

Comparative Analysis of the Various Generalized Ohm's Law Terms in Magnetosheath Turbulence as Observed by Magnetospheric Multiscale

Julia E. Stawarz,¹ L. Matteini,¹ T. N. Parashar,² L. Franci,³ J. P. Eastwood,¹ C. A. Gonzalez,⁴ I. L. Gingell,⁵ J. L. Burch,⁶ R. E. Ergun,⁷ N. Ahmadi,⁷ B. L. Giles,⁸ D. J. Gershman,⁸ O. Le Contel,⁹ P.-A. Lindqvist,¹⁰ C. T. Russell,¹¹ R. J. Strangeway,¹¹ and R. B. Torbert¹²



¹Imperial College London, ²Victoria University of Wellington, ³Queen Mary University of London, ⁴University of Texas at Austin, ⁵University of Southampton, ⁶Southwest Research Institute, ⁷University of Colorado Boulder, ⁸NASA Goddard Space Flight Center, ⁹Laboratoire de Physique des Plasmas, ¹⁰KTH Royal Institute of Technology, ¹¹University of California Los Angeles, ¹²University of New Hampshire

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High-resolution, multi-spacecraft measurements from Magnetospheric Multiscale (MMS) in Earth's magnetosheath provide a unique opportunity to explore the role of generalized Ohm's law in shaping the electric field spectrum of turbulent plasmas

- Nearly all of the terms in generalized Ohm's law are measurable with MMS well into the sub-ion scales
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- E_{MHD} provides the main contribution at large scales and a combination of E_{Hall} and E_{Pe} dominate at scales smaller than the ion inertial length
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- E_{Pe} is larger than expected from Alfvén waves and partially anti-aligns with E_{Hall} at sub-ion scales
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- Ratio of nonlinear to linear terms in E_{MHD} and E_{Hall} are dictated by $\delta b_{rms}/B_0$ and nonlinear contribution to E_{MHD} becomes increasingly important at sub-ion scales due to changes in alignment of $\delta \mathbf{u}$ and $\delta \mathbf{b}$
[jump to slide](#)

Examining Turbulent Electric Fields

Fig. 1

$$\begin{aligned}
 \mathbf{E}_{Ohm} = & \underbrace{-\mathbf{u} \times \mathbf{B}}_{\mathbf{E}_{MHD}} + \underbrace{\frac{1}{en} \mathbf{J} \times \mathbf{B}}_{\mathbf{E}_{Hall}} - \underbrace{\frac{1}{en} \nabla \cdot \mathbf{P}_e}_{\mathbf{E}_{Pe}} + \underbrace{\frac{m_e}{e^2 n} \nabla \cdot \left(\mathbf{u} \mathbf{J} + \mathbf{J} \mathbf{u} - \frac{\mathbf{J} \mathbf{J}}{en} \right) + \frac{m_e}{e^2 n} \frac{\partial \mathbf{J}}{\partial t}}_{\mathbf{E}_{Inertia}} + \underbrace{\sum_{\ell=1}^{\infty} \left(-\frac{m_e}{m_i} \right)^{\ell} \mathcal{M}_{\ell}}_{\mathbf{E}_{\delta m_e}}
 \end{aligned}$$

Magnetic Field "Frozen-In" to ion fluid
 Magnetic Field "Frozen-In" to electron fluid
 Correction due to electron thermal motions
 Correction due to difference between ion and electron inertia
 Finite electron mass corrections

$$\mathcal{M}_{\ell} = \frac{2}{en} \mathbf{J} \times \mathbf{B} - \frac{1}{en} \nabla \cdot (\mathbf{P}_e + \mathbf{P}_i) + \frac{m_e}{e^2 n} \left[\nabla \cdot \left(\mathbf{u} \mathbf{J} + \mathbf{J} \mathbf{u} - (1 + 2\ell) \frac{\mathbf{J} \mathbf{J}}{en} \right) + \frac{\partial \mathbf{J}}{\partial t} \right]$$

Electric fields are required to exchange energy between the magnetic field and particles in a plasma

In collisionless plasmas, $\mathbf{E}$ is governed by a generalised Ohm's law in which the different terms correspond to different dynamical effects (Fig. 1)

These terms can provide insight into both nonlinear and dissipative dynamics of a turbulent system

Almost all of the terms in Ohm's law can be directly measured with MMS observations from Earth's magnetosheath

Measuring Ohm's Law with MMS

With MMS, the terms in Ohm's law can be measured using a combination of single and multi-spacecraft techniques

- $\mathbf{E}_{MHD}$ and $\mathbf{E}_{Hall}$ with single-spacecraft measurements
- $\mathbf{E}_{Pe}$, $\mathbf{E}_{inertia}$, and $\mathbf{E}_{\delta m_e}$ with multi-spacecraft gradients

Single-spacecraft measurements are averaged to barycenter of the MMS formation in order to combine with multi-spacecraft measurements

- Several averaging methods are attempted and $\mathbf{E}_{Hall}$ is also computed using curlometer technique, all giving similar results

Explicit time derivative term in $\mathbf{E}_{inertia}$ is omitted since timeseries appears consistent with advected spatial structures

$\mathbf{E}$ is examined in the frame of the background plasma flow

$$\mathbf{E}_{plasma} = \mathbf{E}_{sc} + \mathbf{U}_0 \times \mathbf{B}$$

Results from one magnetosheath interval are presented here (Fig. 2), but in the [paper](#) several additional intervals are examined

Fig. 2

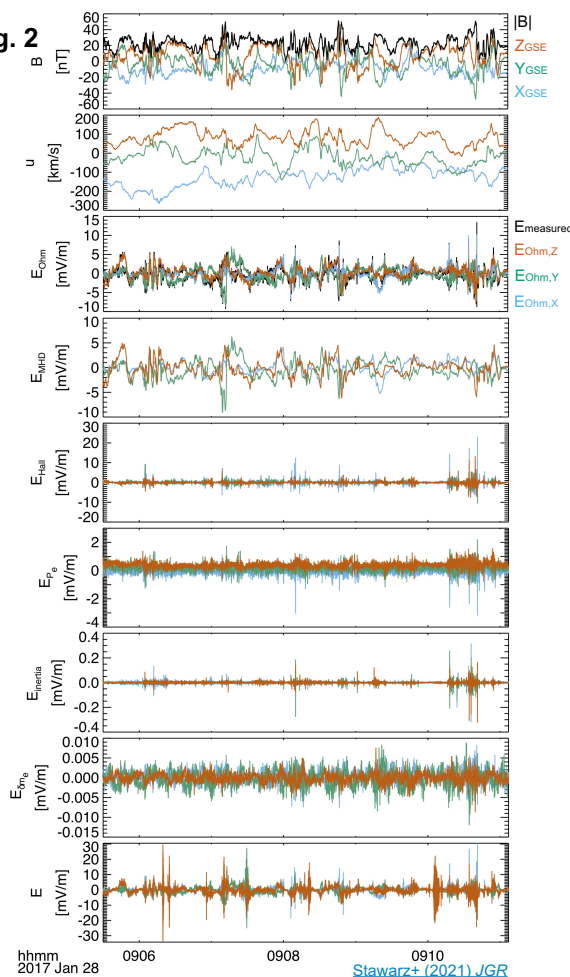
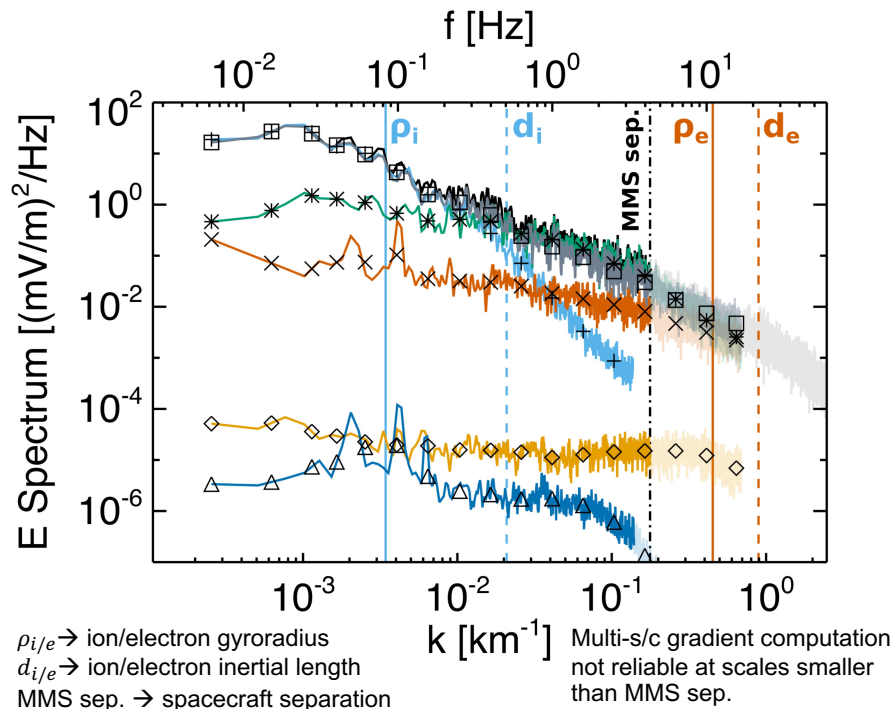


Fig. 3 — E — E_{Ohm} — E_{MHD} — E_{Hall} — E_{Pe} — E_{Inertia} — $E_{\delta m_e}$



Time series of each Ohm's law term computed and then power spectra are compared to spectrum of directly measured E (Fig. 3)

Frequencies converted to wavenumbers using average flow velocity $\rightarrow k = 2\pi f / U_0$

Results

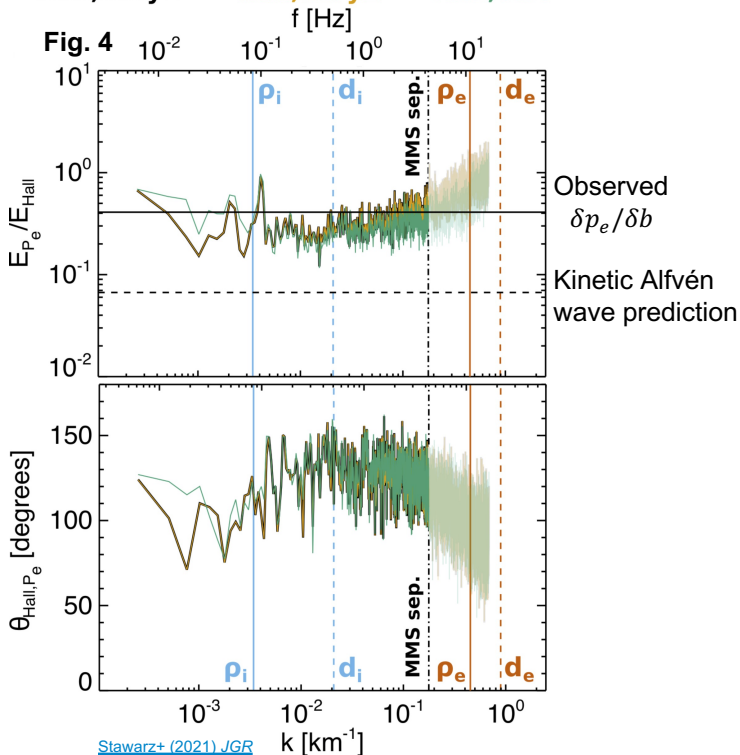
E_{MHD} provides the dominant contribution at large-scales as expected

At sub-proton scales, both E_{Hall} and E_{Pe} contribute to the electric field

E_{inertia} and $E_{\delta m_e}$ are negligible across observable scales

Interaction between Hall and Electron Pressure Terms

$E_{Hall, bary1}$ $E_{Hall, bary2}$ $E_{Hall, curl}$



Comparing the the relative amplitudes and alignments of E_{Hall} and E_{P_e} provides insight into the underlying dynamics

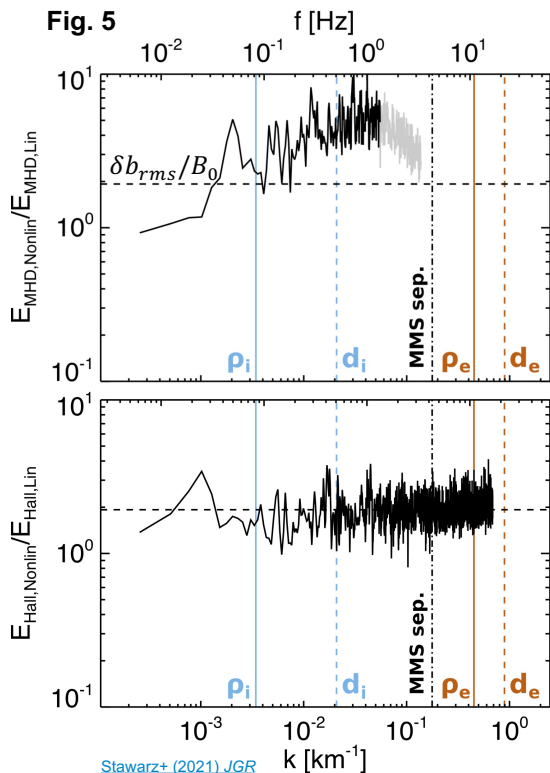
E_{Hall} dominates over E_{P_e} , but E_{P_e} is stronger than expected from linear kinetic Alfvén waves (Fig. 4 top) [Boldyrev+ (2013) ApJ]

→ Excess E_{P_e} could be associated with reconnection events [Phan+ (2018) Nature; Stawarz+ (2019) ApJ], compressive fluctuations, and nonlinear effects

Partial anti-alignment between E_{Hall} and E_{P_e} fluctuations at sub-ion scales observed (Fig. 4 bottom)

→ Perfect anti-alignment expected if currents entirely associated with electron diamagnetic drift

→ Partial anti-alignment can be caused by additional influence of ion dynamics and $E_{||}$ at sub-ion scales



$$E_{MHD} = \underbrace{-\delta \mathbf{u} \times \mathbf{B}_0}_{\text{(Linear Term)}} - \underbrace{\delta \mathbf{u} \times \delta \mathbf{b}}_{\text{(Nonlinear Term)}} \quad E_{Hall} = \underbrace{\delta \mathbf{j} \times \mathbf{B}_0 / en}_{\text{(Linear Term)}} + \underbrace{\delta \mathbf{j} \times \delta \mathbf{b} / en}_{\text{(Nonlinear Term)}}$$

In E_{MHD} (Fig. 5 top)

- Ratio of nonlinear to linear terms $\sim \delta b_{rms}/B_0$ at large scales
- Nonlinear term becomes increasingly important at sub-ion scales due to decrease in alignment between $\delta \mathbf{u}$ and $\delta \mathbf{b}$

[see also [Parashar+ \(2018\) PRL](#)]

In E_{Hall} (Fig. 5 bottom)

- Ratio of nonlinear to linear terms is $\sim \delta b_{rms}/B_0$ across all scales
- Results may suggest a balance between linear and nonlinear timescales