

Implementation of higher-order advection schemes in a numerical ice-sheet model (for ice-core studies)

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Background

Saito et al., 2020, GMD — An exercise for a semi-Lagrangian scheme

- Implementation of RCIP scheme for **1-D age** computations in ice-sheet models
- RCIP(Xiao et al., 1996) $\equiv$ a Rational function based Constrained Interpolation Profile scheme
- The RCIP scheme preserves reasonably well the pattern of input surface mass balance histories in terms of the vertical profile of internal annual layer thickness

This study $\equiv$ an Extension of Saito et al. (2020) on Thermodynamics

A typical formulation of ice temperature (T : temperature; $\vec{v}$: velocity; σ : stress; ϵ : strain rate)

$$(1) \quad \rho c \left[\frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla) T \right] = \nabla \cdot (k \nabla T) + \epsilon \sigma + \Phi$$

Temperature evolution is affected by advection, diffusion, strain heating, phase changes terms. (while the age is solved by a pure advection equation)

$\Rightarrow$ How the temperature solution is affected by the RCIP scheme?

$\Rightarrow$ (Future goal) How the simulated thermodynamical-coupling features are affected by the RCIP scheme?

Methods: RCIP scheme on vertical 1-D temperature computation

Governing equations:

$$(2) \quad \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \Psi = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} + \epsilon \sigma + \Phi \right) \quad \frac{\partial T'}{\partial t} + w \frac{\partial T'}{\partial z} = \frac{\partial \Psi}{\partial z} - \frac{\partial w}{\partial z} T' \quad \left(T' = \frac{\partial T}{\partial z} \right),$$

where the latter is obtained by derivative of the former.

Two-step time integration is adopted to solve the equations:

$$(3) \quad \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = 0, \quad \frac{\partial T'}{\partial t} + w \frac{\partial T'}{\partial z} = 0, \quad \text{Advection phase,}$$
$$(4) \quad \frac{\partial T}{\partial t} = \Psi, \quad \frac{\partial T'}{\partial t} = \frac{\partial \Psi}{\partial z} - \frac{\partial w}{\partial z} T', \quad \text{Non-adv. phase.}$$

The advection phase is solved by a semi-Lagrangian scheme, while the non-advection phase by a normal (e.g., FD) scheme. Upstream value is obtained by the interpolation profile $F(z)$, evaluated as either a polynomial or a rational function:

$$(5) \quad F(z) = C_3 z^3 + C_2 z^2 + C_1 z + C_0 \quad \text{or} \quad F(z) = \frac{C_2 z^2 + C_1 + C_0}{1 + D_1}$$

References

Parrenin, F., G. Dreyfus, G. Durand, S. Fujita, O. Gagliardini, F. Gillet, J. Jouzel, K. Kawamura, N. Lhomme, V. Masson-Delmotte, C. Ritz, J. Schwander, H. Shoji, R. Uemura, O. Watanabe, & N. Yoshida (2007): 1-d-ice flow modelling at EPICA dome C and dome Fuji, east Antarctica. *Clim. Past*, **3**, 2, 243–259. URL <https://cp.copernicus.org/articles/3/243/2007/>.

Saito, F., T. Obase, & A. Abe-Ouchi (2020): Implementation of the RCIP scheme and its performance for 1-D age computations in ice-sheet models. *Geosci. Model Dev.*, **13**, 11, 5875–5896. URL <https://gmd.copernicus.org/articles/13/5875/2020/>.

Vialov, S. (1958): Regularities of glacial shields movement and the theory of plastic viscous flow. *Int. Assoc. Hydrol. Sci. Pub.*, **47**, 266–275. URL <http://hydrologie.org/redbooks/a047/04729.pdf>.

Xiao, F., T. Yabe, & T. Ito (1996): Constructing oscillation preventing scheme for advection equation by rational function. *Comp. Phys. Comm.*, **93**, 1, 1–12. URL <http://www.sciencedirect.com/science/article/pii/0010465995001247>.

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Exercises: RCIP vs first-order upwind (UP1) on 1d

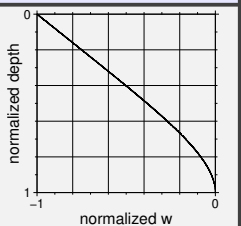
Boundary and other conditions

Cosine-curve surface mass balance; No basal melting; Constant surface temperature; Constant thickness 3000 m; Constant geothermal heat flux = 42 mW m⁻²; Zero strain heating;

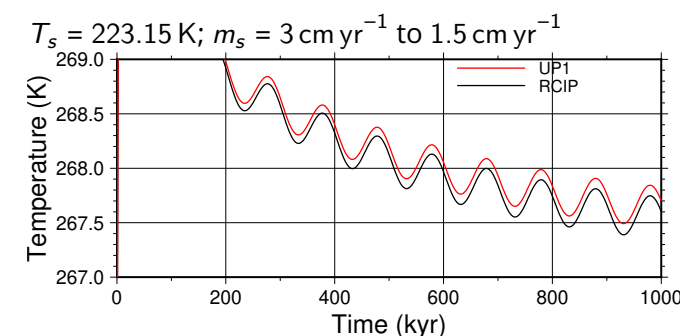
Vertical velocity formulation (Parrenin et al., 2007)

$$(6) \quad w(\zeta) = - \left\{ \left[m_s + m_b - \frac{\partial H}{\partial t} \right] w(\zeta) - m_b \right\}, \quad w(\zeta) = 1 - \frac{p+2}{p+1} (1-\zeta) + \frac{1}{p+1} \zeta^{p+2},$$

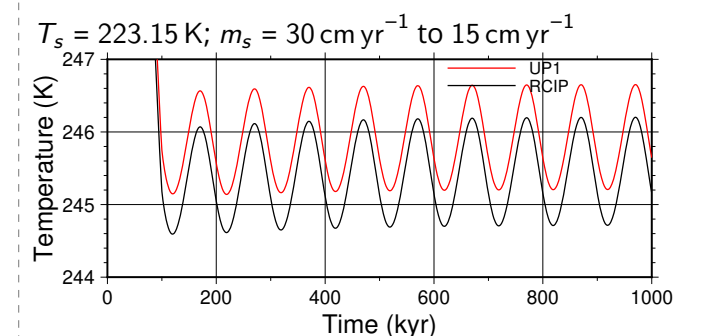
where m_s , m_b is the surface and basal mass balance, respectively. Parameter p controls the w shape, where $p = n = 3$ (Glen's flow-law exponent) corresponds to the vertical velocity at divide under the Vialov profile (Vialov, 1958).



Results: basal temperature time evolution under 100-kyr cycle surface mass balance forcing

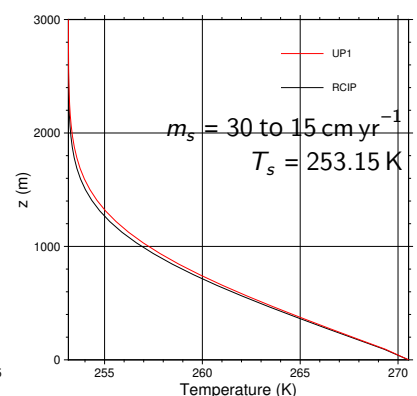
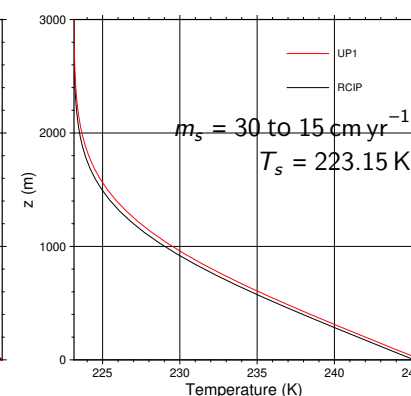
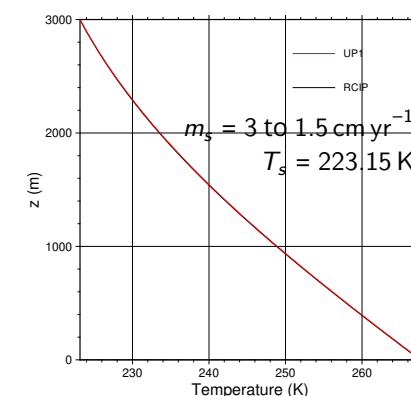


amplitudes: 0.358 K (RCIP); 0.352 K (UP1)



amplitudes: 1.49 K (RCIP); 1.45 K (UP1)

Results: vertical temperature profiles ($t = 100$ kyr snap shots)



Summary and Prospects

- Changes in the computed solution using RCIP scheme are at most 0.5 K, and the amplitude of the oscillation is less than 0.1 K, thus they are rather minor (not surprising, though)
- How about thermodynamical coupling features? — wait for future development toward 3d extensions