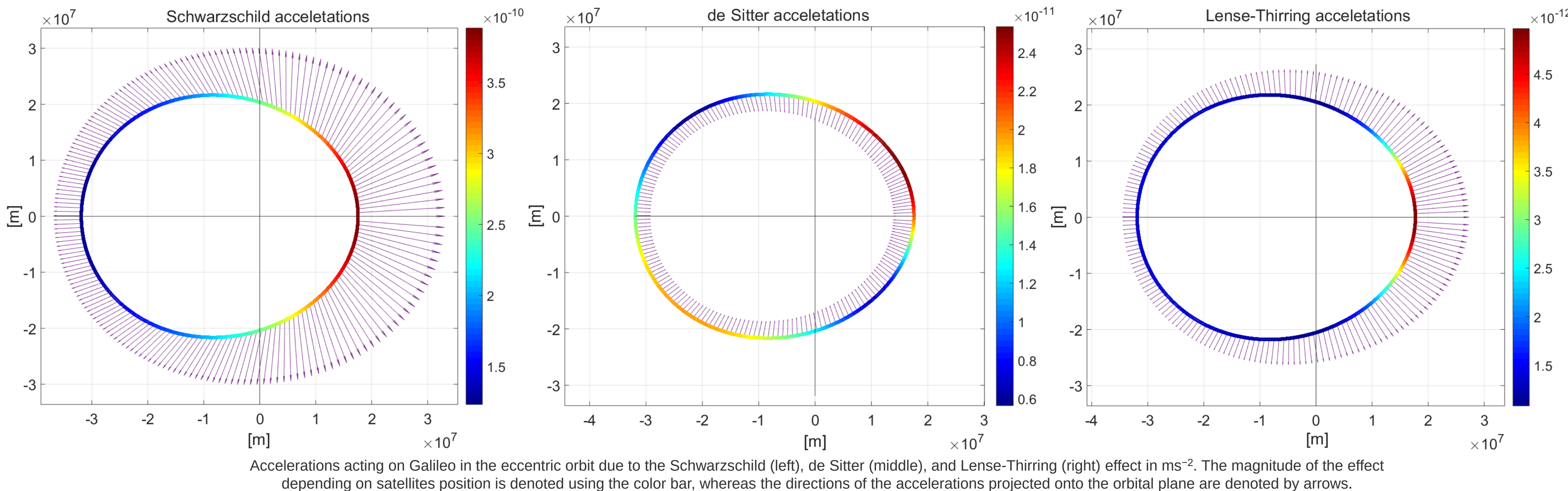


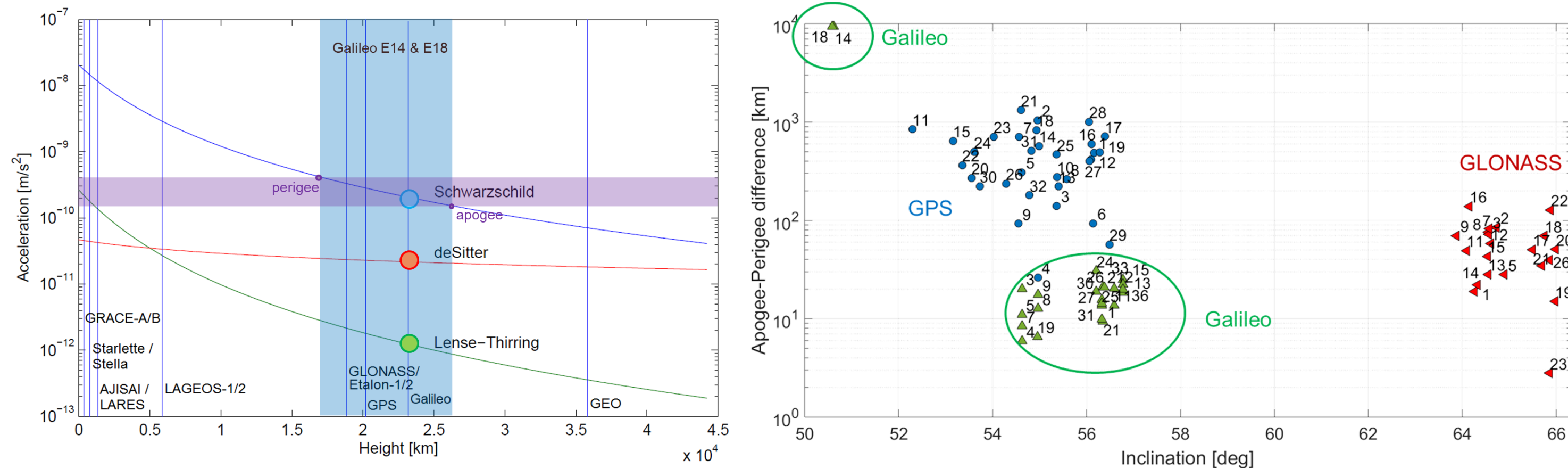


General Relativistic effects acting on GNSS orbits with a focus on Galileo satellites launched into incorrect orbital planes



Three orbital effects emerging from general relativity are typically considered for Earth-orbiting satellites: the **Schwarzschild** effect, **Lense-Thirring** effect or frame-dragging, and the **de Sitter** or geodetic precession effect. For circular orbits and short satellite orbital arcs, the dominating Schwarzschild effect is difficult to determine, because it causes a constant radial acceleration which can be absorbed by a small modification in the gravitational constant GM term or a constant offset in the estimated semi-major axis of a satellite orbit. To separate the effects caused by the Schwarzschild effect from other orbital effects, especially those emerging from orbit modeling issues of non-gravitational accelerations, eccentric satellite orbits should be used.

The first pair of satellites belonging to the Galileo satellite system was accidentally launched into non-circular orbits with height variations between from 17,180 km for the perigee to 26,020 km for the apogee. The eccentric orbits introduced new opportunities for the verification of the effects emerging from general relativity when employing the Galileo constellation. We derive analytical formulas for perturbations of Earth satellite orbits due to general relativity effects and verify which effects are detectable using the current accuracy of GNSS orbits.



Mean offset of the semi-major axis (assuming the same value of the gravitational constant G for Newtonian and relativistic orbits):

$$\Delta a_{Sch} = -4 \frac{GM}{c^2} = -17.74 \text{ mm}$$

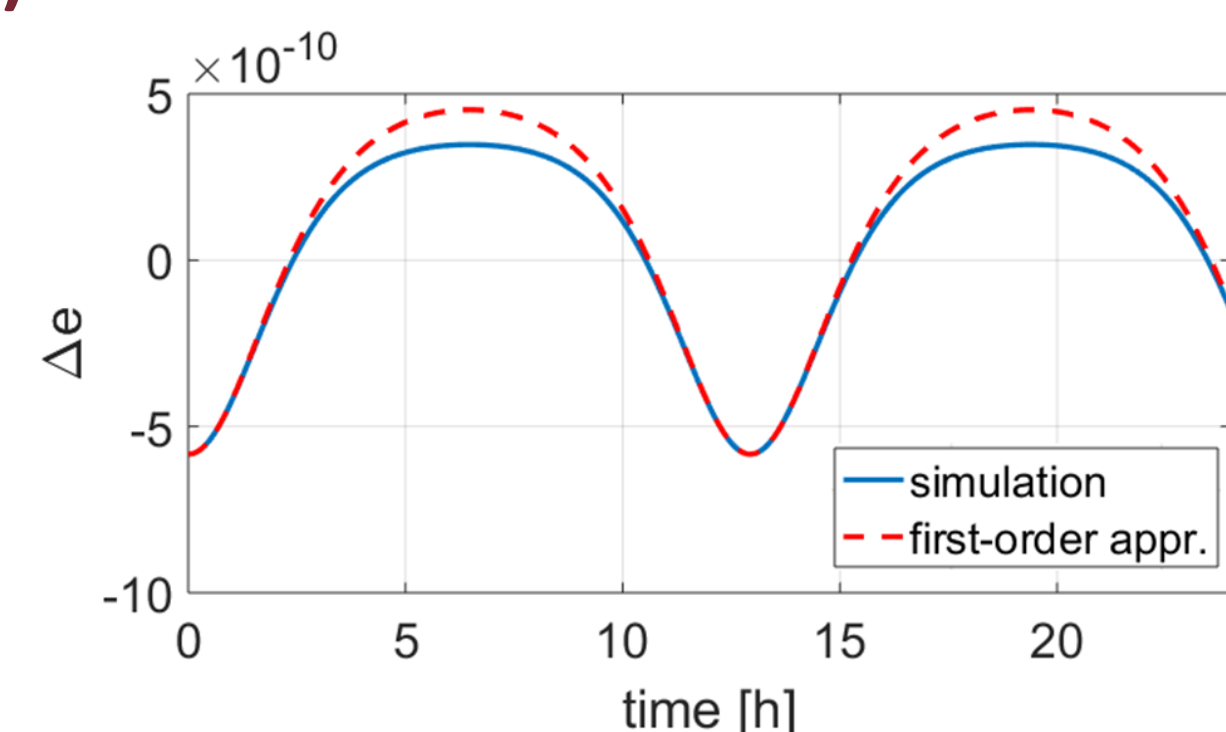
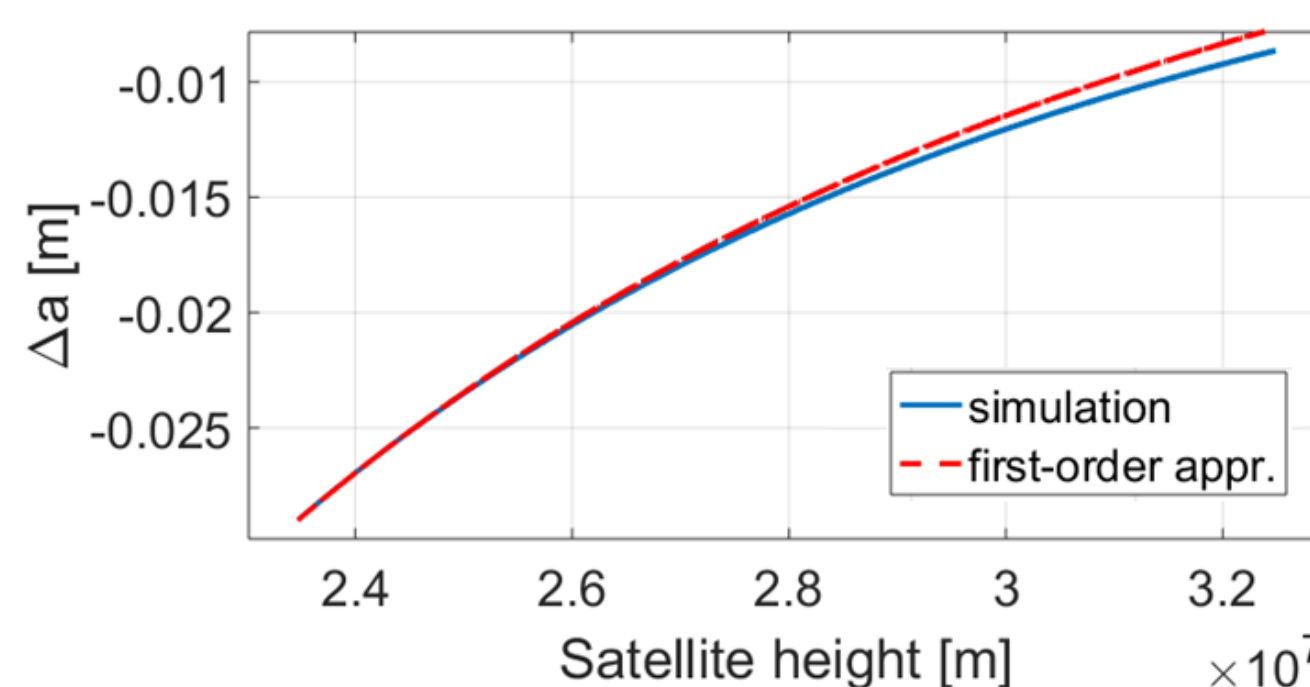
Periodical variations of the semi-major axis (based on first-order Gaussian perturbations):

$$\Delta a_{Sch} = \frac{GM}{c^2(1-e^2)^2} [(-14-6e^2)e \cos u - 5e^2 \cos 2u]$$

Periodical variations of the eccentricity (based on first-order Gaussian perturbations):

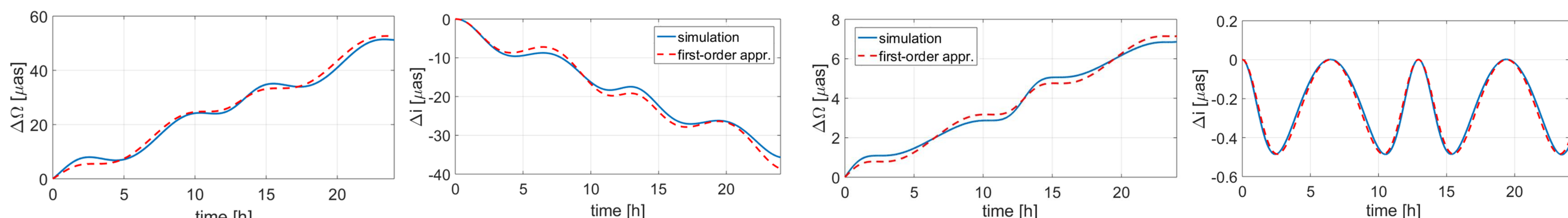
$$\Delta e_{Sch} = -\frac{GM}{c^2 a e (1-e^2)} [(3+7e^2)e \cos u + \frac{5}{2}e^2 \cos 2u]$$

Orbit shape & size changes (Schwarzschild)



Semi-major axis (left) and eccentricity (right) changes for Galileo E14 in the eccentric orbit due to the Schwarzschild effect.

Orbit orientation change (de Sitter, Lense-Thirring)



Secular rates and periodical variations of the right ascension of the ascending node (Ω) and inclination (i) due to de Sitter effect (dS, left) and Lense-Thirring (LT, right).

For more explanation, see:

Sośnica, K., *et al.* General relativistic effects acting on the orbits of Galileo satellites. *Celest Mech Dyn Astr* **133**, 14 (2021). <https://doi.org/10.1007/s10569-021-10014-y>

Summary:

We conducted simulations of orbit perturbations due to the main effects emerging from general relativity. We derive the analytical formulas of temporal variations of Keplerian parameters based on the first-order Gaussian perturbations and compare how well they agree with the observed perturbations from numerical simulations. The Schwarzschild effect introduces a constant and periodic changes of **semi-major axis and revolution period**, periodic variations of **eccentricity**, and a secular motion of **perigee**, all of which exceed the level of 1 mm. The secular rate of $\omega=1.3$ mas/day corresponds even to 167 mm/day at GPS heights.

For Galileo E14/E18 in eccentric orbits, the change of the semi-major axis due to the Schwarzschild effect is **-29.02 mm in perigee and -8.65 mm in apogee**. This gives a difference of more than 20 mm, thus, fully detectable using current GNSS precise orbit determination techniques. We found that the **geodetic precession** strongly depends on the orientation of the ecliptic with respect to the orbital plane and may introduce a secular change of the inclination angle. The mean secular rate of the ascending node, which equals **52.53 $\mu\text{s/day}$** corresponds to a drift of **7.2 mm/day at the Galileo heights after 1 day**.

General Relativistic effects acting on GNSS orbits with a focus on Galileo satellites launched into incorrect orbital planes

Krzysztof Sośnica, Grzegorz Bury, Radosław Zajdel, Kamil Kaźmierski,
Javier Ventura-Traveset, Roberto Prieto Cerdeira, and Luis Mendes

General Relativity and Earth artificial satellites – IERS Conventions 2010

Orbit effects

Schwarzschild

Lense-Thirring

De Sitter (Geodetic precession)

Time systems & Clocks

Differences between Barycentric Coordinate Time (TCB), Geocentric Coordinate Time (TCG), and Terrestrial Time (TT)

„Proper time” for satellite clocks estimated using the satellite position and velocity and considering Earth’s oblateness (J_2)

Doppler frequency & time shifts of clocks due to Earth’s rotation and sat. motion (incl. Sagnac)

Signal propagation

Gravitational delay (Shapiro effect)

Secondary effects (J_2 and higher-term effects)

PPN: parameterized post-Newtonian approach is typically employed for artificial Earth satellites; Corrections for VLBI and Lunar Laser Ranging are performed in Barycentric Celestial Reference Frame.

IERS Conventions 2010 and the orbital effects

$$\Delta \vec{r} = \frac{GM}{c^2 r^3} \left\{ \left[2(\beta + \gamma) \frac{GM}{r} - \gamma \vec{r} \cdot \vec{r} \right] \vec{r} + 2(1 + \gamma)(\vec{r} \cdot \vec{r}) \vec{r} \right\} +$$
$$(1 + \gamma) \frac{GM}{c^2 r^3} \left[\frac{3}{r^2} (\vec{r} \times \vec{r})(\vec{r} \cdot \vec{J}) + (\vec{r} \times \vec{J}) \right] +$$
$$\left\{ (1 + 2\gamma) \left[\vec{R} \times \left(\frac{-GM_S \vec{R}}{c^2 R^3} \right) \right] \times \vec{r} \right\}$$

Schwarzschild

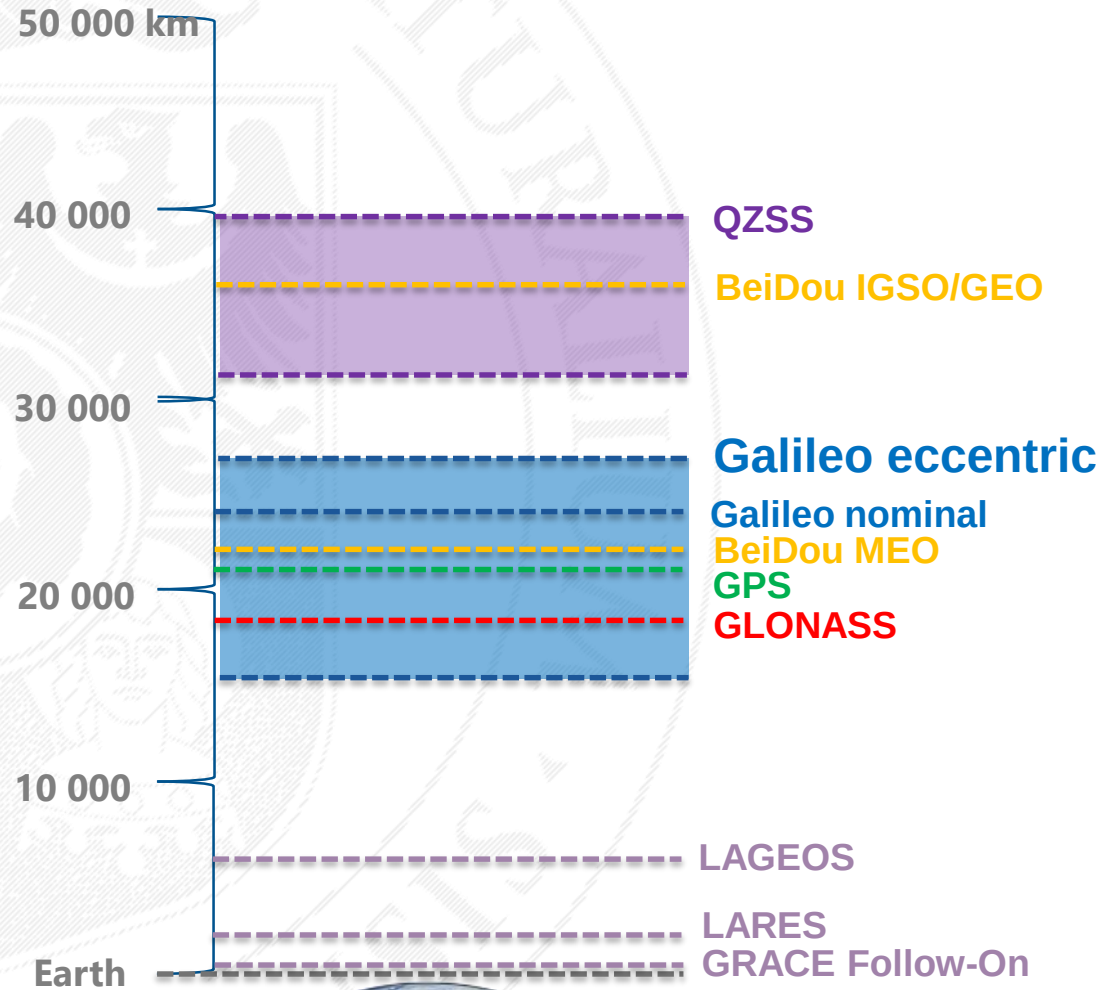
**Lense-Thirring
frame dragging**

**De Sitter
geodetic precession**

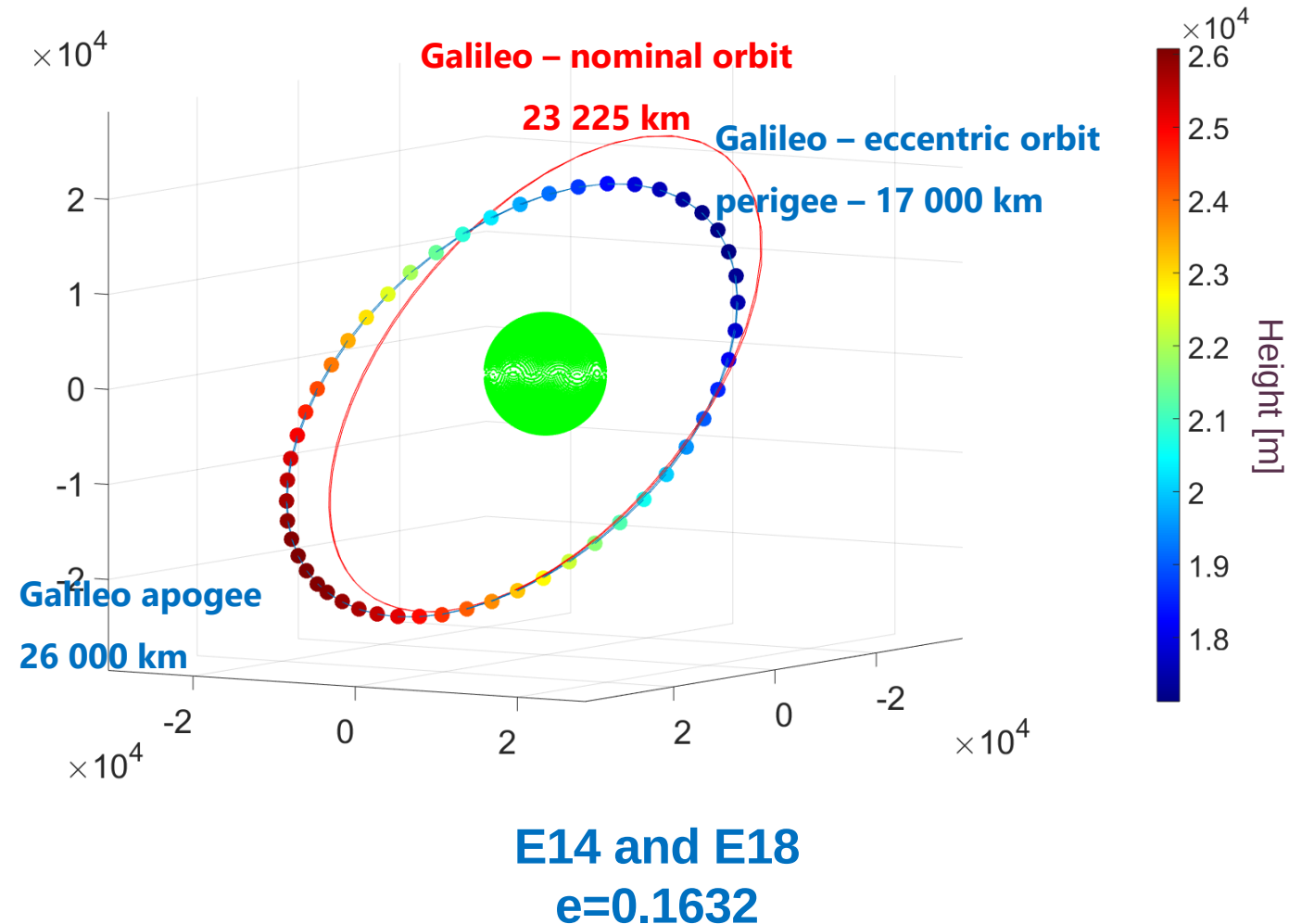
where: GM - standard Earth gravitation product; GM_S - standard Sun gravitation product; β, γ - PPN parameters equal to 1 in general relativity; c - speed of light; $\vec{r}, \dot{\vec{r}}$ - position and velocity of a satellite with respect to the geocenter; $\vec{R}, \dot{\vec{R}}$ - position and velocity of the Earth with respect to the Sun; $\vec{J}$ - Earth's angular momentum per unit mass.

Effects are
for Galileo eccentric:
 $2.422 \cdot 10^{-10}$,
 $1.5 \cdot 10^{-12}$,
and **$3.06 \cdot 10^{-11} \text{ m}\cdot\text{s}^{-2}$.**

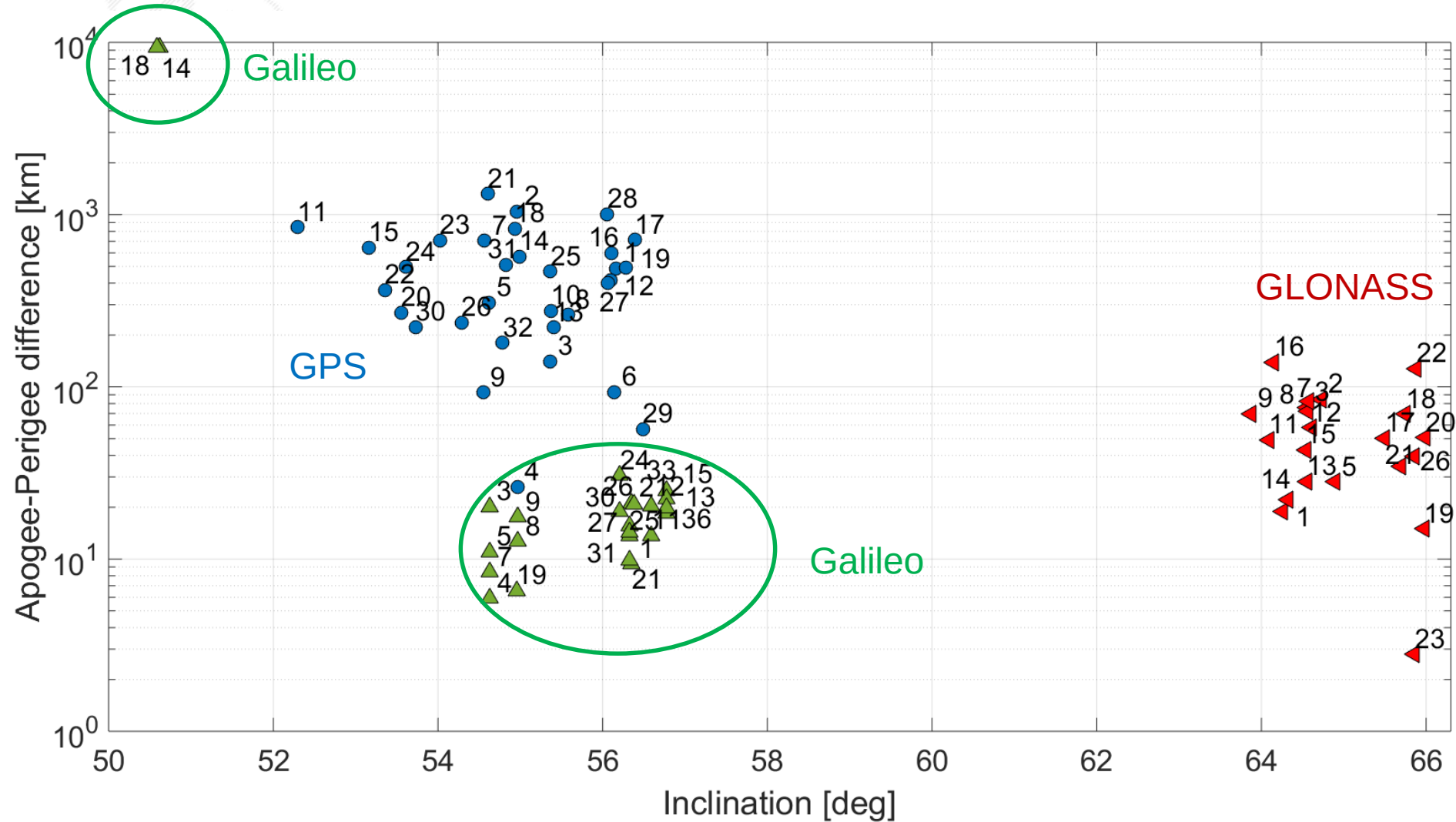
GNSS Constellations



Galileo E14 and E18 in eccentric orbits

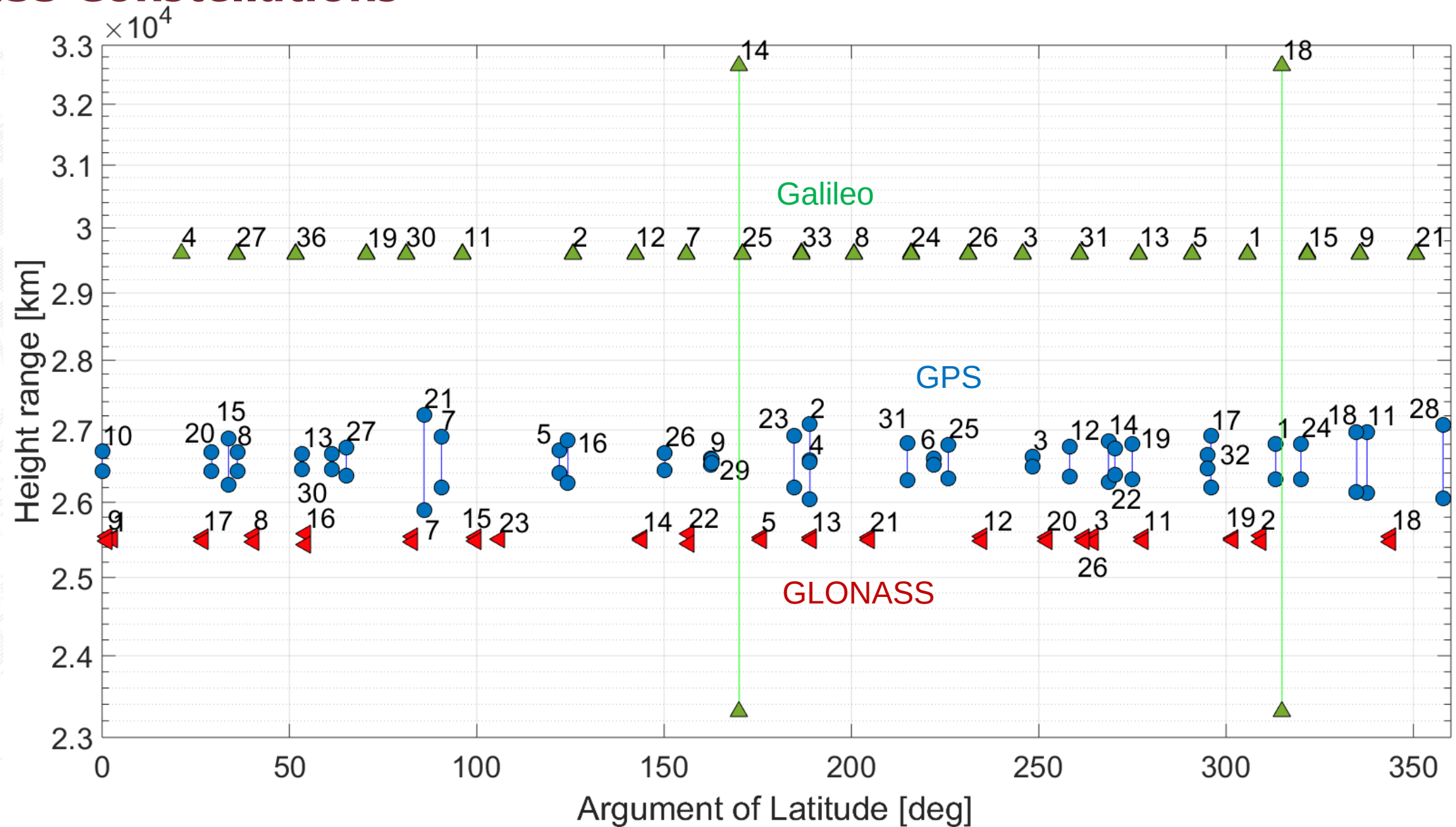


GNSS Constellations – eccentricity and inclination

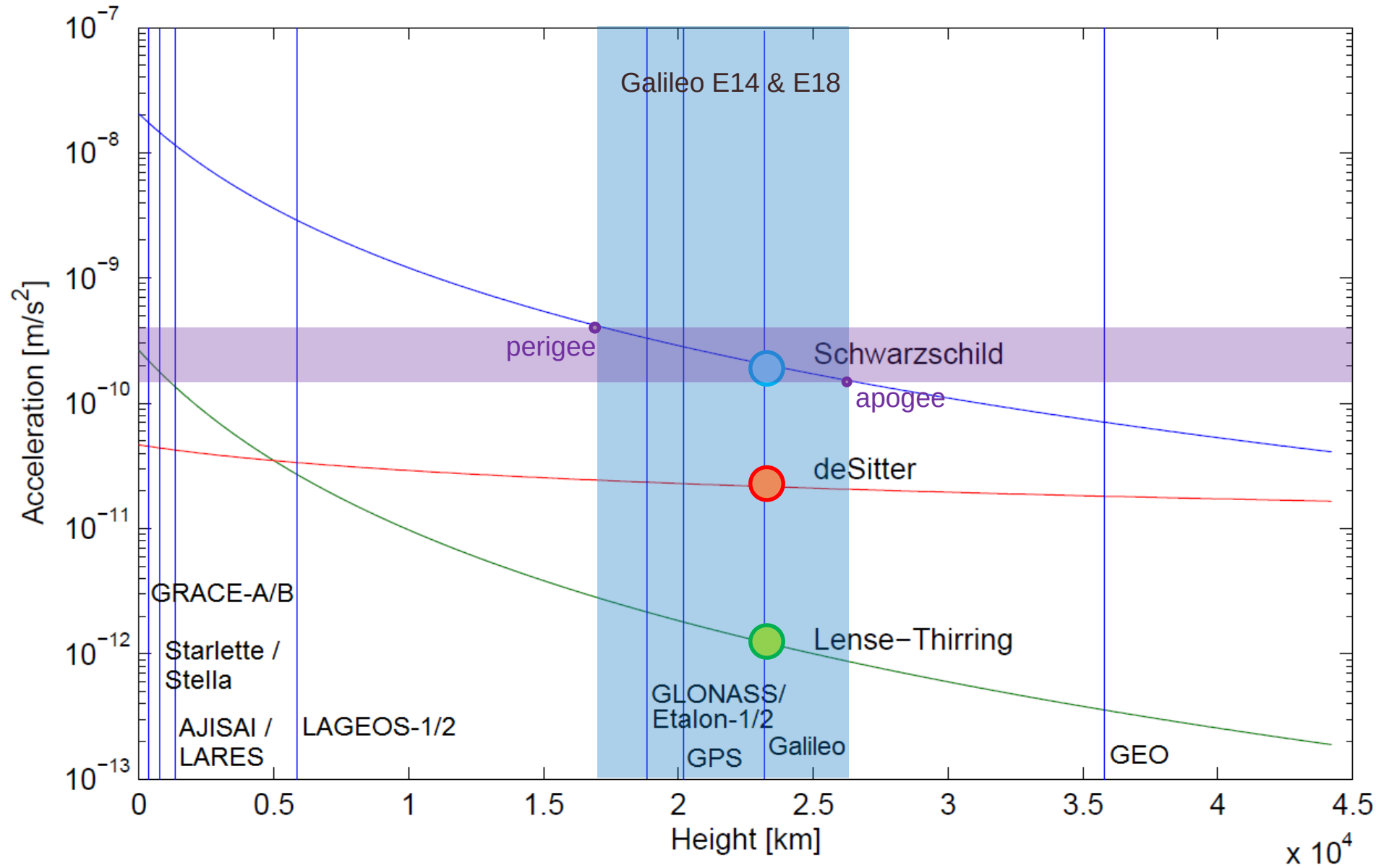


Galileo Satellites are good proxies of circular orbits with a regular distribution of satellites, except for E14 and E18 which are good proxies of eccentric orbits.

GNSS Constellations

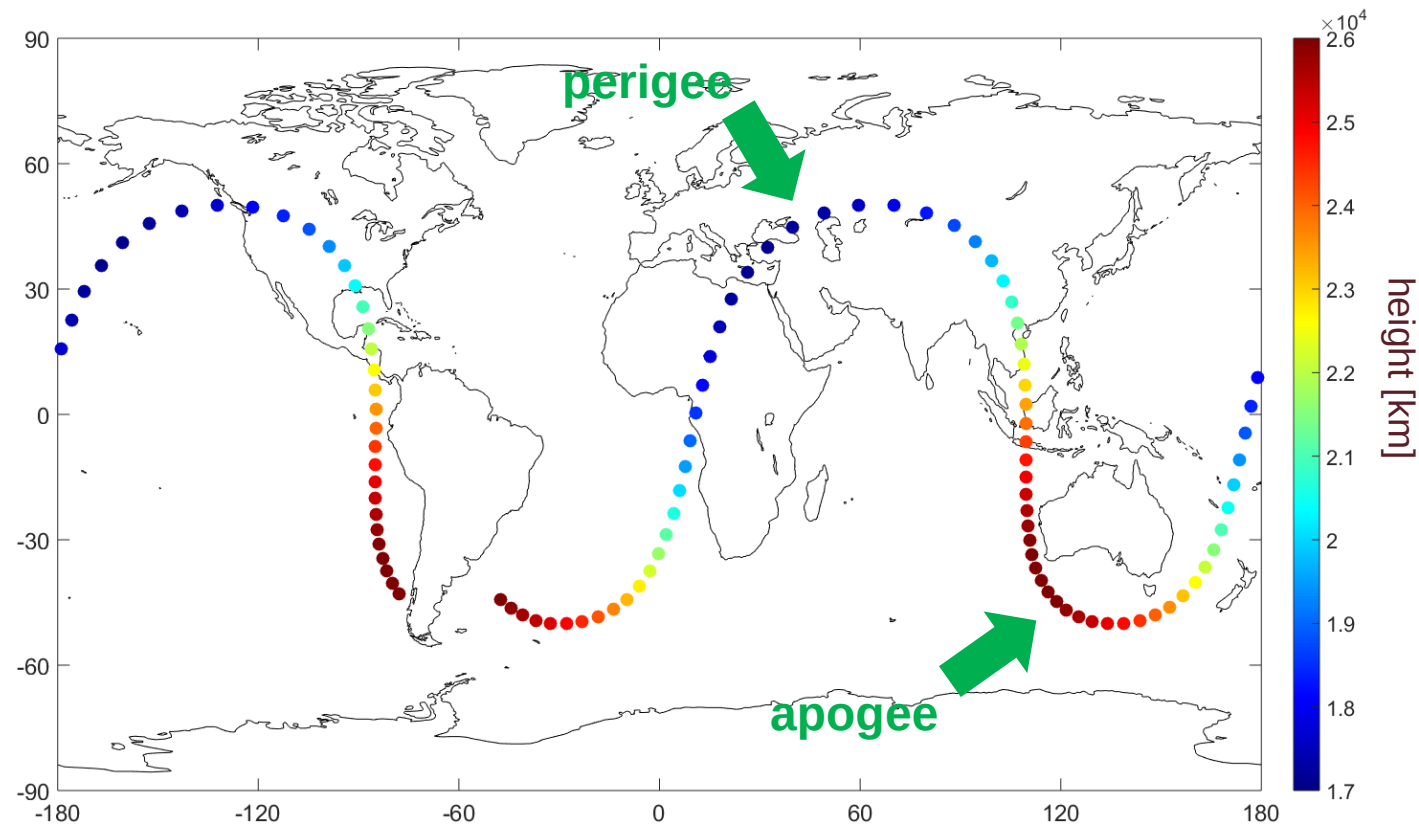


IERS Conventions 2010 and the orbital effects



Modified version, after
Hugentobler (2008)
*Orbit Perturbations due to
Relativistic Corrections. IERS
2010 Conventions, Chapter 10
supplementary material.*

Systematic errors may occur – perigee is above southern hemisphere



Satellite positions of E14 every 15 minutes.

Systematic effects may occur, because different ground GNSS (and SLR) stations observe the satellites in perigee and apogee. A full revolution of perigee takes 31 years for E14&E18 (and 65-90 years for Galileo satellites in circular orbits)

Sośnica, K., Prange, L., Kaźmierski, . *et al.* Validation of Galileo orbits using SLR with a focus on satellites launched into incorrect orbital planes. *J Geod* **92**, 131–148 (2018). <https://doi.org/10.1007/s00190-017-1050-x>

First-order Gaussian perturbations

Gaussian perturbations due to accelerations in radial (R), along-track (S), and cross-track (W)

$$\dot{a} = \sqrt{\frac{p}{GM}} \frac{2a}{1-e^2} \left[e \sin \nu \cdot R + \frac{p}{r} \cdot S \right]$$

$$\dot{e} = \sqrt{\frac{p}{GM}} \left[\sin \nu \cdot R + (\cos \nu + \cos E) \cdot S \right]$$

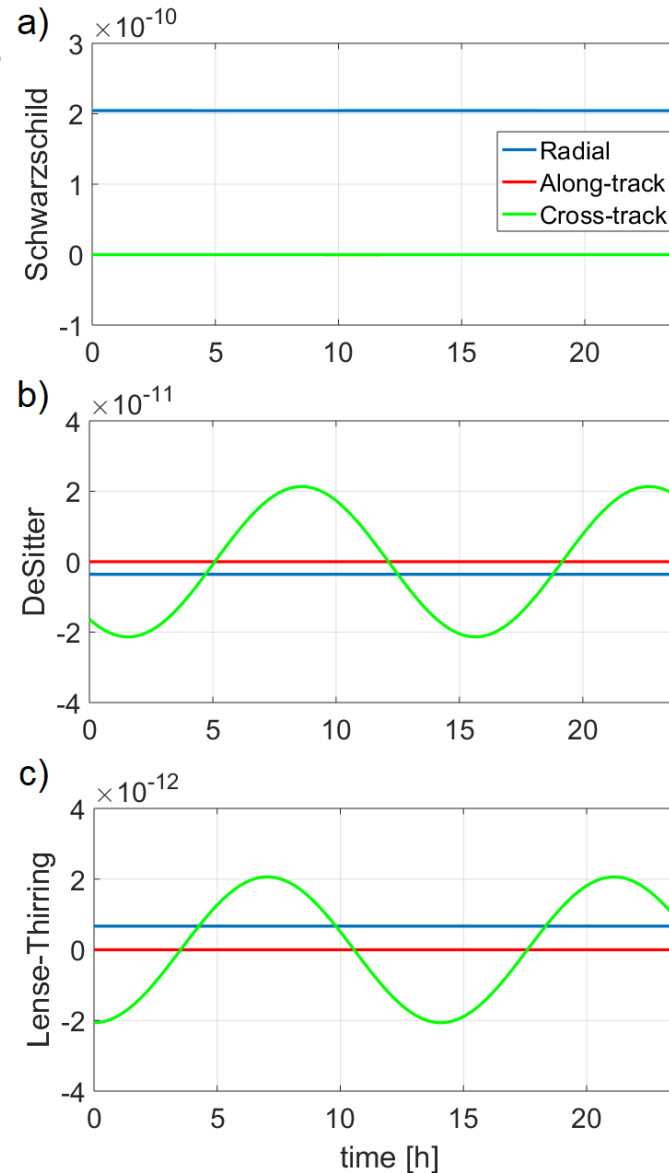
$$\dot{i} = \frac{r \cos u}{na^2 \sqrt{1-e^2}} \cdot W$$

$$\dot{\Omega} = \frac{r \sin u}{na^2 \sqrt{1-e^2} \sin i} \cdot W$$

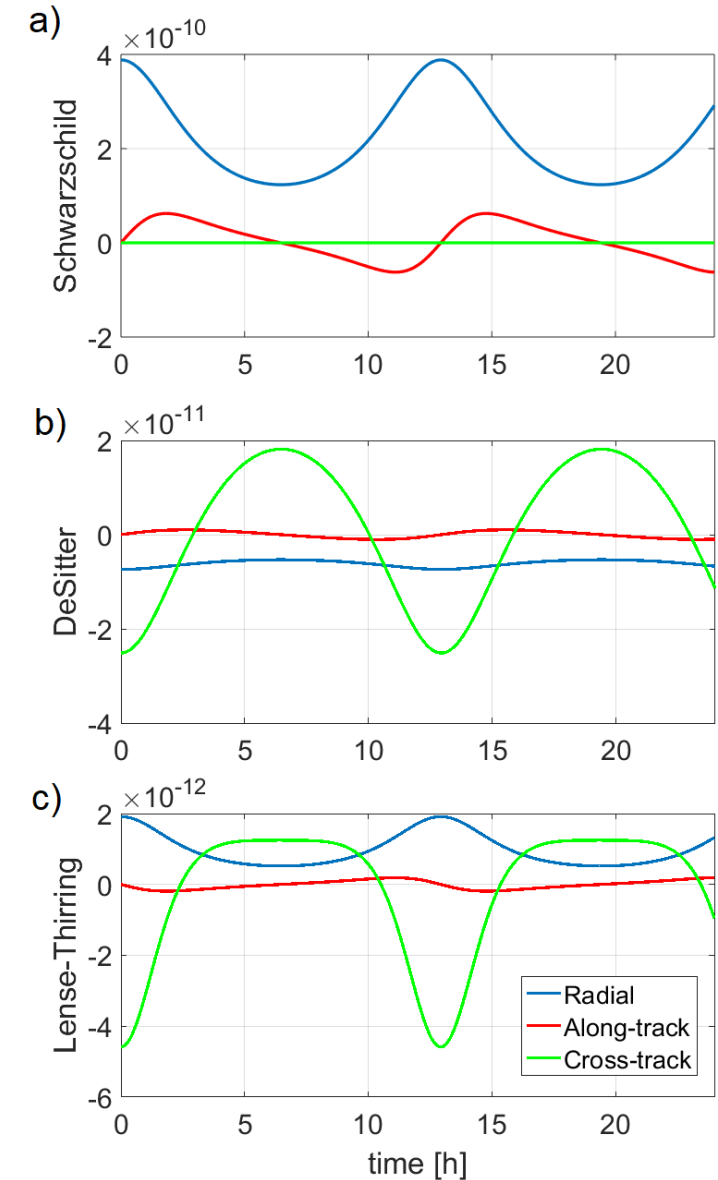
$$\dot{\omega} = \frac{1}{e} \sqrt{\frac{p}{GM}} \left[-\cos \nu \cdot R + \left(1 + \frac{r}{p} \right) \sin \nu \cdot S \right] - \dot{\Omega} \cos i$$

$$\dot{M}_0 = \frac{1-e^2}{nae} \left[\left(\cos \nu - 2e \frac{r}{p} \right) \cdot R - \left(1 + \frac{r}{p} \right) \sin \nu \cdot S \right] - \frac{3n}{2a} (t - t_0) \dot{a}$$

Solution of the differential equations does not provide information on constant offsets (if they appear). Offsets must be derived on the basis of celestial mechanics.

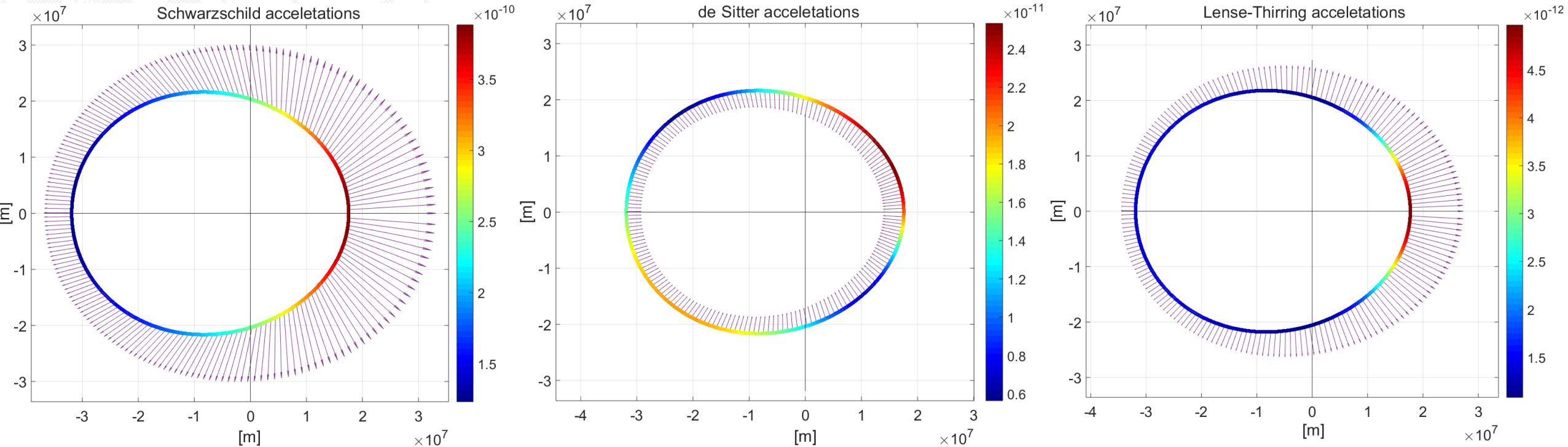


E08 – circular orbit



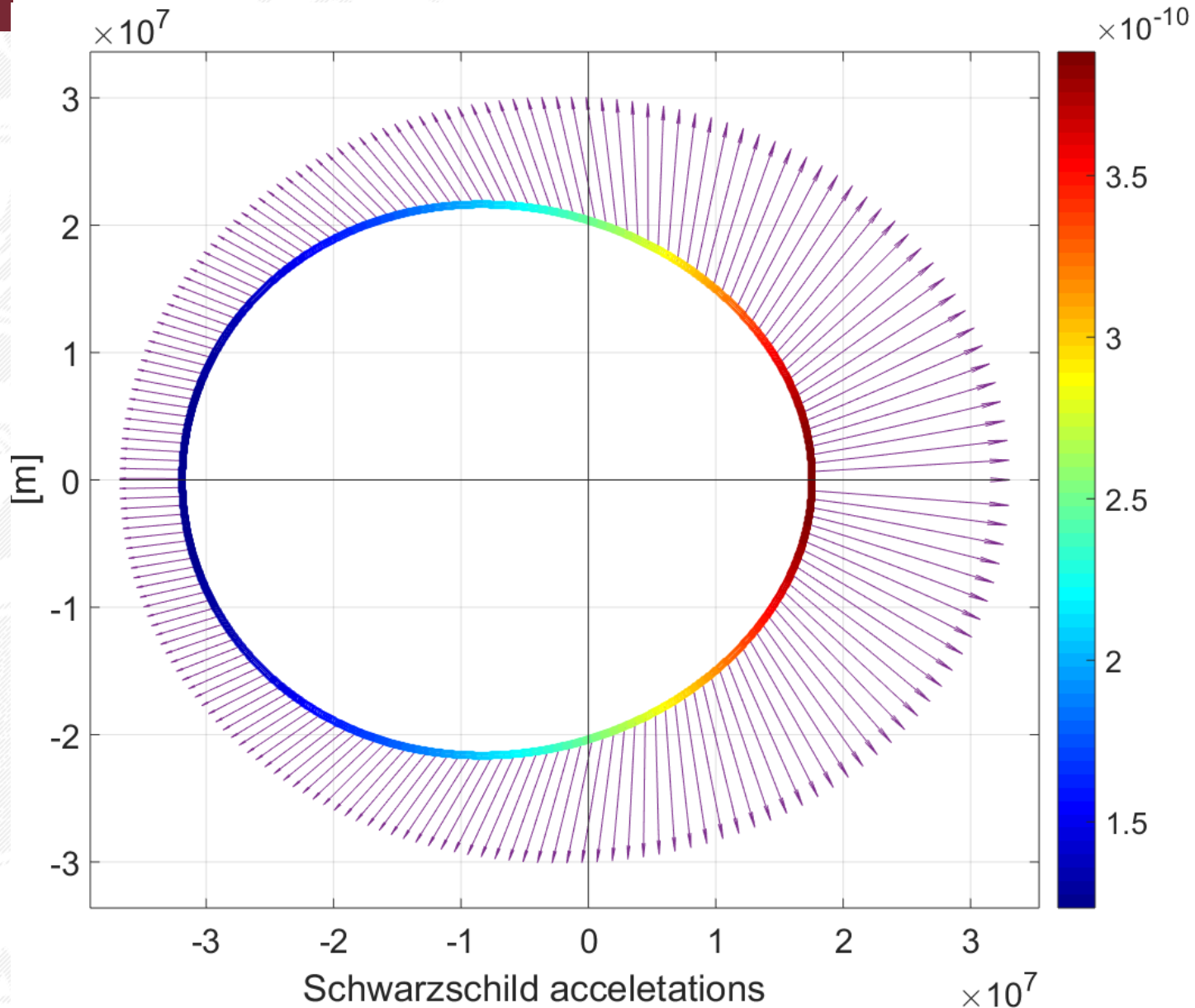
E14 – eccentric orbit

IERS Conventions 2010 and the orbital Effects on Galileo E14



Sośnica, K., Bury, G., Zajdel, R. *et al.* General relativistic effects acting on the orbits of Galileo satellites. *Celest Mech Dyn Astr* **133**, 14 (2021). <https://doi.org/10.1007/s10569-021-10014-y>

IERS Conventions 2010 and the orbital Effects on Galileo E14

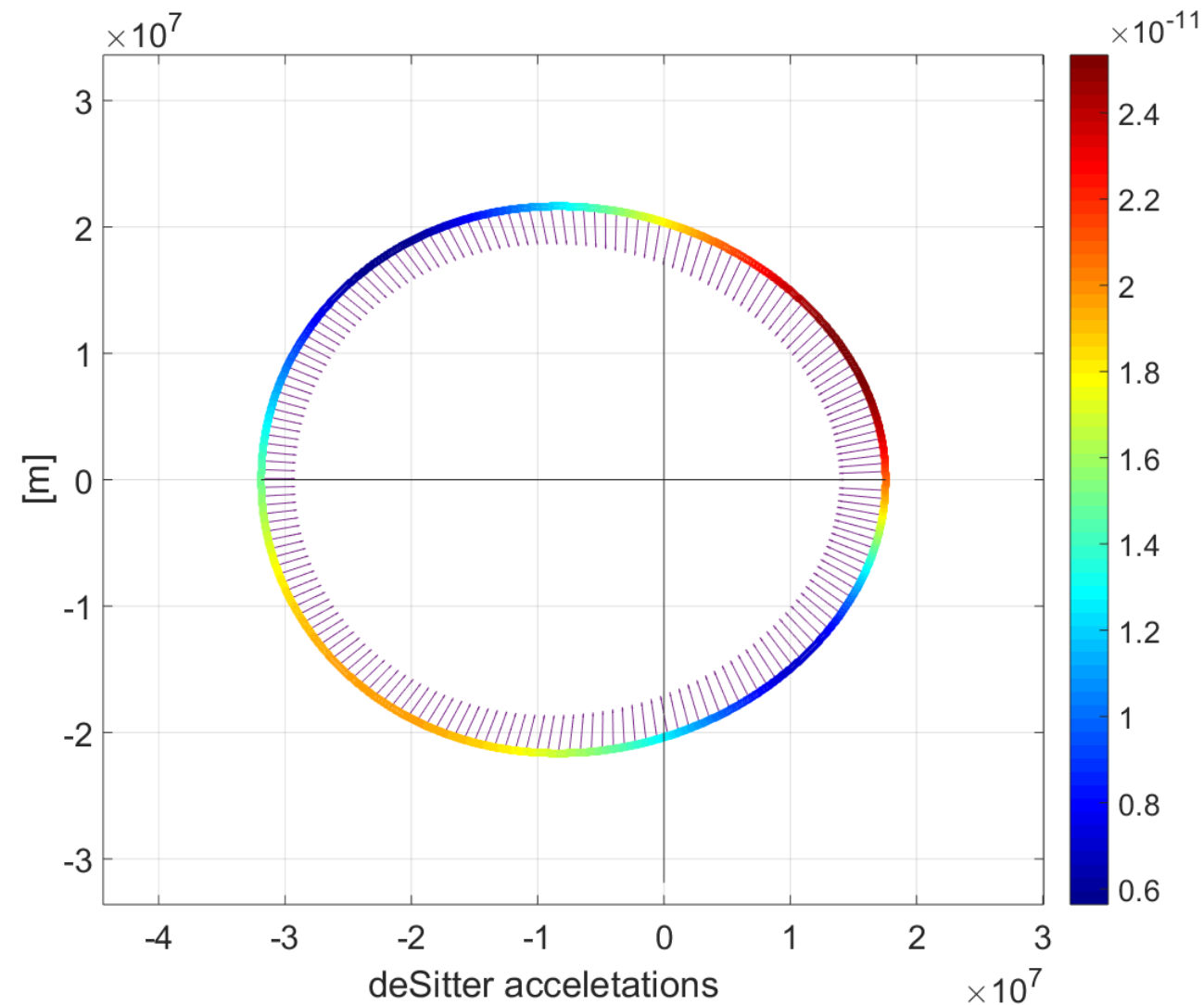
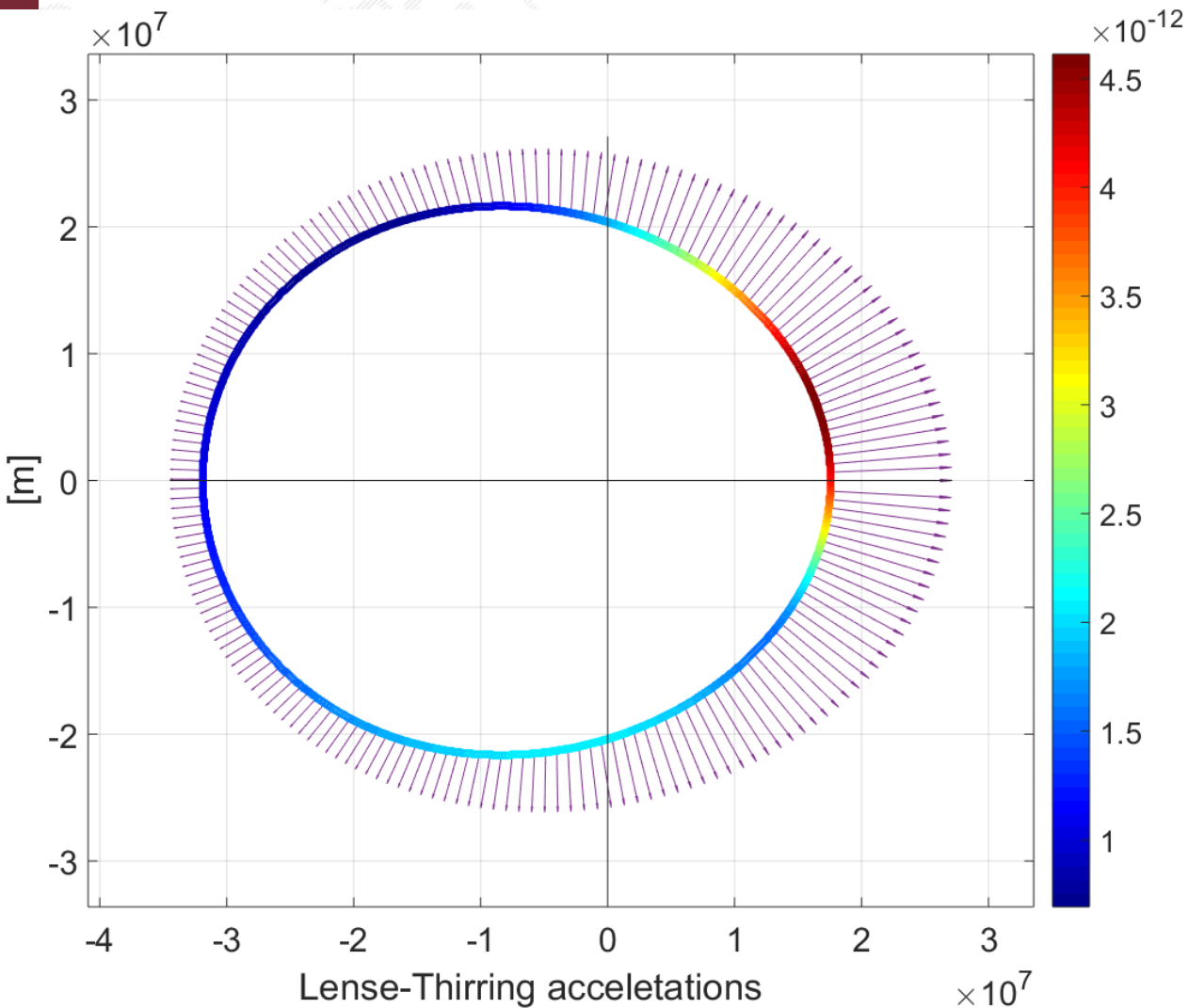


All Schwarzschild accelerations are in-plane (no out-of-plane accelerations occur). For circular orbits only radial accelerations occur. For elliptical orbits, both radial and along-track accelerations take place (Schwarzschild effect depends not only on the position but also on the velocity).

$$\Delta \ddot{\vec{r}}_{\text{Sch}} = 3 \frac{(GM)^2}{c^2 a^4} \cdot \vec{r}$$

Sośnica, K., Bury, G., Zajdel, R. *et al.* General relativistic effects acting on the orbits of Galileo satellites. *Celest Mech Dyn Astr* **133**, 14 (2021). <https://doi.org/10.1007/s10569-021-10014-y>

IERS Conventions 2010 and the orbital Effects on Galileo E14



Lense-Thirring and de Sitter are in-plane and out-of-plane accelerations.

Schwarzschild effect – semi-major axis

$$\Delta a_{Sch}^{Tot} = \Delta a_{Sch} + \Delta a_{Sch}$$

Constant part:

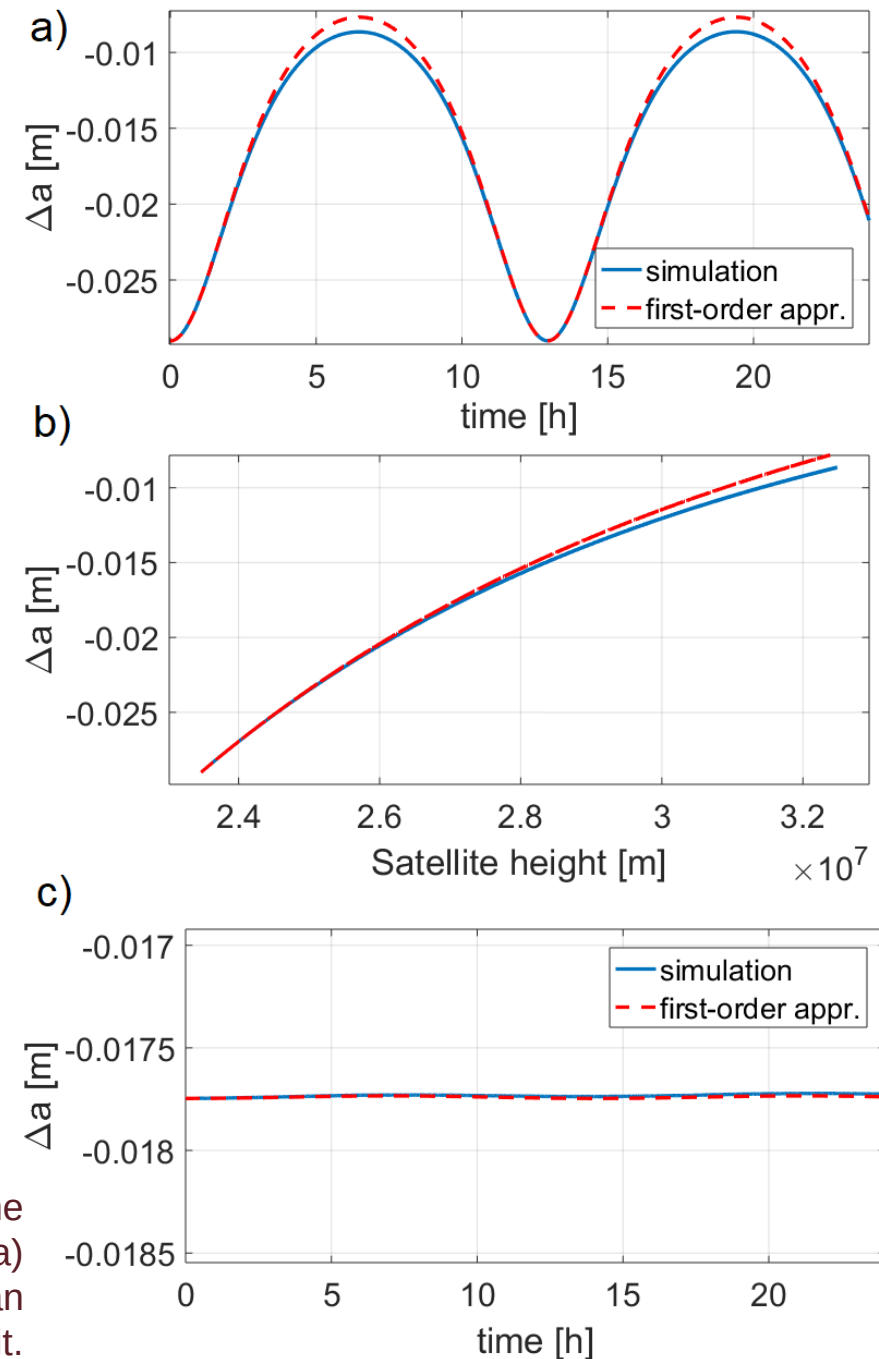
$$\Delta a_{Sch} = -4 \frac{GM}{c^2} = -17.74 \text{ mm}$$

Periodical part:

$$\Delta a_{Sch} = \frac{GM}{c^2(1-e^2)^2} [(-14-6e^2)e \cos u - 5e^2 \cos 2u]$$

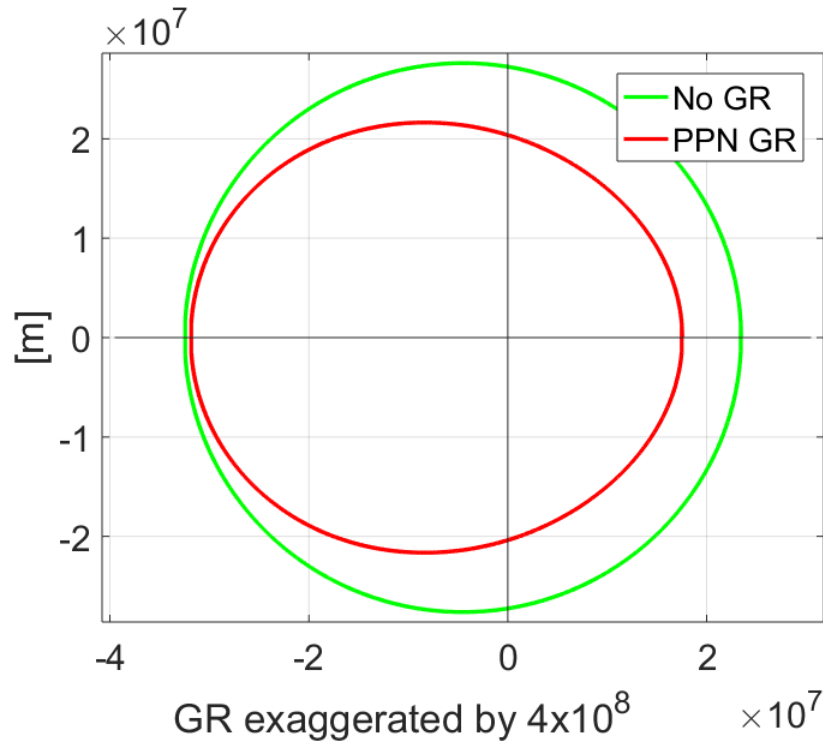
Sośnica, K., Bury, G., Zajdel, R. *et al.* General relativistic effects acting on the orbits of Galileo satellites. *Celest Mech Dyn Astr* **133**, 14 (2021).
<https://doi.org/10.1007/s10569-021-10014-y>

Semi-major axis change of E14 due to the Schwarzschild acceleration as a function of a) time and b) the satellite height for E14 in an elliptical orbit and c) for E08 in a circular orbit.

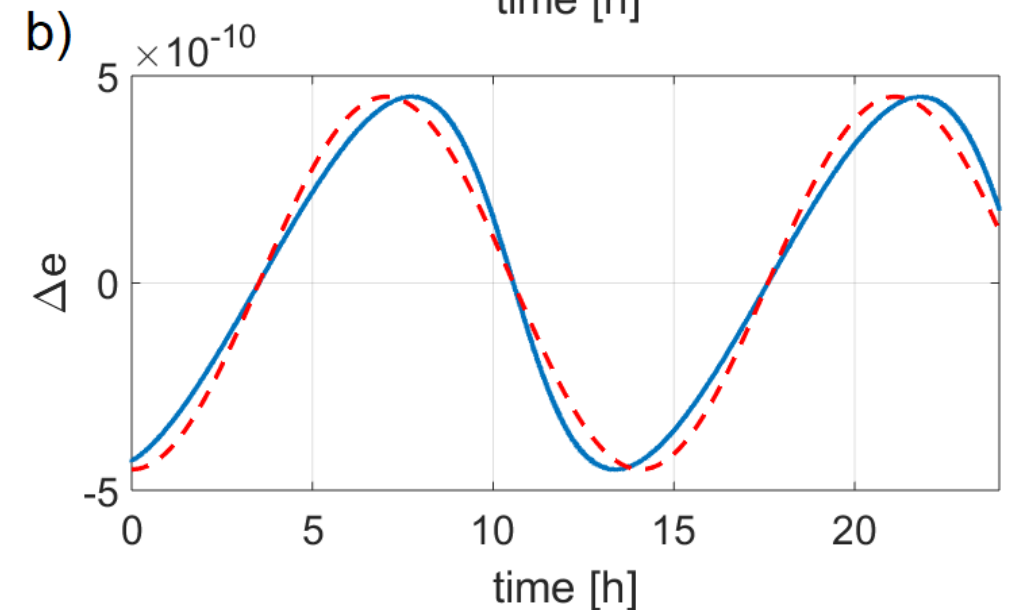
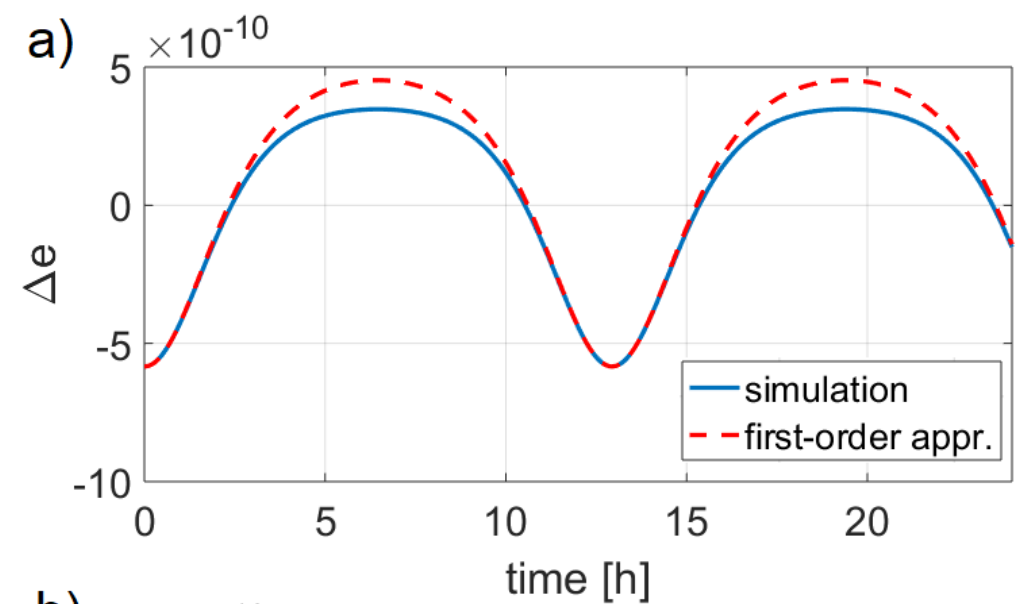


Schwarzschild effect – eccentricity

$$\Delta e_{Sch} = -\frac{GM}{c^2 a e (1 - e^2)} \left[(3 + 7e^2) e \cos u + \frac{5}{2} e^2 \cos 2u \right]$$



Comparison between E14 orbit without and with the Schwarzschild accelerations with the exaggeration of the total displacement effect in the geocentric reference frame



Eccentricity changes due to the Schwarzschild acceleration as a function of time for a) E14 and b) E08. The initial point coincides with orbit's perigee.

Schwarzschild effect – revolution period

Revolution period

$$T = \frac{2\pi}{n}, \quad n_{Sch} = \left(\frac{GM}{a^3} \right)^{1/2} \left(1 + \frac{6GM}{c^2 a} \right)$$

Areal velocity:

$$A = \frac{1}{2} |\vec{h}| = \frac{1}{2} |\vec{r} \times \dot{\vec{r}}|,$$

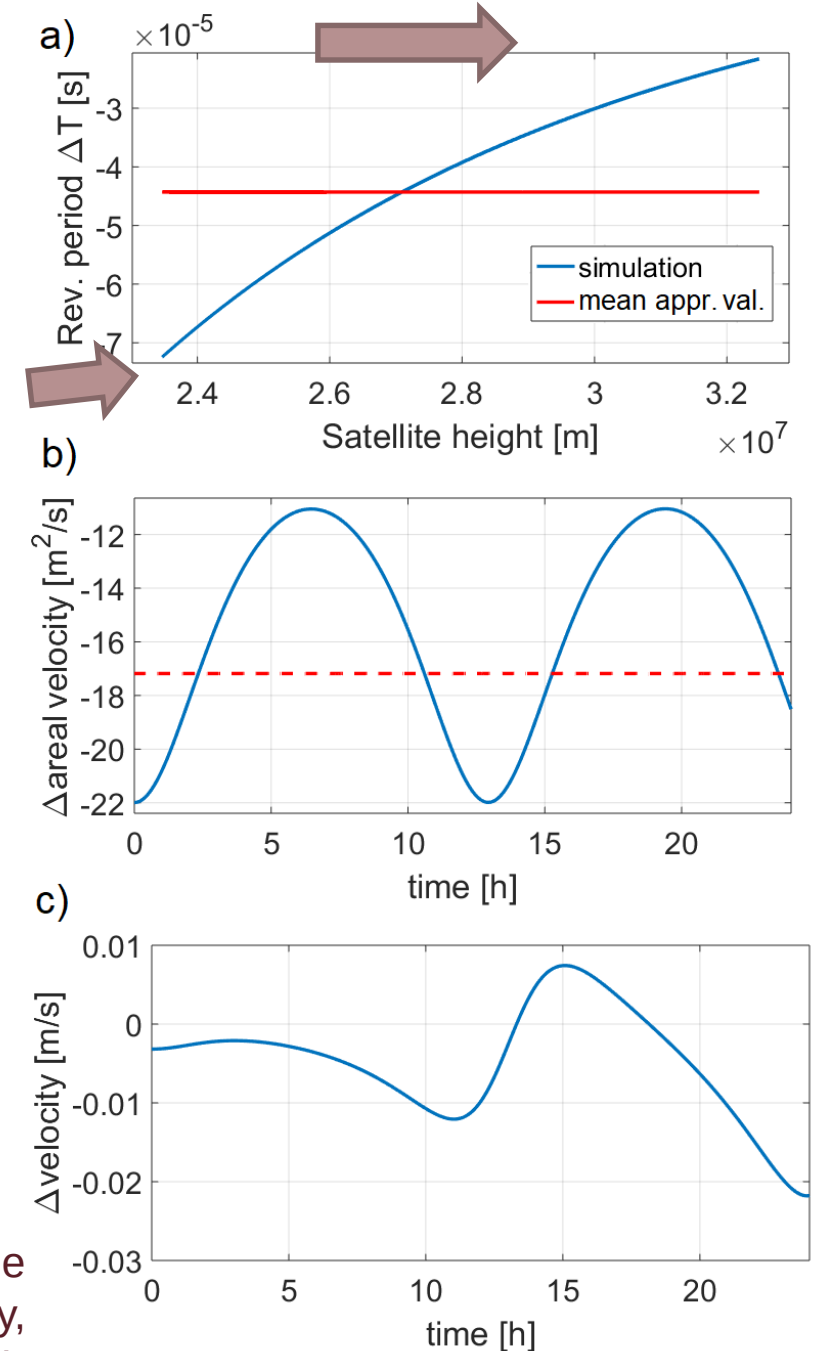
Angular momentum:

$$|\vec{h}_{Sch}| = \left[GMa(1 - e^2) \right]^{1/2} \left[1 - \frac{GM}{2c^2 a} + \frac{4GM}{c^2 a(1 - e^2)} \right]$$

Impact of the Schwarzschild term on the
a) revolution period, b) areal velocity,
and c) linear velocity of E14.

$$\Delta T_{apogee} = -21.57 \mu s$$

$$\Delta T_{perigee} = -72.44 \mu s$$



Schwarzschild effect – perigee

Einstein's term

$$\Delta\omega_{Schw} = \frac{3GM}{c^2 a(1-e^2)} n\Delta t$$

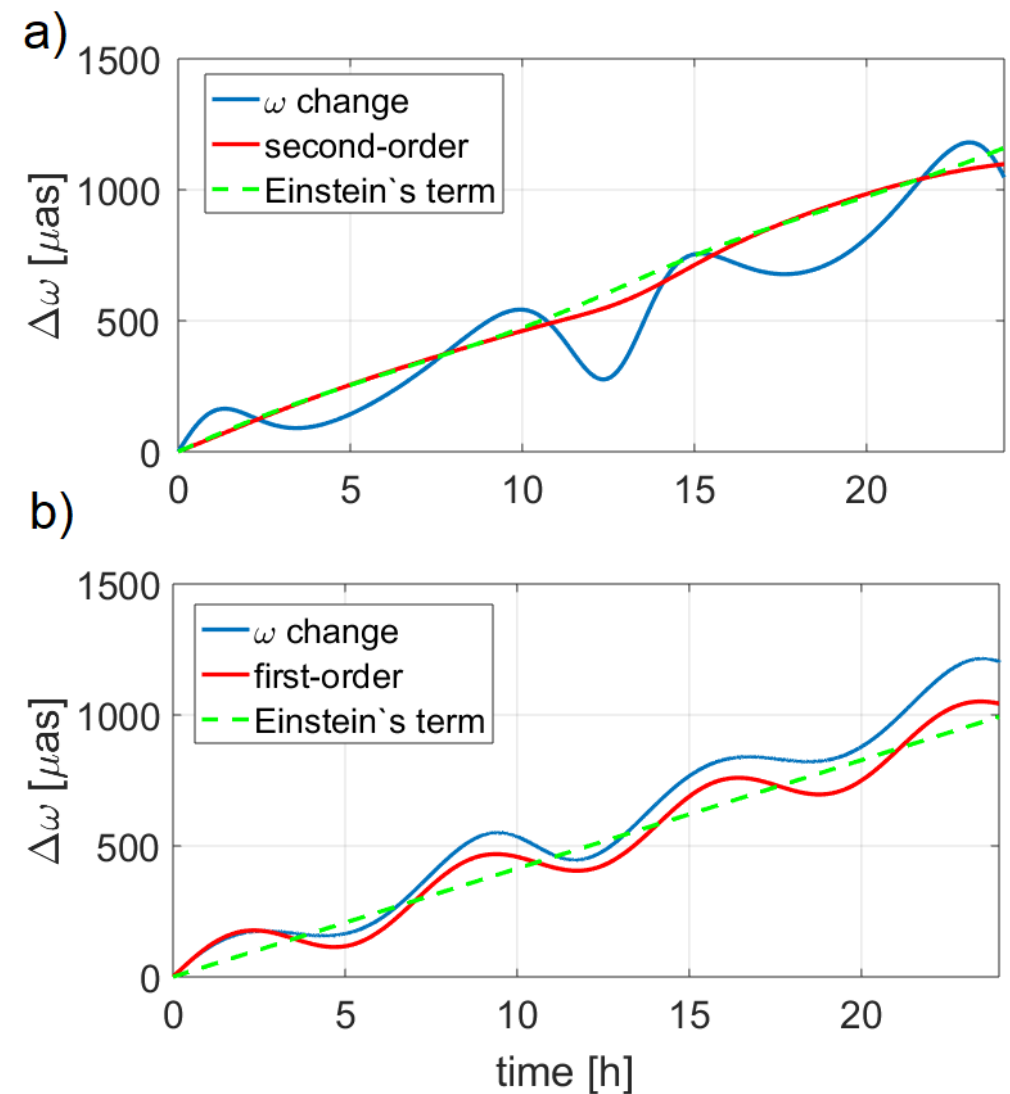
First-order perturbations:

$$\Delta\omega_{Schw} = \frac{3GM}{c^2 a(1-e^2)} (n\Delta t + \sin 2u)$$

Second-order according to Iorio (2020):

$$\Delta\omega_{Schw} = \frac{(GM)^2 [5(23 + 20e^2 - 4e^4)]}{2n^2 c^4 a^2 (1-e^2)^3} + \frac{(GM)^2 6e [(34 + 26e^2) \cos u + 15e \cos(2u)]}{2n^2 c^4 a^2 (1-e^2)^3}$$

For Galileo E14, the mean formal error of perigee determination is 223 μs from 1-day solutions, whereas for E08 the perigee error is 95 468 μs .



Impact of the Schwarzschild term on the perigee of Galileo a) E14 and b) E08 from the numerical simulation (corrected by a change of the mean anomaly), first and second-order approximations, and the Einstein's term.

De Sitter – semi-major axis, eccentricity

Constant offset of the semi-major axis

$$\Delta a_{dS} = +4 \frac{GM_S}{\sqrt{GM}c^2} \frac{a^{\frac{5}{2}}}{R_S} n_S \sqrt{1 - e_S^2} \cos \beta_{ele}$$

Periodic perturbations of the semi-major axis:

$$\Delta a_{dS} = -6 \frac{e}{\sqrt{1 - e^2}} \frac{GM_S}{c^2 R_S} n_S \sqrt{1 - e_S^2} \cos \beta_{ele} \sin u$$

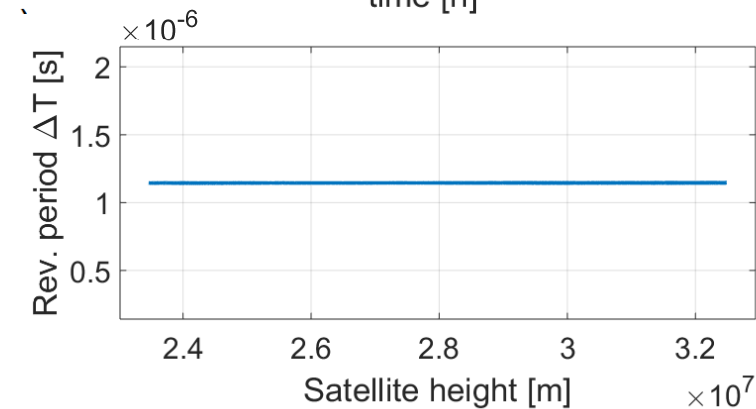
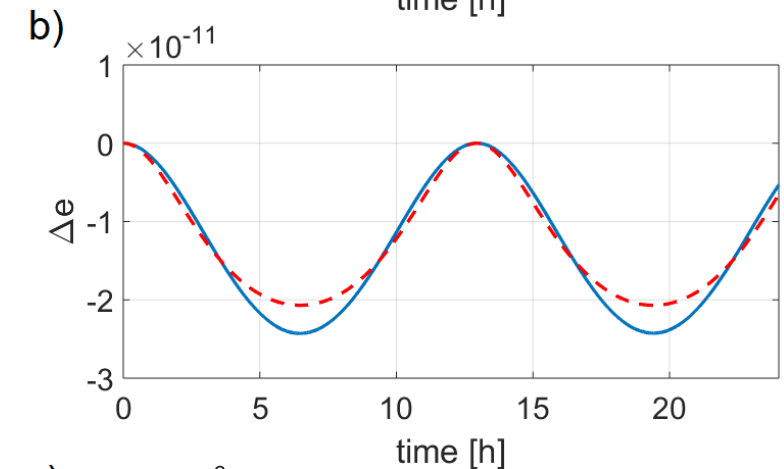
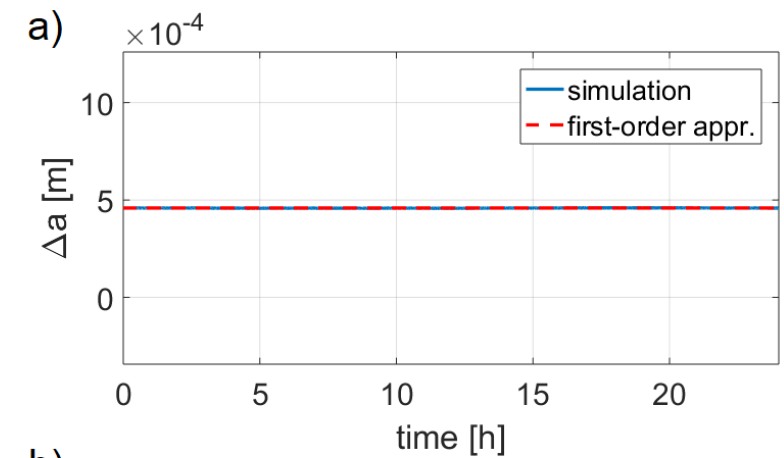
Eccentricity variations:

$$\Delta e_{dS} = \frac{3}{4} \frac{GM_S}{c^2 R_S} \frac{n_S}{n} \sqrt{1 - e_S^2} \sqrt{1 - e^2} \sin \beta_{ele} (\cos u - 1)$$

Mean motion:

$$n_{dS} = \left(\frac{GM}{a^3} \right)^{1/2} \left(1 - \frac{6GM_S}{ac^2 R_S} \frac{n_S}{n} \sqrt{1 - e_S^2} \cos \beta_{ele} \right)$$

For GEO, the impact on a is 4 mm.



Impact of the de Sitter effect on the semi-major axis, eccentricity, revolution period of Galileo E14 from the numerical simulations (blue line) and first-order Gaussian perturbations (red line).

De Sitter – node, inclination, perigee for E14

Ascending node

$$\Delta\Omega_{dS} = \frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{n_S}{n} \sqrt{\frac{1-e_S^2}{1-e^2}} \left(n\Delta t - \frac{1}{2} \sin 2u \right) \frac{\sin(\beta_{ele})}{\sin i}$$

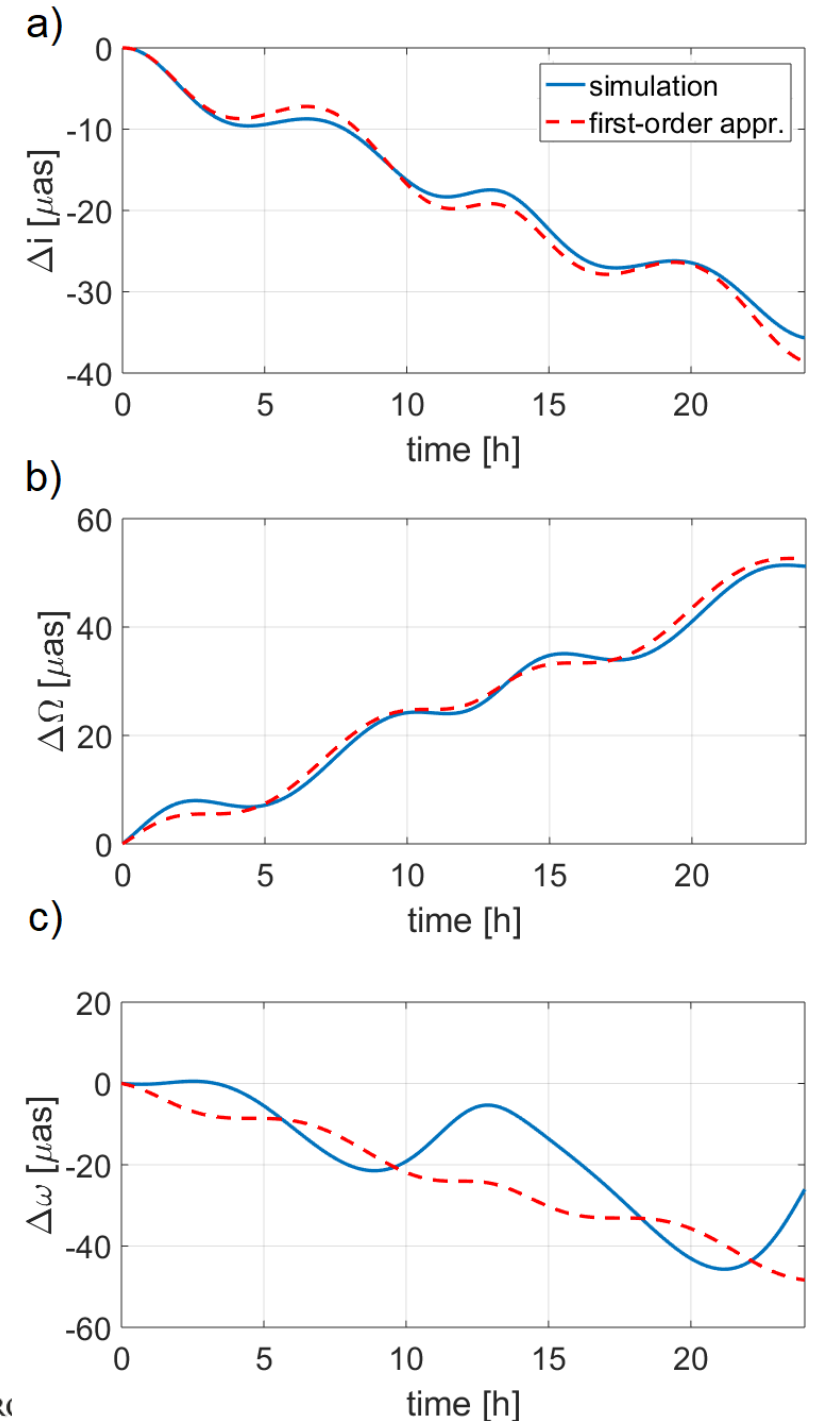
Inclination

$$\Delta i_{dS} = -\frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{n_S}{n} \sqrt{\frac{1-e_S^2}{1-e^2}} \left(n\Delta t \sin(\beta_{ele} - i) - \frac{1}{2} \sin 2u \right) \cdot \sin(\beta_{ele})$$

Argument of perigee:

$$\Delta\omega_{dS} = \frac{3}{4} \frac{GM_S}{ec^2 R_S} \frac{n_S}{n} \sqrt{1-e_S^2} \sqrt{1-e^2} \sin(\beta_{ele} - i) \sin u - \Delta\Omega_{dS} \cos i.$$

Nodal drift equals 52.53 $\mu\text{as/day}$ for circular polar orbits over long periods that is 7.2 mm at the Galileo heights after one day.
Over long periods the inclination drift is equal to zero.
Over shorter periods, the nodal drift may be very large.



De Sitter – node, inclination, perigee for GEO

Ascending node

$$\Delta\Omega_{dS} = \frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{n_S}{n} \sqrt{\frac{1-e_S^2}{1-e^2}} \left(n\Delta t - \frac{1}{2} \sin 2u \right) \frac{\sin(\beta_{ele})}{\sin i}$$

Inclination

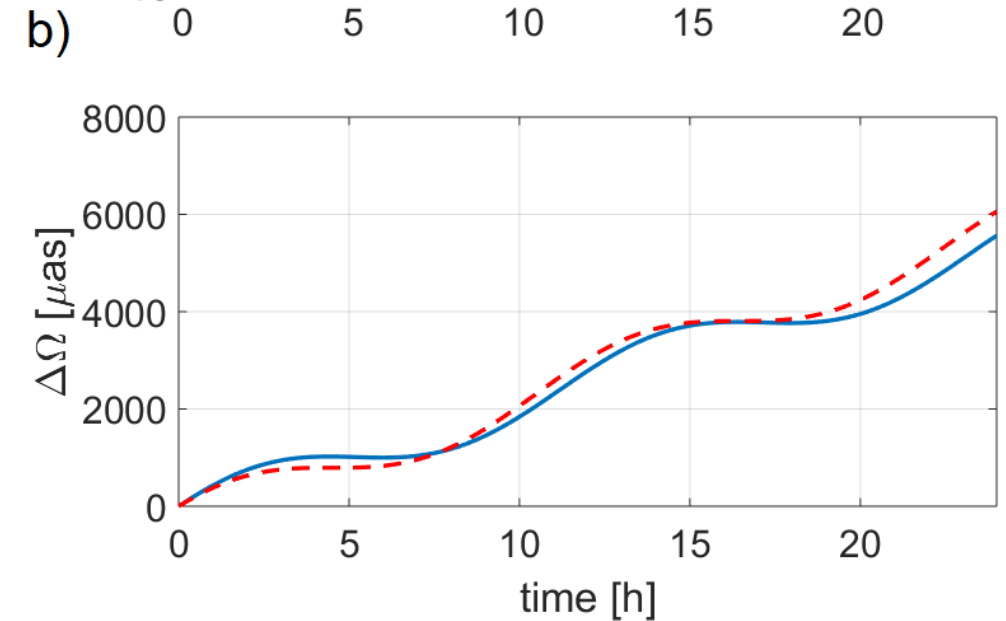
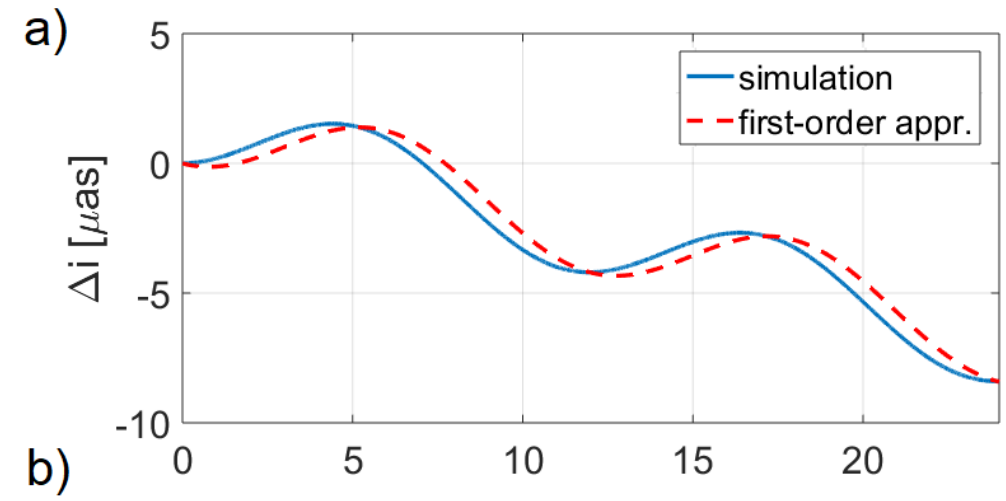
$$\Delta i_{dS} = -\frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{n_S}{n} \sqrt{\frac{1-e_S^2}{1-e^2}} \left(n\Delta t \sin(\beta_{ele} - i) - \frac{1}{2} \sin 2u \right) \cdot \sin(\beta_{ele})$$

For GEO satellites and for specific Sun configurations, the nodal drift may reach over 1 meter per day because of the coefficient

$$\frac{\sin(\beta_{ele})}{\sin i}$$

$$\cos \beta_{ele} = \cos \varepsilon \cos i + \sin \varepsilon \sin i \cos(\Omega - \Omega_S)$$

$$|i - \varepsilon| < \beta_{ele} < |i + \varepsilon|$$



Impact of the de Sitter effect on the a) inclination, b) right ascension of ascending node of a geostationary orbit with $i = 0.2^\circ$

De Sitter – node, inclination, perigee for GEO

Sośnica, K., Prange, L., Kaźmierski, . *et al.* Validation of Galileo orbits using SLR with a focus on satellites launched into incorrect orbital planes. *J Geod* **92**, 131–148 (2018).
<https://doi.org/10.1007/s00190-017-1050-x>

Table 1 Characteristics of IOV and FOC Galileo satellites and the orbits thereof

PRN number	E11, E12	E19, E20	E18, E14	E26, E22	E24, E30	E08, E09
Name	Galileo-101, 102	Galileo-103, 104	Galileo-201, 202	Galileo-203, 204	Galileo-205, 206	Galileo-208, 209
Type	IOV	IOV	FOC	FOC	FOC	FOC
Launch date	October 21, 2011	October 12, 2012	August 22, 2014	March 27, 2015	September 11, 2015	December 17, 2015
Retroreflectors (corner cubes)	84	84	60	60	60	60
Corner cube dimensions (mm)	33.0 × 23.3	33.0 × 23.3	28.2 × 19.1	28.2 × 19.1	28.2 × 19.1	28.2 × 19.1
Rear/front coating (Y—yes, N—no)	N/Y	N/Y	N/N	N/N	N/N	N/N
Mass (kg)	695	697	661/662	706	709	708
Orbital plane	B	C	incorrect	B	A	C
Area-to-mass ratio (m ² kg ^{−1})	0.019	0.019	0.020	0.019	0.019	0.019
Semi-major axis (km)	29,600	29,600	27,978	29,602	29,601	29,600
Altitude (km)	23,225	23,225	17,178–26,019	23,227	23,226	23,225
Eccentricity	0.0001	0.0002	0.1585/ 0.1584	0.0003	0.0004	0.0001
Revolution period (h)	14.08	14.08	12.94	14.08	14.08	14.08
Inclination (°)	55.57	54.93	50.10/50.16	55.36	57.25	54.94
$\beta_{\max}$ (°)	63.5	75.5	48.0/48.5	58.2	39.0	75.0
Draconitic year (days)	355.7	355.6	351.6	355.6	356.1	355.6
Revolution of node (years)	37.5	36.9	25.9/25.8	37.4	39.1	36.9
Revolution of perigee (years)	70.9	65.3	31.4/31.3	70.0	92.2	65.3

Orbit parameters of E18 and E14 refer to the situation after satellite maneuvers reduced the orbit eccentricity

Lense-Thirring – semi-major axis, eccentricity

Semi-major axis – constant offset

$$\Delta a_{LT} = -\frac{8}{3} \frac{J}{c^2} \sqrt{\frac{GM}{a}} \cos i$$

Semi-major axis – periodical variations

$$\Delta a_{LT} = 4 \frac{e}{\sqrt{1-e^2}} \frac{Jn}{c^2} \cos i \sin u$$

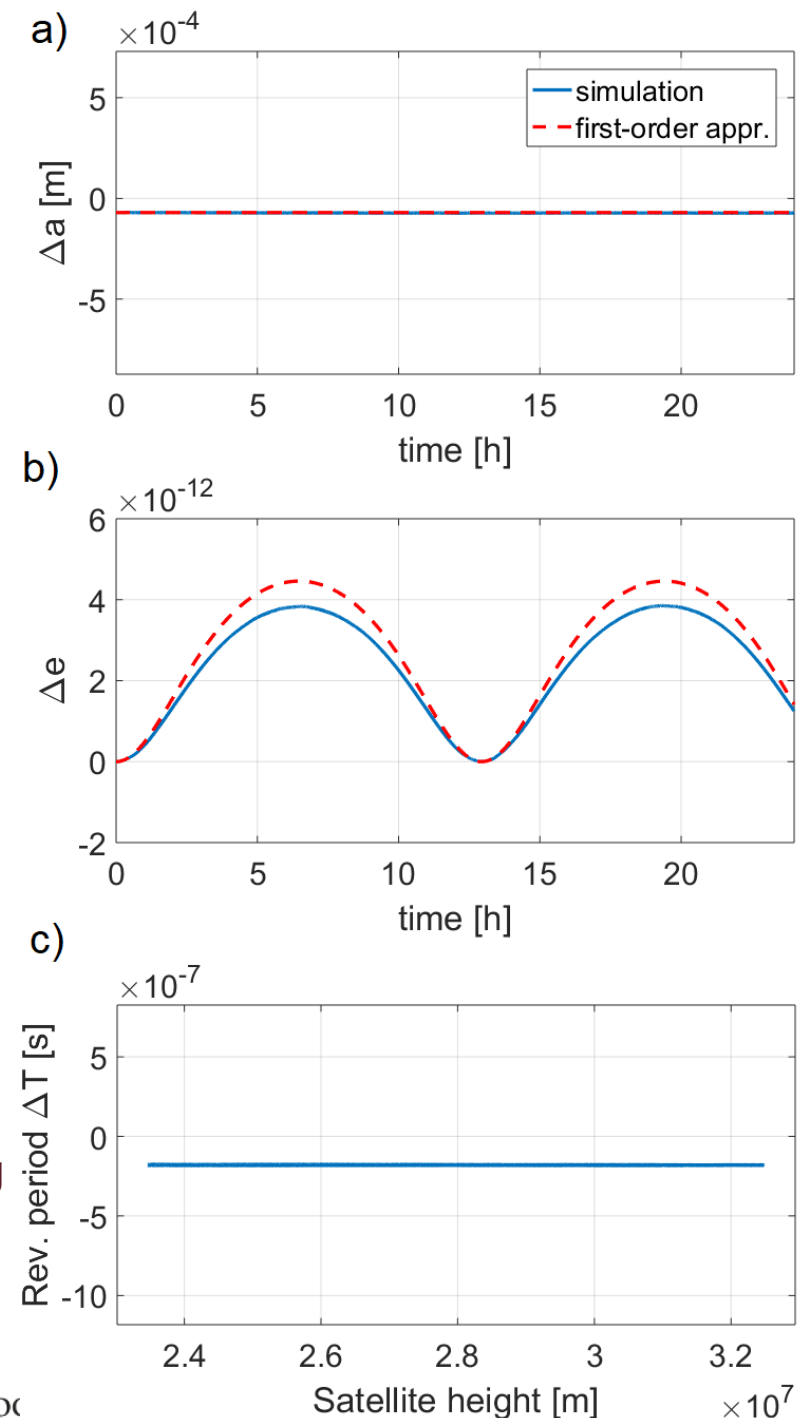
Eccentricity

$$\Delta e_{LT} = -2\sqrt{1-e^2} \frac{Jn}{c^2} \cos i (\cos u - 1)$$

Mean motion

$$n_{LT} = \left(\frac{GM}{a^3} \right)^{1/2} \left(1 + 4 \frac{GM}{c^2 a^3} J \cos i \right)$$

Impact of the Lense-Thirring on a) the semi-major axis, b) eccentricity, and c) the revolution period of E14.



Lense-Thirring – node, inclination, perigee

Ascending node

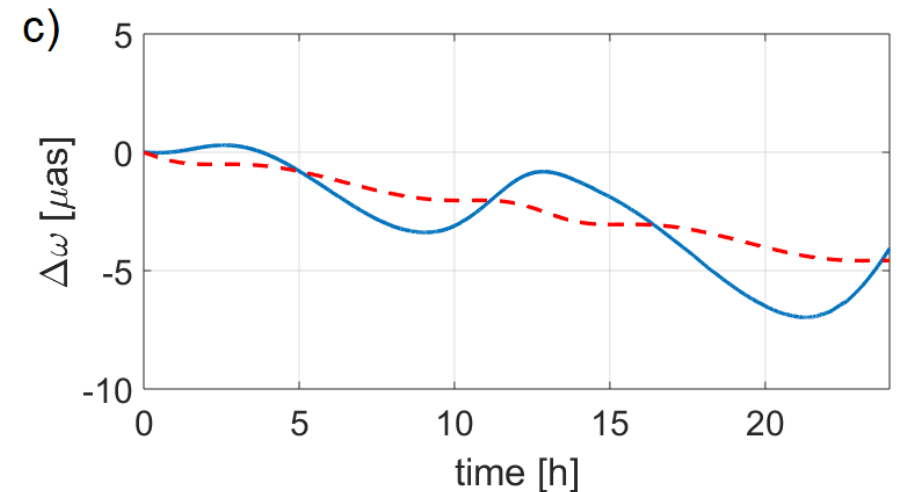
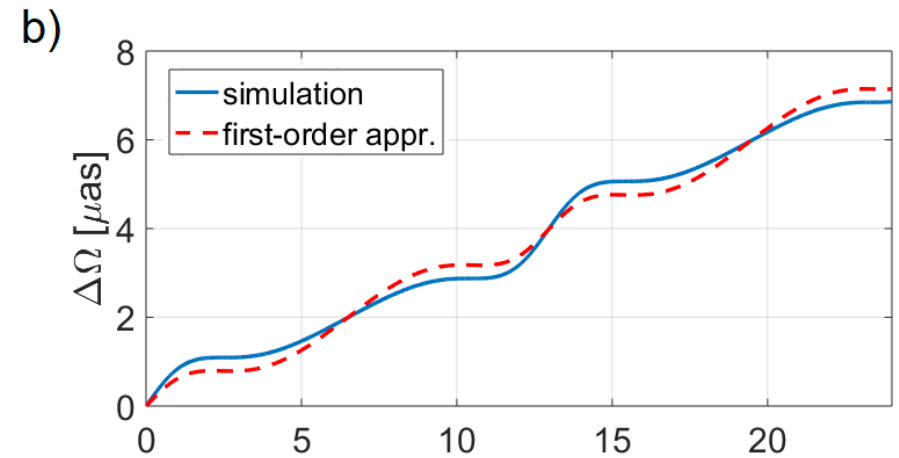
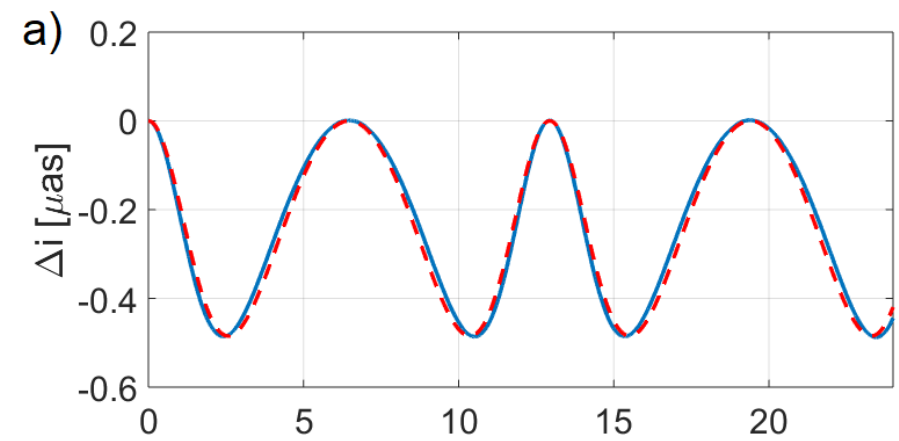
$$\Delta\Omega_{LT} = 2 \frac{GMJ}{c^2 a^3 (1 - e^2)^{\frac{3}{2}} n} (n\Delta t - \frac{1}{2} \sin 2u)$$

Inclination

$$\Delta i_{LT} = \frac{GMJ \sin i}{c^2 a^3 (1 - e^2)^{\frac{3}{2}} n} (-\frac{1}{2} + \frac{1}{2} \sin 2u)$$

Argument of perigee

$$\Delta\omega_{LT} = -2 \frac{GMJ \cos i}{c^2 a^3 (1 - e^2)^{\frac{1}{2}} n} (n\Delta t - \frac{3}{2} \sin 2u)$$



General relativity effects - summary

	GEO BeiDou C01	Galileo E08	Galileo E14	GPS G18	LARES-2	LAGEOS-1
Height [km]	35 793	23 230	17 097 – 26 117	20 190	12 270	12 271
Schwarzschild						
a [mm]	-17.74	-17.75 to -17.73	-29.02 to -8.65	-18.87 to -16.65	-17.74	-17.74
e [$\times 10^{-10}$]	-3.19 to 3.18	-4.49 to 4.49	-5.85 to 3.47	-5.09 to 4.87	-11.09 to 11.08	-11.23 to 11.23
i [mas/day]	-	-	-	-	-	-
Ω [mas/day]	-	-	-	-	-	-
ω' [mas/day]	0.4102	1.0471	1.2034	1.3031	8.9771	8.9830
T [μ s]	-54.39 to -54.37	-45.54 to -45.56	-72.45 to -21.57	-45.91 to -40.45	-25.28	-25.28
De Sitter						
a [mm]	+4.16	+0.31	+0.46	+0.26	-0.01	-0.15
e [$\times 10^{-10}$]	-0.15 to 0.00	-0.19 to 0.00	-0.24 to 0.00	-0.15 to 0.00	-0.06 to 0.00	-0.06 to 0.00
i [mas/day]	-0.0084	-0.0178	-0.0176	-0.0196	-0.0208	-0.0151
Ω [mas/day]	+5.6501	+0.0556	+0.0556	+0.0556	+0.0520	+0.0365
ω' [mas/day]	-0.1108	-0.0328	-0.0395	-0.0259	+0.0081	+0.0080
T [μ s]	+12.75	+0.80	+1.14	+0.63	-0.02	-0.24
Lense-Thirring						
a [mm]	-0.09	-0.06	-0.07	-0.07	-0.06	+0.06
e [$\times 10^{-10}$]	0.00	0.00 to 0.04	0.00 to 0.04	0.00 to 0.04	0.00 to 0.19	0.00 to 0.19
i [mas]	+/-0.0000	+/-0.0005	+/-0.0005	+/-0.0005	+/-0.0019	+/-0.0019
Ω [mas/day]	+0.0021	+0.0061	+0.0068	+0.0084	+0.0827	+0.0827
ω' [mas/day]	-0.0024	-0.0045	-0.0046	-0.0052	-0.0320	+0.0321
T [μ s]	-0.27	-0.15	-0.18	-0.16	-0.09	+0.09

Summary

We analyzed three main effects emerging from general relativity: Schwarzschild, Lense-Thirring, and De Sitter. We studied the impact on GNSS orbits with a focus on two Galileo satellites accidentally launched into eccentric orbits. We derive analytical formulas of temporal variations of Keplerian parameters based on the first-order Gaussian perturbations and celestial mechanics; and then, we compare how well they agree with the observed perturbations from numerical simulations.

The Schwarzschild effect introduces a constant **shift of the semi-major axis of -17.7 mm** for all Earth satellites, independently of the satellite height. Schwarzschild causes also periodic changes of **semi-major axis and revolution period**, periodic variations of **eccentricity**, and a secular motion of **perigee**. The secular rate of perigee corresponds to almost 17 cm/day at GNSS heights.

For Galileo E14/E18 in eccentric orbits, the change of the semi-major axis due to the Schwarzschild effect is **-29 mm in perigee and -9 mm in apogee**. This gives a difference of more than 20 mm, thus, is fully detectable using current GNSS precise orbit determination techniques. We found that the **de Sitter effect** strongly depends on the orientation of the ecliptic with respect to the orbital plane and may introduce a secular change of the inclination angle. The mean secular rate of the ascending node, caused by de Sitter, corresponds to a drift of **7.2 mm/day at the Galileo heights after 1 day**.

Thus, the general relativistic effects play a crucial role in satellite geodesy as their impact reaches the level from 1 mm to 17 cm after one day for GNSS heights.



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Thank you for your attention!

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