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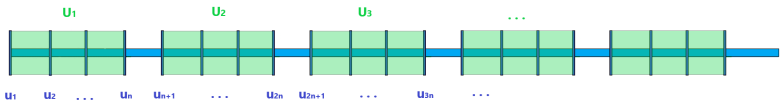
# Subgrid-scale parametrization of unresolved scales in forced Burgers equation using Generative Adversarial Networks (GAN)

## Finite-Volume Discretization

$$\frac{d}{dt}u_i = \frac{F_{i+1/2} - F_{i-1/2}}{\Delta x} + \rho_i, \quad i \in \{1, \dots, N\}$$

## Define Coarse Variables

"Box" filter: Average over $n$ points




$$U_I = \frac{1}{n}\sum_{k=n(I-1)+1}^{nI} u_k, \qquad \text{sub-grid: } y_i = u_i - U_{I(i)}$$

## Equation for Coarse Variables

$$\frac{d}{dt}U_I = -\frac{F^{tot}_{I+1/2} - F^{tot}_{I-1/2}}{n\Delta x} + \rho^I, \quad F^{tot}_{I+1/2}(\mathbf{U}, \mathbf{y}) = \frac{1}{n}\sum_{i \in Box\ I} F_{i+1/2}$$

## Flux Decomposition

$$F^{tot}_{I+1/2}(\mathbf{U}, \mathbf{y}) = F_{I+1/2}(\mathbf{U}) + G_{I+1/2}(\mathbf{U}, \mathbf{y})$$

## Generative Adversarial Networks

Neural Networks Designed to reproduce arbitrary probability distributions

$$G_{I+1/2}(\mathbf{U}, \mathbf{y}) \sim \text{Conditional GAN } \tilde{G}_{I+1/2}(U_I, U_{I+1})$$

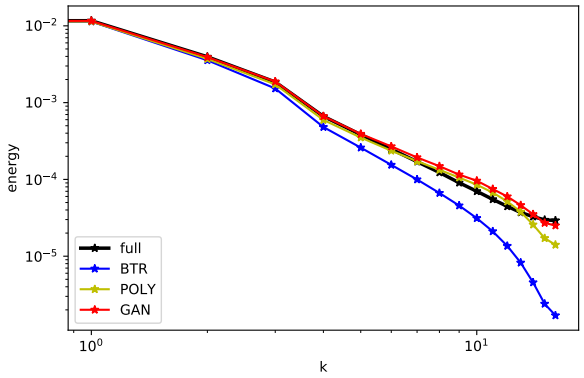
$\tilde{G}_{I+1/2}(U_I, U_{I+1})$: NN trained on the DNS data; can be used far outside of the training regime

## Test Case: Forced Burgers Equation

$$\partial_t u + \frac{1}{2}\partial_x u^2 = \nu \partial_{xx} u + \rho \qquad SGS = -\frac{1}{2}\partial_x(\overline{uu} - UU)$$

## Compare Equilibrium Properties

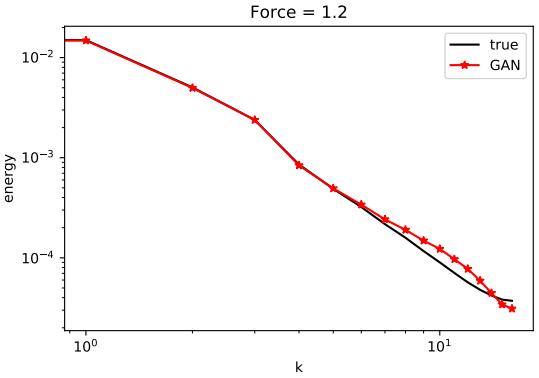
Energy Spectra:




## Advantages of GAN Parametrization

1. Accurately reproduces equilibrium properties of the DNS
2. Adequate Response to Increased Forcing (10-20%) (regression completely fails)
3. Robustness with respect to changes in the averaging window $n = 4$, 8, 16

Energy Spectra:
(20% increase in forcing; no re-training)

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