Fluid model of the plasma flow in the magnetic tail of a planet

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Background and motivation

- Question: how is the planetary plasma enclosed by the magnetopause accelerated in the tail of planetary magnetospheres ?
- We address the question by considering a symmetrized configuration of Uranus at solstice time. The configuration may also apply to the case of Mercury.
- We write the MHD equations for the plasma on the symmetry axis tailwards of the planet.
- The fluid velocity is found to obey a equation similar to that for a compressible gas in a duct (Hugoniot type equation).
- However, unlike a compressible gas in a duct, the plasma may be accelerated through the expanding duct defined by the magnetopause.

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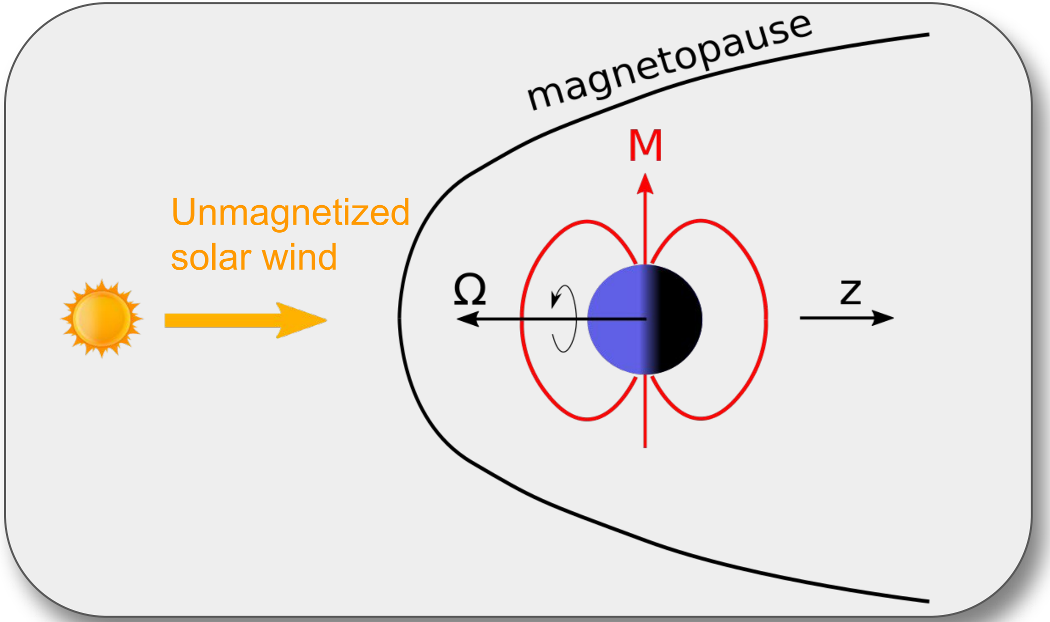
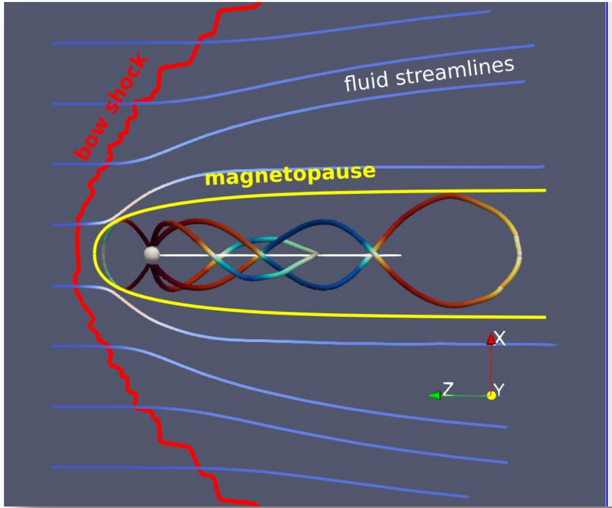
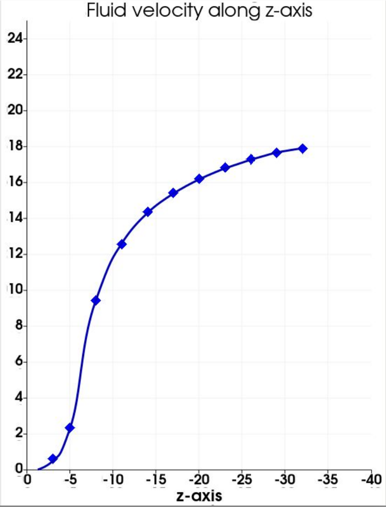
Inspired by the magnetosphere of Uranus at solstice

But we adopt a symmetrized version (see e.g. Pantellini, A&A, 2020):



What do we observe in 3D MHD numerical simulations ?





- Planetary field lines are transverse on z-axis (consequence of symmetry)
- Plasma is accelerated tailwards. File lines are stretched by the flow, twisted by rotation .

  Question: Is there a simple link between the shape of the magnetopause acceleration and the acceleration of the plasma along the *z* axis ?

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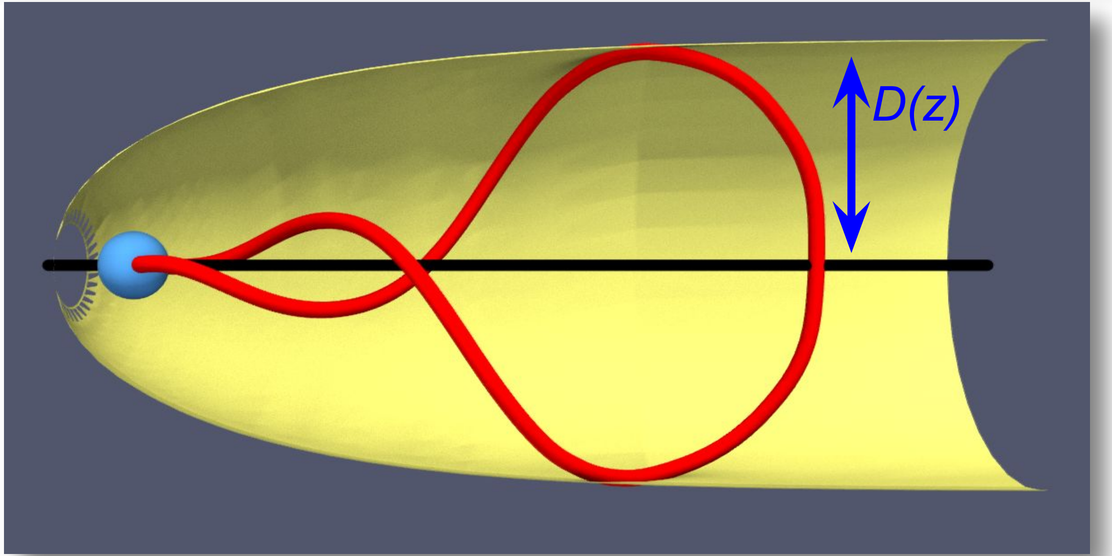
We suppose an impermeable magnetopause with prescribed shape *D(z)*.
By definition, planetary field lines “slide” along the magnetopause.



We assume radius of curvature of field lines crossing the z-axis is defined locally as $R=\alpha^{-1}\, D/D'$ where α is an ad-hoc parameter which summarizes the action of the magnetopause on the plasma.

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Fluid equations on the symmetry axis (the z axis)

NB: We assume no magnetic field in the surrounding solar wind so that the system is steady-state in the rotating frame (cf e.g. Toth et al, JGR, 2004; Pantellini, A&A 2020).

Setting $' = \partial/\partial z$:

- $(D^2 u \rho)' = 0$ … continuity
- $\rho u u' = -(p + \frac{1}{2} B^2)' - \alpha B^2 D'/D$ … momentum
- $(DuB)' = 0$ … induction: rot($\mathbf{u}$x$\mathbf{B}$)=0)
- $(p\rho^{-\gamma})'=0$ … energy

Where p, ρ, u, B are the pressure, mass density, fluid velocity and magnetic field strength on the z axis. γ is the plasma adiabatic index.

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“Solution”

$$(M^2 - 1)\frac{u'}{u} = -2\,\frac{\alpha - \alpha_{\rm cr}}{1 + \alpha_{\rm cr}}\,\frac{D'}{D}$$

where $\alpha_{\rm cr} \equiv 1 + \gamma\beta$ and $\beta \equiv 2p/B^2$

$$M \equiv u/c_{\rm f},\ c_{\rm f}^2 = \frac{B^2}{\varrho}(1 + \gamma\beta/2)$$

NB: for $\alpha > \alpha_{cr}$ and for sub-fast flow ($M<1$) u' and D' have the same sign → plasma is accelerated inside an expanding magnetopause

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Special limits

$$\beta \gg 1: \qquad (M^2 - 1)\frac{u'}{u} = 2\,\frac{D'}{D}$$

$$\beta \ll 1: \qquad (M^2 - 1)\frac{u'}{u} = -(\alpha - 1)\,\frac{D'}{D}$$

NB: The case $\beta \gg 1$ is the classical Hugoniot equation (Hugoniot, CRHSAS, 1886) describing the steady state motion of a compressible fluid in a duct of varying section *D* (in the subsonic regime the fluid decelerates in an expanding duct).

In the case $\beta \ll 1$ and for $\alpha > \alpha_{cr}=1$, the fluid accelerates in an expanding duct.

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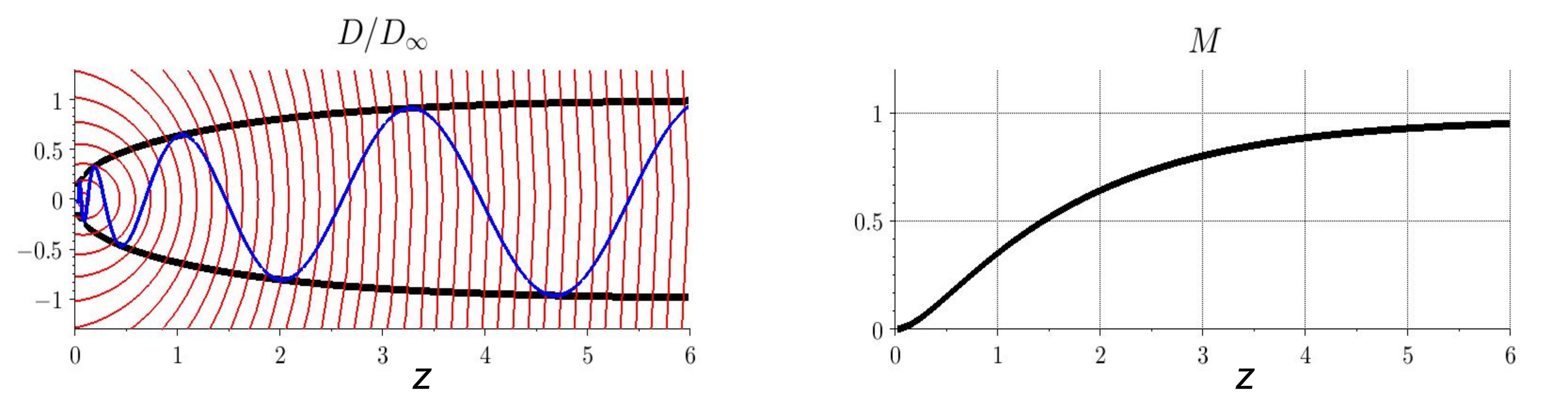
An example



NB: Fluid equations have been integrated numerically for the magnetopause profile *D(z)* in the left panel and α=1.1α_{cr} . Conditions at right boundary are *M*=0.95 and β=0.1. Red lines show circles with radius = curvature of field lines on *z* axis (note that plot is not isometric). Blue line indicate phase of stretched field lines assuming a constant planetary rotation period.

Conclusions

- The motion of the plasma along the axis of symmetry in the tail of a symmetrized version of the magnetosphere of Uranus at solstice is described by a Hugoniot type equation.
- The plasma on the axis of symmetry is accelerated tailwards in the expanding duct defined by the magnetopause provided the radius of curvature of the planetary field lines exceed a threshold which depends on β.
- This result is non-intuitive. First, the behavior is opposite to that of an ordinary gas in a duct of varying cross section. Second, paradoxically, the tailward acceleration of the plasma increases with increasing magnetic tension despite the tension acting in the opposite direction.

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