Emergence of transverse size in electric streamers

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Presentation plan

Parametric model of streamers

Validation of the parametric model using steady-state streamers

Modification of the parametric model for interacting streamers

Results and discussion

Outline

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Parametric model of streamers

Goal

To describe a **streamer** with a finite but sufficient set of parameters, and to find a way to efficently calculate them

Approach

- we search for a solution in the shape of a column-like ionization front
- we simplify the hydrodynamic equations and obtain a finite system of algebraic equations for the finite set of streamer parameters
- solve it!

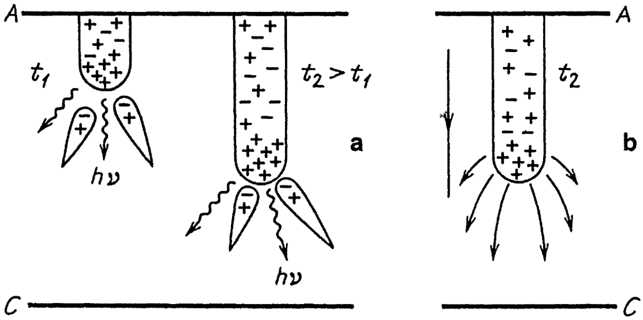
For details, see Lehtinen [2020] (link).

External parameters and assumed streamer shape

External parameters (determined by observation conditions):

- Uniform electric field E_e
- Streamer length L

Streamer is modeled as an ionized column in the shape of a cylinder with a hemispherical cap:



Parameters and relations between them

Streamer parameters

- Radius a
- Velocity V
- Channel electron density n_s
- Maximum field at the tip E_m
- Field inside the channel E_s

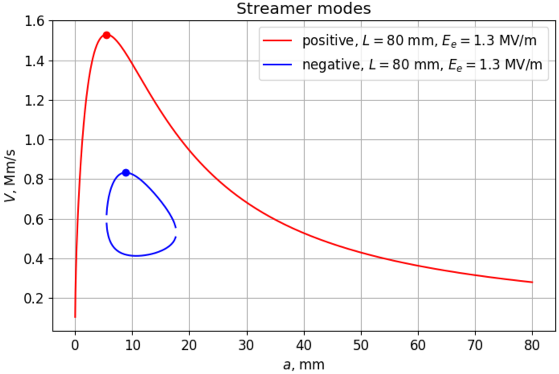
Relations between streamer parameters

1. Electrostatic
2. Current continuity
3. Ionization/relaxation balance
4. Photo- and impact ionization balance

Streamer parameters **cannot** be determined uniquely from these relations!

The streamer "modes" and the preferred mode

However, we can determine the streamer "modes": all parameters are functions of radius a. This is analogous to the linear modes of small harmonic perturbations of a flat ionization front.



$V(a)$ has a maximum at a certain "preferred" radius a. We propose that this mode corresponds to the physical streamer parameters.

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The "steady-state streamer" concept

Goal

To validate the parametric model, at least against another type of calculations.

We want to demonstrate that only one streamer "mode" is chosen with an optimal radius a corresponding to the maximum of $V(a)$, by the means of straightforwardly solving the hydrodynamic (HD) equations using a finite-difference time-domain computer simulation.

Problem: the streamer is an inherently unstable phenomenon with L growing all the time.

We solve it by modeling a "steady-state" streamer: we synchronize the electrode with the streamer front by moving the electrode so that L (the distance between the streamer tip and the electrode) stays constant.

This synchronization may be tricky because the streamer accelerates as L grows.

Numerical solution for a steady-state streamer

A streamer with the electrode motion synchronized to the motion of its tip evolves to a steady state **independently** of initial conditions.

Results for $E_e = 2$ MV/m, $L = 5$ cm. The side boundary is at infinity, bottom boundary (at $z = 0$) is the positive electrode, top boundary is at infinity, electric field is vertically up, units are mm.

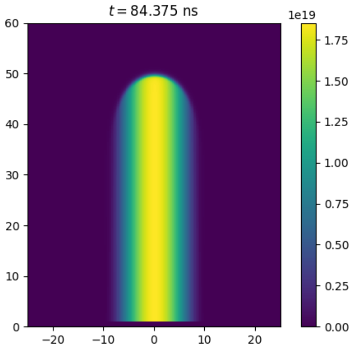

Figure: Electron density n_e

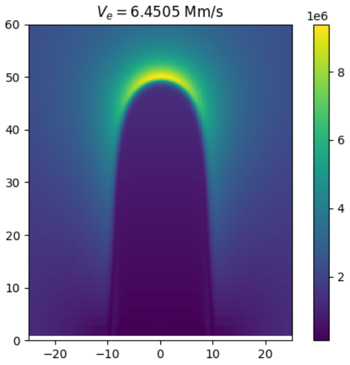

Figure: Electic field E

Numerical solution for a steady-state streamer: Velocity V

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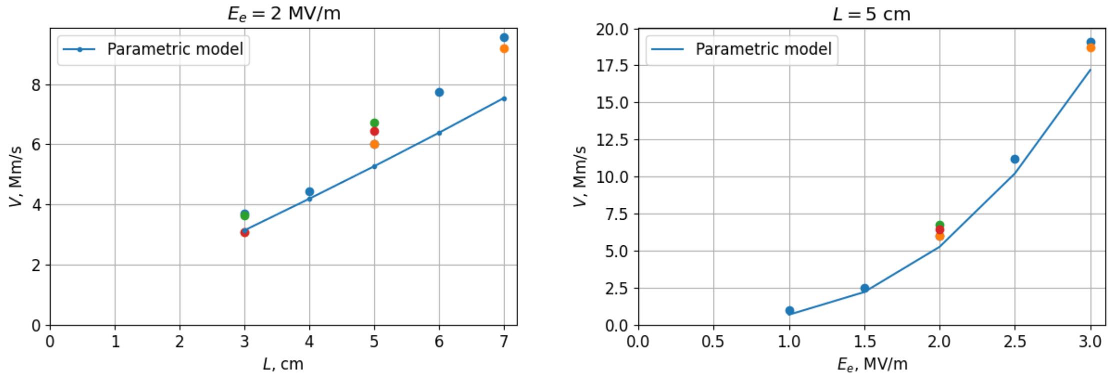
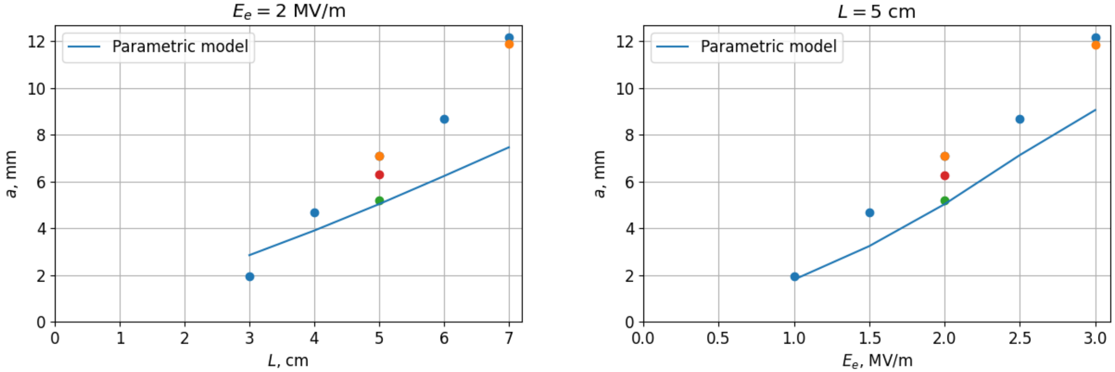
The system always achieves the same stable state independently of the initial conditions.

The parameters obtained by hydrodynamic (HD) simulations do in fact approximately agree with the "preferred mode" from parametric model calculations (HD simulations are dots)



Numerical solution for a steady-state streamer: Radius a

The radius is a bit tricky because the radius of the curvature of the ionization front (plotted) turned out to be different that the radius of the channel.



Some caveats

- The parametric model of Lehtinen [2020] assumed $E_s = const$ in the channel. However, the HD simulations produce E_s decreasing from the tip of the streamer towards electrode. We took this into account by modifying the parametric model for these calculations: the voltage drop along the channel is $U_s = 0.65 E_s L$ instead of $E_s L$.
- Multiple different results in HD model for same parameters are due to inaccuracies in the HD model. E.g., smaller grid step sizes lead to higher streamer velocities. More accurate results are coming soon!

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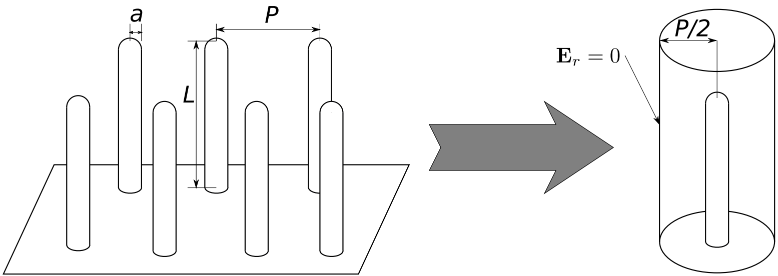
What do we want to model

We want to study the effect of streamers propagating parallel to each other at transverse distance P from each other.

We can approximately model the effect of streamer interaction by having only one streamer, but have it propagate in a box with side walls with Neumann boundary conditions at transverse distance $P/2$.



Approach

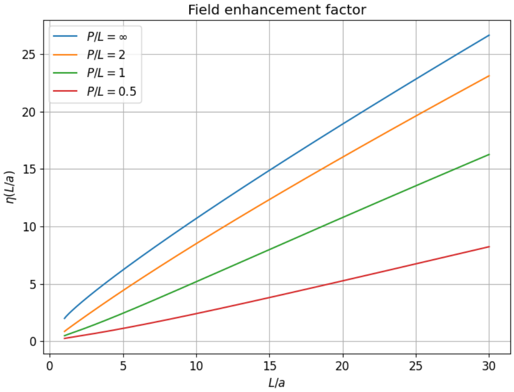
The electrostatic relation (1) is modified. The field at the tip of a static conducting column in external field E0 is enhanced by the field of the surface charges $\eta(L/a)E_0$. Previously, we used

$$\eta = 2 + 0.56(2L/a)^{0.92}$$

given by Raizer [1991, p. 356].
However, the presence of the walls (or, equivalently, neighbor streamers) makes factor η, now dependent also on P, smaller
We also have corrected the Raizer formula for $P = \infty$ to

$$\eta(L/a) = 2 + 1.23(L/a - 1)^{0.89}$$

This was calculated by modeling a cylinder shape using finite-difference calculations:



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Results for positive streamers in a bunch

The results show that when the streamers are close to each other, their velocity and radius become smaller.

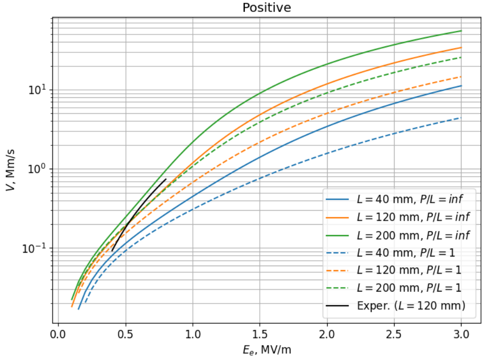

Figure: Velocity as a function of length E_e for finite P (transverse distance between streamers). Experimental data are from Allen and Mikropoulos [1999].

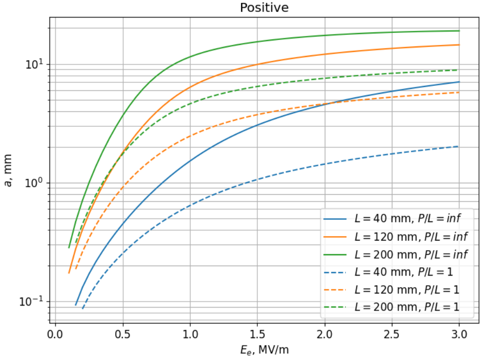

Figure: Radius as a function of E_e for finite P.

Stabilization of radius and velocity with length

The propagation stabilizes, i.e., the velocity and radius tend to stationary values, if the inter-streamer distance P is kept constant:

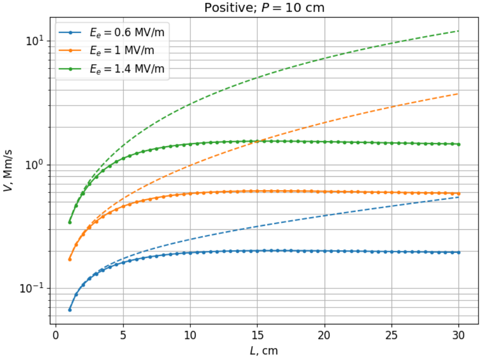

Figure: Velocity as a function of length (dashed is $P = \infty$).

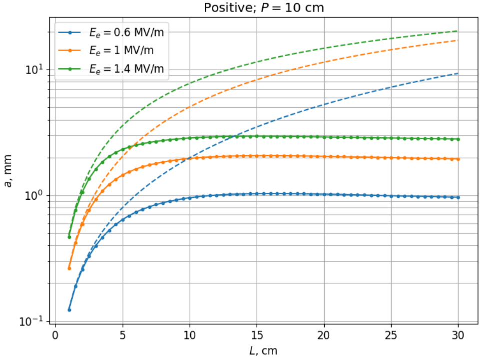

Figure: Radius as a function of length (dashed is $P = \infty$).

Discussion: The reason for propagation stabilization

- Propagation is determined by the high field at the tip $E_m \sim \Delta U/a$, where ΔU is the available potential drop at the tip.
- Single streamer: $\Delta U = U_e - U_s = LE_e - LE_s$, where $U_e = LE_e$ is the external field potential drop along the channel, and $U_s = LE_s$ is the potential drop inside the channel.
- Streamers in a bunch at transverse distance P: the field between streamers is $\sim E_e$ only in the interval of the length the order of P behind the tips, and deeper than that it is $\sim E_s$. Thus, $U_e = PE_e + (L - P)E_s$, while $U_s = LE_s$ is unchanged. Then $\Delta U = U_e - U_s = PE_e - PE_s$
- Streamers in a bunch: ΔU is independent of L, hence the "stabilization" with $P < L$ now being the "effective" length.

The situation, when all streamers in a bunch travel with the same speed, is unstable. If one of the streamers gets ahead of others, it will accelerate and the gap between it and the rest will grow further. All the streamers in a bunch end up having different speeds, so the question is not as easy as it first seems.

References

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