



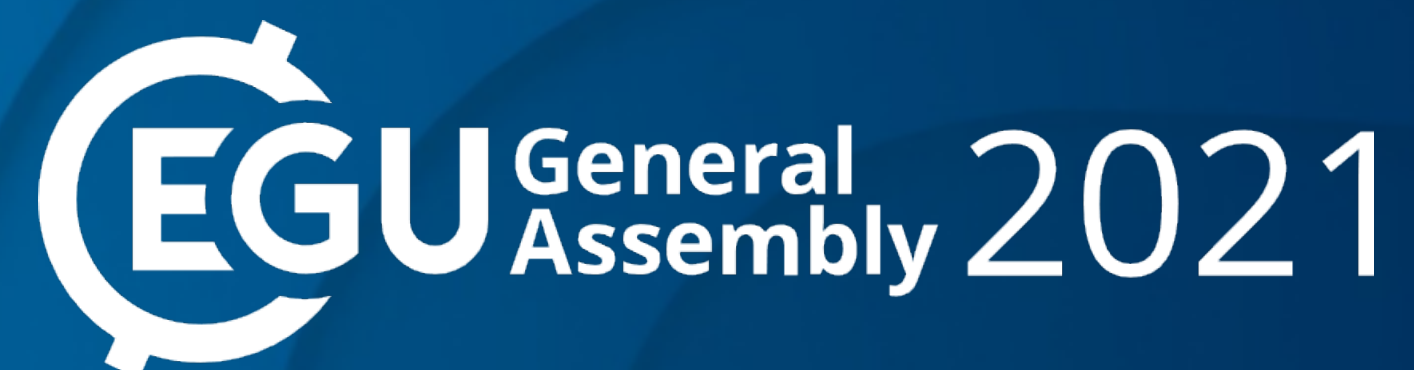
Numerical simulations of tsunami generation in caldera lakes by subaqueous explosive volcanism

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Climate, Freshwater & Ocean Science



Background

Volcanic... tsunami?

- NZ has long history of tsunamis

Tsunamis of (1855, 1868, 1947, 1960)

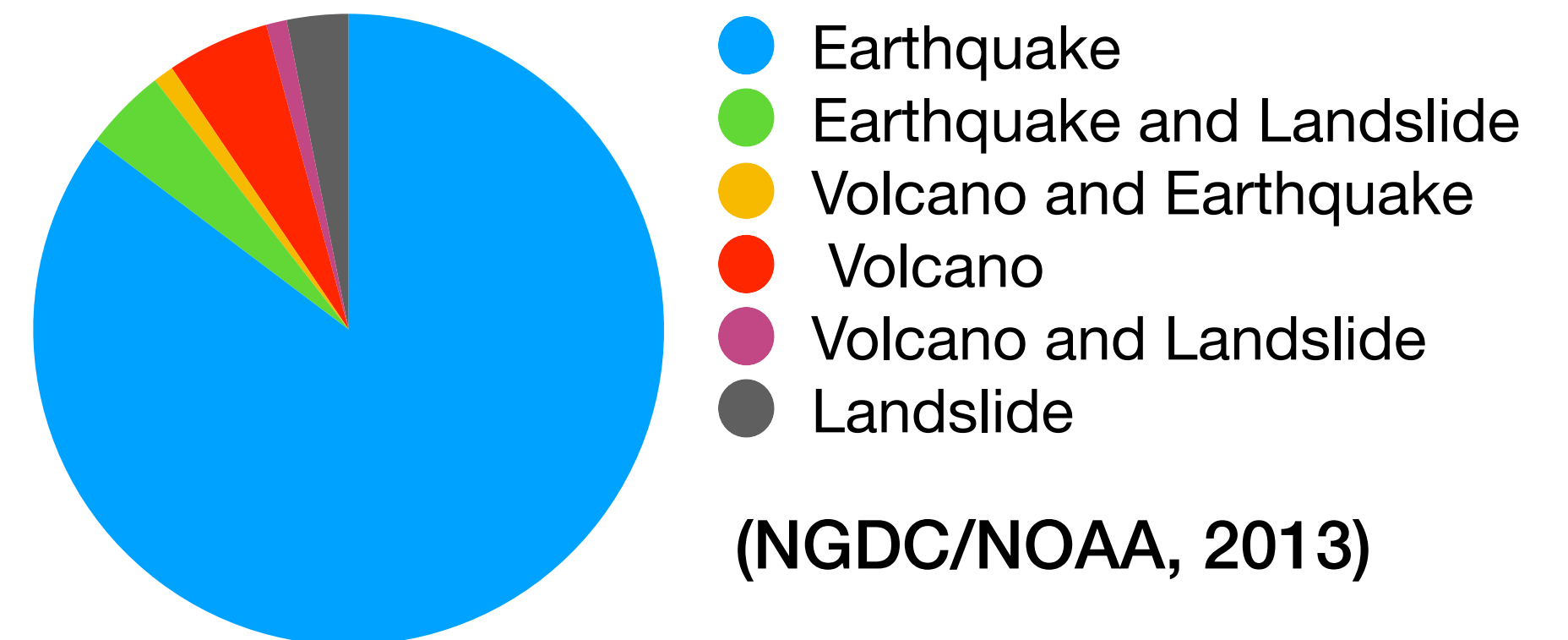
Māori oral traditions

Archaeology and geologic record

- Tsunamis aren't just about earthquakes!



(Bisnis, 2018)

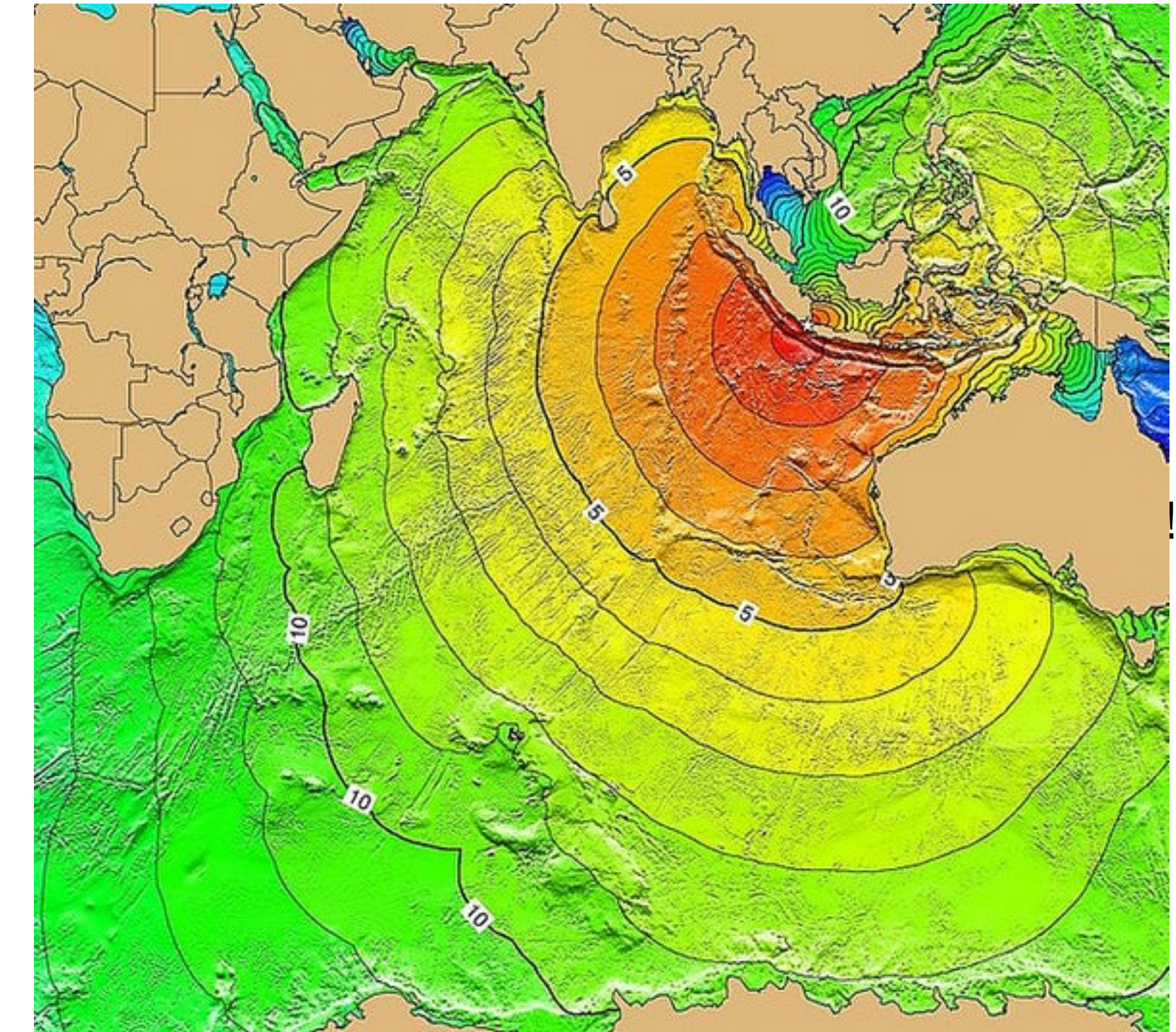


(NGDC/NOAA, 2013)



Volcanic Tsunami!

- Over 26% of all direct volcanic fatalities
- Larger eruptions contribute more towards this total such as...



(NOAA)

Krakatau, 1883

- The 3rd blast on 27 Aug largest of four that day
- Blew away most of the island; loudest sound heard ever
- Over 36,000 killed, almost all by tsunami within 800km
- Waves locally up to 40m high

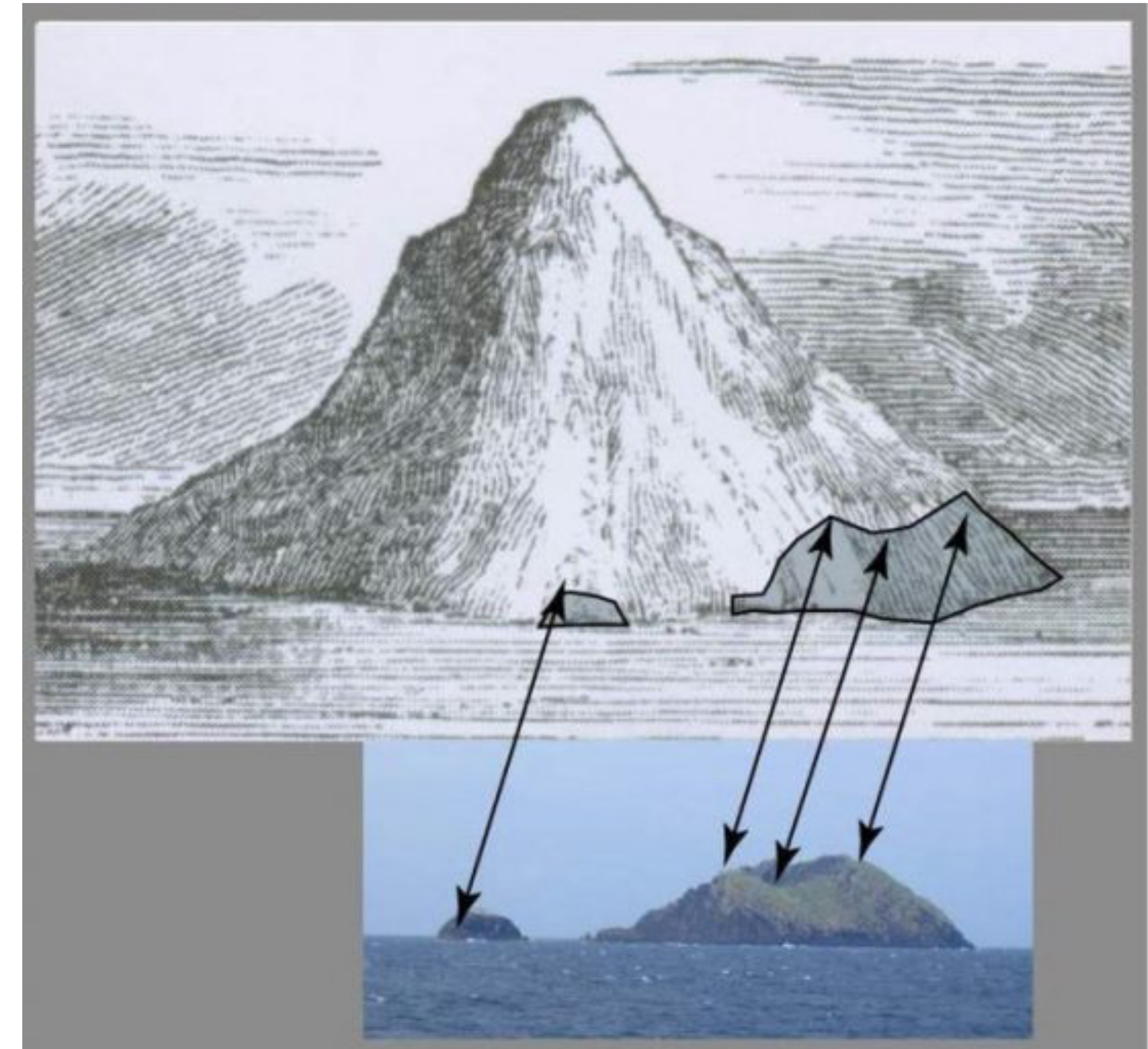


Ritter Island, 1888

- Major lateral collapse, 5km³ mobilised
- Over 15m run ups within 50km
- Likely over 1500 deaths

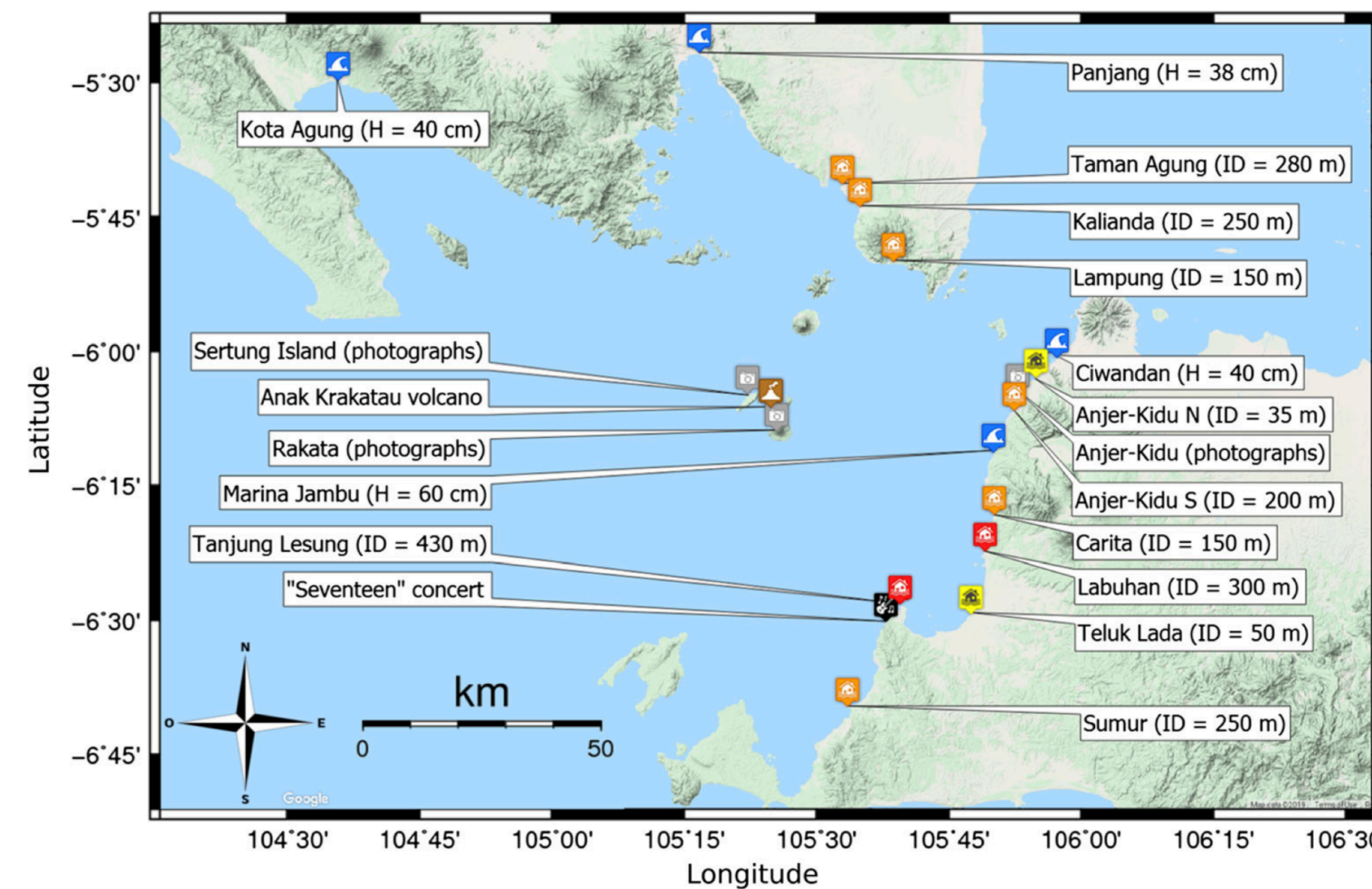
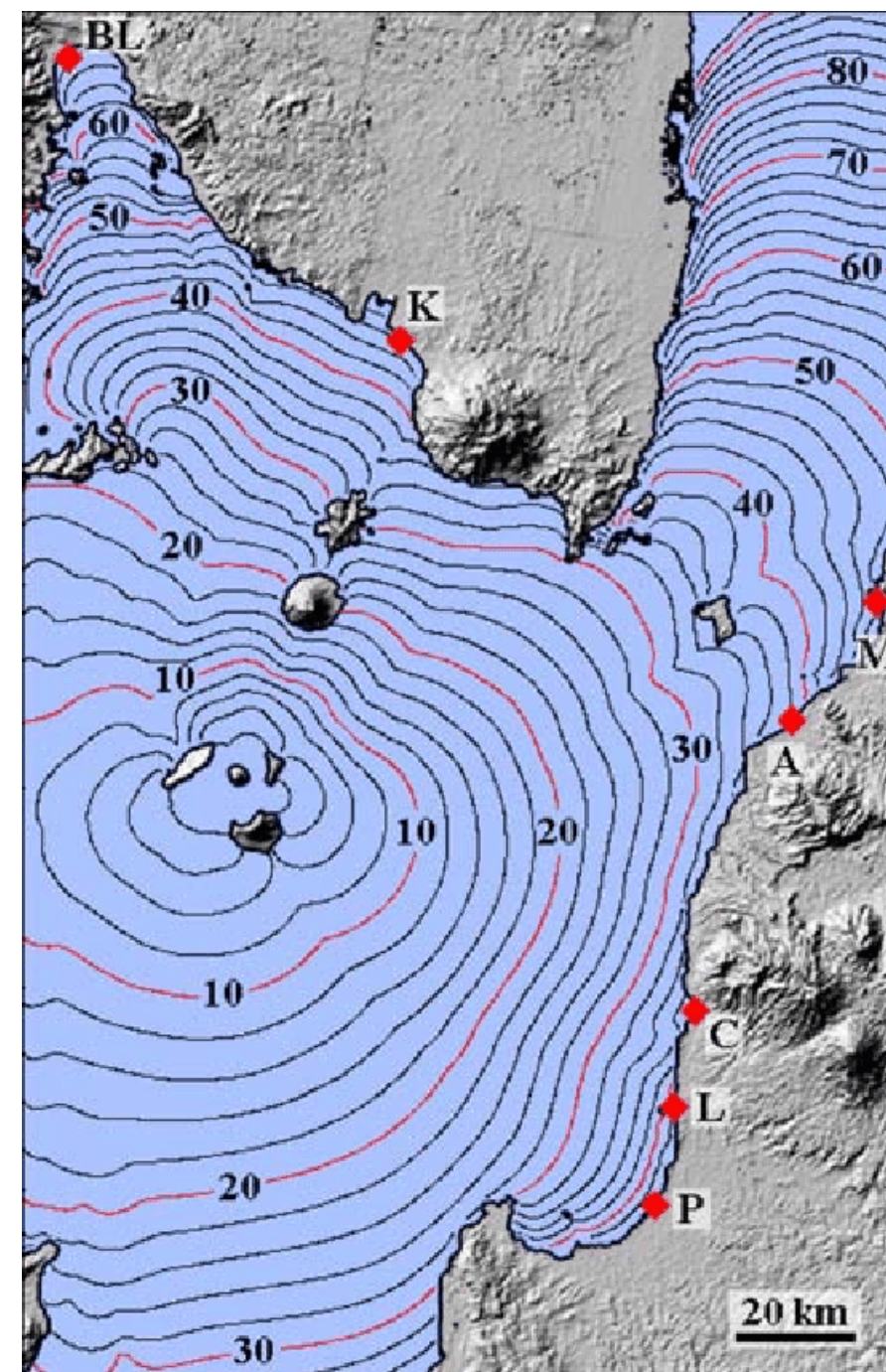
Kick 'em Jenny, 1939

- Submarine eruptive sequence
- Amplitudes ~2m besides coast of northern Grenada (10km)
- Sinking hazards persist

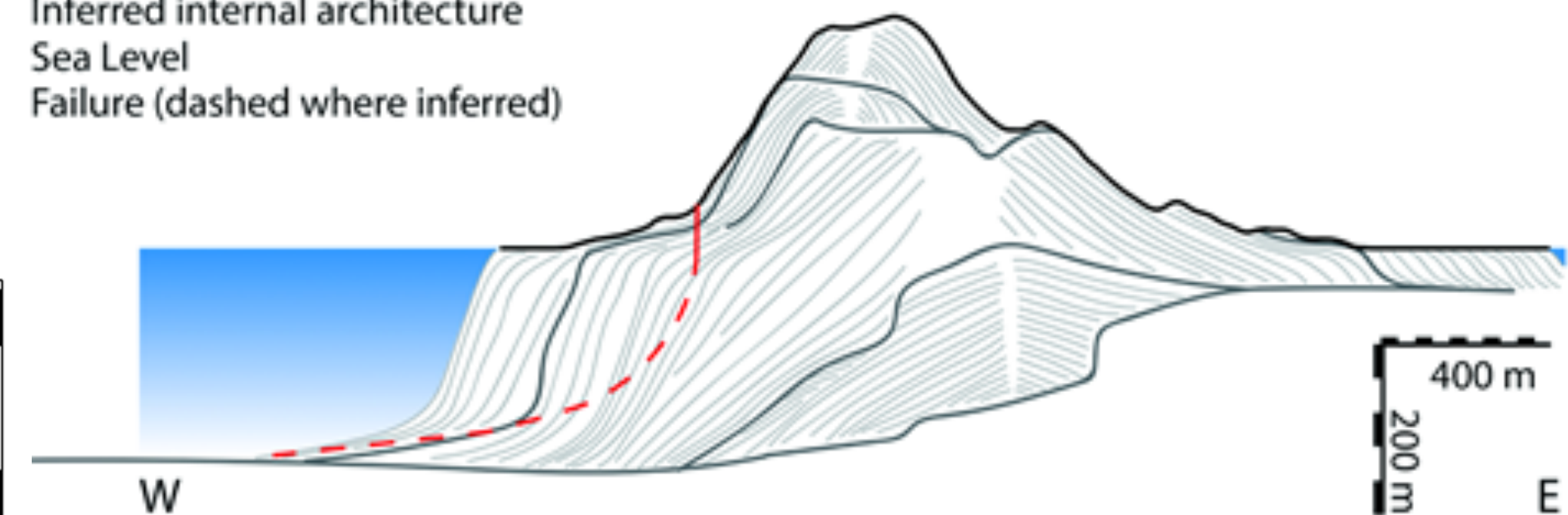


Anak Krakatau, Dec. 2018

- Eruption and partial collapse ($<0.2\text{km}^3$)
- 426 fatalities, 14,000+ injuries across Java & Sumatra
- 13m run ups on west Java coast.

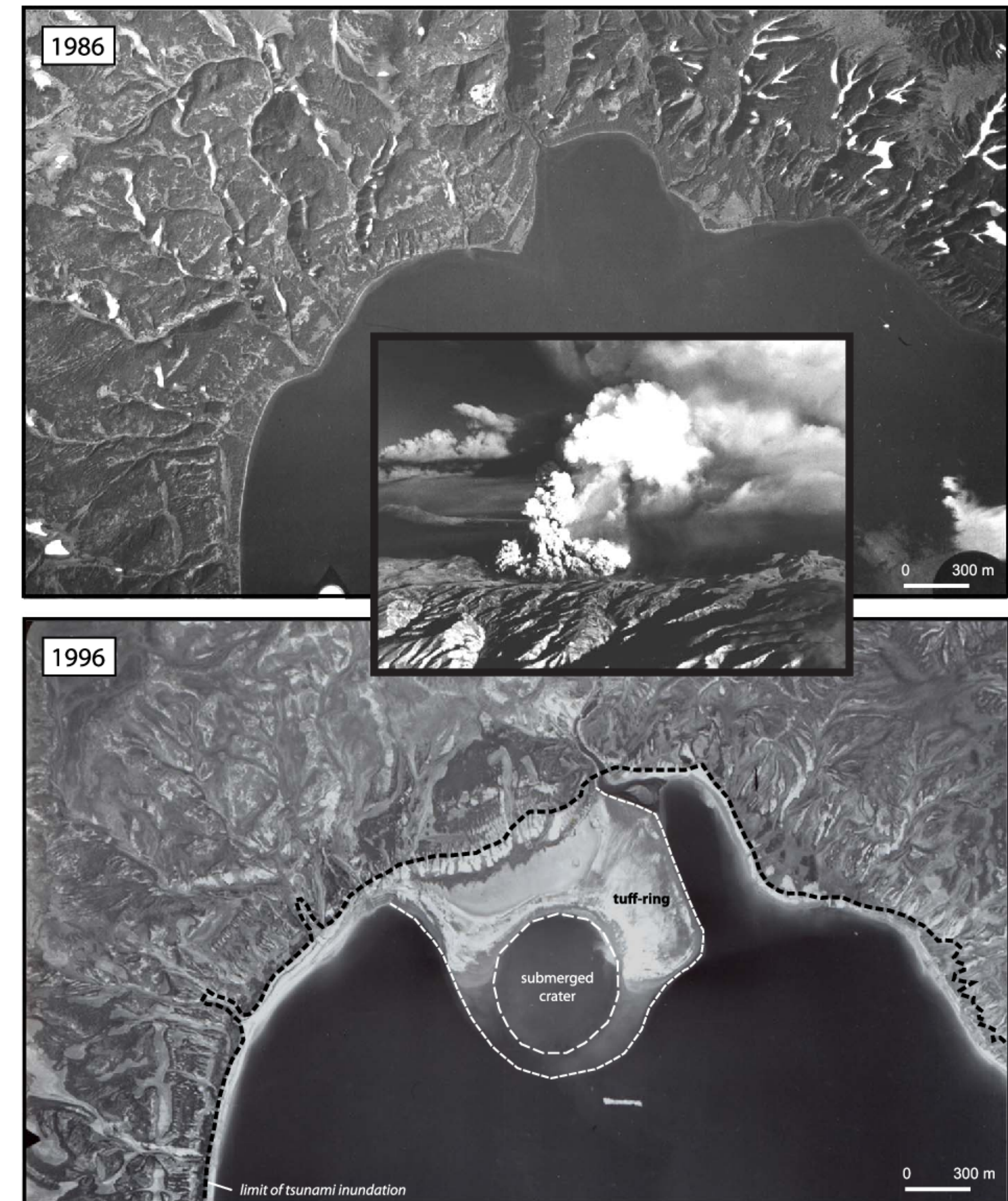
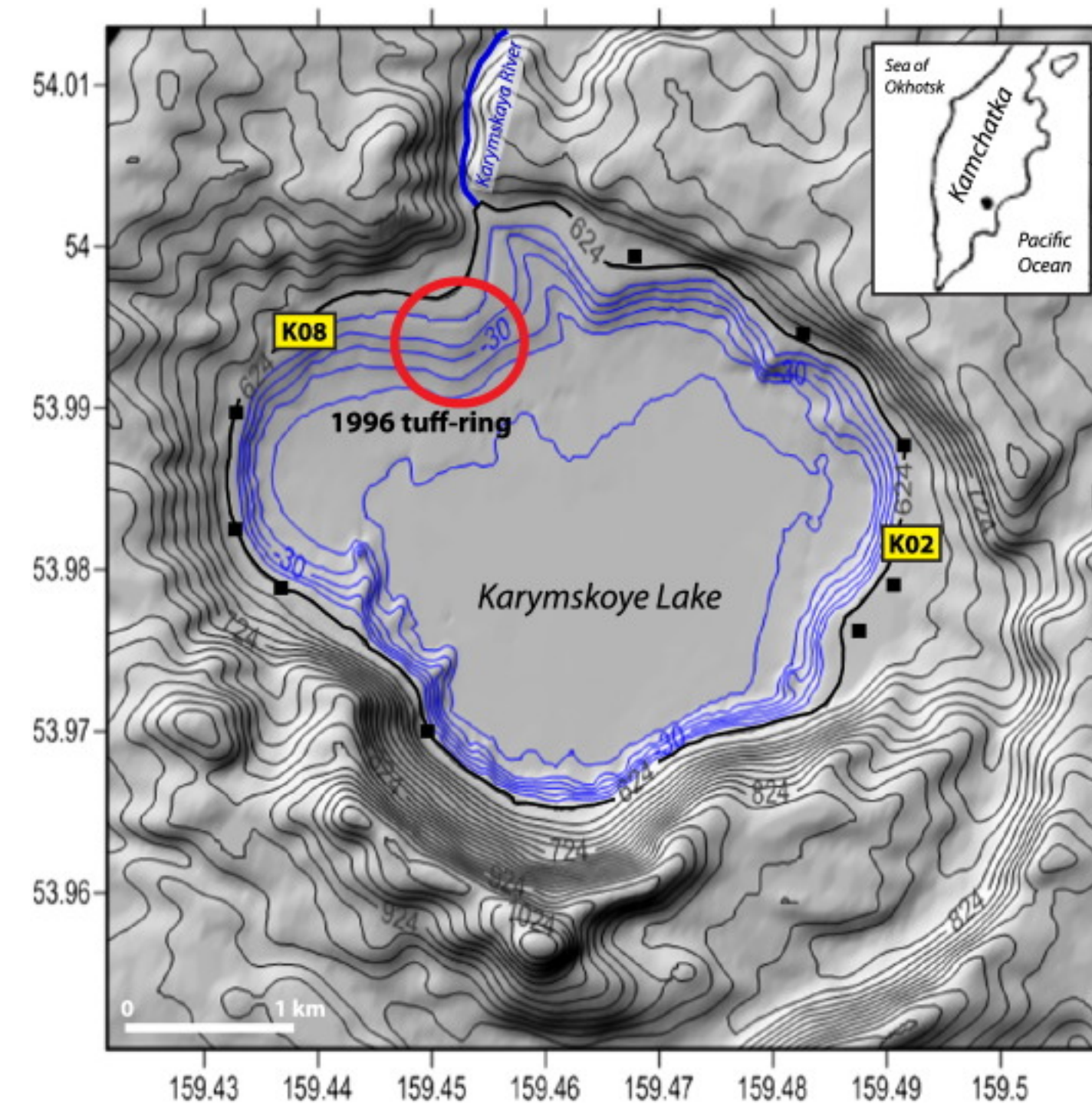


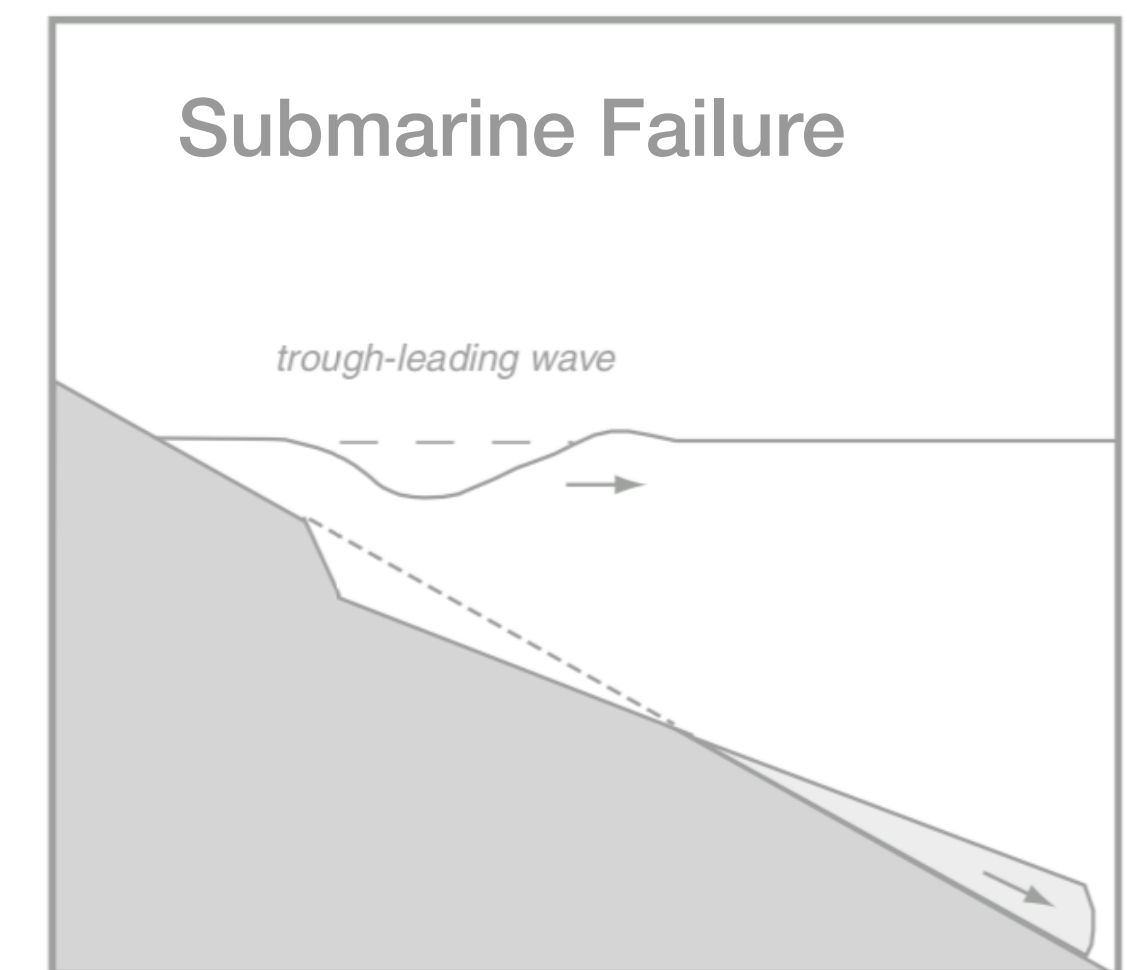
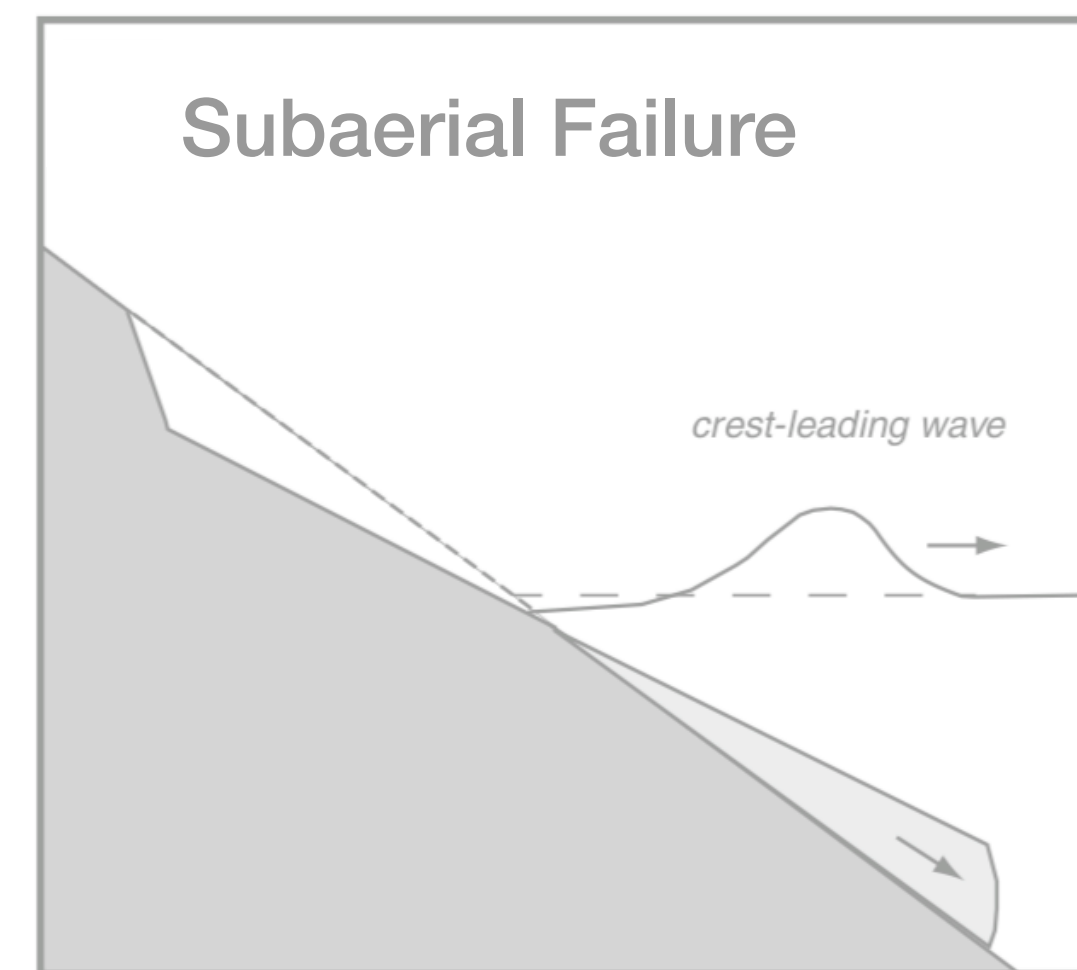
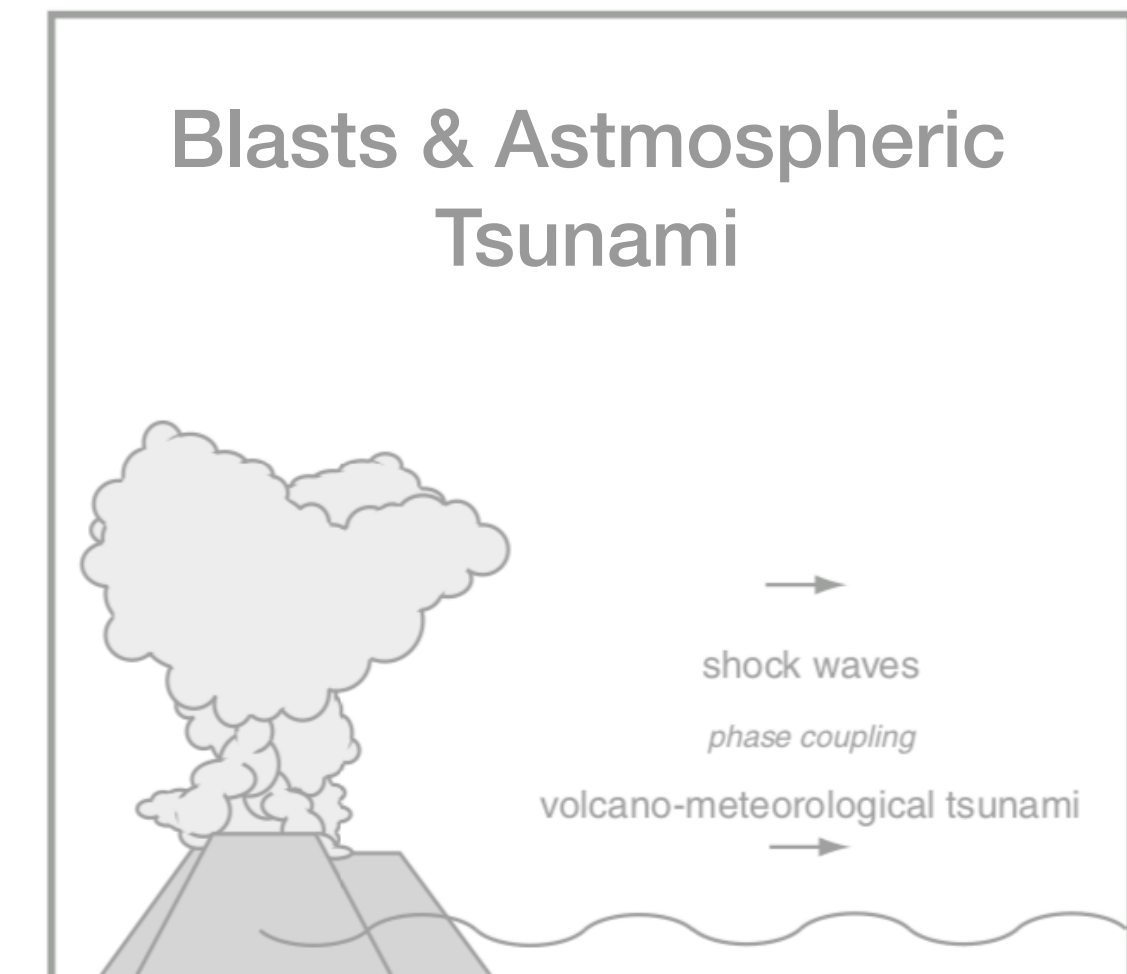
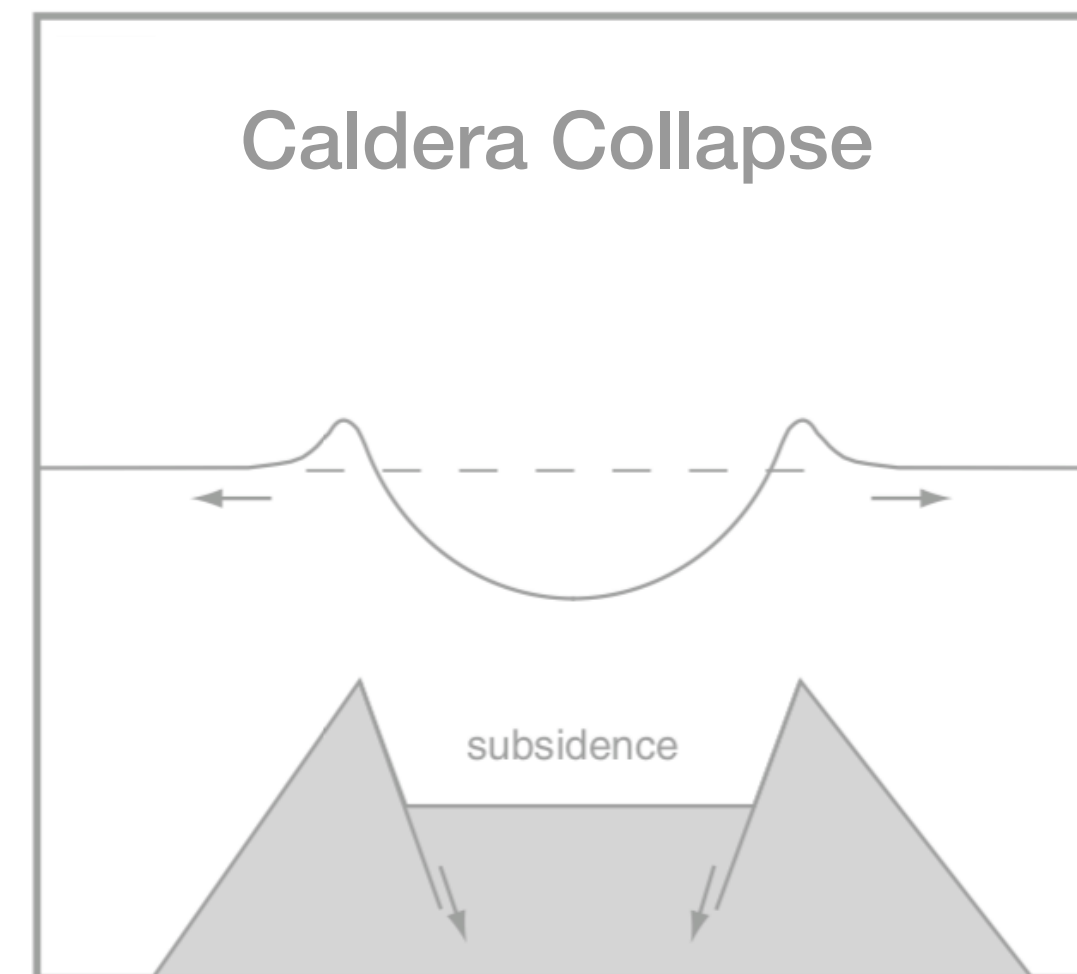
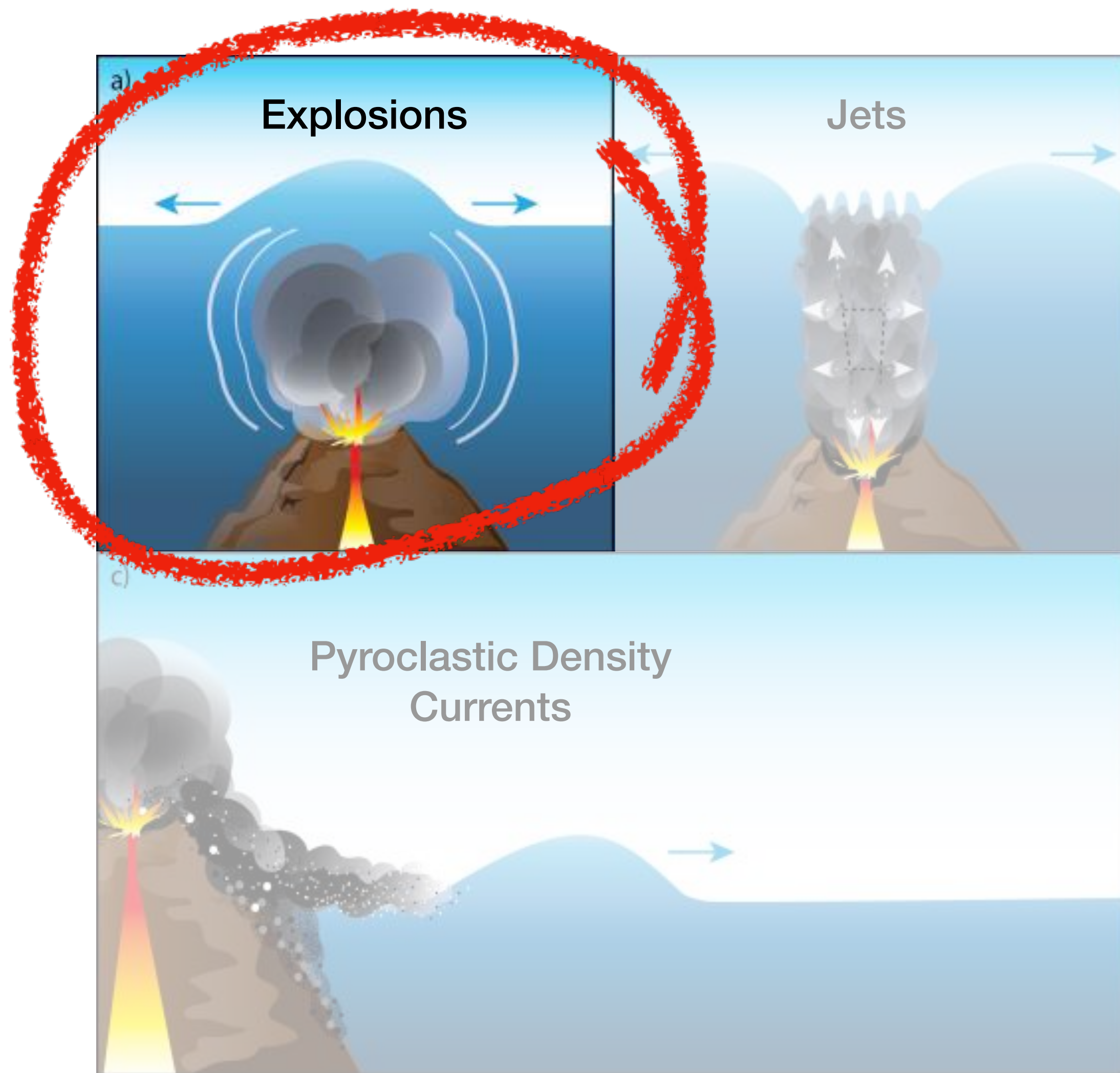
— Pre-failure topography
— Historic topographies & bathymetries
— Inferred internal architecture
— Sea Level
— Failure (dashed where inferred)



Karymskoye Lake, 1996

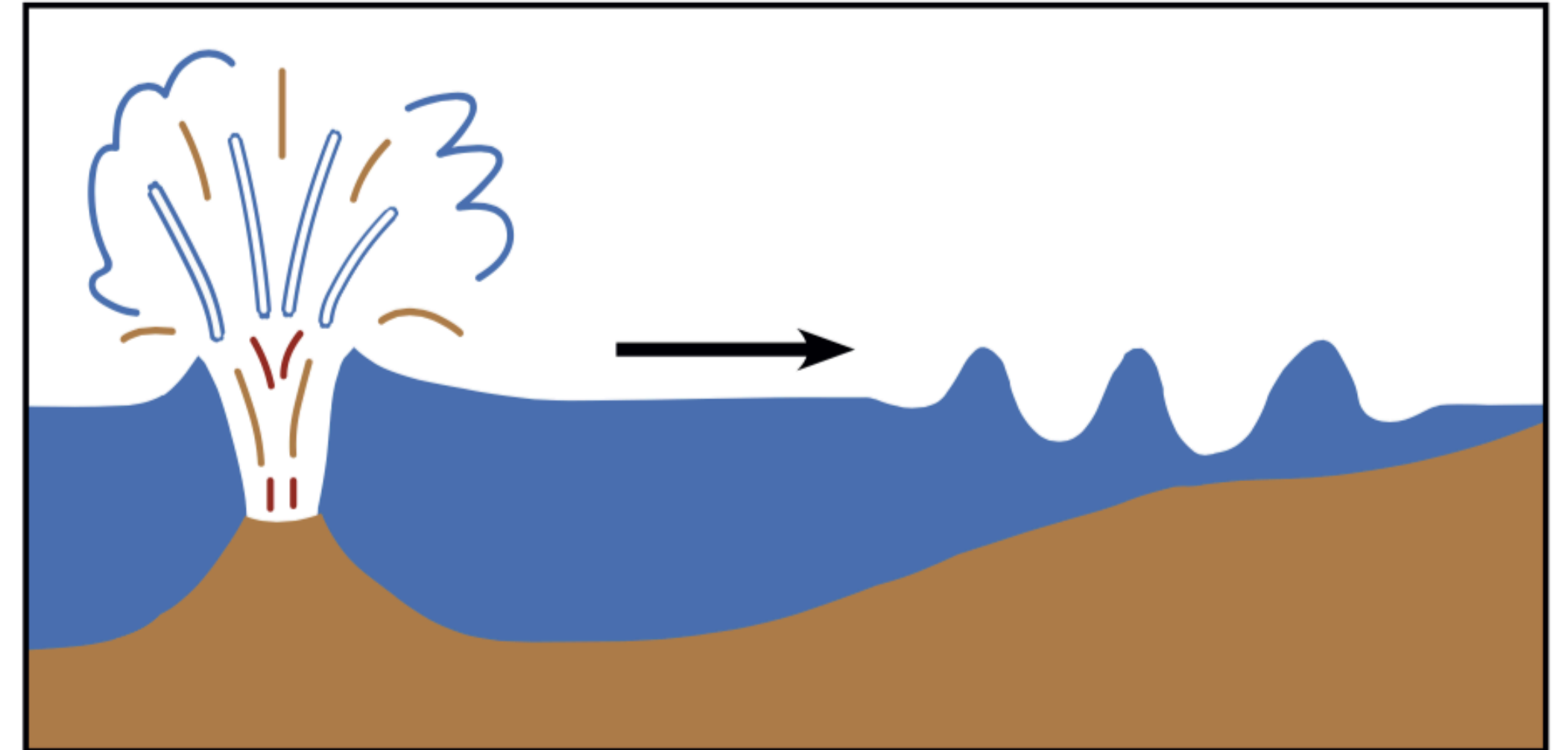
- Sighting of waves generated by ongoing eruption from air
- Two main waves inundated the coastal terrace up to 100 m inland on the eastern shore of the lake





Submarine Explosions

- Early research as a result of wartime necessity, expanded with military research into nuclear testing. Usually focused on shock waves and effects on the ship structures and hulls.
- Work involving explosive bubbles and the free-surface identified depths of charge and seabed and explosive energy as modelling parameters.
- Mathematical models developed for simplified initial conditions which have dominated research since.



Limitations

- Experimental modelling and investigation of explosive submarine processes is very difficult!
- Ideal circumstances for wave generation are not well constrained
- More complications such as ejected material, complex water-magma interactions + condensation not considered

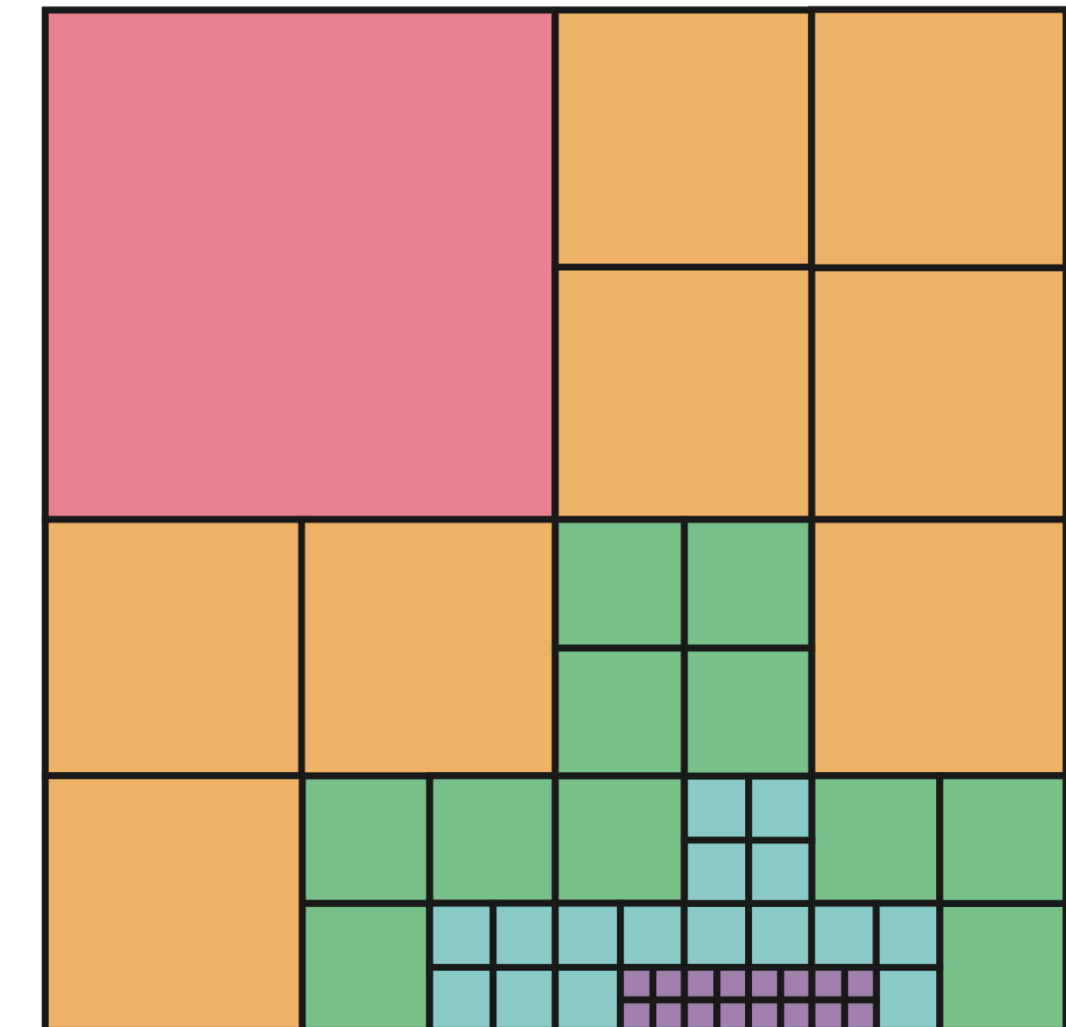
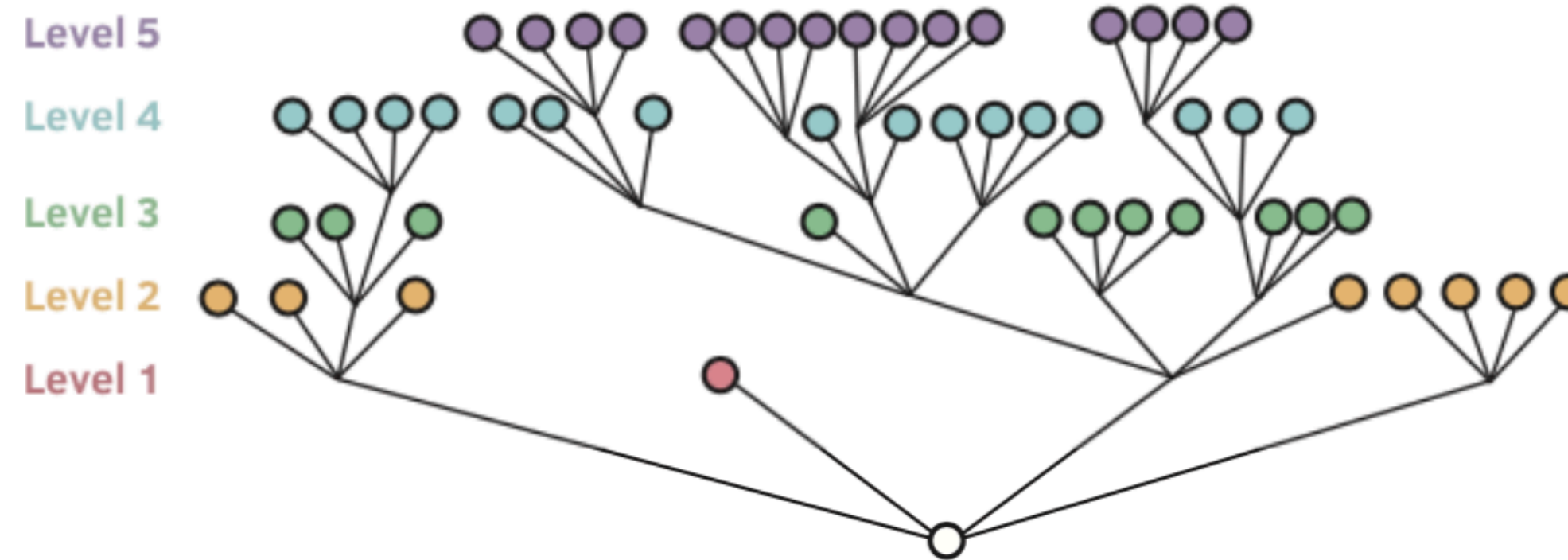
....This lack of understanding limits the usefulness of tsunami propagation modelling to identify the mechanism's hazard.



Numerical Method

Basilisk

- Open-source software
- Parallelised
- Variety of numerical schemes



- Spatial adaptivity using tree-grids



<http://basilisk.fr>

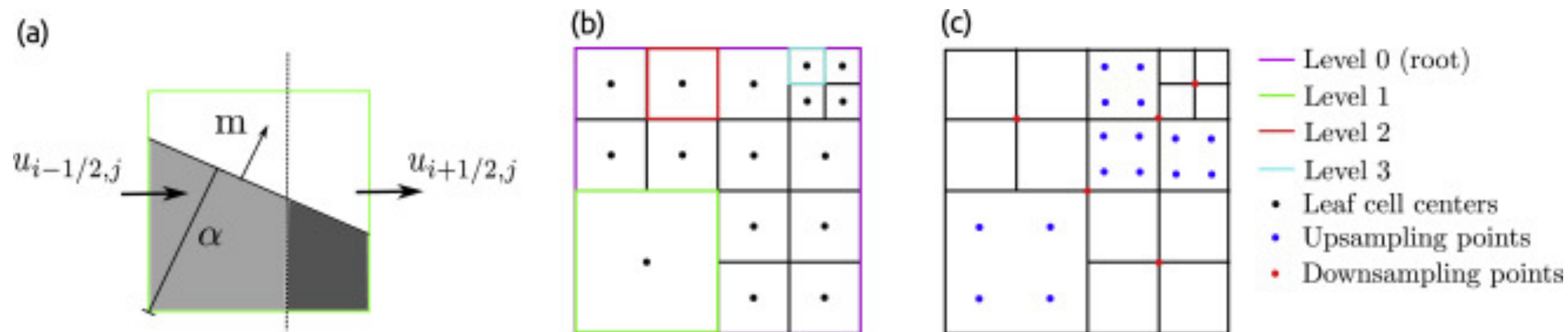


- **Incompressible** variable-density, **Naiver-Stokes** equations with surface tension

$$\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot (2\mu \mathbf{D}) + \sigma \kappa \delta_s \mathbf{n}$$

$$\nabla \cdot \mathbf{u} = 0$$

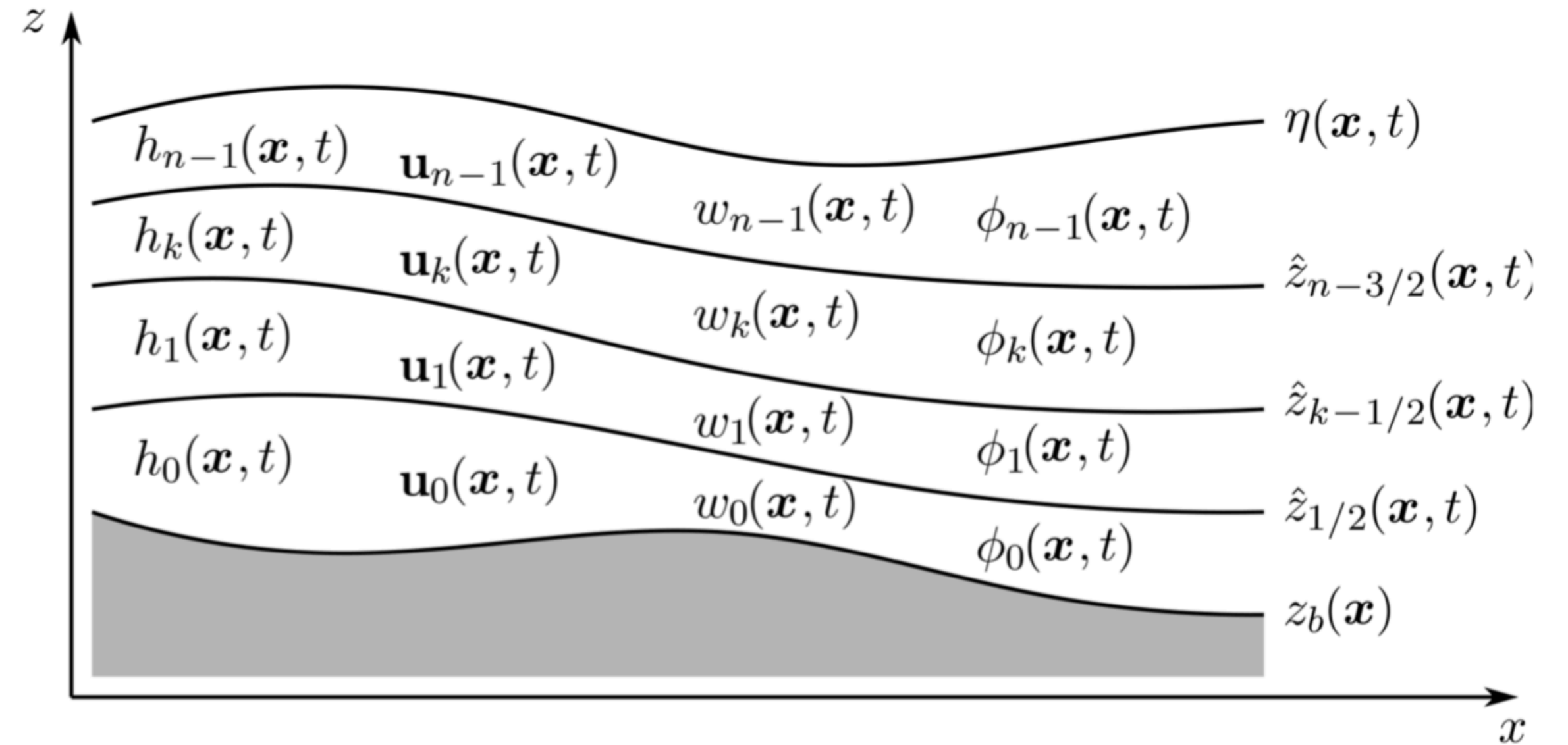
- Volume of fluid (**VOF**) method $\partial_t C + \mathbf{u} \cdot \nabla C = 0$
- Balanced-force surface tension discretisation



(Lopez-Herrera, 2019)



- **Multilayer** scheme recently developed
- Non-hydrostatic, vertically Lagrangian



$$\partial_t h_k + \nabla \cdot (h \mathbf{u})_k = 0,$$

$$\partial_t (h \mathbf{u})_k + \nabla \cdot (h \mathbf{u} \mathbf{u})_k = -g h_k \nabla \eta - \nabla (h \phi)_k + [\phi \nabla \hat{z}]_k,$$

$$\partial_t (h w)_k + \nabla \cdot (h w \mathbf{u})_k = -[\phi]_k,$$

$$\nabla \cdot (h \mathbf{u})_k + [w - \mathbf{u} \cdot \nabla \hat{z}]_k = 0,$$

Evolution of layer thickness

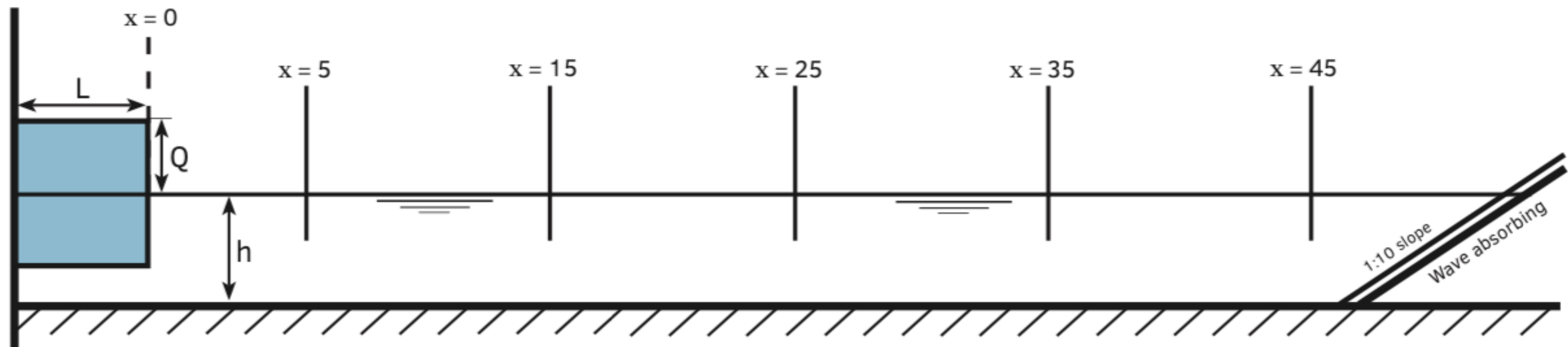
Conservation of momentum

Conservation of volume / mass

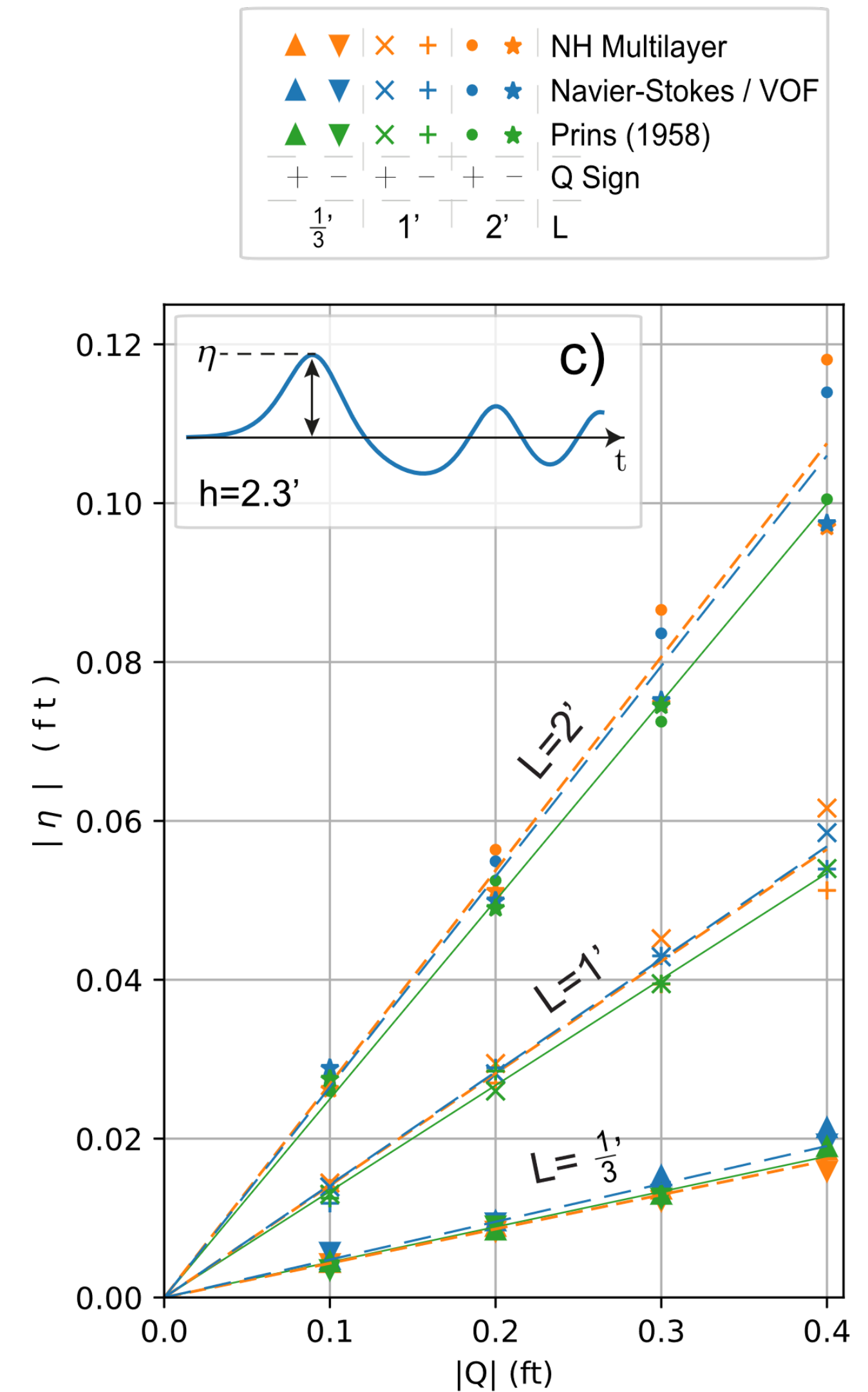
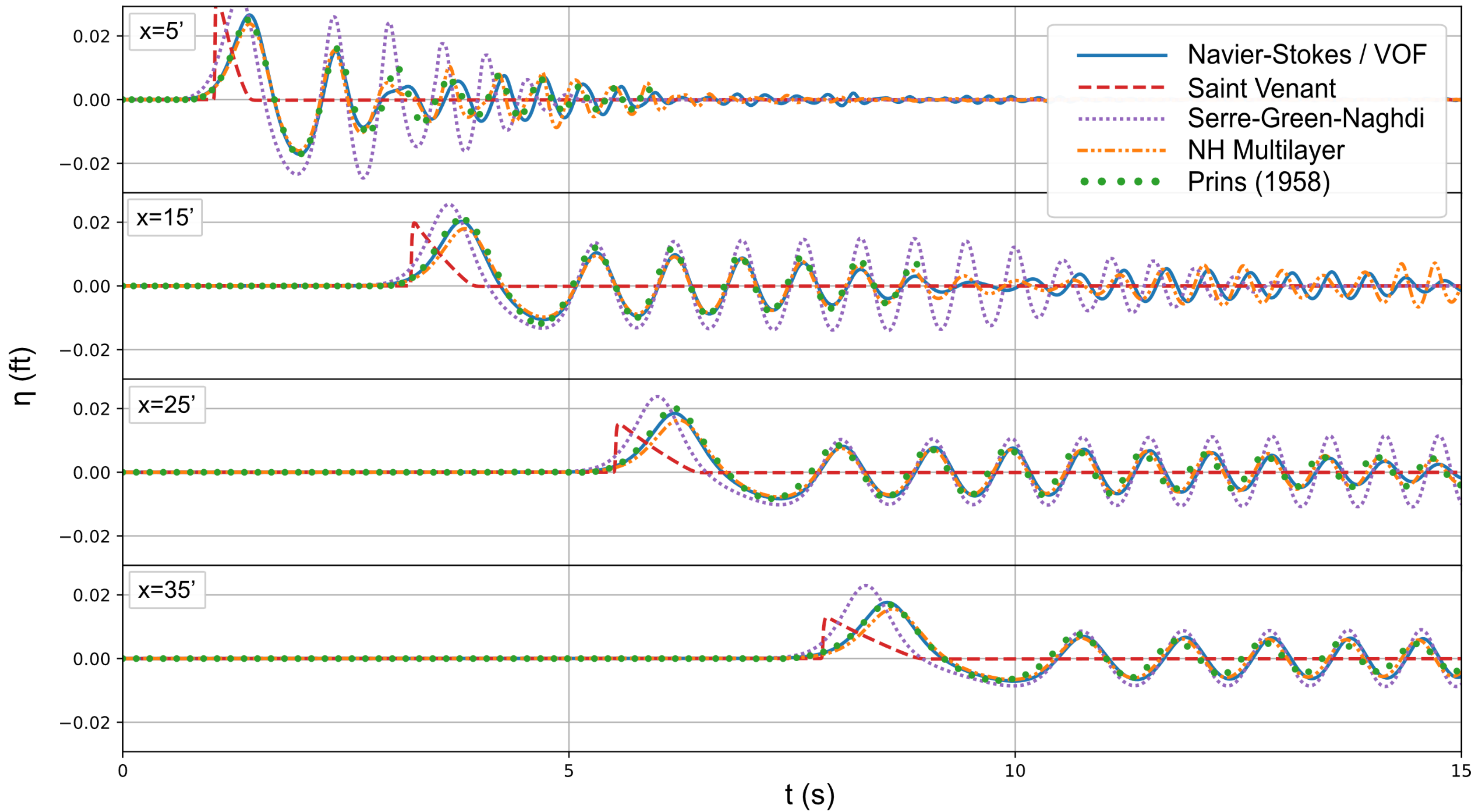


Lab-scale

- Flume experiment of waves generated by elevated / depressed disturbance
- Varied volume disturbance L , Q , and water levels h
- To model different surface wave regimes

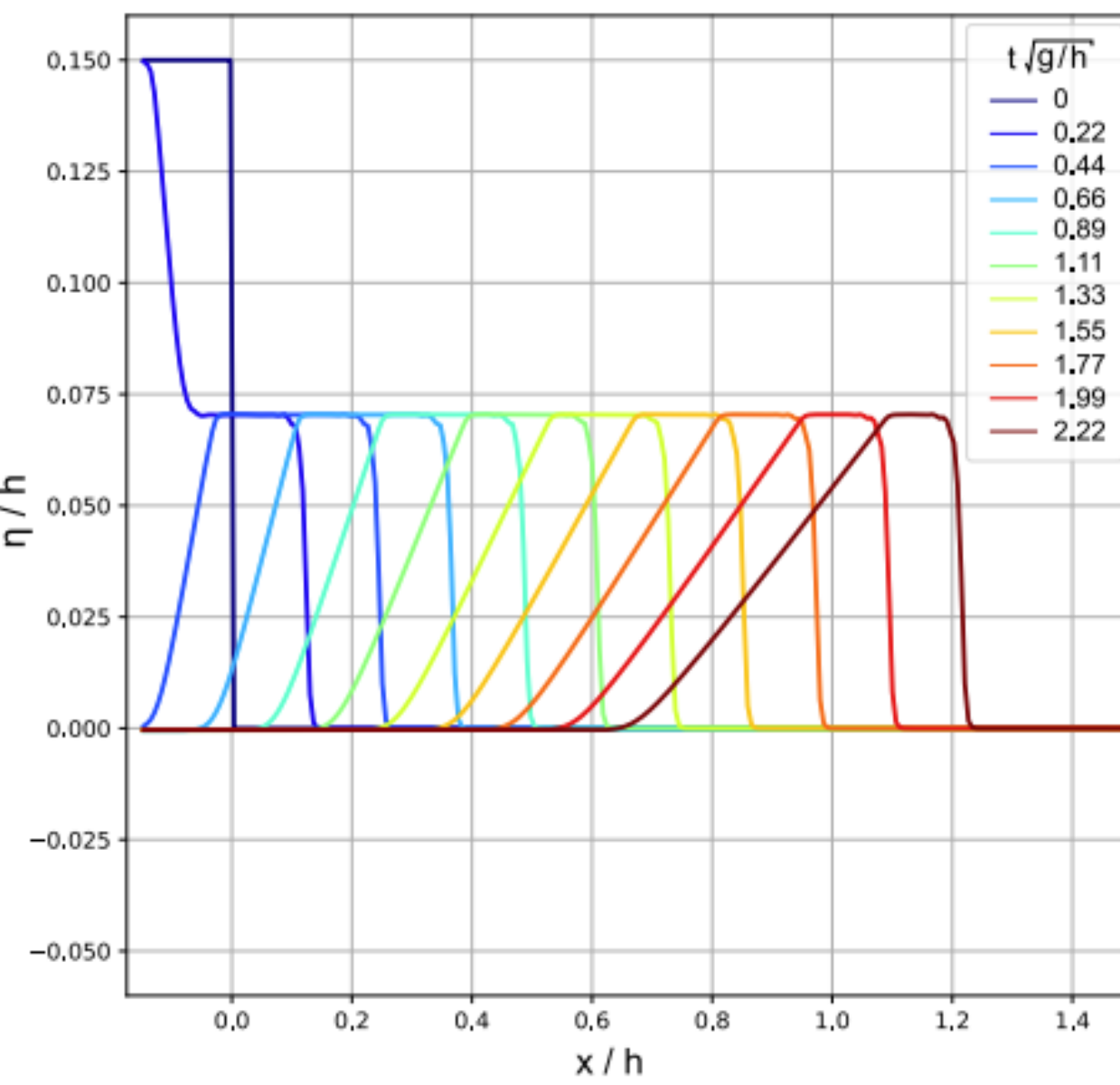


- Model comparison



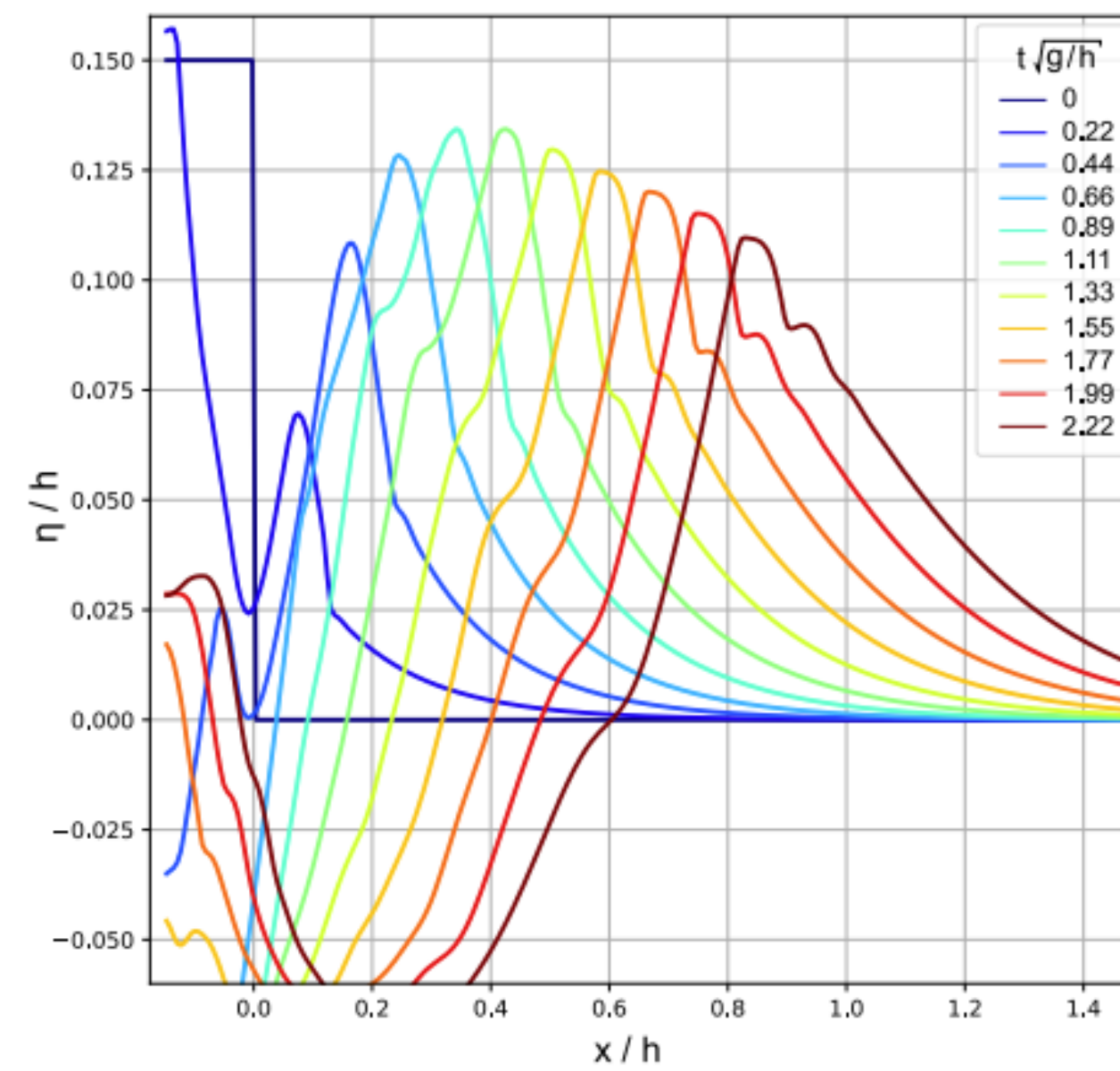
Shallow Water Equations

a) Saint Venant

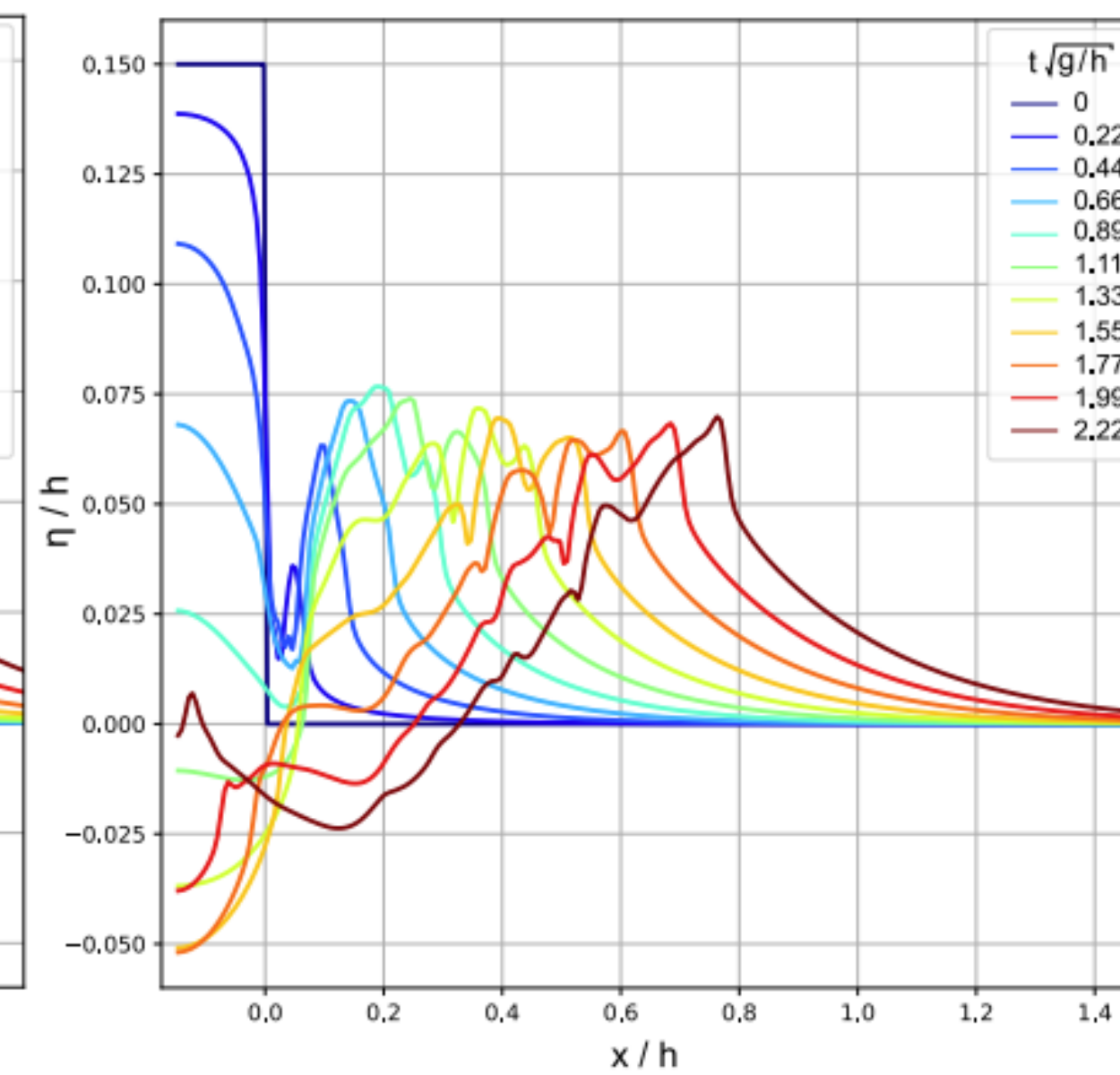


Boussinesq-type

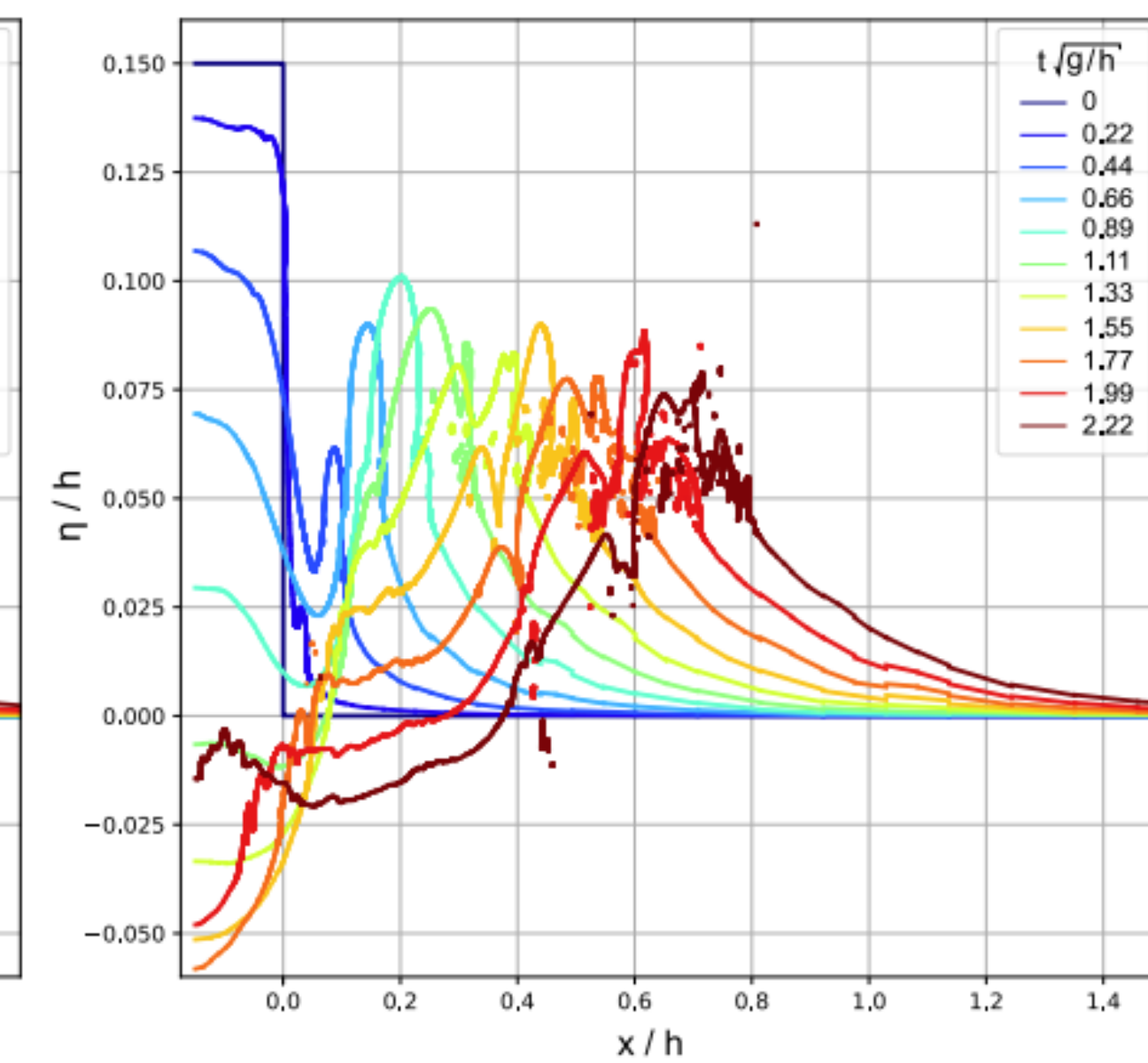
b) Serre-Green-Naghdi

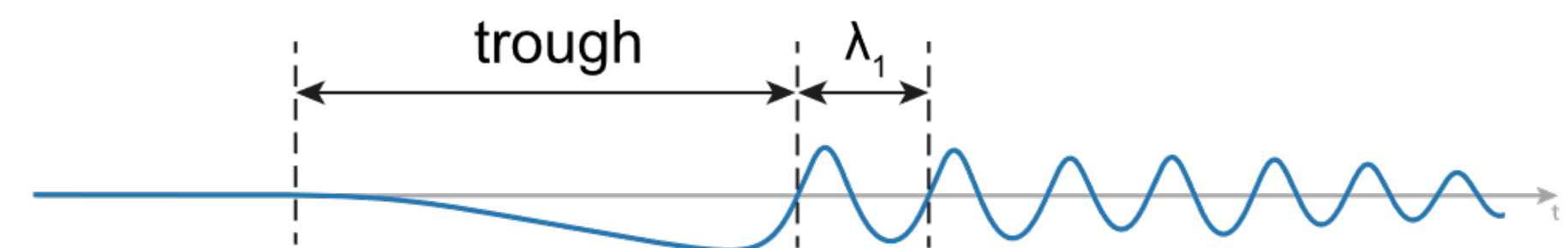
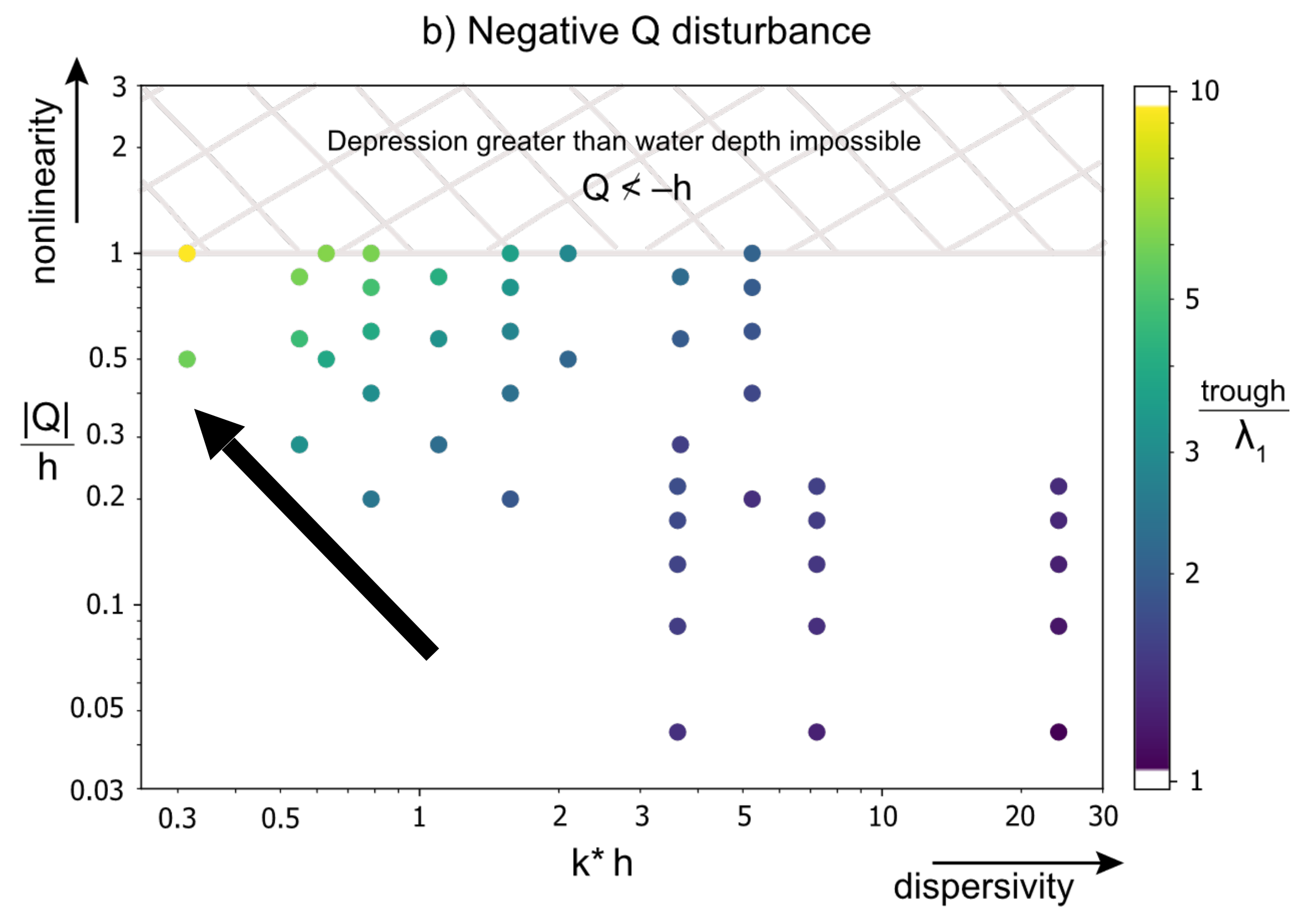
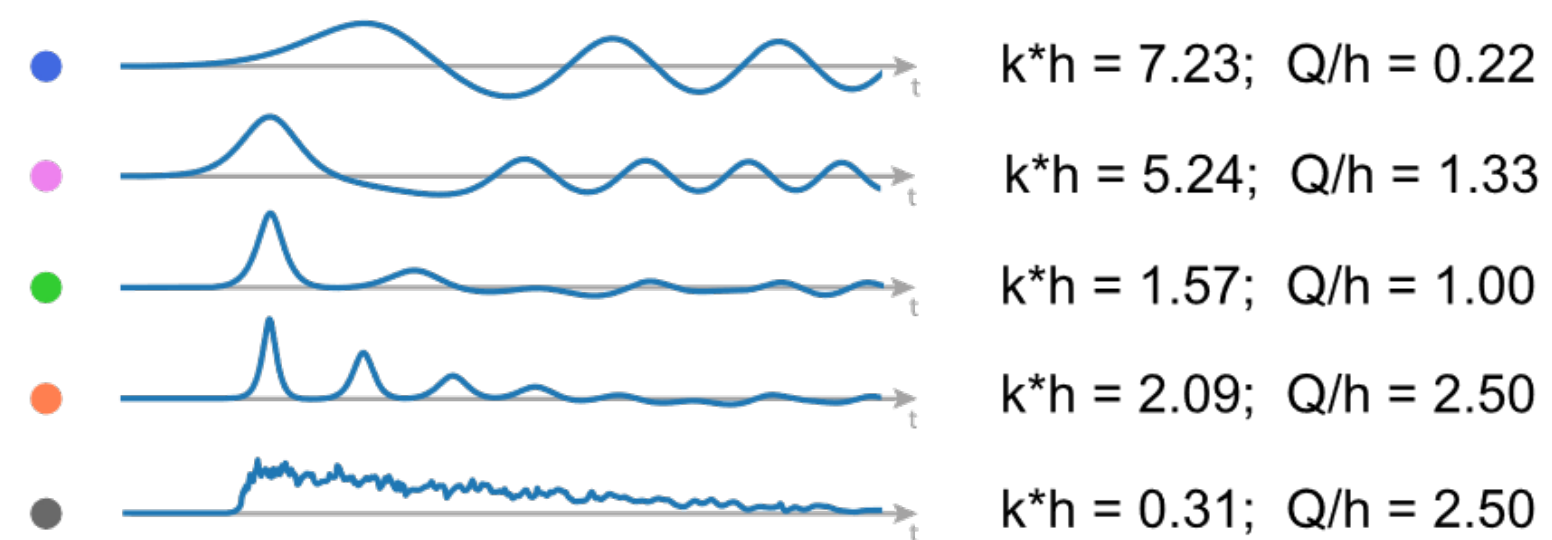
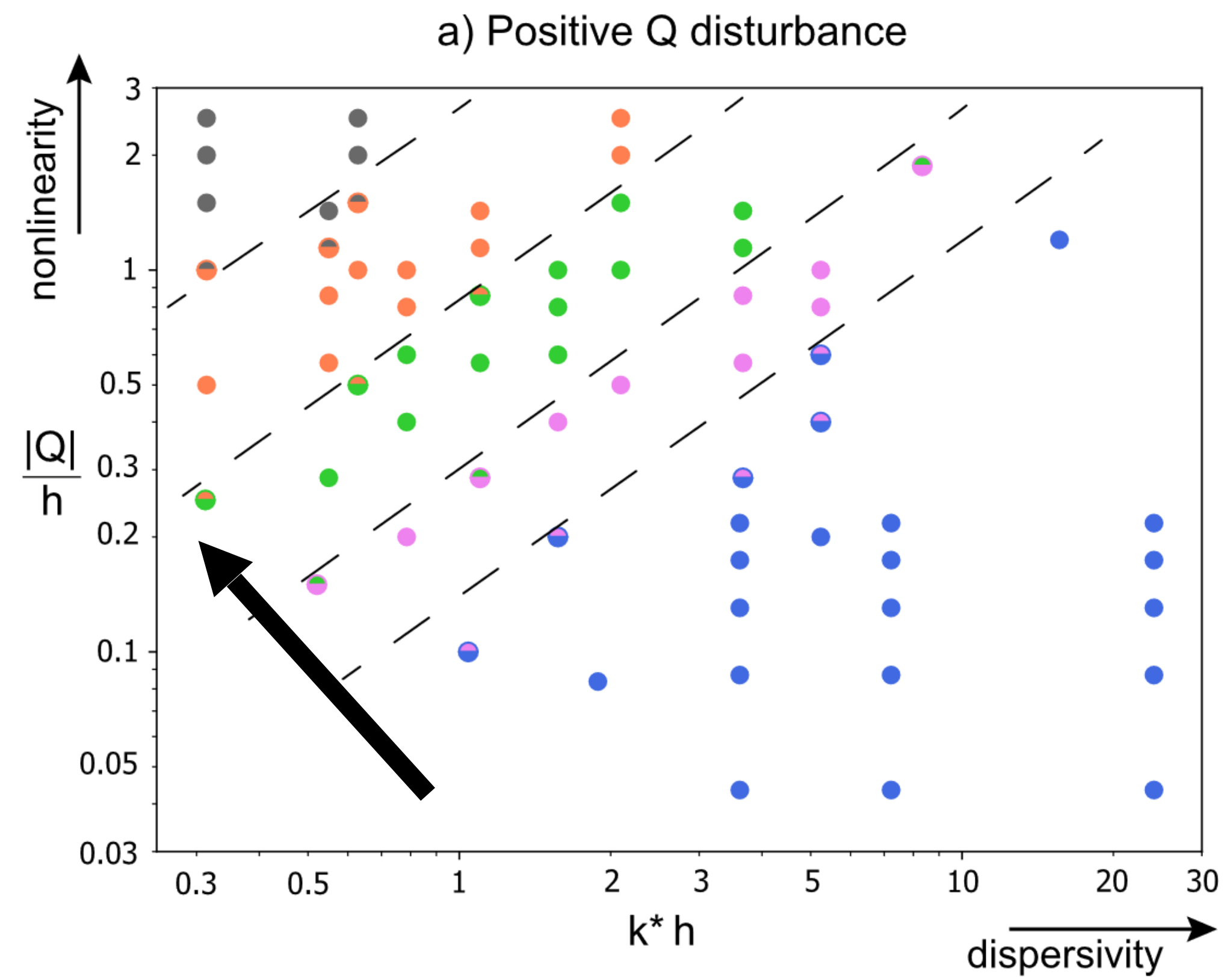


c) Non-Hydrostatic Multilayer



d) Navier-Stokes / VOF

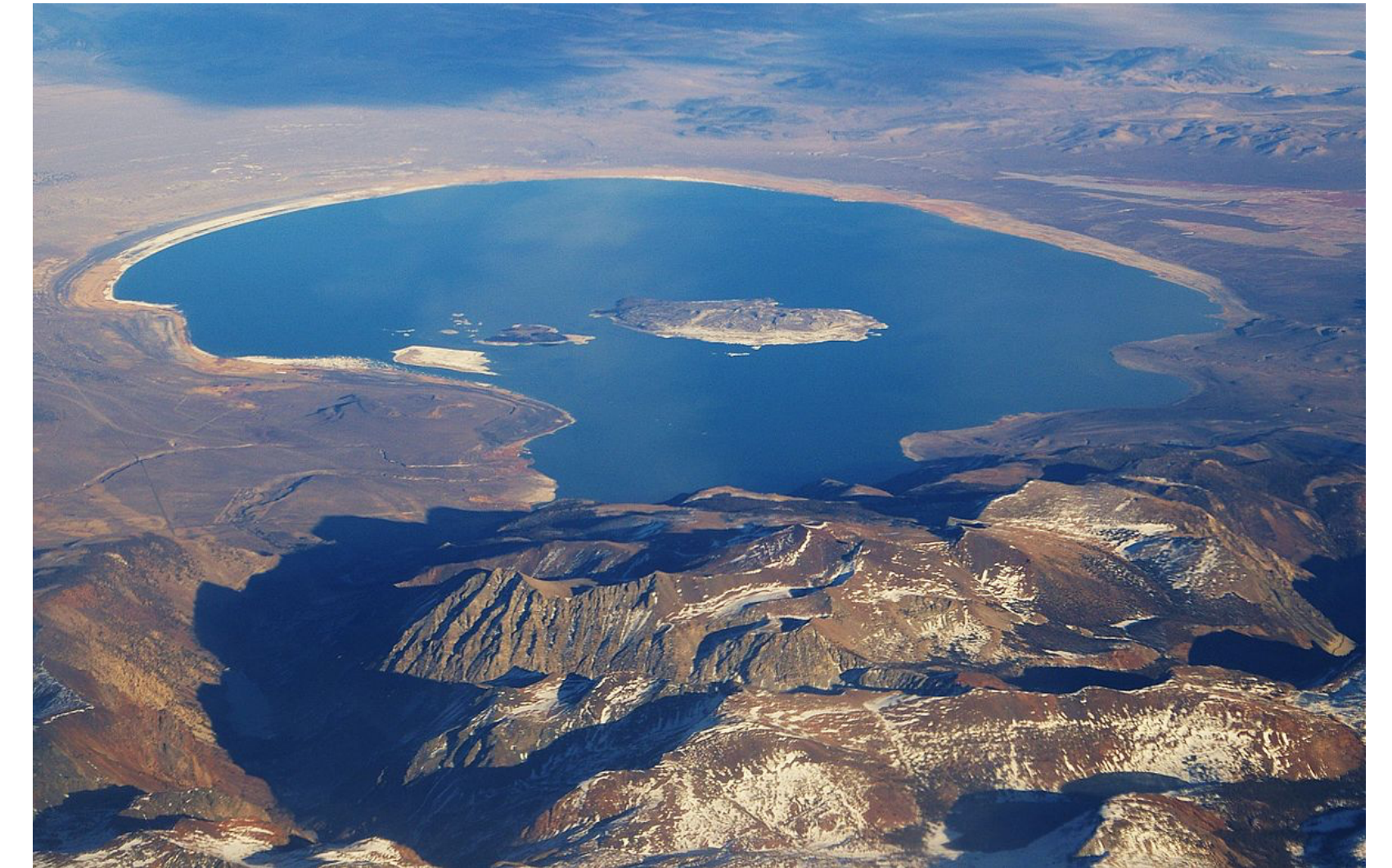
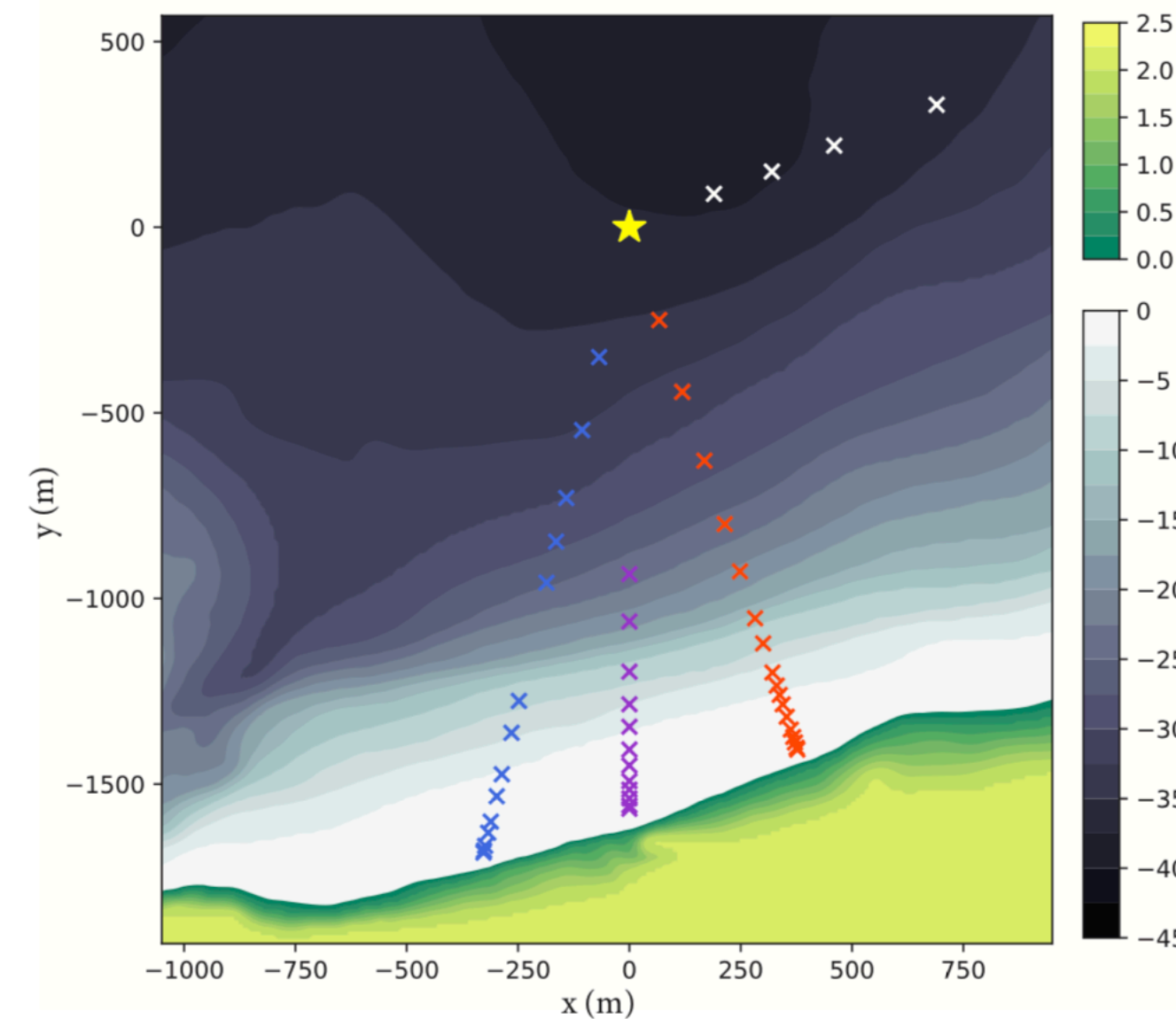
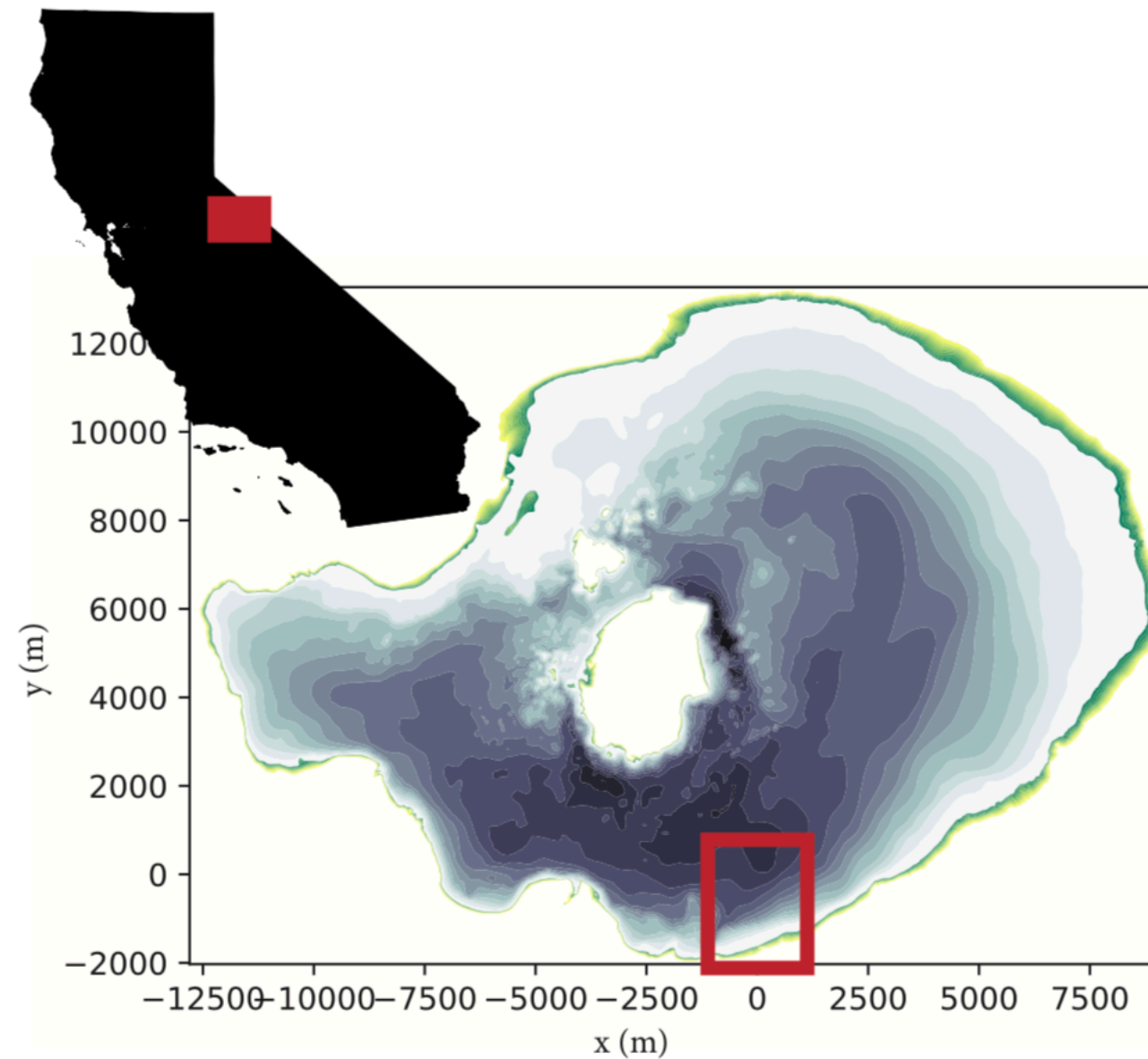




Field-scale

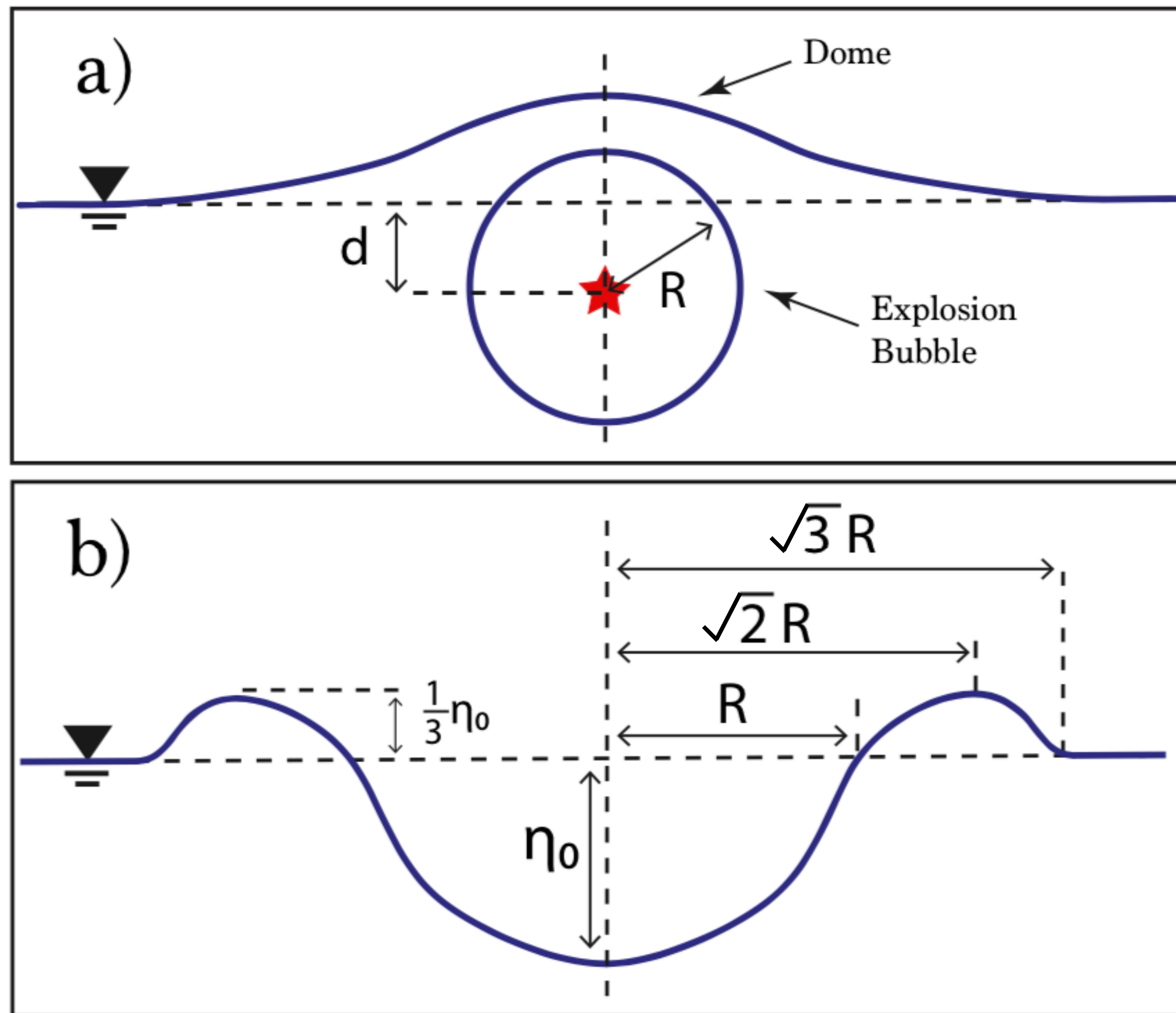
Mono Lake

- U.S. military test series investigating explosive wave generation in 1965

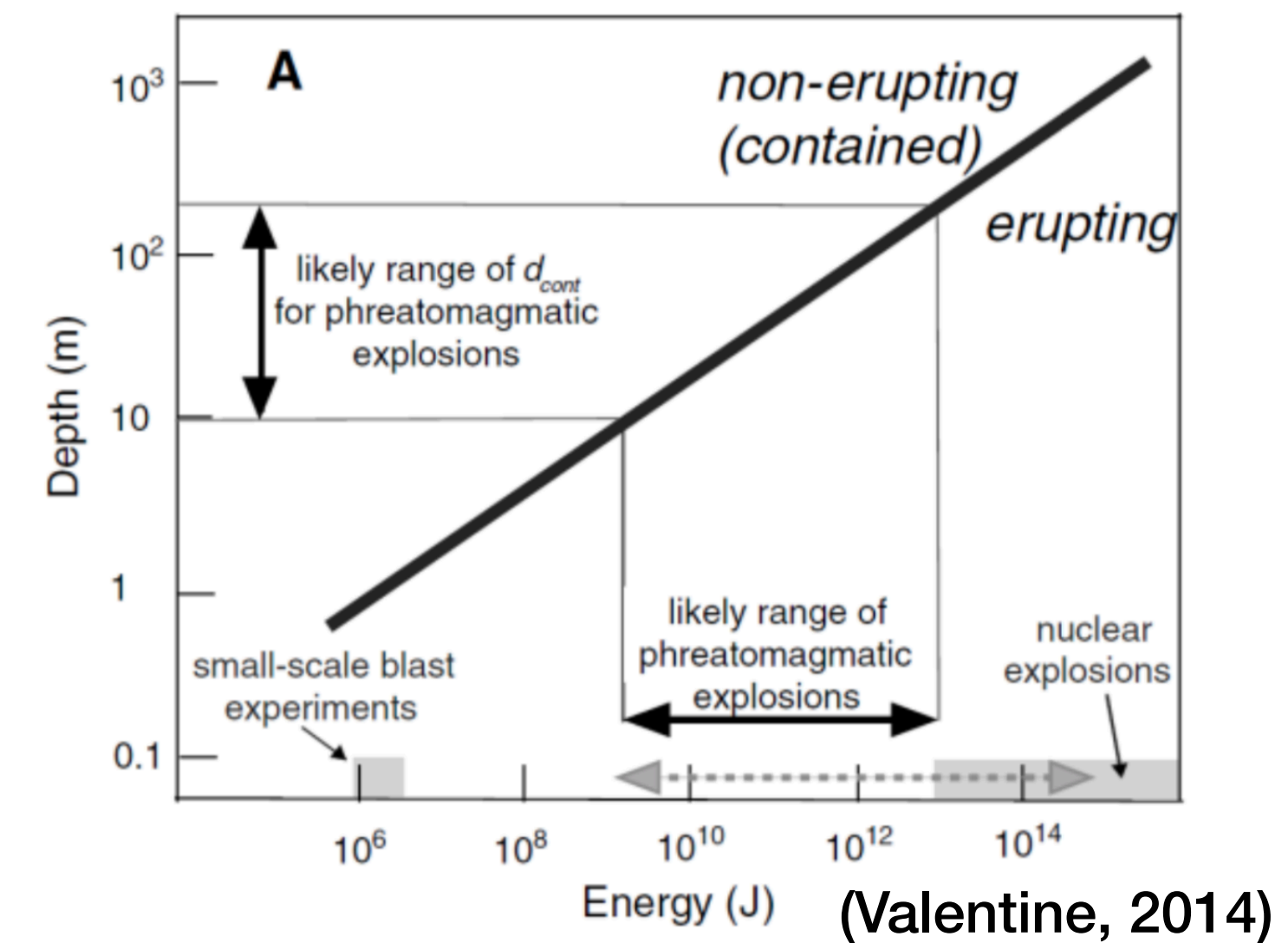


- Ten 9,250 lb charges at range of depths





(from Safari et al., 2019)



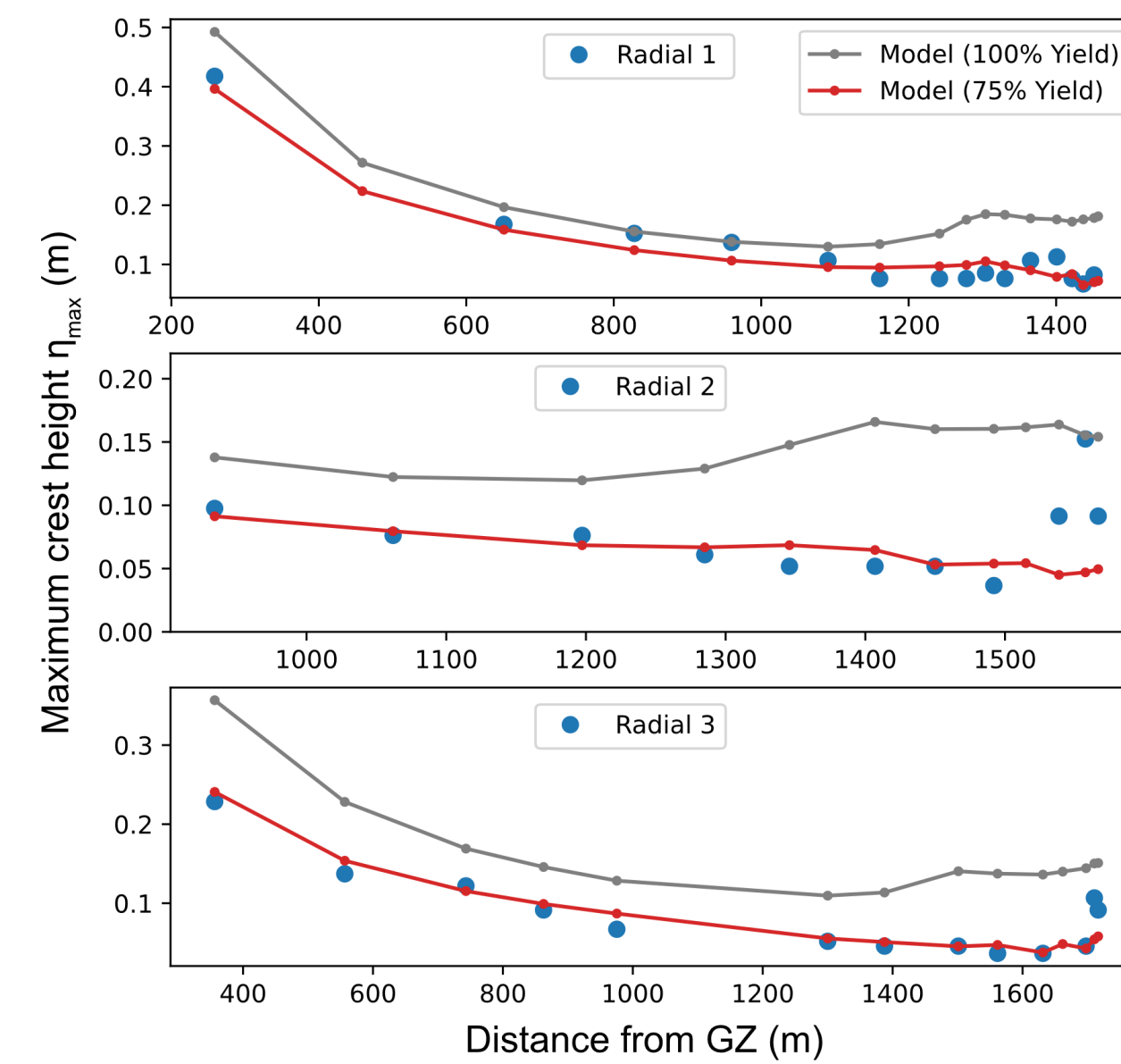
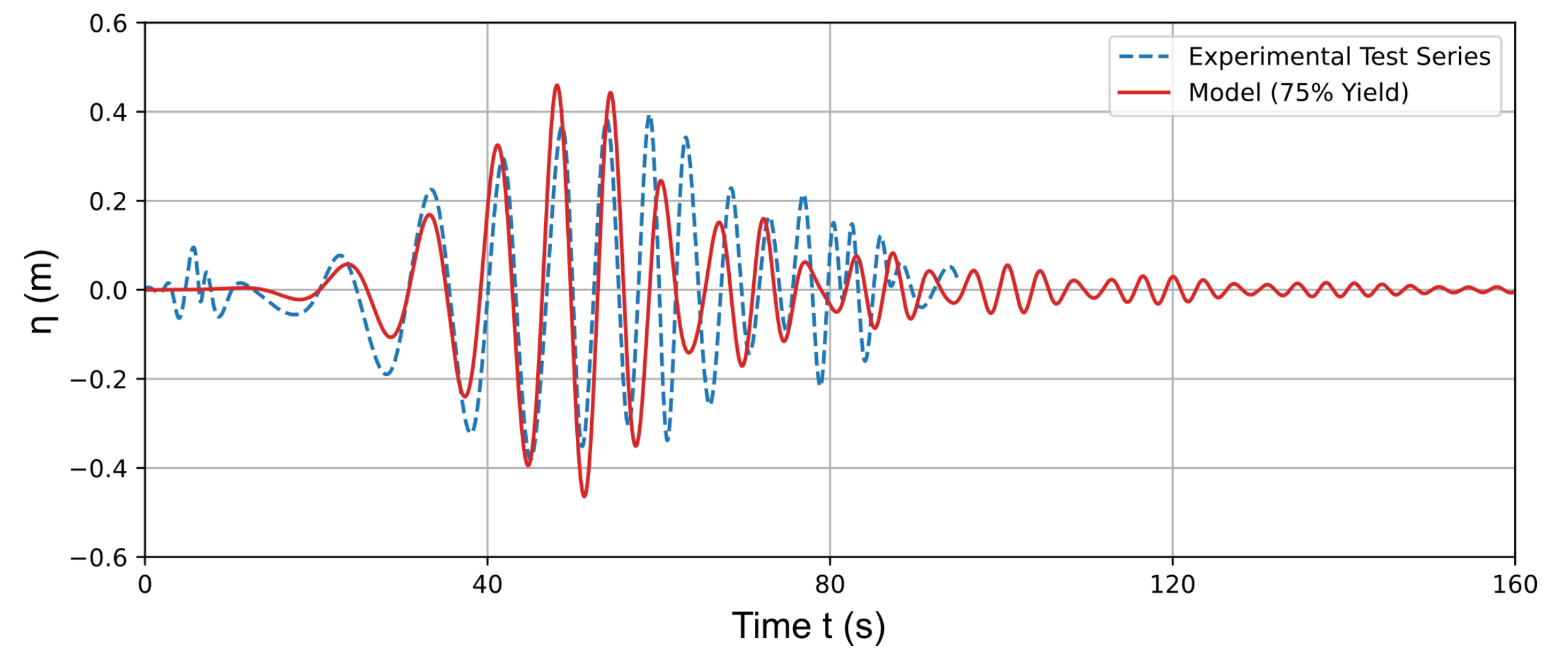
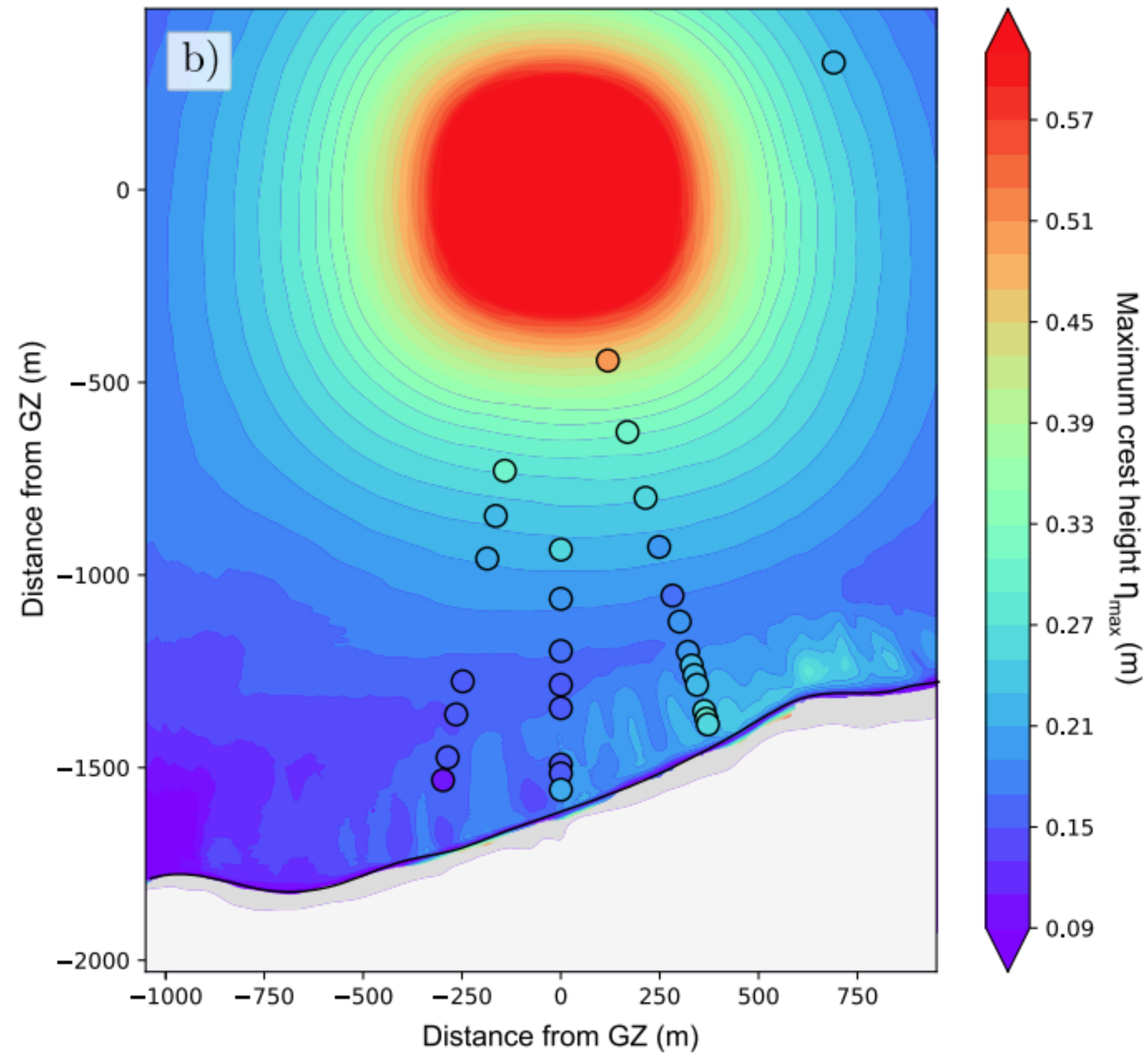
$$\eta_c = aE^{\frac{6}{25}}$$

$$R = bE^{\frac{3}{10}}$$

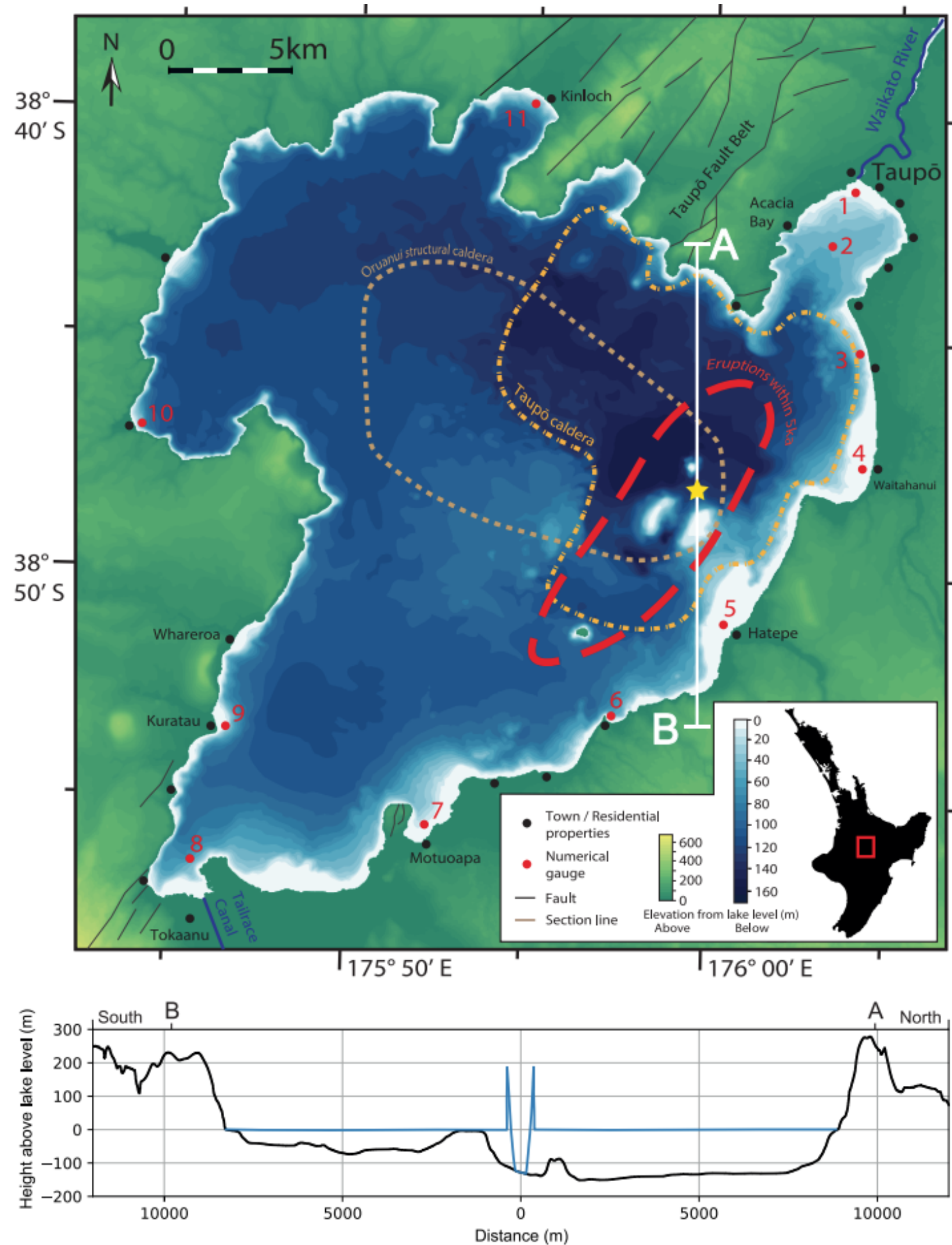
Depth classification

$$a = \begin{cases} 0.0956, & 0 < \frac{d}{E^{\frac{3}{10}}} \leq 10^{-3} \\ 0.0470, & 10^{-3} < \frac{d}{E^{\frac{3}{10}}} \leq 3 \times 10^{-2} \end{cases}$$

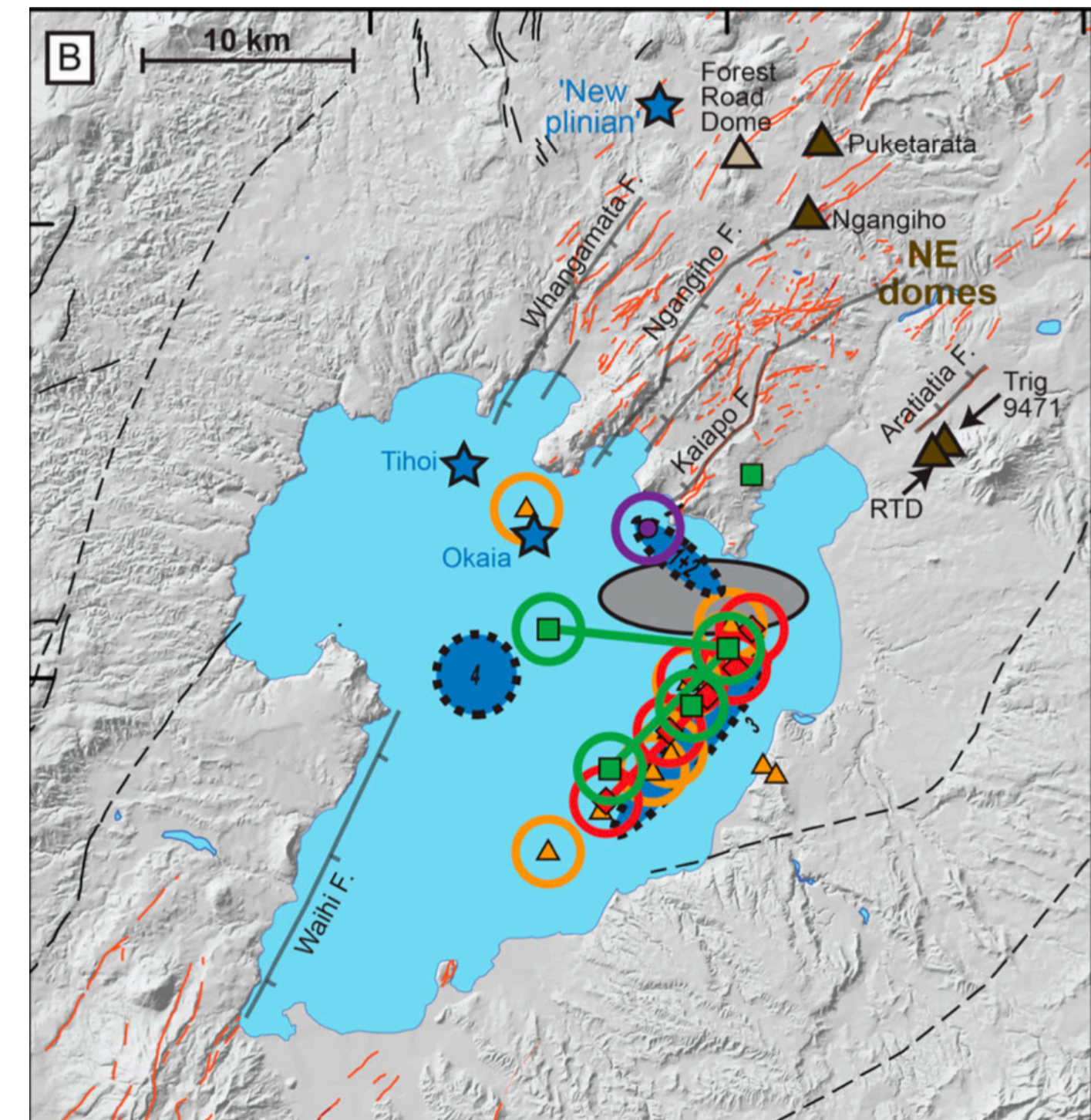
$$b = \begin{cases} 0.1248, & 0 < \frac{d}{E^{\frac{3}{10}}} \leq 10^{-3} \\ 0.1408, & 10^{-3} < \frac{d}{E^{\frac{3}{10}}} \leq 3 \times 10^{-2} \end{cases}$$



Lake Taupō



- Terrain of combined bathymetry model and foreshore LiDAR
- Source location selected in region of Holocene activity (12-1.7 ka)



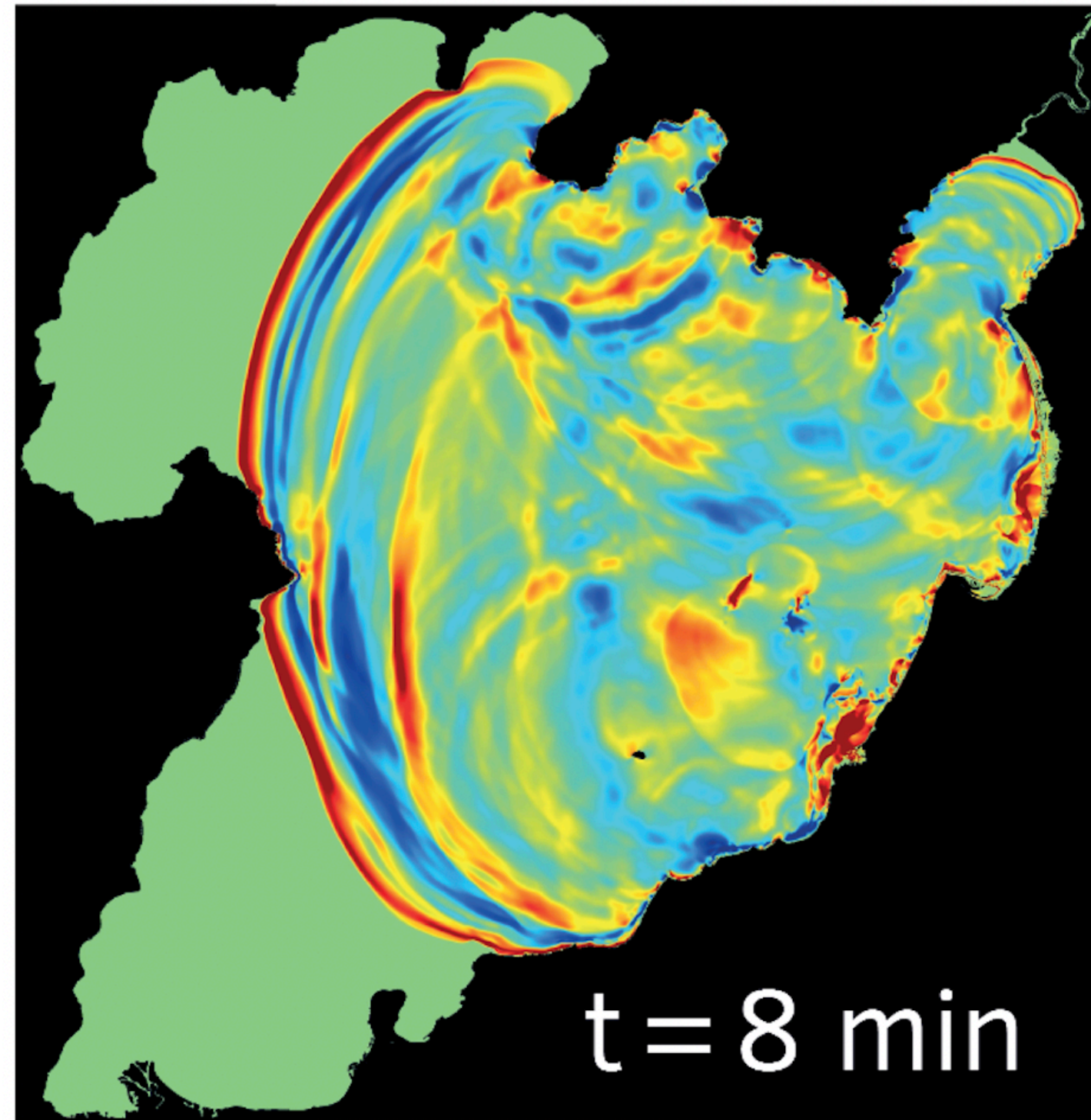
(Barker *et. al*, 2020)

Animation of example multilayer simulation:

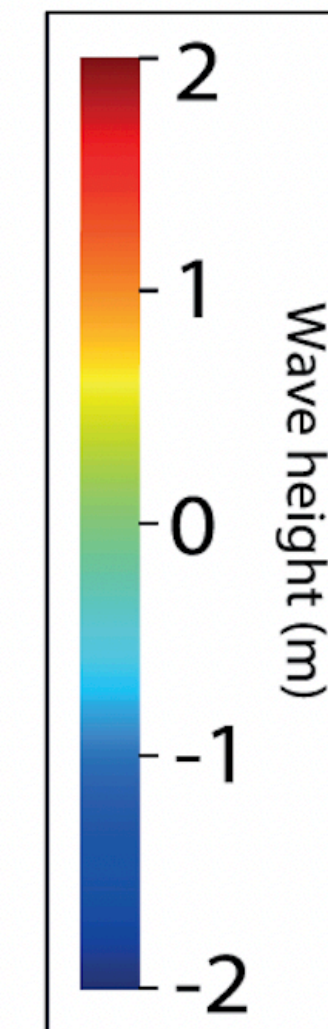
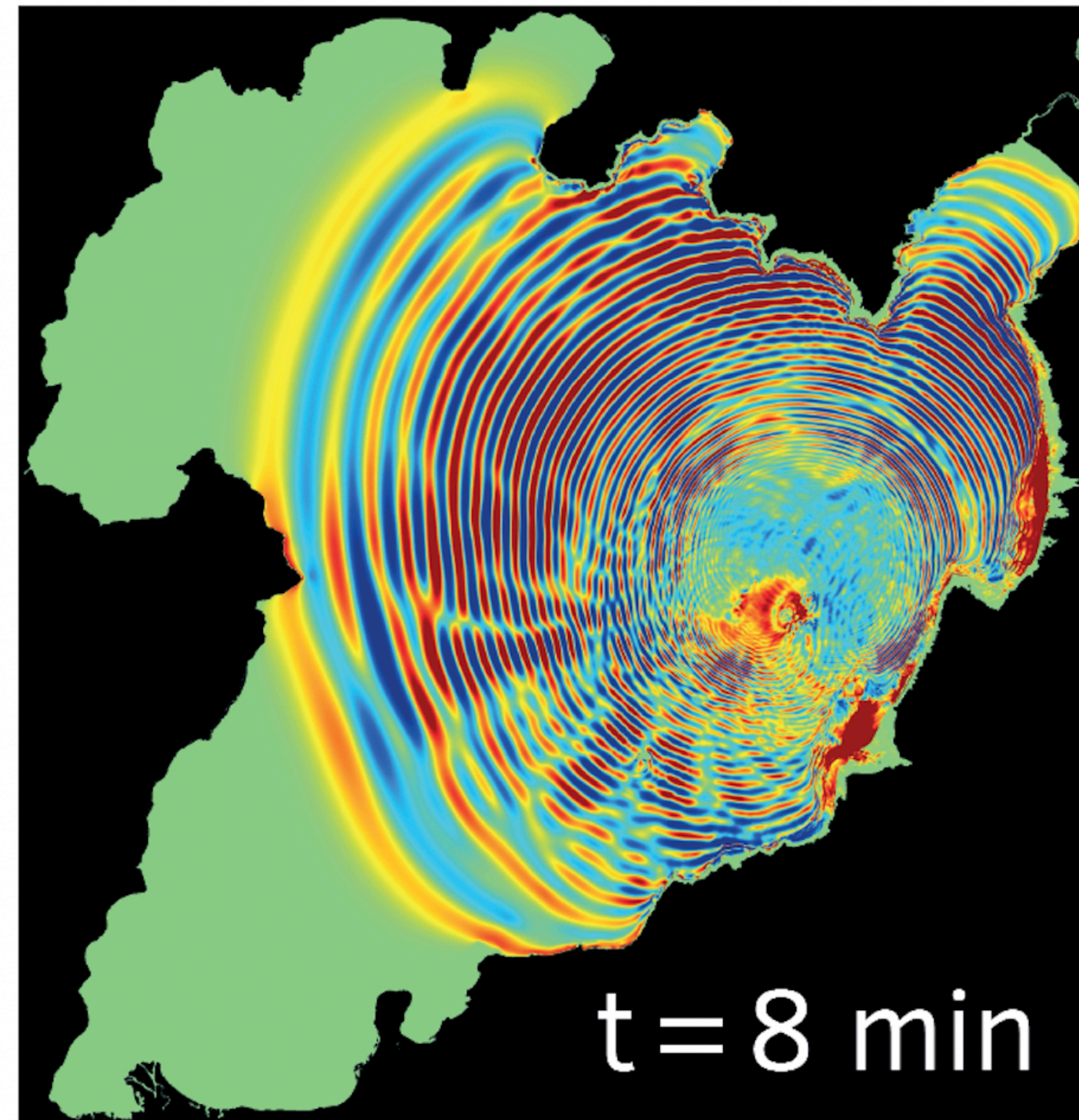
High quality video linked on EGU21 under this abstract.

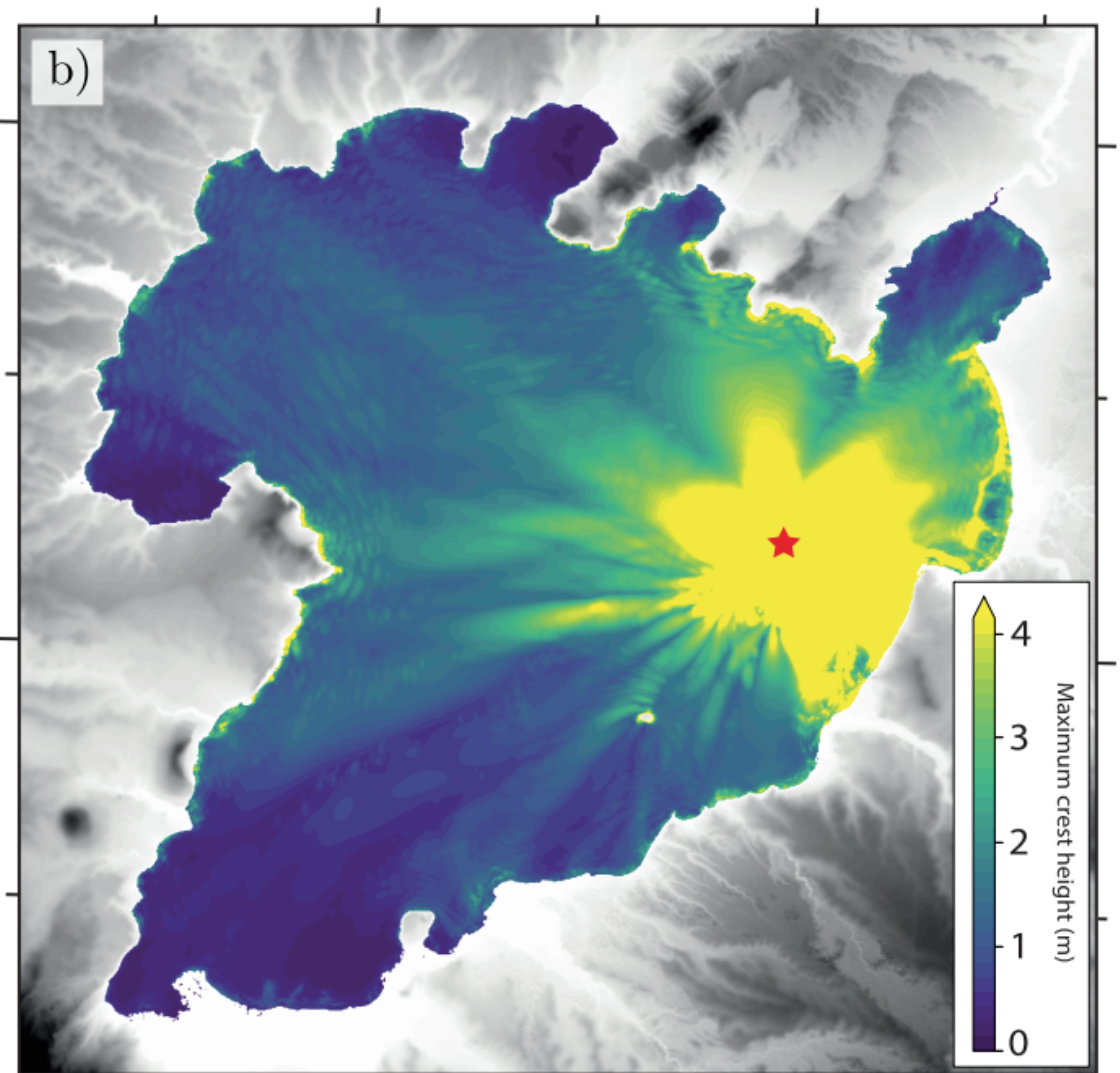
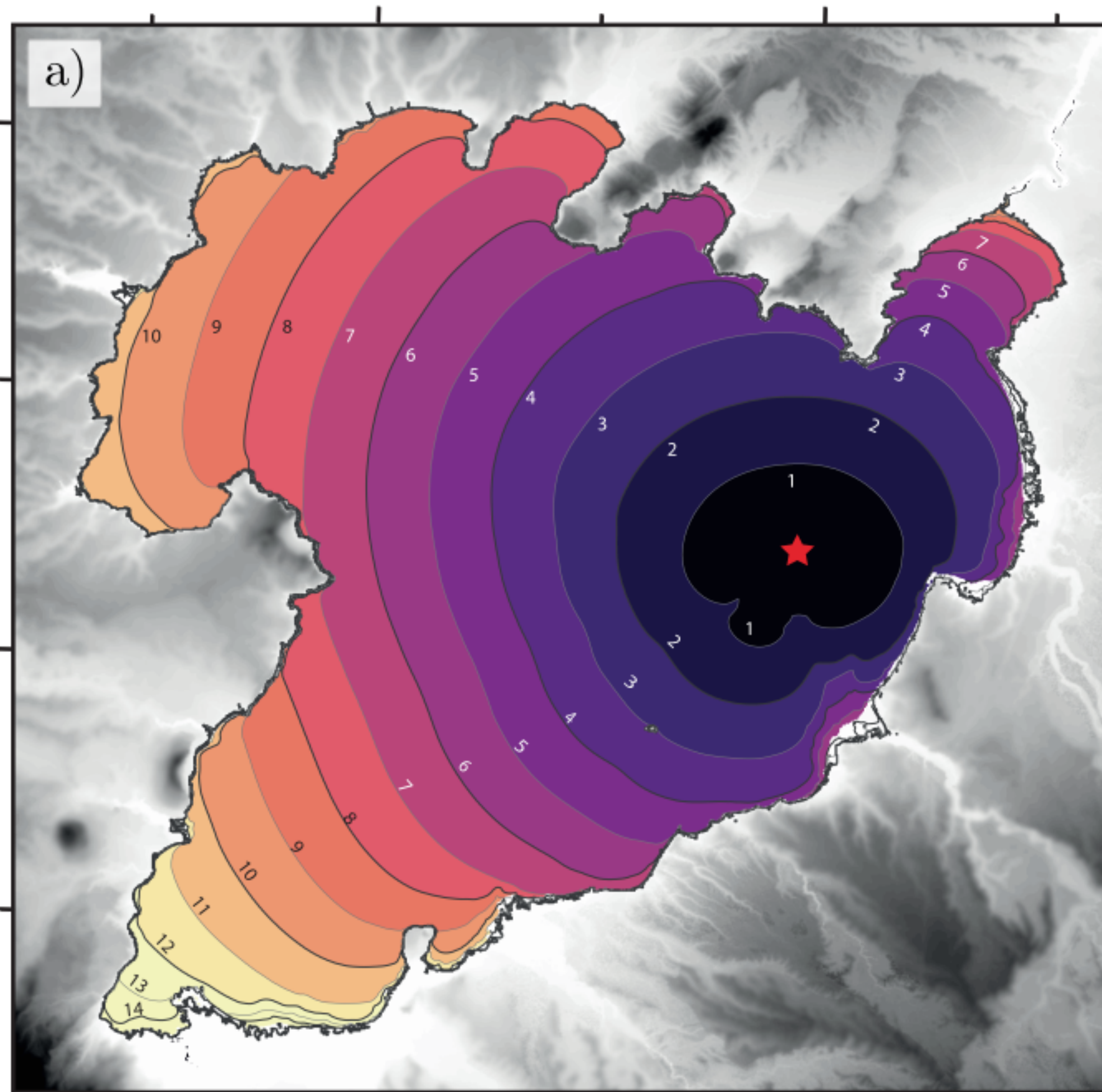
<https://meetingorganizer.copernicus.org/EGU21/EGU21-7080.html>

Saint Venant



Non-Hydrostatic Multilayer





- Outputs include maximum wave heights; numerical gauges; inundated zones

Upcoming

- Study depth
 - Model sensitivity
 - Hazard analysis and implications
- Towards more realistic volcanic scenarios
 - e.g. time dependent sources such as pulsing, jets, etc.



(D.B. Townsend)

