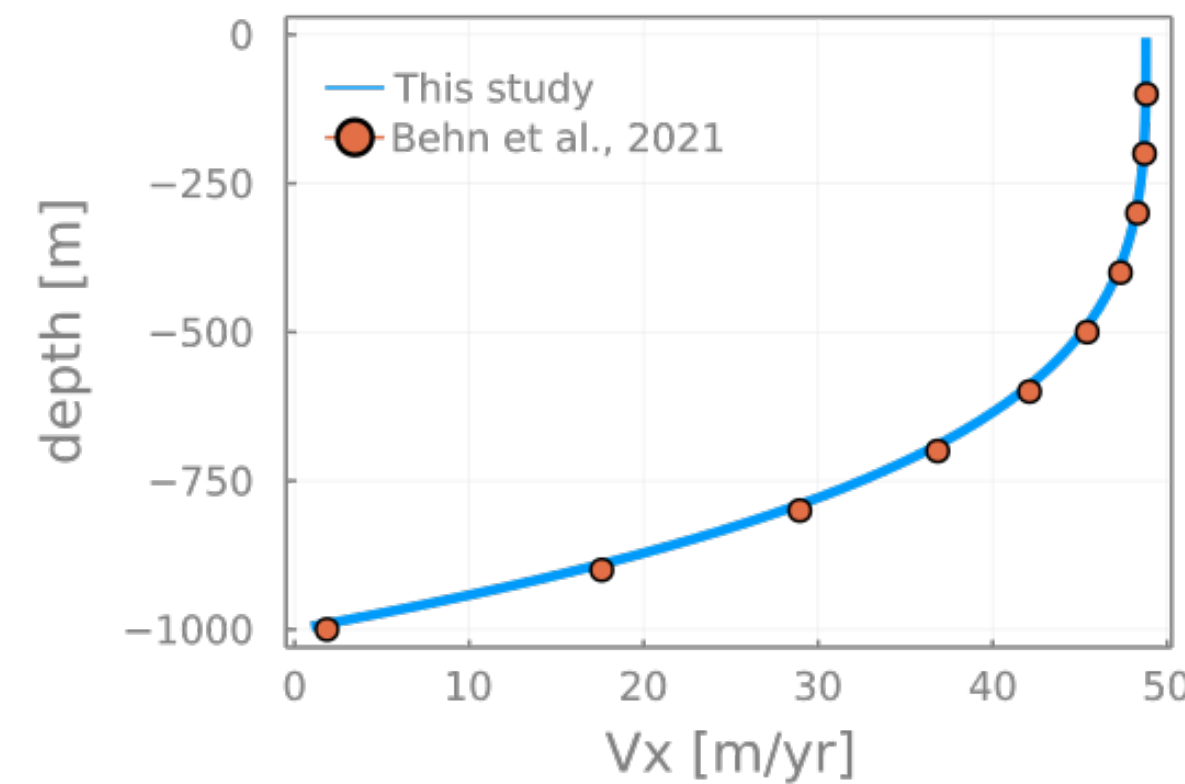
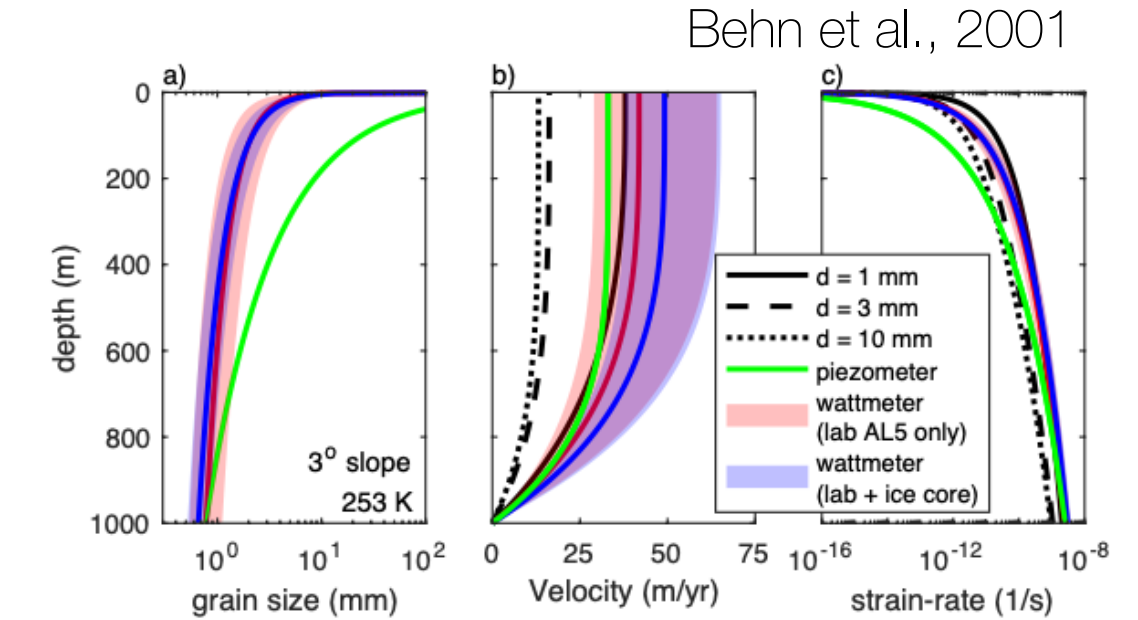
$$d_{\text{gg}} = p^{-1} d^{1-p} K_{\text{gg}} \exp\left(-\frac{Q_{\text{gg}}}{RT}\right)$$

Grain size reduction (wattmeter)

$$d_{\text{gg}} = -\frac{d^2}{c\gamma} \left(\lambda_{\text{disl}} Q_{\text{disl}} + \lambda_{\text{gbs}} Q_{\text{gbs}} \right)$$

Rate of mechanical work per unit volume

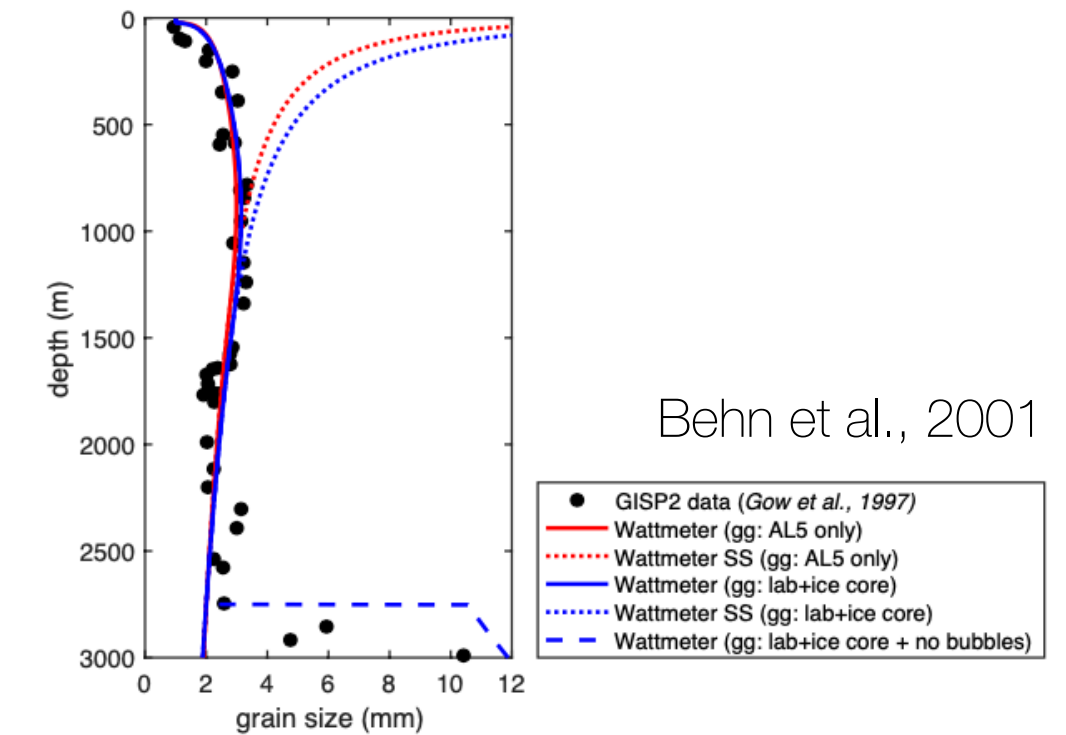
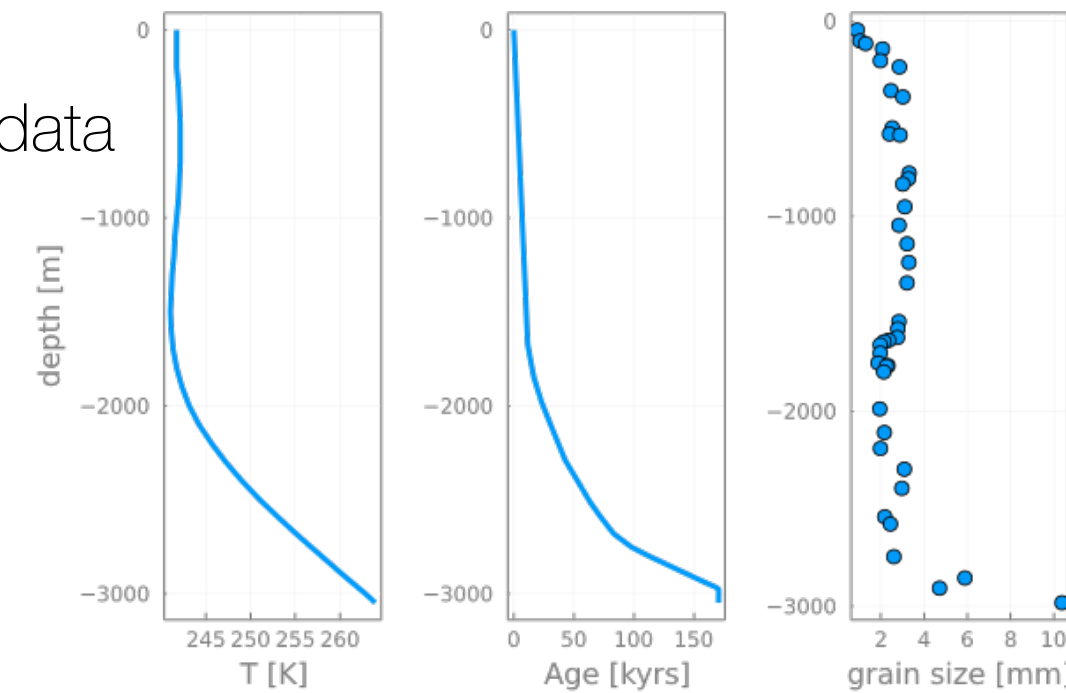
$$Q = \dot{W}_{ij} \dot{W}_{ij} \quad \dot{W}_{ij} = \tau_{ij} \dot{\epsilon}_{ij}$$



Grain size evolution - GSE

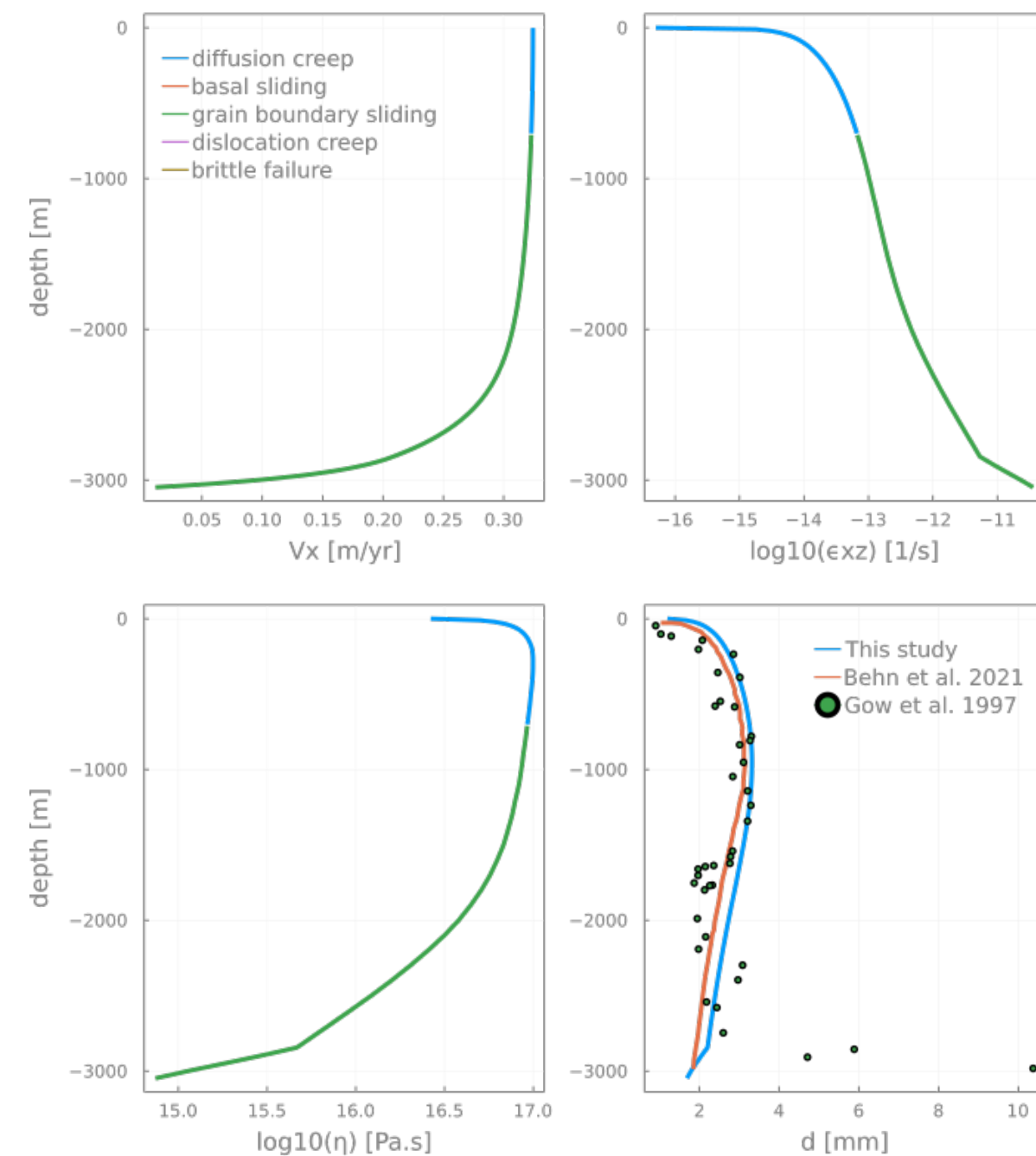
Including grain-size evolution (GSE) and full
Goldsby rheology to fit the GISP2 ice core data

GISP2 data

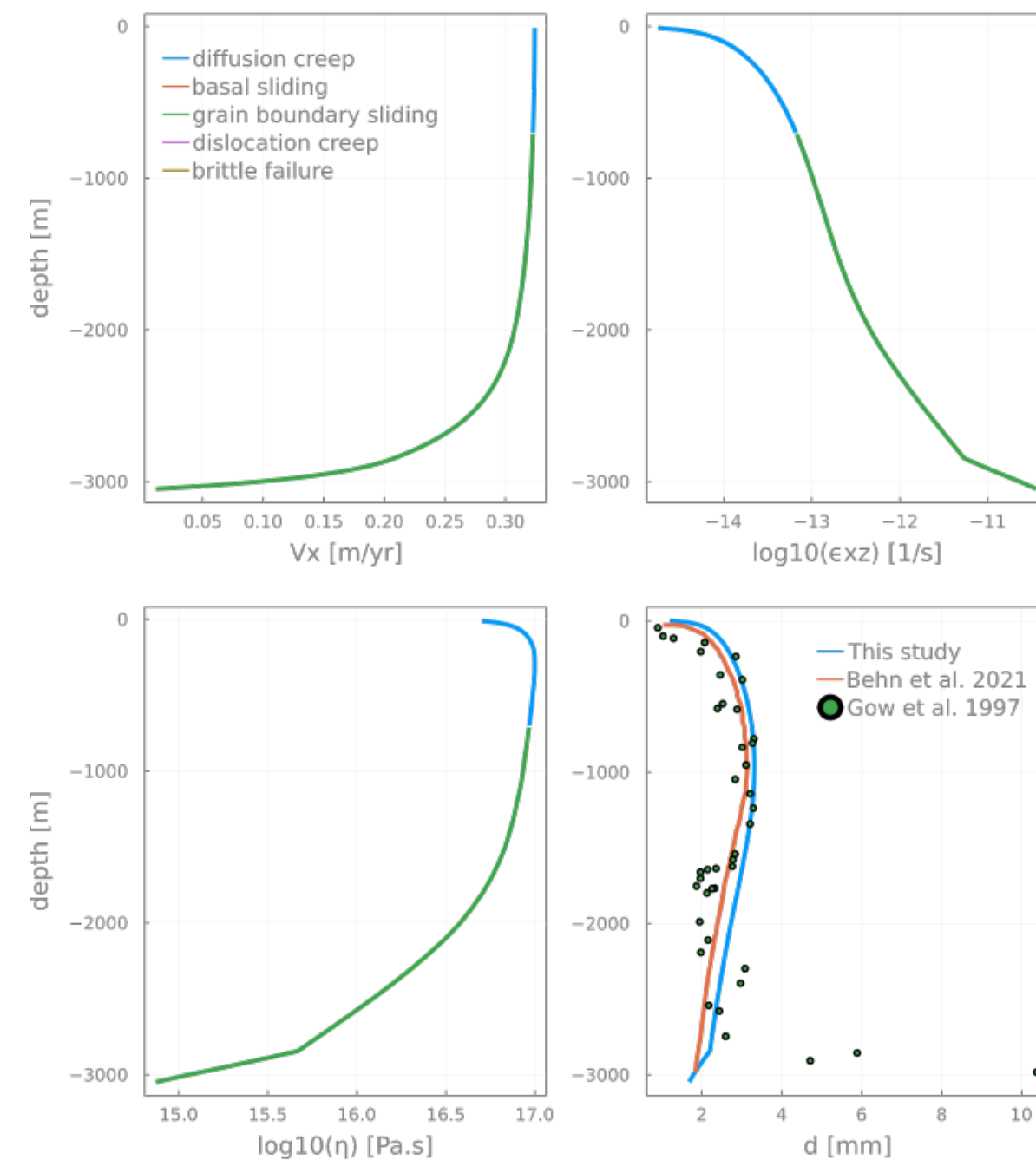


Behn et al., 2001

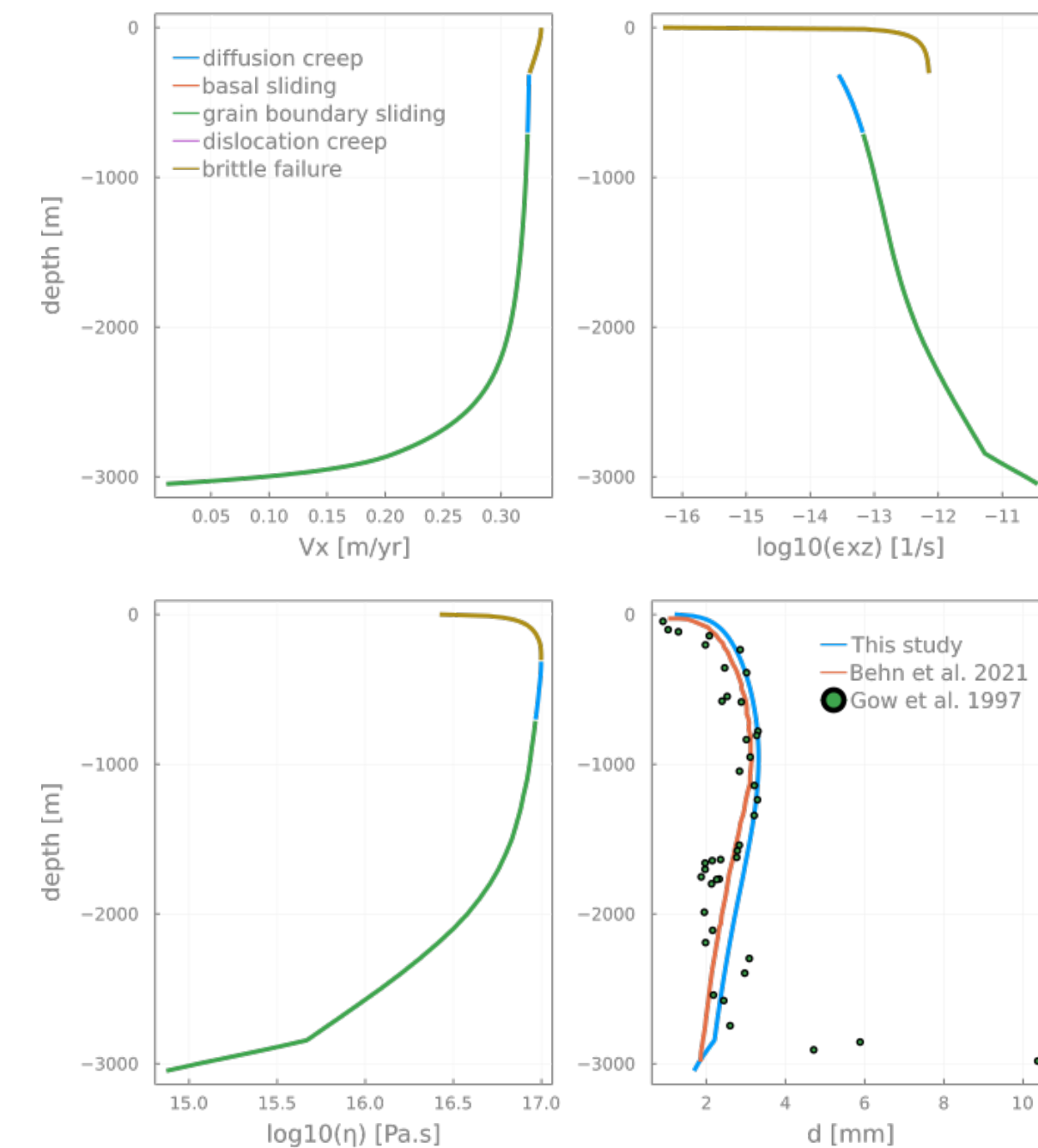
Goldsby viscous



Goldsby visco-elastic



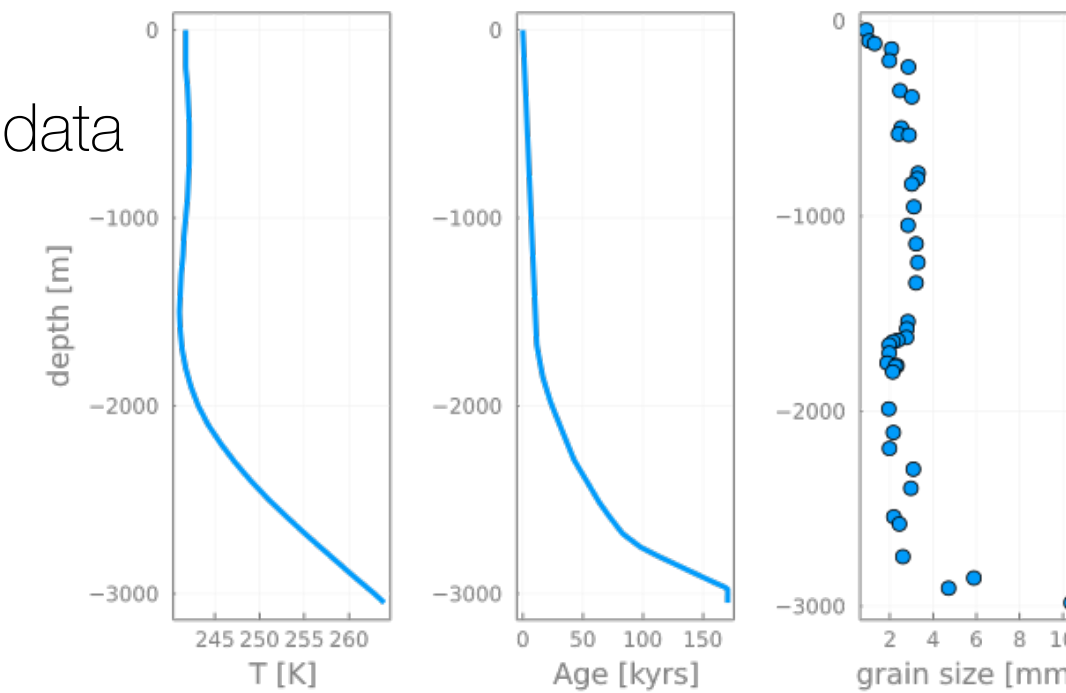
Goldsby visco-elasto-plastic



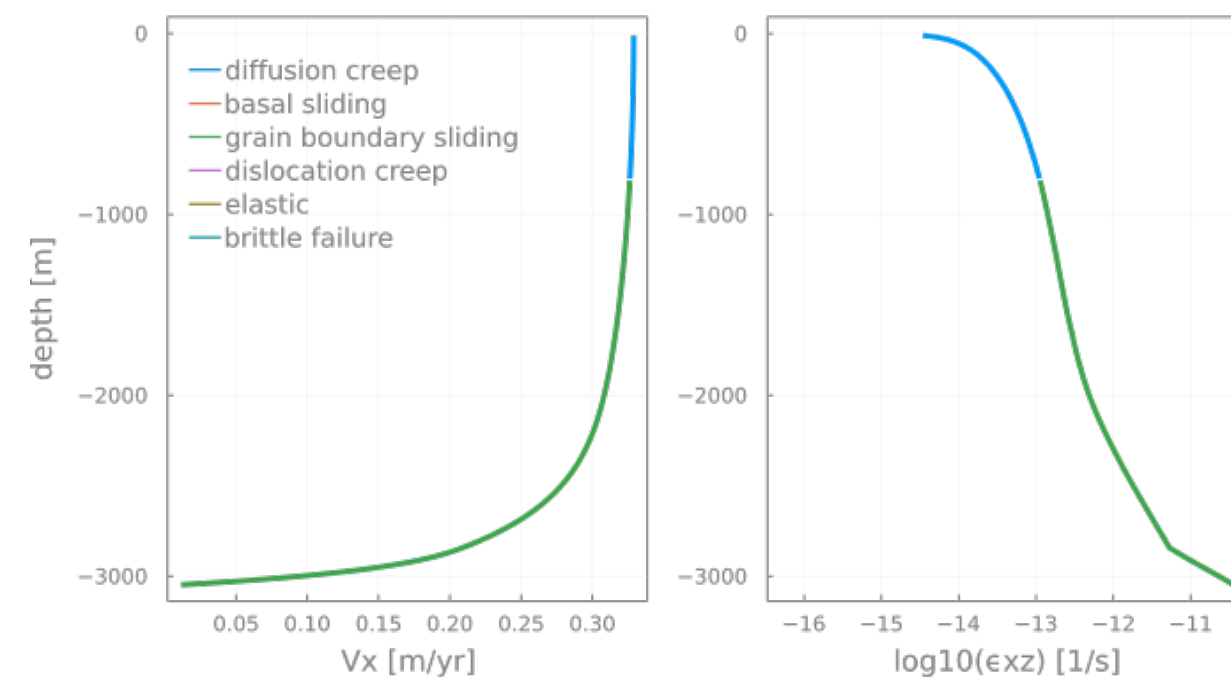
Grain size evolution - GSE

Effect of visco-elasticity and temporal accuracy on grain size distribution (1, 10 and 50 steps).

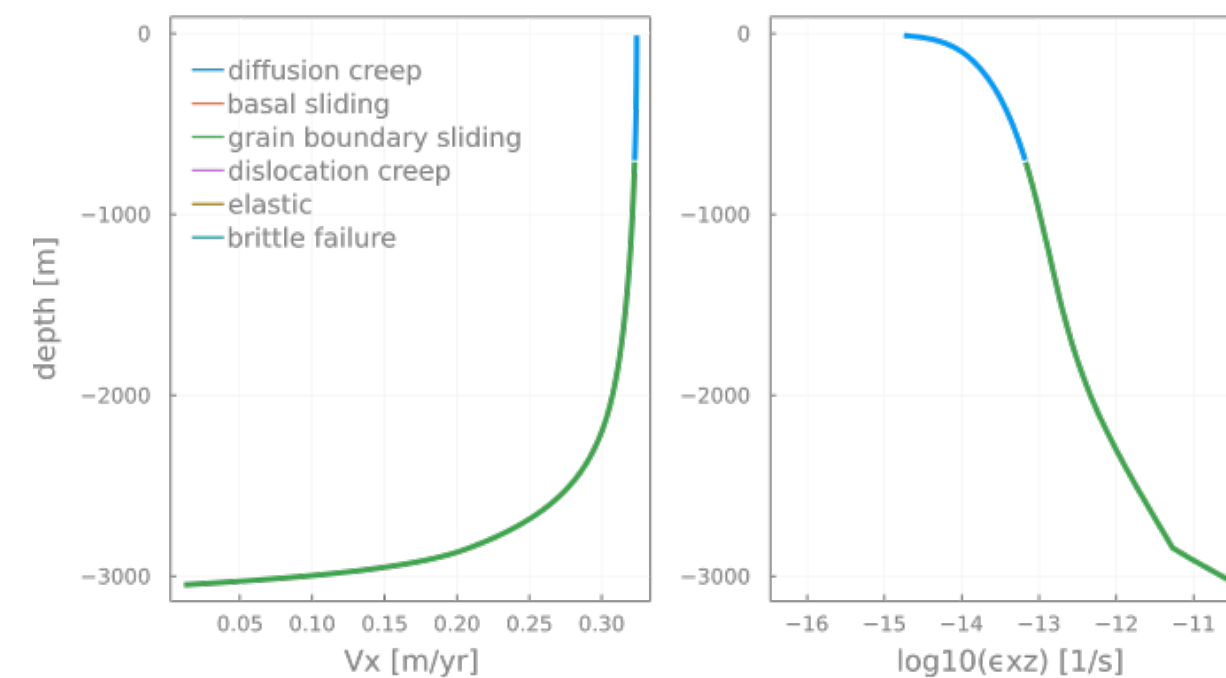
GISP2 data



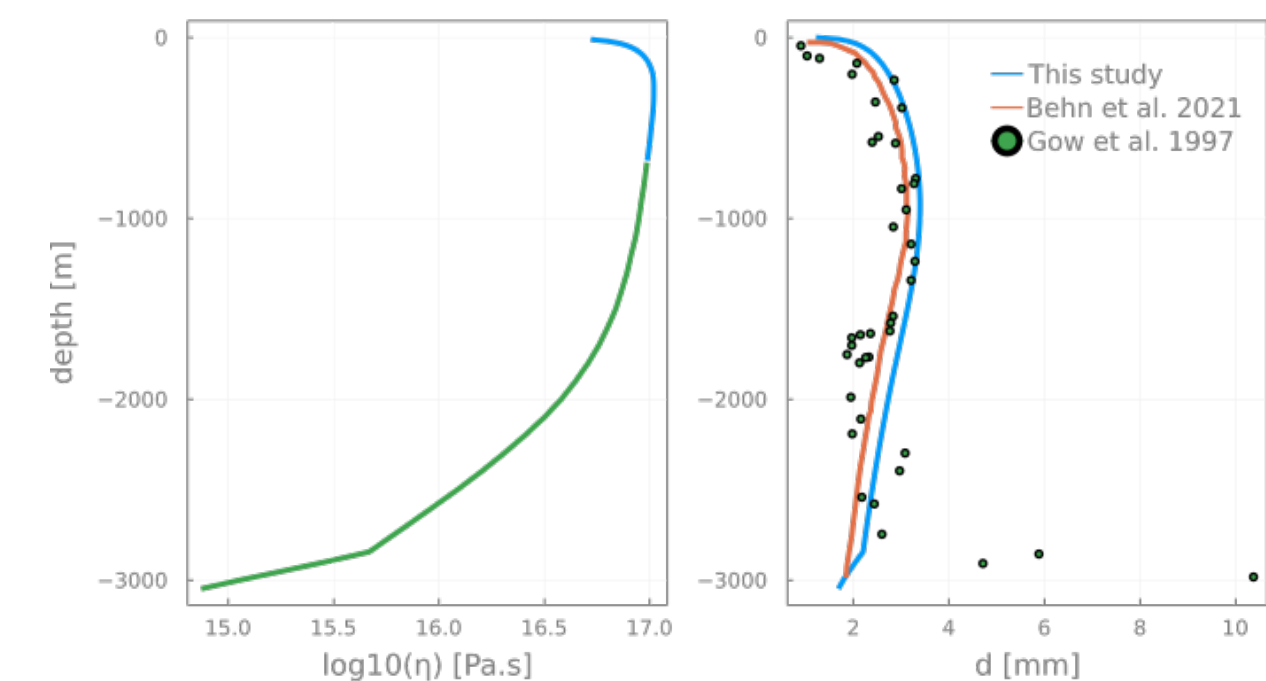
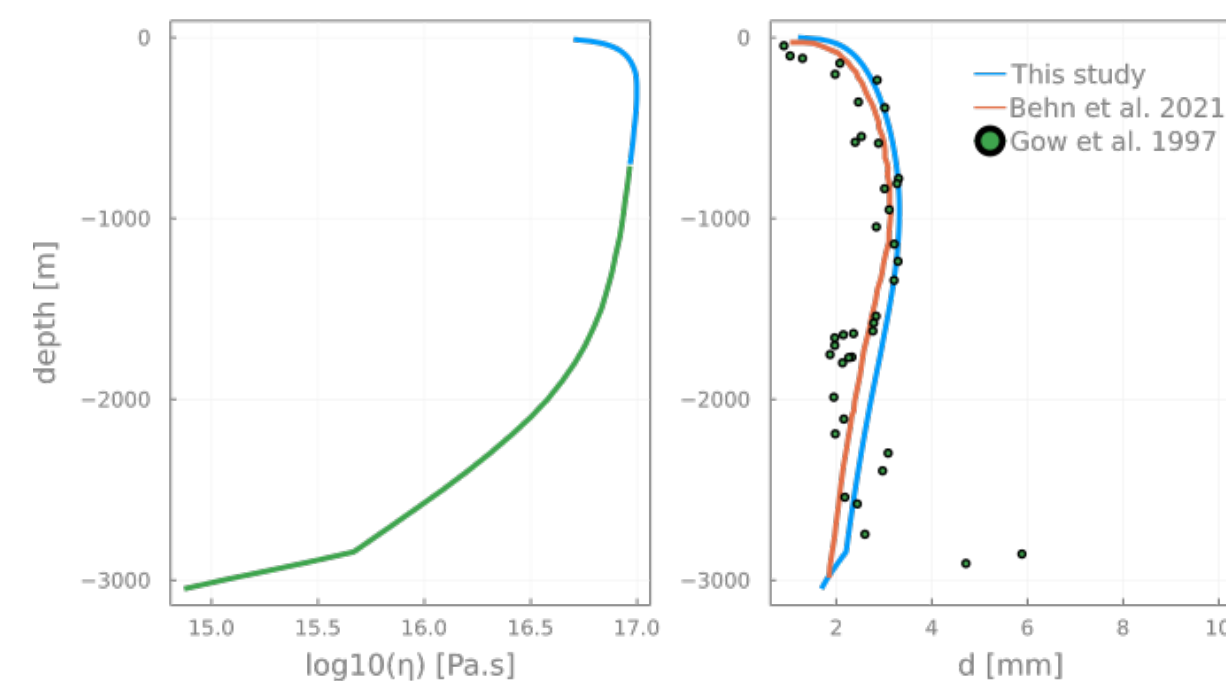
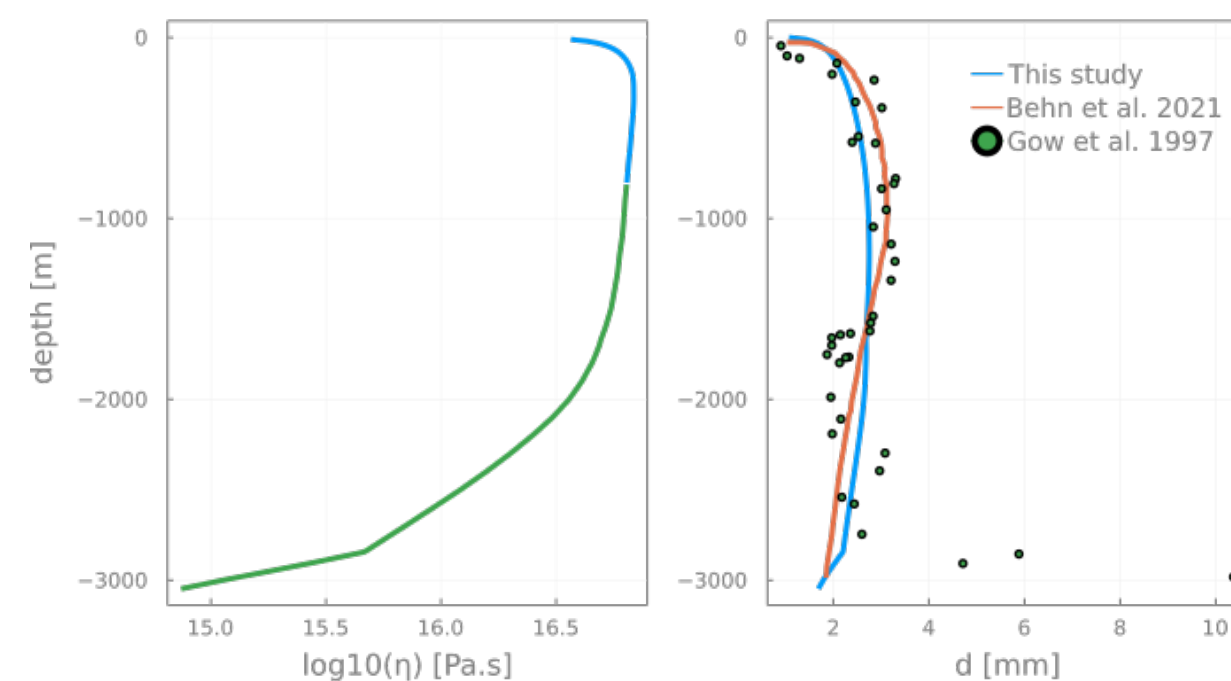
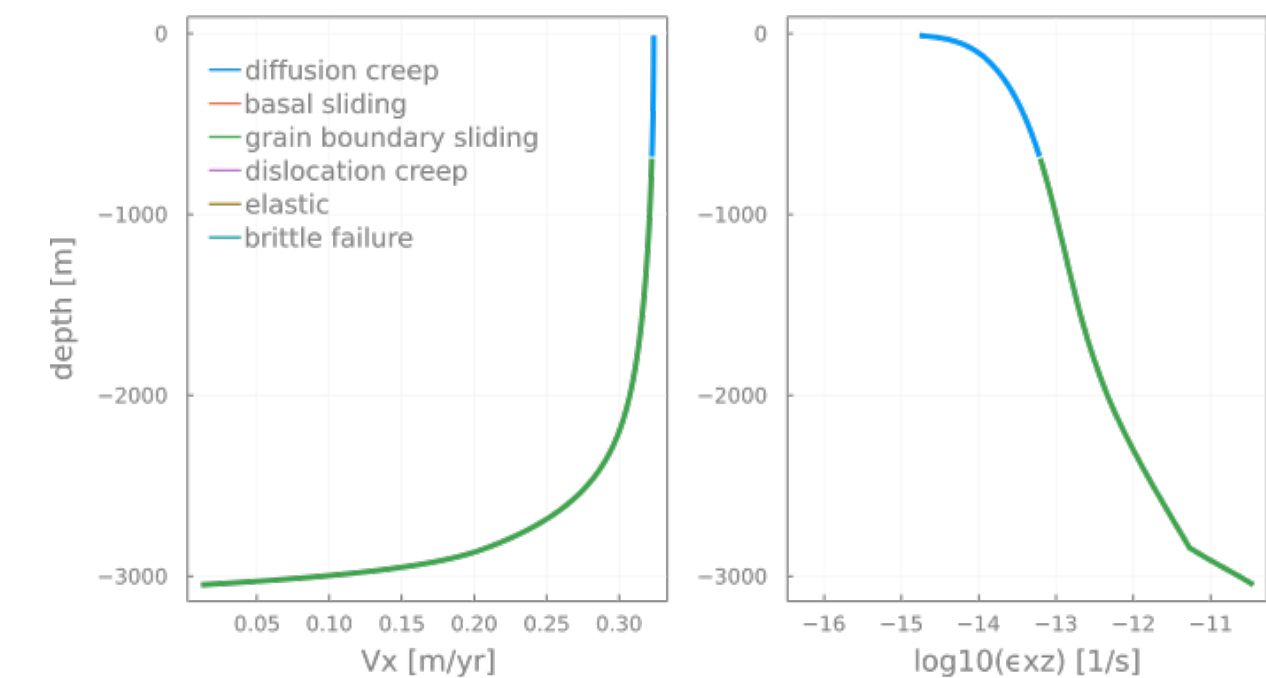
Steady-state in 1 step



Steady-state in 10 steps

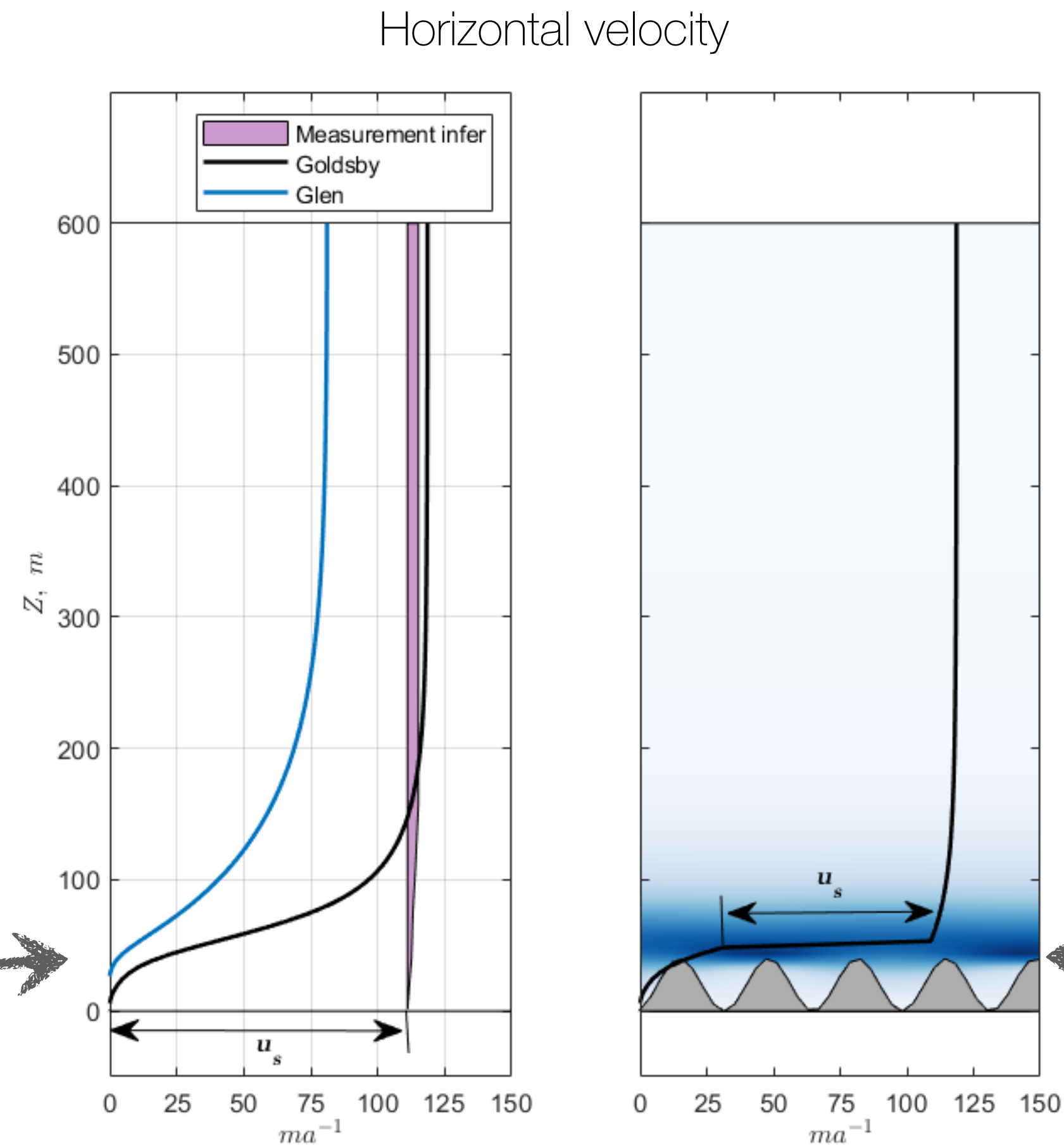
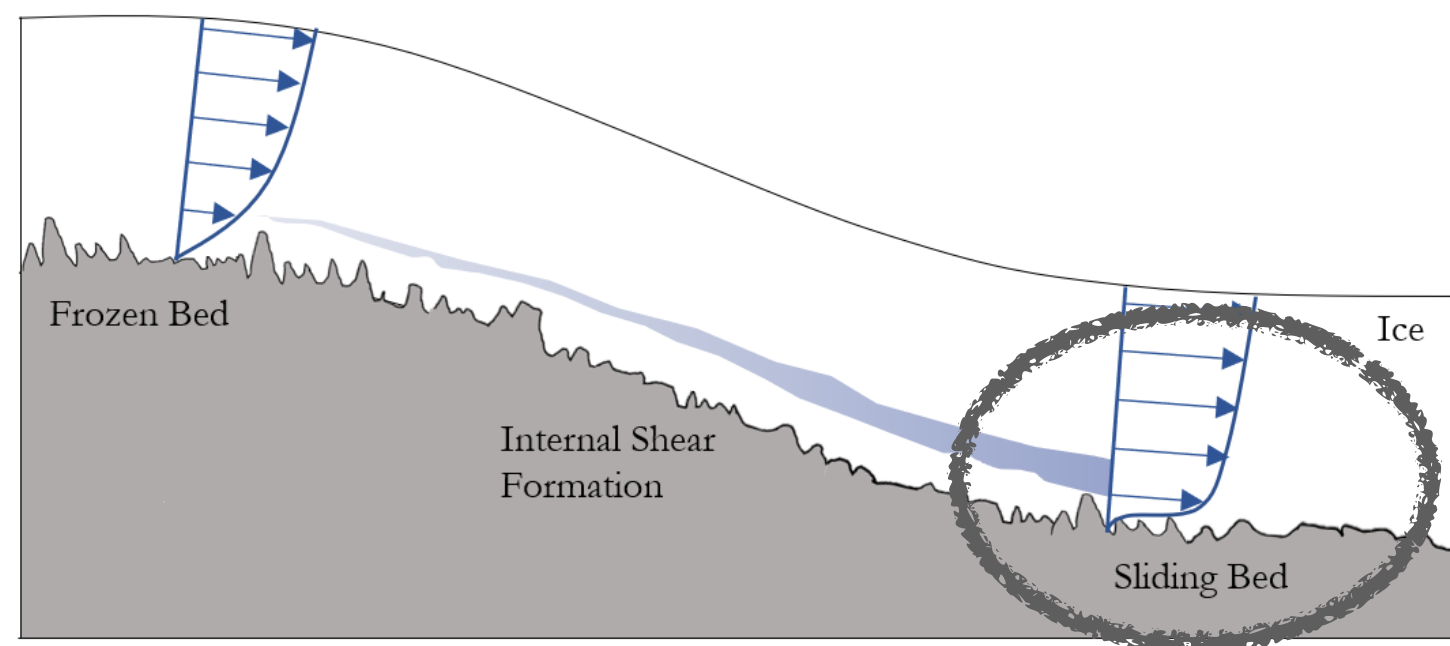


Steady-state in 50 steps

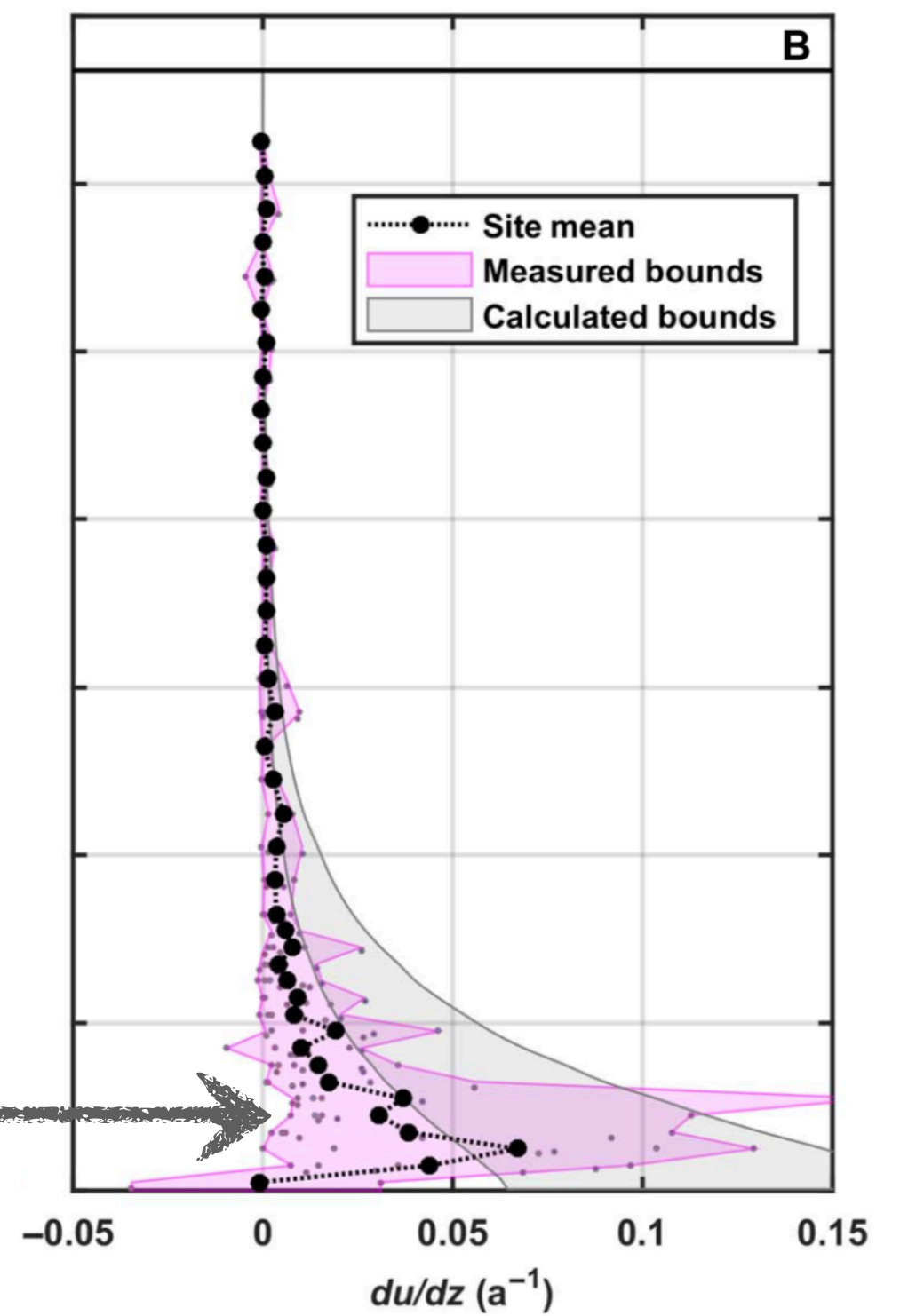


Next steps

- From 1D to 2D and 3D calculations in a full thermo-mechanical (Stokes) model
- Understanding ice-flow and sliding mechanisms
- Basal sliding vs dynamical “shear-interface”
- Application to ice streams and alpine glaciers



Shear deformation



Maier et al. 2019

Take home message

- Non-linear and complex rheology may play a key role in spontaneous localisation of deformation and ice flow.
- Generalisation and implementation of laboratory derived flow laws is challenging (invariant formulation, correction factors, tensors).
- Our general invariant-based formulation including all 4 regimes from Goldsby's flow law reproduces recent results.
- Our wattmeter-based grain size evolution reproduces recent results and fits GISP2 ice core data for variable total time.
- Transient part seems to play a role when considering visco-elastic rheology and GSE.