

Comparison of different topographic mass integration schemes in topographic reduction and its impact on geoid modelling: a case study in the Colorado area

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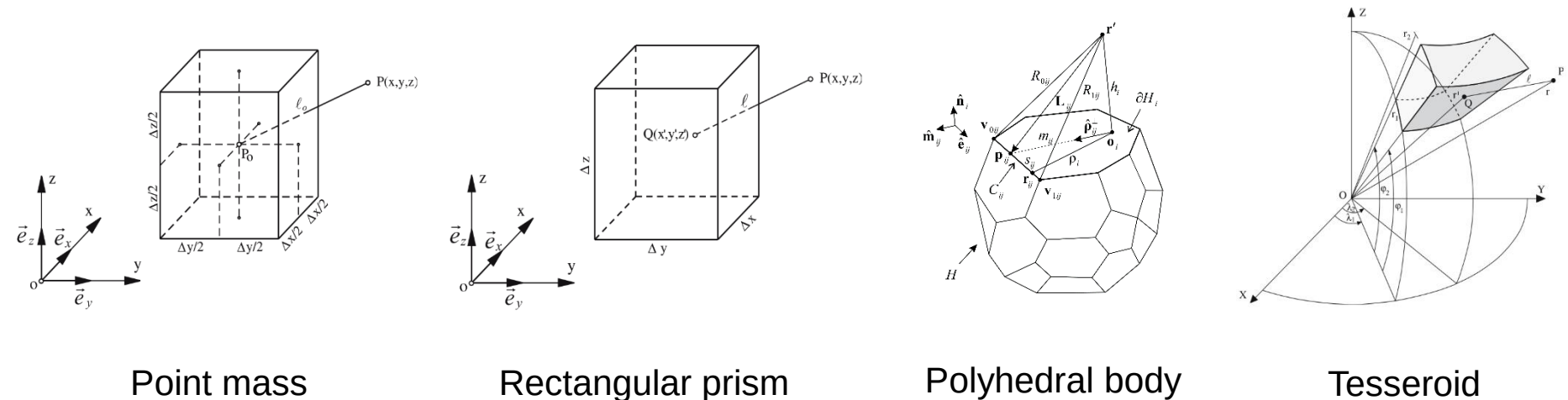
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Gravity forward modeling

Why do we forward model the gravity effect of mass distribution in geodesy and geophysics?

- Gravity reduction in geoid modeling (e.g., gravimetric terrain correction, residual terrain model, topographic/isostatic reduction, atmospheric correction)
- Removing temporal gravity signals from ground gravimetric measurements
- Revealing geological structures inside the Earth and planets
- Gravity inversion (e.g., subsurface density anomalies, crustal-mantle structures)

Mass elements for representing mass distribution



Gravity forward modeling

Mass element comparison in gravity effect approximation

Point Mass:

- **Pros:** analytical solution, fast computation
- **Cons:** low accuracy, very poor representation of mass distribution

Rectangular Prism:

- **Pros:** analytical solutions, high accuracy in near zone computation, small-scale computation in Cartesian coordinates
- **Cons:** large-scale computation in spherical coordinates, slow computation, lower accuracy in far zone computation, poor representation of mass distribution at high latitudes

Polyhedral Body:

- **Pros:** analytical solutions, high accuracy in near zone computation, small-scale computation in Cartesian coordinates, well representation of complex mass geometry
- **Cons:** large-scale computation in spherical coordinates, slow computation, lower accuracy in far zone computation

Tesseroïd:

- **Pros:** any scale computation in spherical coordinates, high accuracy in far zone computation, well representation of mass distribution
- **Cons:** no analytical solution, mid accuracy in near zone computation but can be improved

Gravity forward modeling

Numerical tesseroid approaches

$$T(r, \phi, \lambda) = G\rho \int_{\lambda_1}^{\lambda_2} \int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} H(P, Q) dr' d\phi' d\lambda'$$

with integral kernel $H(P, Q) = \frac{r'^2 \cos \phi'}{\sqrt{r^2 + r'^2 - 2rr' \cos \psi}}$. Here, $\rho = \text{constant}$.

- Numerical integration, using three-dimensional Gauss-Legendre quadrature (**GLQ3D**), see e.g., in [Grombein et al. \(2013\)](#), or one-dimensional analytical integral along radial direction + two-dimensional GLQ (**GLQ2D**), see e.g., in [Wild-Pfeiffer \(2008\)](#), or GLQ3D along with three-dimensional adaptive subdivision (**GLQ3D_3DAD**), see e.g., in [Uieda et al. \(2016\)](#), or GLQ3D along with two-dimensional adaptive subdivision (**GLQ3D_2DAD**), see e.g., in [Lin and Denker \(2019\)](#), or Double Exponential quadrature (**DEQ2D**), see e.g., in [Fukushima \(2017, 2018\)](#)
- Taylor series expansion of integral kernel $H(P, Q)$ up to zeroth (**TSE0**) or second order (**TSE2**), similar to MacMillan's idea for rectangular prism, see e.g., in [Heck and Seitz \(2007\)](#), [Grombein et al. \(2013\)](#)
- Prism approximation with coordinate transformation, see e.g., in [Wild-Pfeiffer \(2008\)](#)
- Point mass approximation, see e.g., in [Wild-Pfeiffer \(2008\)](#)

Gravity forward modeling

State-of-art method used in topographic reduction

- The mass elements derived from digital terrain models (DTMs) are treated as rectangular prisms
- A computational strategy of using the exact rectangular prism formula, the MacMillan's formula, and the point mass formula is applied to gravity forward modeling
- A constant density of 2.67 g/cm^3 is assumed in each prismatic body

What will we do in this study?

- Replacing prisms by tesseroids and applying precise numerical tesseroid approaches
- Considering lateral density variation in the topographic mass modeling

Main questions intended to be answered

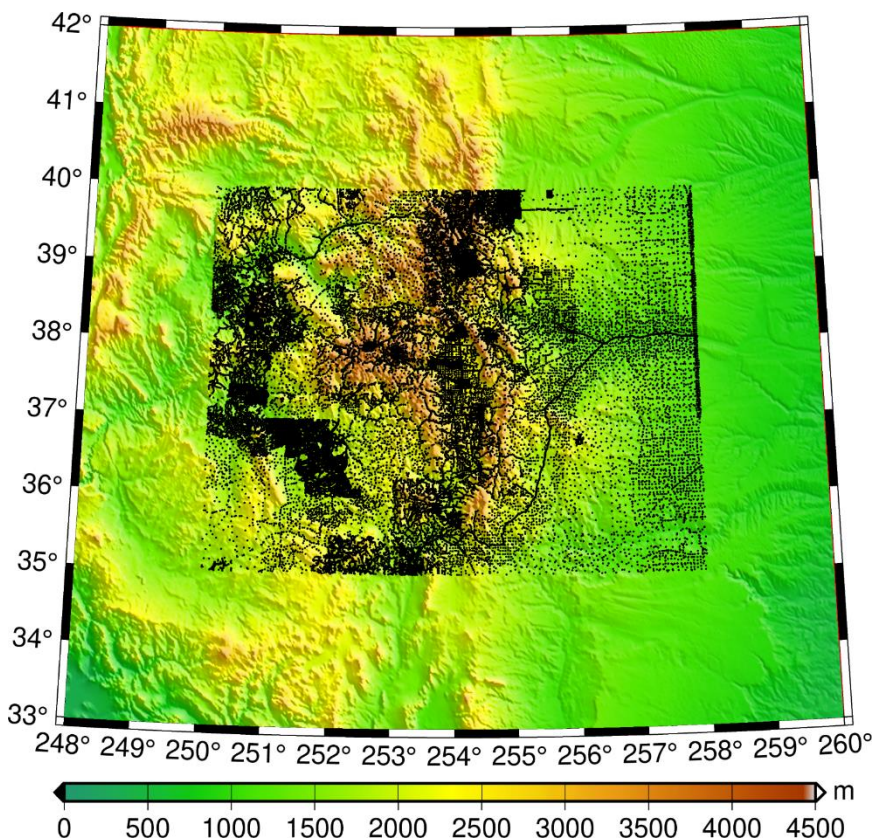
- What is the contribution of using rigorous topographic mass modeling methods to topographic reduction and geoid modeling?
- What is the contribution of considering lateral density variation to topographic reduction and geoid modeling?

Modeling methods

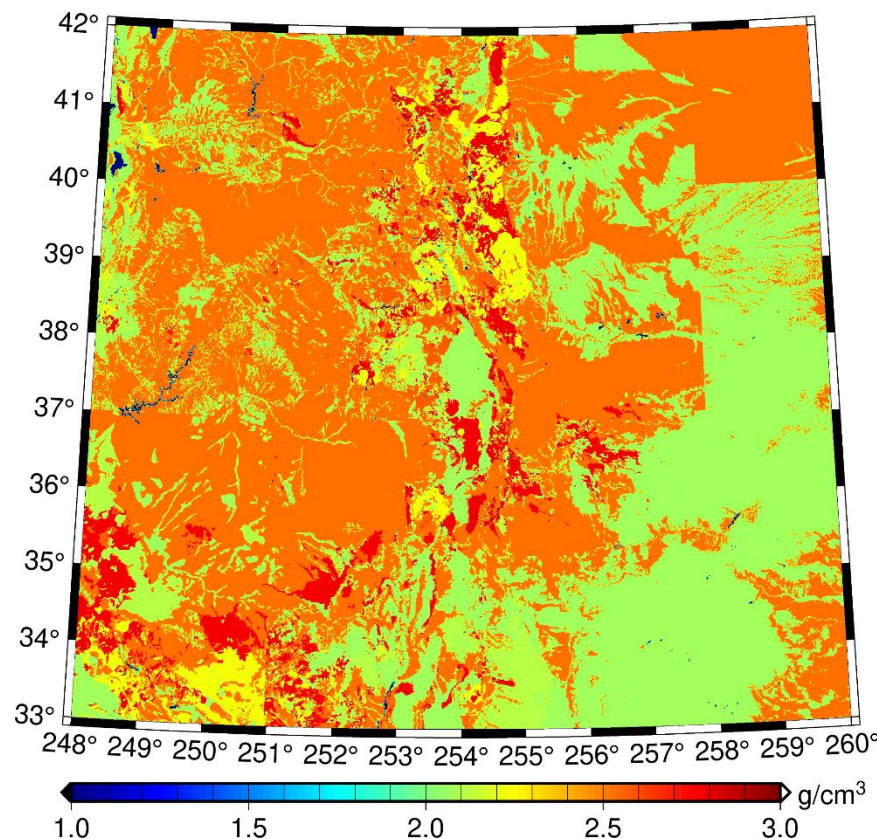
- Different computational strategies of combining various numerical approaches
- Numerical approach selection controlled by the “distance-size” ratio D

Method	“distance-size” ratio D	Used numerical approach
TC/TC90	Nearest 9/225 prisms	Exact rectangular prism formula (Forsberg 1984)
	$D < D_1$	Exact rectangular prism formula
	$D_1 \leq D < D_2$	MacMilan’s formula (MacMilan 1930)
	$D \geq D_2$	Point mass formula (Wild-Pfeiffer 2008)
TCTESSV1	Nearest 9/225 tesserooids	GLQ3D_2DAD (Lin and Denker 2019 , Lin et al. 2020)
	$D < D_1$	GLQ3D_2DAD
	$D_1 \leq D < D_2$	TSE2 (Heck and Seitz 2007)
	$D \geq D_2$	TSE0 (Heck and Seitz 2007)
TCTESSV2	Nearest 9/225 tesserooids	Exact rectangular prism formula ($-80^\circ \leq \phi \leq 80^\circ$) GLQ3D_2DAD ($\phi < -80^\circ$ or $\phi > 80^\circ$)
	$D < D_1$	GLQ3D_2DAD
	$D_1 \leq D < D_2$	TSE2
	$D \geq D_2$	TSE0
TCTESSV3	Nearest 9/225 tesserooids	DEQ2D (Fukushima 2017, 2018)
	$D < D_1$	GLQ3D_2DAD
	$D_1 \leq D < D_2$	TSE2
	$D \geq D_2$	TSE0

Data sets



Topography of research area (units in meter:
mean = 1733, min = 314, max = 4385) based
SRTM DTM and the distribution of 59303
computation points



2D mass-density distribution (units in g/cm^3 :
mean = 2.356, min = 1.000, max = 2.854)
derived from the UNB density model
"30s_UNB_TopoDensT_2v01.grd"

Method assessment

Method comparison

- A constant density model with a value of 2.67 g/cm^3 (Constant_2.67)
- Three types of topographic reduction
- TCTESSV1a: 3 GLQ nodes are used for evaluating the nearest 9/225 tesserooids
- TCTESSV1b: 10 GLQ nodes are used for evaluating the nearest 9/225 tesserooids

Residual terrain model (RTM) effect_ζ (Units: m)

Method	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	-0.010	-0.158	0.206	0.030	0.031
(b) TCTESSV1a	-0.010	-0.158	0.206	0.030	0.031
(c) TCTESSV1b	-0.010	-0.158	0.206	0.030	0.031
(d) TCTESSV2	-0.010	-0.158	0.206	0.030	0.031
(e) TCTESSV3	-0.010	-0.158	0.206	0.030	0.031
(a)-(b/c/d/e)	0.000	0.000	0.000	0.000	0.000

RTM effect_Δg (Units: mGal)

Method	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	-5.126	-221.361	80.504	14.904	15.761
(b) TCTESSV1a	-5.156	-222.741	79.693	14.912	15.778
(c) TCTESSV1b	-5.126	-221.631	81.305	14.906	15.763
(d) TCTESSV2	-5.127	-221.355	81.323	14.907	15.764
(e) TCTESSV3	-5.126	-221.354	81.331	14.907	15.763
(a)-(b)	-0.030	-0.265	2.774	0.120	0.123
(a)-(c)	0.001	-0.800	0.587	0.016	0.016
(a)-(d)	0.001	-0.819	0.586	0.017	0.017
(a)-(e)	0.000	-0.827	0.581	0.017	0.017

Method assessment

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- A constant density model with a value of 2.67 g/cm^3 (Constant_2.67)
- Three types of topographic reduction
- TCTESSV1a: 3 GLQ nodes are used for evaluating the nearest 9/225 tesserooids
- TCTESSV1b: 10 GLQ nodes are used for evaluating the nearest 9/225 tesserooids

GTC_Δg (Units: mGal)

Method	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	2.691	-1.290	101.476	3.464	4.387
(b) TCTESSV1a	2.689	-1.289	103.070	3.464	4.385
(c) TCTESSV1b	2.689	-1.289	101.712	3.463	4.384
(d) TCTESSV2	2.690	-1.289	101.711	3.463	4.385
(e) TCTESSV3	2.690	-1.289	101.714	3.463	4.385
(a)-(b)	0.002	-1.746	0.110	0.010	0.011
(a)-(c)	0.002	-0.425	0.090	0.003	0.003
(a)-(d)	0.001	-0.425	0.090	0.003	0.003
(a)-(e)	0.001	-0.428	0.088	0.003	0.003

Gravimetric terrain correction (GTC)_ζ (Units: m)

Method	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	-1.275	-16.371	37.505	6.019	6.153
(b) TCTESSV1a	-1.276	-16.394	37.565	6.028	6.161
(c) TCTESSV1b	-1.276	-16.394	37.565	6.028	6.161
(d) TCTESSV2	-1.276	-16.394	37.565	6.028	6.161
(e) TCTESSV3	-1.276	-16.394	37.565	6.028	6.161
(a)-(b/c/d/e)	0.001	-0.060	0.023	0.008	0.009

Method assessment

Method comparison

- A constant density model with a value of 2.67 g/cm^3 (Constant_2.67)
- Three types of topographic reduction
- TCTESSV1a: 3 GLQ nodes are used for evaluating the nearest 9/225 tesserooids
- TCTESSV1b: 10 GLQ nodes are used for evaluating the nearest 9/225 tesserooids

DTE_Δg (Units: mGal)

Method	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	224.107	79.241	439.125	60.301	232.078
(b) TCTESSV1a	224.171	79.862	432.789	60.231	232.122
(c) TCTESSV1b	228.925	82.596	441.992	61.837	237.130
(d) TCTESSV2	228.940	82.603	442.098	61.853	237.148
(e) TCTESSV3	228.944	82.604	442.107	61.855	237.152
(a)-(b)	-0.064	-20.536	38.092	1.454	1.455
(a)-(c)	-4.818	-25.132	33.760	2.172	5.285
(a)-(d)	-4.833	-25.066	33.773	2.150	5.289
(a)-(e)	-4.837	-25.073	33.771	2.151	5.293

Direct topographic effect (DTE)_ζ (Units: m)

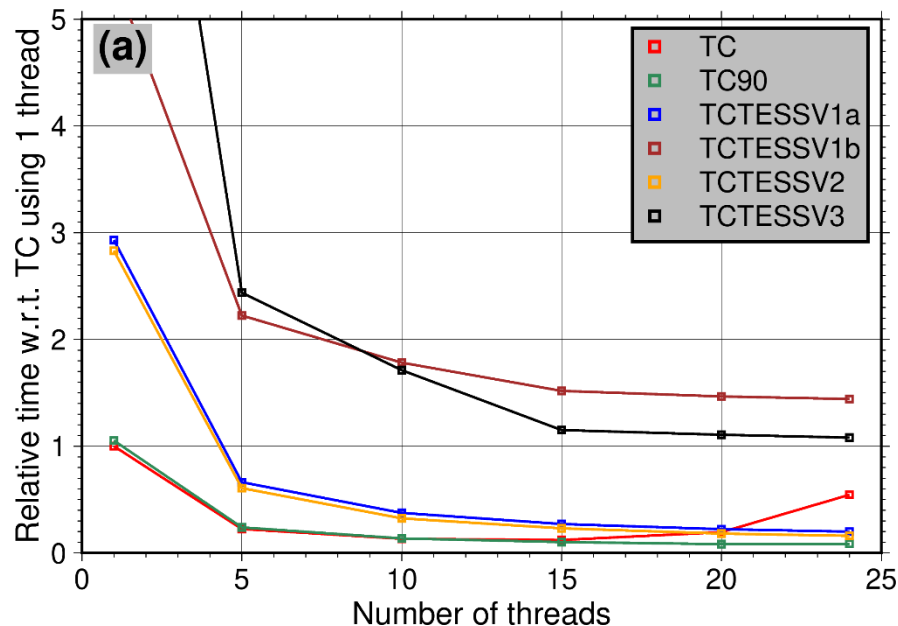
	Mean	Min.	Max.	STD	RMS
(a) TC/TC90	41.480	20.545	54.105	7.316	42.121
(b) TCTESSV1a	41.530	20.568	54.174	7.326	42.171
(c) TCTESSV1b	41.531	20.568	54.175	7.327	42.172
(d) TCTESSV2	41.531	20.568	54.175	7.327	42.172
(e) TCTESSV3	41.531	20.568	54.175	7.327	42.172
(a)-(b)	-0.050	-0.075	-0.022	0.011	0.051
(a)-(c/d/e)	-0.050	-0.076	-0.022	0.011	0.051

Method assessment

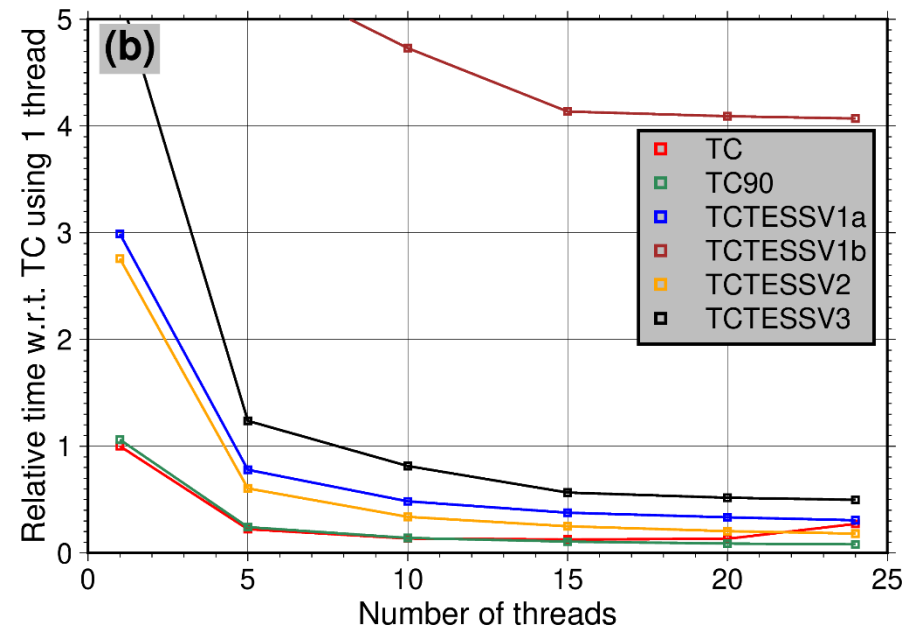
Parallel computation comparison

- In the RTM case (3''x3'' dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
- Used density model: Constant_2.67
- Relative computational time w.r.t. the method TC using 1 thread

Relative time of computing $RTM_{\Delta g}$



Relative time of computing RTM_{ζ}



Method assessment

Density model comparison

- In the RTM case (3''x3'' dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
- Only using the method TCTESSV3
- Used density model: UNB_density and Constant_2.67
- Three types of topographic reduction

ζ differences at 59303 points (units: m)

Reduction type	Mean	Min.	Max.	STD	RMS
RTM	0.020	-0.017	0.070	0.013	0.024
GTC	0.023	-4.522	1.529	0.576	0.576
DTE	-3.870	-6.359	-2.465	1.048	4.009

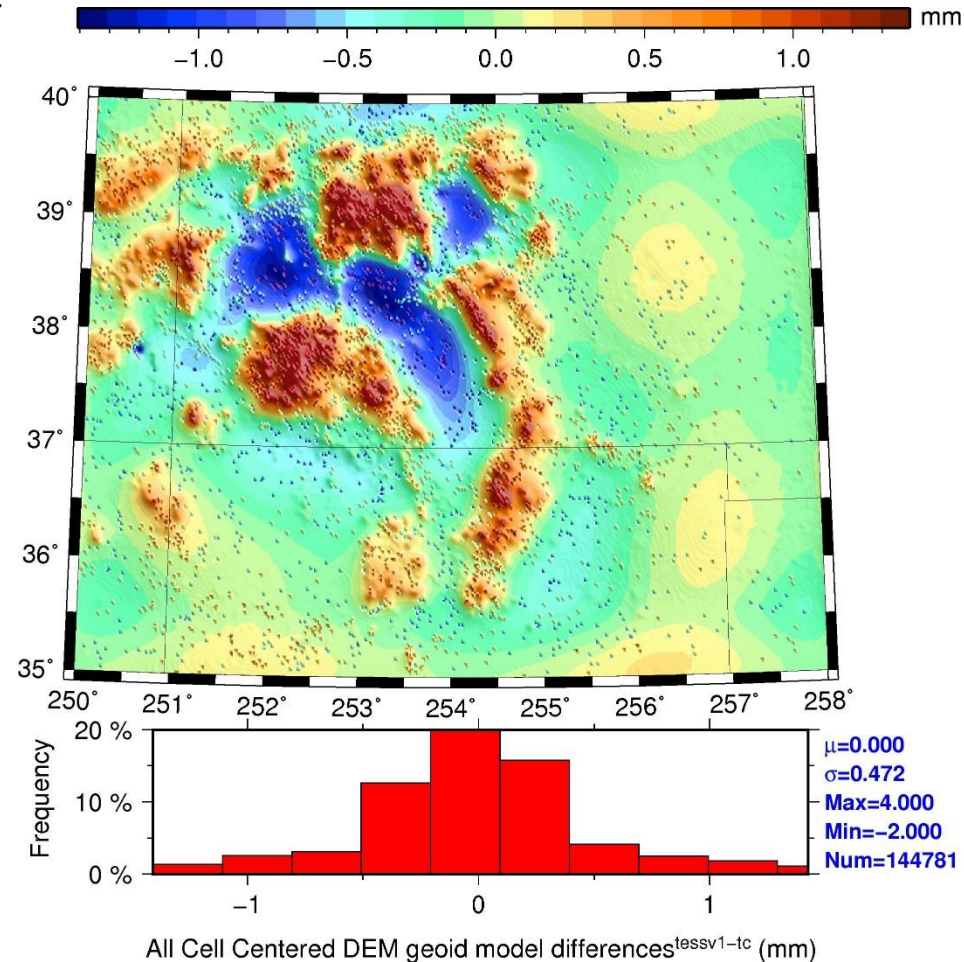
Δg differences at 59303 points (units: mGal)

Reduction type	Mean	Min.	Max.	STD	RMS
RTM	0.635	-13.525	46.219	2.069	2.165
GTC	-0.211	-16.145	1.403	0.387	0.441
DTE	-24.229	-114.038	10.443	16.979	29.585

Method effect on the geoid

- In the RTM case (3"x3" dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
- Modeling method: TCTESSV1a .vs. TC
- Used density model: Constant_2.67

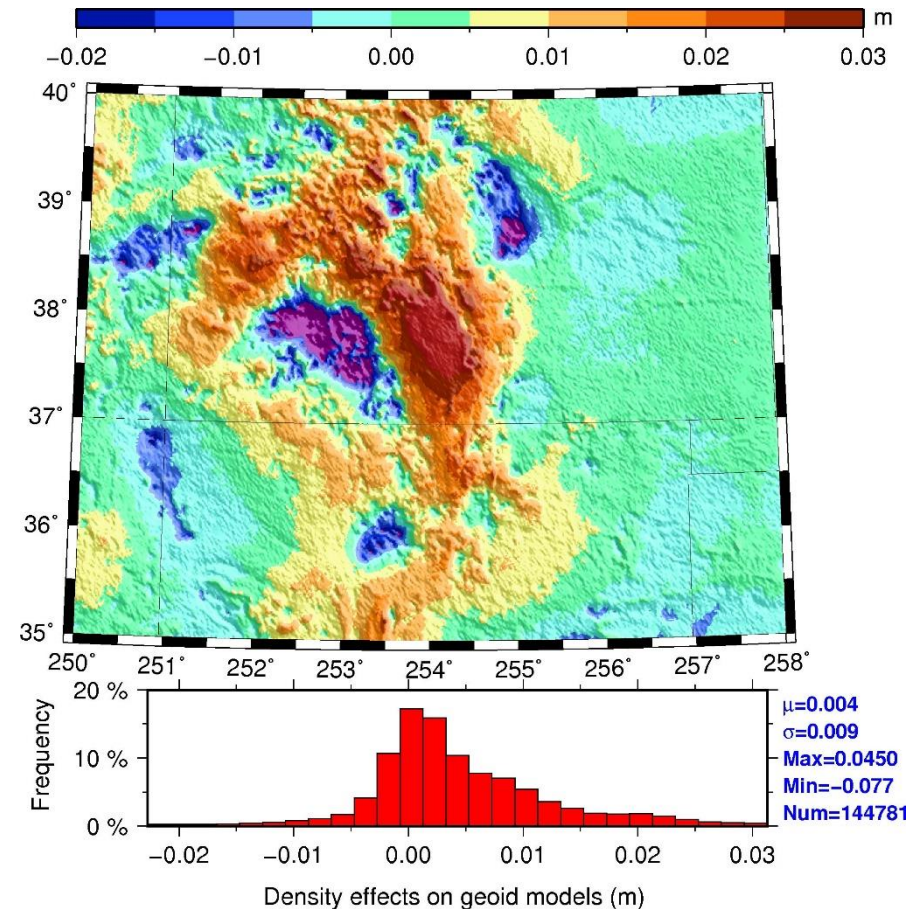
- The geoid changes are at mm level
- Large changes mainly exist in the mountainous area due to different RTM effects computed by the two modeling methods



Density variation effect on the geoid

- In the RTM case (3''x3'' dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
- Modeling method: TCTESSV3
- Used density model: UNB_density .vs. Constant_2.67

- The geoid changes are at cm level
- Large changes mainly exist in the mountainous area due to significantly different RTM effects computed by the two density models
- If the UNB density model reflects the real density distribution in the research area, its impact on the geoid modeling must be taken into account

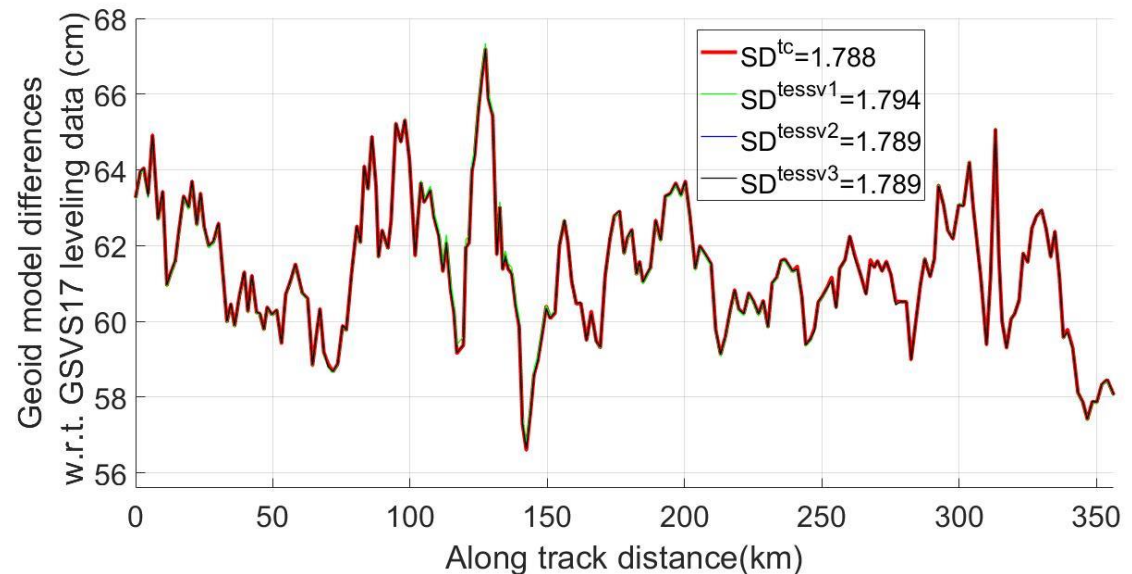


Geoid validation

Modeling method comparison

- In the RTM case (3"x3" dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
- Modeling method: TC, TCTESSV1a, TCTESSV2, TCTESSV3
- Used density model: Constant_2.67
- Validation data: GSVS17 leveling data

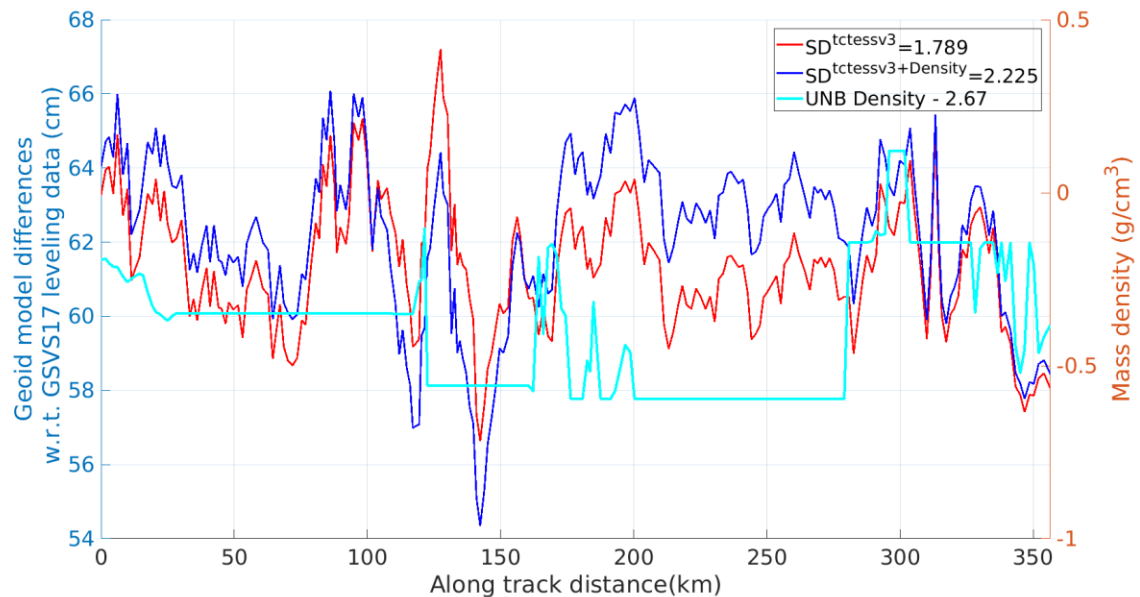
- Nearly the same geoid difference profiles are obtained for 4 used modeling methods
- The SD of the geoid differences is about 1.79 cm



Geoid validation

Density model comparison

- In the RTM case (3''x3'' dense DTM, 1'x1' coarse DTM, and 5'x5' reference DTM)
 - Modeling method: TCTESSV3
 - Used density model: UNB_density, Constant_2.67
 - Validation data: GSVS17 leveling data
-
- The geoid difference profile based on UNB model deviates from the one based on a constant density, having a maximum deviation of about 2 cm
 - The deviations are mainly due to the smaller densities of the UNB model (Cyan line)
 - The SD of the geoid differences is about 2.23 cm for the UNB model which is slightly larger than the SD of 1.79 cm for the constant density



Summary

- The implementation of OpenMP parallelization can significantly improve the computational efficiency. For each method, the computational speed in the case of using 24 threads is about 10 times faster than that using a single thread
- Application of rigorous modeling methods provides different results w.r.t. those computed by the prismatic method (TC or TC90) in the Colorado area. The magnitudes of the differences are comparable to those in the White Sands area shown in our previous work (Lin and Denker 2019). The impact of these differences on the geoid is at mm level in the RTM case
- In the RTM case, the inclusion of UNB density model can cause cm level geoid changes w.r.t. a constant density assumption in the Colorado area, where the large changes mainly exist in the mountainous area
- The validation of geoid models based on GSVS17 leveling data shows that the use of rigorous modeling methods provides nearly the same geoid difference profiles as that using the prismatic method, yielding an SD of about 1.79 cm. The geoid difference profile based on UNB density model deviates from the one using a constant density of 2.67 g/cm^3 , having a maximum deviation of about 2 cm. The SD of the former is about 2.23 cm which is slightly larger than the SD of 1.79 cm for the latter. The deviations are mainly due to the smaller densities from the UNB density model

Thank you for your attention!