

Nonequilibrium scaling vs. the temporal evolution of turbulence

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Equilibrium Taylor's law:

$$\epsilon = C_\epsilon \frac{\mathcal{U}^3}{L}$$

where $\mathcal{U}$ is the characteristic velocity scale of turbulent eddies (typically square root of the turbulence kinetic energy $\mathcal{U} = \sqrt{2k/3}$), L is the integral length scale of turbulence.

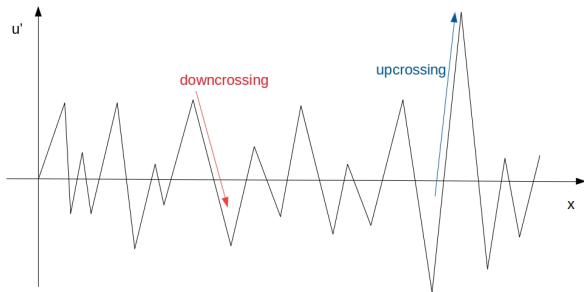
Is this law universal?

Goto & Vassilicos "The dissipation rate coefficient of turbulence is not universal and depends on the internal stagnation point structure" Phys. Fluids, 21, 035104, 2009

Taylor's law vs. number density of zero-crossings.

Mazellier & Vassilicos "The turbulence dissipation constant is not universal because of its universal dependence on large-scale flow topology" Phys. Fluids, 20, 015101, 2008

The constant C_ϵ is related to the scaling constant of the number density of zero-crossings n_s of turbulent fluctuations



Taylor's law vs. number density of zero-crossings.

The scaling constant C_s of the number density of zero-crossings

n_s

$$n_s = \frac{C_s}{L} \left(\frac{L}{\eta_c} \right)^{2/3},$$

η_c is the cut-off scale.

For large Re

$$C_\epsilon \approx 0,26 C_s^3$$

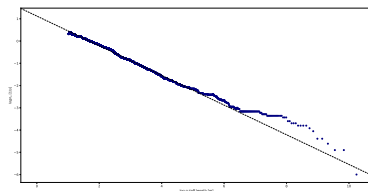
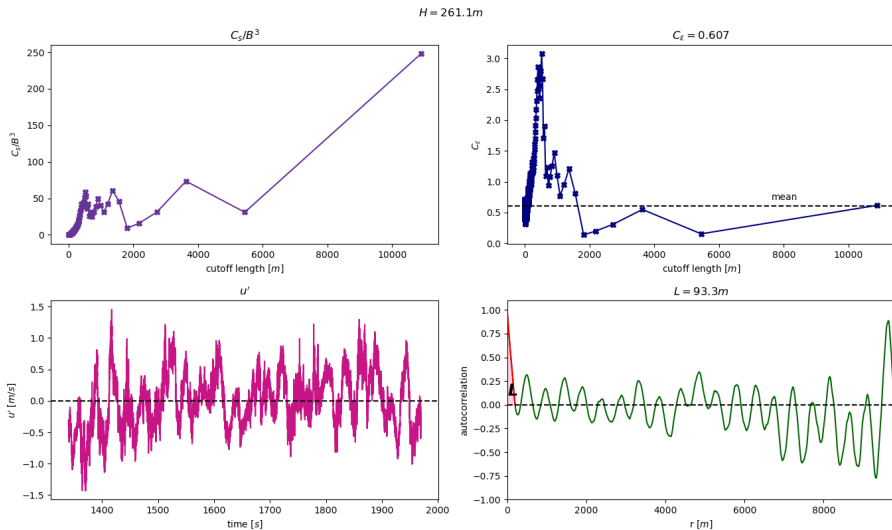
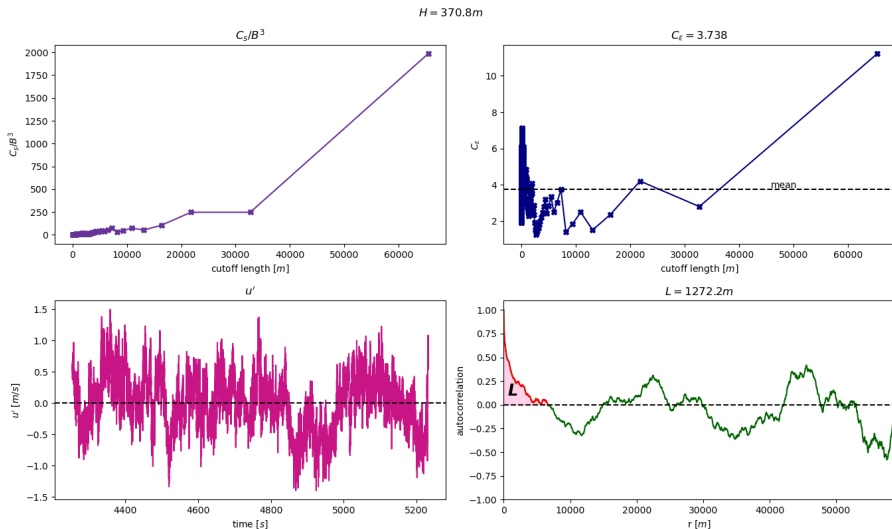


Figure: Profile of $n_s(\eta_c)$

Taylor's law vs. number density of zero-crossings.



Taylor's law vs. number density of zero-crossings.



C_ϵ in decaying turbulence.

In the low wavenumber range energy spectrum scales as

$$E(\kappa, t) \sim \kappa^q.$$

Both C_ϵ and C_s are in fact dependent on the Taylor-scale based Reynolds number Re_λ and on the scaling coefficient q [Goto & Vassilicos, 2009].

Turbulence decay [Bos et al. Phys. Fluids 19, 045101, 2007]

$$k \sim t^{-n}, \quad \epsilon \sim nt^{-(n+1)}, \quad L \sim t^{1-n/2}, \quad n \approx \frac{2(q+1)}{q+3}$$

It follows that there is a finite time $\mathcal{T}$ needed for the energy injected at large scales to reach the dissipative end of the energy cascade.

$$\epsilon(t + \mathcal{T}) = C_\epsilon \frac{[\mathcal{U}(t)]^3}{L(t)} = C_\epsilon \frac{[\mathcal{U}(t + \mathcal{T})]^3}{L(t + \mathcal{T})} \left(1 + \frac{\mathcal{T}}{t}\right)^{-(n+1)}$$

Cocequence: In the decaying turbulence $C_\epsilon \approx 1$ is typically larger than in the forced turbulence, where $C_\epsilon \approx 0.6$

Turbulence kinetic energy (TKE) budget

Consider TKE budget for horizontally homogeneous shear flow

$$\frac{\partial k}{\partial t} = \underbrace{-\frac{1}{2} \frac{\partial \overline{u'_i u'_i w'}}{\partial z}}_{\text{turbulent transport } T} - \underbrace{\frac{1}{\rho} \frac{\partial \overline{p' w'}}{\partial z} - \overline{w' u'_i} \frac{\partial \overline{u_i}}{\partial z}}_{\text{production } P} - \underbrace{\epsilon}_{\text{dissipation}} + \underbrace{\overline{b' w'}}_{\text{buoyancy flux } B}$$

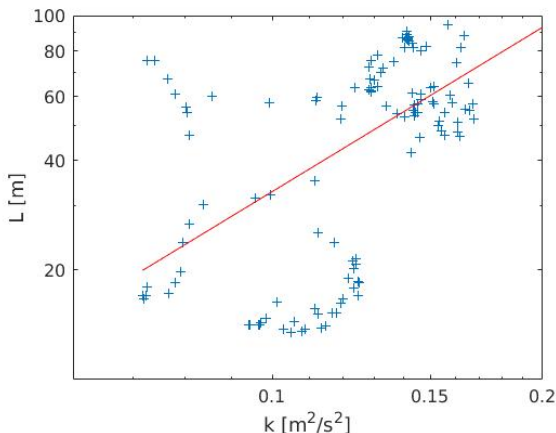
$$\frac{\partial k}{\partial t} = T + P - \epsilon + B$$

Consider leading order terms for the following cases

- stationary, shear-produced turbulence: $0 = P - \epsilon$
- stationary, buoyancy-produced turbulence: $0 = -\epsilon + B$
- stationary, advected turbulence: $0 = T - \epsilon$
- decaying turbulence: $\partial k / \partial t = -\epsilon$

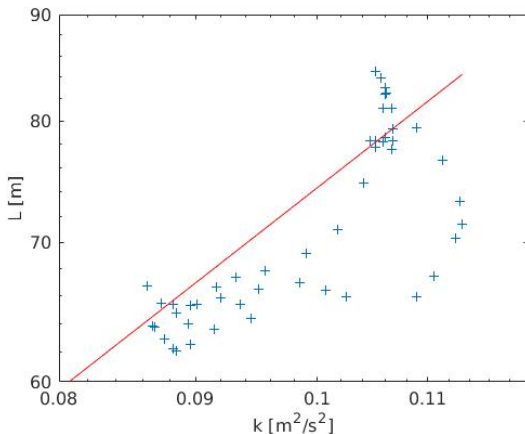
Stationary, shear-produced turbulence: $0 = P - \epsilon$

For the balance $P = \epsilon$ assuming a constant shear and with the use of the Taylor's law: $\epsilon \sim k^{3/2}/L$ we obtain $L \sim k$



Stationary, buoyancy-produced turbulence: $0 = B - \epsilon$

Assuming $B = \overline{w'b'} = \text{const}$ and using the Taylor's law: $\epsilon \sim k^{3/2}/L$. we obtain $L \sim k^{3/2}$



Decaying turbulence: $dk/dt = -\epsilon$

While k decreases in time, the length scale L increases in time, hence

$$kL^a = \text{const}, \quad a > 0, \quad k \sim 1/L^a$$

