Modeling and analysis of LOFAR scintillation data

M. Grzesiak, Mariusz Pożoga, Barbara Matyjasiak, Dorota Przepiórka, Hanna Rothkaehl, Katarzyna Budzińska, and Barbara Atamaniuk

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Scintillation - fluctuations of wave characteristics after passing through medium with variable refractive index

Interferometry in low frequencies (10-240 MHz)

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LOFAR (Low Frequency Array)

Interferometry in low frequencies (10-240 MHz)

Consists of:

- Two antenna fields with 96 dipole antennas in each
- LBA (Low Band Antenna) – observations in 10-90 MHz range
- HBA (High Band Antenna) – observations in 110-240 MHz range
- Directional observations possible (beamforming)
Scintillation
(fluctuations of wave characteristics after passing through medium with variable refractive index)
Modeling pattern evolution – dispersion analysis

\[ \psi(r, t) = \int dr' K^t(r - r') \psi(r', 0) \]
\[ \hat{\psi}(k, t) = \hat{\psi}(k, 0) e^{\Omega(k)t} \]

\[ \mathbb{E}[\psi(r_1, t_1) \psi(r_2, t_2)] = \int dk P(k) e^{\Omega(k)\tau} e^{ik \cdot \zeta} = C(\zeta, \tau), \]
\[ \zeta = r_2 - r_1, \tau = t_2 - t_1 \]

\[ P(\zeta, \omega) = \int d\tau C(\zeta, \tau) e^{i\omega \tau} = \int d\tau e^{-i\omega \tau} \int dk P(k) e^{\Omega(k)\tau} e^{ik \cdot \zeta} = \int dk P(k) e^{ik \cdot \zeta} \delta(\omega - \Omega(k)) \]

\[ < \zeta > = \frac{\int d\zeta \zeta C(\zeta, \tau)}{\int d\zeta C(\zeta, \tau)} \]
\[ \frac{\partial}{\partial \tau} < \zeta > = \nabla_k \Omega(k)|_{k=0} \]

M. Grzesiak, A. W. Wernik, Dispersion analysis of spaced antenna measurements, Annales Geophysicae, 27, 2843-2849, 2009
Basic example – frozen flow

\[ K(r' - r, t) = \delta(r' - (r - v dt)) \]

\[ \frac{\partial}{\partial t} C - \mathbf{v} \cdot \nabla \xi C = 0 \]

\[ \xi_\alpha = r_k - r_l \]

Simulation

Example from LOFAR

\[ \tau_\alpha = 0 < \tau_\alpha \]

\[ \tau_\alpha = 0 < \tau_\alpha < \tau_\alpha \]
Correlation function features recognition

\[ C(\xi - v \tau) \]

Estimation of geometry and drift velocity

\[ C(\xi) = \rho(\xi^T Q \xi) \]

\[ \frac{\partial}{\partial \tau} C(\xi - vt) = 0 \]

\[ \nabla_{\xi} \tau_m = \frac{Qv}{v^T Qv} \]

\[ \xi^T Qv = \tau_m v^T Qv \rightarrow \tau_m = \frac{\xi^T Qv}{v^T Qv} \]

\[ v = \frac{Q^{-1} \nabla_{\xi} \tau_m}{(\nabla_{\xi} \tau_m)^T Q^{-1} \nabla_{\xi} \tau_m} \]
Estimation of geometry and drift velocity

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</table>
Space-frequency analysis

\[ f(x, t) = \exp(ikx) \quad x \rightarrow x - vt \]

\[ f(x_1, t_1) f^*(x_2, t_2) = \exp(ikd) \exp(ikv \tau), \ d = x_1 - x_2, \ \tau = t_1 - t_2 \]

\[ \omega = kv, \ \Delta \phi = kd \rightarrow \Delta \phi = \frac{d}{v} \omega \]
Frequency domain picture

amplitude PSD (Cassiopeia, set L547449)

amplitude PSD (Cassiopeia, set L547785)

amplitude PSD (Cassiopeia, set L552177)

phase histograms vs. frequency (Cassiopeia, set L547449)

phase histograms vs. frequency (Cassiopeia, set L547785)

phase histograms vs. frequency (Cassiopeia, set L552177)
Phase screen approach & Fresnel filtering

\[ \sqrt{\lambda L} \ll r_0 \]

\[
u(r_\perp, z) = u_0 \frac{ik_0}{2\pi z} \int dr'_\perp \exp \left[ -i \left( \frac{k_0 (r_\perp - r'_\perp)^2}{2z} + \delta \phi(r'_\perp) \right) \right]
\]

\[
\delta \phi(r_\perp) = -r_e \lambda \int dl \Delta N_e(r_\perp, l)
\]
Double phase screen
Problems with frozen flow model

Temporal decorrelation + frozen flow: \[ Q' = T^T QT \]

\[ Q = \begin{bmatrix} Q_s & 0 & 0 \\ 0 & 0 & \beta \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & v_x \\ 0 & 1 & v_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} I \\ 0^T \\ 1 \end{bmatrix}, \quad Q' = \begin{bmatrix} Q_s & Q_s v \\ v^T Q_s & \beta + v^T Q_s v \end{bmatrix} \]

\[ v_T = Q_s^{-1} Q_{[1:2;3]} \]
\(\gamma\) – the velocity scaling factor from fitting the temporal correlation to the spatial correlation

\[ \mathbf{v}_F = \left(1 + \beta / \mathbf{v}_T^T \mathbf{Q}_s \mathbf{v}_T\right) \mathbf{v}_T \]

\(\gamma\) – the velocity scaling factor from fitting the temporal correlation to the spatial correlation

\(v_F\) – the velocity when taken the frozen flow approximation

Estimated velocity doesn’t follow model with separable temporal decorrelation
Conclusions

LOFAR network provides consistent scintillation data of coverage both in time and space that equips us with new possibilities of Spatio-temporal analysis.

The method presented gives an estimate of drift velocity, taking into account possible anisotropy of irregularities. It turns out that the magnitude of drift velocity depends on geomagnetic activity: the larger the Kp index, the greater velocity, which is in agreement with previous observations.

Similar scales of irregularities revealed by correlation analysis at a given time instant in conjunction with velocity estimates explain the broadening of frequency power spectra - larger drift velocity shifts spatial structures in the frequency domain according to the Doppler effect.

The observed nonlinear dependence of time lag on separation can be attributed to propagation through a multilayer ionosphere with different evolution properties.

There are inconsistencies between the observations and the frozen flow model that cannot be accounted for by the temporal decorrelation.