General Relativistic effects acting on GNSS orbits with a focus on Galileo satellites launched into incorrect orbital planes

Three orbital effects emerging from general relativity are typically considered for Earth-orbiting satellites: the Schwarzschild effect, Lense-Thirring effect or frame-dragging, and the de Sitter or geodetic precession effect. For circular orbits and short satellite orbital arcs, the dominating Schwarzschild effect is difficult to determine, because it causes a constant radial acceleration which can be absorbed by a small modification in the gravitational constant GM term or a constant offset in the estimated semi-major axis of a satellite orbit. To separate the effects caused by the Schwarzschild effect from other orbital effects, especially those emerging from orbit modeling issues of non-gravitational accelerations, eccentric satellite orbits should be used.

The first pair of satellites belonging to the Galileo satellite system was accidentally launched into non-circular orbits with height variations between from 17,180 km for the perigee to 26,020 km for the apogee. The eccentric orbits introduced new opportunities for the verification of the effects emerging from general relativity when employing the Galileo constellation. We derive analytical formulas for perturbations of Earth satellite orbits due to general relativistic effects and verify which effects are detectable using the current accuracy of GNSS orbits.

Mean offset of the semi-major axis (assuming the same value of the gravitational constant G for Newtonian and relativistic orbits):

\[ \Delta a = -\frac{3GM}{c^2} = -17.74 \text{ mm} \]

Periodical variations of the semi-major axis (based on first-order Gaussian perturbations):

\[ \Delta a_{G/c} = \frac{GM}{c^2(1-e^2)^2} \left( -14 - 6\epsilon e \cos u - 5\epsilon^2 \cos 2u \right) \]

Periodical variations of the eccentricity (based on first-order Gaussian perturbations):

\[ \Delta e_{G/c} = \frac{GM}{c^2e(1-e^2)^{3/2}} \left( 3 + 7\epsilon^2 \cos u + \frac{5}{2} \epsilon^3 \cos 2u \right) \]

Orbit shape & size changes (Schwarzschild)

Orbit orientation change (de Sitter, Lense-Thirring)


We conducted simulations of orbit perturbations due to the main effects emerging from general relativity. We derive the analytical formulas of temporal variations of Keplerian parameters based on the first-order Gaussian perturbations and compare how well they agree with the observed perturbations from numerical simulations. The Schwarzschild effect introduces a constant and periodic changes of semi-major axis and revolution period, periodic variations of eccentricity, and a secular motion of perigee, all of which exceed the level of 1 mm. The secular rate of \( \omega = 1.3 \) mas/day corresponds even to 167 mm/day at GPS heights.

For Galileo E14/E18 in eccentric orbits, the change of the semi-major axis due to the Schwarzschild effect is \(~29.02\ mm\ in\ perigee\ and\ ~8.65\ mm\ in\ apogee\). This gives a difference of more than 20 mm, thus, fully detectable using current GNSS precise orbit determination techniques. We found that the geodetic precession strongly depends on the orientation of the ecliptic with respect to the orbital plane and may introduce a secular change of the inclination angle. The mean secular rate of the ascending node, which equals 52.53 mas/day corresponds to a drift of 7.2 mm/day at the Galileo heights after 1 day.
General Relativistic effects acting on GNSS orbits
with a focus on Galileo satellites
launched into incorrect orbital planes

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General Relativity and Earth artificial satellites – IERS Conventions 2010

**Orbit effects**

- **Schwarzschild**
- **Lense-Thirring**
- **De Sitter** (Geodetic precession)

**Time systems & Clocks**

- Differences between Barycentric Coordinate Time (TCB), Geocentric Coordinate Time (TCG), and Terrestrial Time (TT)
- "Proper time" for satellite clocks estimated using the satellite position and velocity and considering Earth's oblateness ($J_2$)
- Doppler frequency & time shifts of clocks due to Earth's rotation and sat. motion (incl. Sagnac)

**Signal propagation**

- Gravitational delay (Shapiro effect)
- Secondary effects ($J_2$ and higher-term effects)

PPN: parameterized post-Newtonian approach is typically employed for artificial Earth satellites; Corrections for VLBI and Lunar Laser Ranging are performed in Barycentric Celestial Reference Frame.
IERS Conventions 2010 and the orbital effects

\[
\Delta \vec{r} = \frac{GM}{c^2r^3} \left\{ \left[ 2(\beta + \gamma) \frac{GM}{r} - \gamma \vec{r} \cdot \vec{r} \right] \vec{r} + 2(1 + \gamma)(\vec{r} \cdot \vec{r})\vec{r} \right\} + \\
(1 + \gamma) \frac{GM}{c^2r^3} \left[ \frac{3}{r^2} (\vec{r} \times \vec{r}) (\vec{r} \cdot \vec{J}) + (\vec{r} \times \vec{J}) \right] + \\
\left\{ (1 + 2\gamma) \frac{\vec{R} \times \left( -\frac{GM_S \vec{R}}{c^2R^3} \right)}{R} \right\} \times \vec{r}
\]

where: \(GM\) - standard Earth gravitation product; \(GM_S\) - standard Sun gravitation product; \(\beta, \gamma\) - PPN parameters equal to 1 in general relativity; \(c\) - speed of light; \(\vec{r}, \vec{r}\) - position and velocity of a satellite with respect to the geocenter; \(\vec{R}, \vec{R}\) - position and velocity of the Earth with respect to the Sun; \(\vec{J}\) - Earth’s angular momentum per unit mass.

Schwarzschild

Lense-Thirring frame dragging

De Sitter geodetic precession

Effects are for Galileo eccentric:
- \(2.422 \cdot 10^{-10}\),
- \(1.5 \cdot 10^{-12}\),
and \(3.06 \cdot 10^{-11}\) m·s\(^{-2}\).
GNSS Constellations

50 000 km
40 000
30 000
20 000
10 000
Earth

QZSS
BeiDou IGSO/GEO

Galileo eccentric
Galileo nominal
BeiDou MEO
GPS
GLONASS

Galileo E14 and E18 in eccentric orbits

Galileo – nominal orbit
23 225 km
Galileo apogee
26 000 km
Galileo – eccentric orbit
perigee – 17 000 km

E14 and E18
e=0.1632
Galileo Satellites are good proxies of circular orbits with a regular distribution of satellites, except for E14 and E18 which are good proxies of eccentric orbits.
GNSS Constellations

- Galileo
- GPS
- GLONASS
IERS Conventions 2010 and the orbital effects

Modified version, after Hugentobler (2008)
*Orbit Perturbations due to Relativistic Corrections. IERS 2010 Conventions, Chapter 10 supplementary material.*
Systematic errors may occur – perigee is above southern hemisphere

Systematic effects may occur, because different ground GNSS (and SLR) stations observe the satellites in perigee and apogee. A full revolution of perigee takes 31 years for E14&E18 (and 65-90 years for Galileo satellites in circular orbits)

First-order Gaussian perturbations

Gaussian perturbations due to accelerations in radial (R), along-track (S), and cross-track (W)

\[ \dot{\alpha} = \sqrt{\frac{p}{GM}} \frac{2a}{1 - e^2} \left[ e \sin \nu \cdot R + \frac{p}{r} \cdot S \right] \]
\[ \dot{e} = \sqrt{\frac{p}{GM}} \left[ \sin \nu \cdot R + \left( \cos \nu + \cos E \right) \cdot S \right] \]
\[ i = \frac{r \cos u}{na^2 \sqrt{1 - e^2}} \cdot W \]
\[ \dot{\Omega} = \frac{\sin u}{na^2 \sqrt{1 - e^2} \sin i} \cdot W \]
\[ \dot{\omega} = \frac{1}{e} \sqrt{\frac{p}{GM}} \left[ - \cos \nu \cdot R + \left( 1 + \frac{r}{p} \right) \sin \nu \cdot S \right] - \dot{\Omega} \cos i \]
\[ M_0 = \frac{1 - e^2}{na e} \left[ \left( \cos \nu - 2e \frac{r}{p} \right) \cdot R - \left( 1 + \frac{r}{p} \right) \sin \nu \cdot S \right] - \frac{3n}{2a} (t - t_0) \dot{\alpha} \]

Solution of the differential equations does not provide information on constant offsets (if they appear). Offsets must be derived on the basis of celestial mechanics.

E08 – circular orbit
E14 – eccentric orbit
IERS Conventions 2010 and the orbital Effects on Galileo E14

All Schwarzschild accelerations are in-plane (no out-of-plane accelerations occur). For circular orbits only radial accelerations occur. For elliptical orbits, both radial and along-track accelerations take place (Schwarzschild effect depends not only on the position but also on the velocity).

\[ \Delta \mathbf{a}_{Sch} = \frac{3 (GM)^2}{c^2 a^4} \cdot \mathbf{r} \]

IERS Conventions 2010 and the orbital Effects on Galileo E14

Lense-Thirring and de Sitter are in-plane and out-of-plane accelerations.
Schwarzschild effect – semi-major axis

\[ \Delta a^{Tot}_{Sch} = \Delta a_{Sch} + \Delta a_{Sch} \]

Constant part:

\[ \Delta a_{Sch} = -4 \frac{GM}{c^2} = -17.74 \, \text{mm} \]

Periodical part:

\[ \Delta a_{Sch} = \frac{GM}{c^2(1 - e^2)^2} \left[ (-14 - 6e^2)e \cos u - 5e^2 \cos 2u \right] \]


Semi-major axis change of E14 due to the Schwarzschild acceleration as a function of a) time and b) the satellite height for E14 in an elliptical orbit and c) for E08 in a circular orbit.
Schwarzschild effect – eccentricity

\[
\Delta e_{Sch} = -\frac{GM}{c^2 ae(1-e^2)} \left[(3+7e^2)e \cos u + \frac{5}{2}e^2 \cos 2u\right]
\]

Comparison between E14 orbit without and with the Schwarzschild accelerations with the exaggeration of the total displacement effect in the geocentric reference frame.

Eccentricity changes due to the Schwarzschild acceleration as a function of time for a) E14 and b) E08. The initial point coincides with the orbit’s perigee.
Schwarzschild effect – revolution period

Revolution period

\[ T = \frac{2\pi}{n} \]

\[ n_{Sch} = \left( \frac{GM}{a^3} \right)^{1/2} \left( 1 + \frac{6GM}{c^2a} \right) \]

Areal velocity:

\[ A = \frac{1}{2} |\vec{h}| = \frac{1}{2} |\vec{r} \times \vec{v}| \]

Angular momentum:

\[ |\vec{h}_{Sch}| = \left[ GMa(1 - e^2) \right]^{1/2} \left[ 1 - \frac{GM}{2c^2a} + \frac{4GM}{c^2a(1 - e^2)} \right] \]

\[ \Delta T_{apogee} = -21.57 \, \mu s \]

\[ \Delta T_{perigee} = -72.44 \, \mu s \]

Impact of the Schwarzschild term on the
a) revolution period, b) areal velocity, and c) linear velocity of E14.
Impact of the Schwarzschild term on the perigee of Galileo a) E14 and b) E08 from the numerical simulation (corrected by a change of the mean anomaly), first and second-order approximations, and the Einstein's term.

For Galileo E14, the mean formal error of perigee determination is 223 μas from 1-day solutions, whereas for E08 the perigee error is 95 468 μas.
De Sitter – semi-major axis, eccentricity

Constant offset of the semi-major axis

\[ \Delta a_{ds} = +4 \frac{GM_S}{\sqrt{GMc^2RS}} \frac{a_S^5}{n_S} \sqrt{1 - e_S^2} \cos \beta_{ele} \]

Periodic perturbations of the semi-major axis:

\[ \Delta a_{ds} = -6 \frac{e}{\sqrt{1 - e^2}} \frac{GM_S}{c^2RS} n_S \sqrt{1 - e_S^2} \cos \beta_{ele} \sin u \]

Eccentricity variations:

\[ \Delta e_{ds} = \frac{3}{4} \frac{GM_S}{c^2RS} n_S \sqrt{1 - e_S^2} \sqrt{1 - e^2} \sin \beta_{ele} (\cos u - 1) \]

Mean motion:

\[ n_{ds} = \left( \frac{GM}{a^3} \right)^{1/2} \left( 1 - \frac{6GM_S n_S}{ac^2RS} \frac{1}{n_S} \sqrt{1 - e_S^2} \cos \beta_{ele} \right) \]

For GEO, the impact on \( a \) is 4 mm.

Impact of the de Sitter effect on
the semi-major axis, eccentricity,
revolution period of Galileo E14
from the numerical simulations
(blue line) and first-order
Gaussian perturbations (red line).
De Sitter – node, inclination, perigee for E14

Ascending node

\[ \Delta \Omega_{DS} = \frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{nS}{n} \sqrt{\frac{1 - e^2}{1 - e^2}} \left( n \Delta t - \frac{1}{2} \sin 2u \right) \frac{\sin(\beta_{ele})}{\sin i} \]

Inclination

\[ \Delta i_{DS} = -\frac{3}{2} \frac{GM_S}{c^2 R_S} \frac{nS}{n} \sqrt{\frac{1 - e^2}{1 - e^2}} \left( n \Delta t \sin(\beta_{ele} - i) - \frac{1}{2} \sin 2u \right) \cdot \sin(\beta_{ele}) \]

Argument of perigee:

\[ \Delta \omega_{DS} = \frac{3}{4} \frac{GM_S}{e c^2 R_S} \frac{nS}{n} \sqrt{1 - e^2} \sqrt{1 - e^2} \sin(\beta_{ele} - i) \sin u - \Delta \Omega_{DS} \cos i. \]

Nodal drift equals 52.53 μas/day for circular polar orbits over long periods that is 7.2 mm at the Galileo heights after one day. Over long periods the inclination drift is equal to zero. Over shorter periods, the nodal drift may be very large.
De Sitter – node, inclination, perigee for GEO

Ascending node

\[
\Delta \Omega_{dS} = \frac{3 GM_S}{2 c^2 R_S} \frac{nS}{n} \sqrt{\frac{1 - e^2_S}{1 - e^2}} \left( n\Delta t - \frac{1}{2} \sin 2u \right) \frac{\sin(\beta_{ele})}{\sin i}
\]

Inclination

\[
\Delta i_{dS} = -\frac{3 GM_S}{2 c^2 R_S} \frac{nS}{n} \sqrt{\frac{1 - e^2_S}{1 - e^2}} \left( n\Delta t \sin(\beta_{ele} - i) - \frac{1}{2} \sin 2u \right) \cdot \sin(\beta_{ele})
\]

For GEO satellites and for specific Sun configurations, the nodal drift may reach over 1 meter per day because of the coefficient

\[
\frac{\sin(\beta_{ele})}{\sin i}
\]

\[
\cos \beta_{ele} = \cos \varepsilon \cos i + \sin \varepsilon \sin i \cos(\Omega - \Omega_S)
\]

\[
|i - \varepsilon| < \beta_{ele} < |i + \varepsilon|
\]

Impact of the de Sitter effect on the a) inclination, b) right ascension of ascending node of a geostationary orbit with \(i = 0.2^\circ\)
De Sitter – node, inclination, perigee for GEO


Table 1 Characteristics of IOV and FOC Galileo satellites and the orbits thereof

<table>
<thead>
<tr>
<th>PRN number</th>
<th>E11, E12</th>
<th>E19, E20</th>
<th>E18, E14</th>
<th>E26, E22</th>
<th>E24, E30</th>
<th>E08, E09</th>
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</thead>
<tbody>
<tr>
<td>Name</td>
<td>Galileo-101, 102</td>
<td>Galileo-103, 104</td>
<td>Galileo-201, 202</td>
<td>Galileo-203, 204</td>
<td>Galileo-205, 206</td>
<td>Galileo-208, 209</td>
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<tr>
<td>Type</td>
<td>IOV</td>
<td>IOV</td>
<td>FOC</td>
<td>FOC</td>
<td>FOC</td>
<td>FOC</td>
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<tr>
<td>Retroreflector (corner cubes)</td>
<td>84</td>
<td>84</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Corner cube dimensions (mm)</td>
<td>33.0 × 23.3</td>
<td>33.0 × 23.3</td>
<td>28.2 × 19.1</td>
<td>28.2 × 19.1</td>
<td>28.2 × 19.1</td>
<td>28.2 × 19.1</td>
</tr>
<tr>
<td>Rear/front coating (Y—yes, N—no)</td>
<td>N/Y</td>
<td>N/Y</td>
<td>N/N</td>
<td>N/N</td>
<td>N/N</td>
<td>N/N</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>695</td>
<td>697</td>
<td>661/662</td>
<td>706</td>
<td>709</td>
<td>708</td>
</tr>
<tr>
<td>Orbital plane</td>
<td>B</td>
<td>C</td>
<td>incorrect</td>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>Area-to-mass ratio (m² kg⁻¹)</td>
<td>0.019</td>
<td>0.019</td>
<td>0.020</td>
<td>0.019</td>
<td>0.019</td>
<td>0.019</td>
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<tr>
<td>Semi-major axis (km)</td>
<td>29,600</td>
<td>29,600</td>
<td>29,798</td>
<td>29,602</td>
<td>29,601</td>
<td>29,600</td>
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<td>Altitude (km)</td>
<td>23,225</td>
<td>23,225</td>
<td>17,178–26,019</td>
<td>23,227</td>
<td>23,226</td>
<td>23,225</td>
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<td>Eccentricity</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.1585/0.1584</td>
<td>0.0003</td>
<td>0.0004</td>
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<td>Revolution period (h)</td>
<td>14.08</td>
<td>14.08</td>
<td>12.94</td>
<td>14.08</td>
<td>14.08</td>
<td>14.08</td>
</tr>
<tr>
<td>Inclination (°)</td>
<td>55.57</td>
<td>54.93</td>
<td>50.10/50.16</td>
<td>55.36</td>
<td>57.25</td>
<td>54.94</td>
</tr>
<tr>
<td>( \beta_{\text{max}} ) (°)</td>
<td>63.5</td>
<td>75.5</td>
<td>48.0/48.5</td>
<td>58.2</td>
<td>39.0</td>
<td>75.0</td>
</tr>
<tr>
<td>Draconic year (days)</td>
<td>355.7</td>
<td>355.6</td>
<td>351.6</td>
<td>356.1</td>
<td>356.1</td>
<td>355.6</td>
</tr>
<tr>
<td>Revolution of node (years)</td>
<td>37.5</td>
<td>36.9</td>
<td>25.9/25.8</td>
<td>37.4</td>
<td>39.1</td>
<td>36.9</td>
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<tr>
<td>Revolution of perigee (years)</td>
<td>70.9</td>
<td>65.3</td>
<td>31.4/31.3</td>
<td>70.0</td>
<td>92.2</td>
<td>65.3</td>
</tr>
</tbody>
</table>

Orbit parameters of E18 and E14 refer to the situation after satellite maneuvers reduced the orbit eccentricity.
Lense-Thirring – semi-major axis, eccentricity

Semi-major axis – constant offset

\[ \Delta a_{LT} = -\frac{8}{3} \frac{J}{c^2} \sqrt{\frac{GM}{a}} \cos i \]

Semi-major axis – periodical variations

\[ \Delta a_{LT} = 4 \frac{e}{\sqrt{1-e^2}} \frac{Jn}{c^2} \cos i \sin u \]

Eccentricity

\[ \Delta e_{LT} = -2 \sqrt{1-e^2} \frac{Jn}{c^2} \cos i (\cos u - 1) \]

Mean motion

\[ n_{LT} = \left( \frac{GM}{a^3} \right)^{1/2} \left( 1 + 4 \frac{GM}{c^2 a^3} J \cos i \right) \]

Impact of the Lense-Thirring on a) the semi-major axis, b) eccentricity, and c) the revolution period of E14.
Lense-Thirring – node, inclination, perigee

**Ascending node**

\[ \Delta \Omega_{LT} = 2 \frac{GMJ}{c^2 a^3 (1 - e^2)^{3/2} n} (n \Delta t - \frac{1}{2} \sin 2u) \]

**Inclination**

\[ \Delta i_{LT} = \frac{GMJ \sin i}{c^2 a^3 (1 - e^2)^{3/2} n} (-\frac{1}{2} + \frac{1}{2} \sin 2u) \]

**Argument of perigee**

\[ \Delta \omega_{LT} = -2 \frac{GMJ \cos i}{c^2 a^3 (1 - e^2)^{1/2} n} (n \Delta t - \frac{3}{2} \sin 2u) \]
## General relativity effects - summary

<table>
<thead>
<tr>
<th></th>
<th>GEO BeiDou C01</th>
<th>Galileo E08</th>
<th>Galileo E14</th>
<th>GPS G18</th>
<th>LARES-2</th>
<th>LAGEOS-1</th>
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<td><strong>Height [km]</strong></td>
<td>35 793</td>
<td>23 230</td>
<td>17 097 – 26 117</td>
<td>20 190</td>
<td>12 270</td>
<td>12 271</td>
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<td><strong>Schwarzschild</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$ [mm]</td>
<td>-17.74</td>
<td>-17.75 to -17.73</td>
<td>-29.02 to -8.65</td>
<td>-18.87 to -16.65</td>
<td>-17.74</td>
<td>-17.74</td>
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<tr>
<td>$e$ [$\times 10^{-10}$]</td>
<td>-3.19 to 3.18</td>
<td>-4.49 to 4.49</td>
<td>-5.85 to 3.47</td>
<td>-5.09 to 4.87</td>
<td>-11.09 to 11.08</td>
<td>-11.23 to 11.23</td>
</tr>
<tr>
<td>$i$ [mas/day]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega$ [mas/day]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\omega'$ [mas/day]</td>
<td>0.4102</td>
<td>1.0471</td>
<td>1.2034</td>
<td>1.3031</td>
<td>8.9771</td>
<td>8.9830</td>
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<tr>
<td>$T$ [$\mu$s]</td>
<td>-54.39 to -54.37</td>
<td>-45.54 to -45.56</td>
<td>-72.45 to -21.57</td>
<td>-45.91 to -40.45</td>
<td>-25.28</td>
<td>-25.28</td>
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<td><strong>De Sitter</strong></td>
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<tr>
<td>$a$ [mm]</td>
<td>+4.16</td>
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<td>-0.15</td>
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<td>$i$ [mas/day]</td>
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<td>+0.0520</td>
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<td>$\omega'$ [mas/day]</td>
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<td>-0.0328</td>
<td>-0.0395</td>
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<td>$T$ [$\mu$s]</td>
<td>+12.75</td>
<td>+0.80</td>
<td>+1.14</td>
<td>+0.63</td>
<td>-0.02</td>
<td>-0.24</td>
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<tr>
<td><strong>Lense-Thirring</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$a$ [mm]</td>
<td>-0.09</td>
<td>-0.06</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.06</td>
<td>+0.06</td>
</tr>
<tr>
<td>$e$ [$\times 10^{-10}$]</td>
<td>0.00</td>
<td>0.00 to 0.04</td>
<td>0.00 to 0.04</td>
<td>0.00 to 0.04</td>
<td>0.00 to 0.19</td>
<td>0.00 to 0.19</td>
</tr>
<tr>
<td>$i$ [mas]</td>
<td>+/-0.0000</td>
<td>+/-0.0005</td>
<td>+/-0.0005</td>
<td>+/-0.0005</td>
<td>+/-0.0019</td>
<td>+/-0.0019</td>
</tr>
<tr>
<td>$\Omega$ [mas/day]</td>
<td>+0.0021</td>
<td>+0.0061</td>
<td>+0.0068</td>
<td>+0.0084</td>
<td>+0.0827</td>
<td>+0.0827</td>
</tr>
<tr>
<td>$\omega'$ [mas/day]</td>
<td>-0.0024</td>
<td>-0.0045</td>
<td>-0.0046</td>
<td>-0.0052</td>
<td>-0.0320</td>
<td>+0.0321</td>
</tr>
<tr>
<td>$T$ [$\mu$s]</td>
<td>-0.27</td>
<td>-0.15</td>
<td>-0.18</td>
<td>-0.16</td>
<td>-0.09</td>
<td>+0.09</td>
</tr>
</tbody>
</table>
Summary

We analyzed three main effects emerging from general relativity: Schwarzschild, Lense-Thirring, and De Sitter. We studied the impact on GNSS orbits with a focus on two Galileo satellites accidentally launched into eccentric orbits. We derive analytical formulas of temporal variations of Keplerian parameters based on the first-order Gaussian perturbations and celestial mechanics; and then, we compare how well they agree with the observed perturbations from numerical simulations.

The Schwarzschild effect introduces a constant shift of the semi-major axis of \(-17.7\) mm for all Earth satellites, independently of the satellite height. Schwarzschild causes also periodic changes of semi-major axis and revolution period, periodic variations of eccentricity, and a secular motion of perigee. The secular rate of perigee corresponds to almost 17 cm/day at GNSS heights.

For Galileo E14/E18 in eccentric orbits, the change of the semi-major axis due to the Schwarzschild effect is \(-29\) mm in perigee and \(-9\) mm in apogee. This gives a difference of more than 20 mm, thus, is fully detectable using current GNSS precise orbit determination techniques. We found that the de Sitter effect strongly depends on the orientation of the ecliptic with respect to the orbital plane and may introduce a secular change of the inclination angle. The mean secular rate of the ascending node, caused by de Sitter, corresponds to a drift of 7.2 mm/day at the Galileo heights after 1 day.

Thus, the general relativistic effects play a crucial role in satellite geodesy as their impact reaches the level from 1 mm to 17 cm after one day for GNSS heights.
Thank you for your attention!